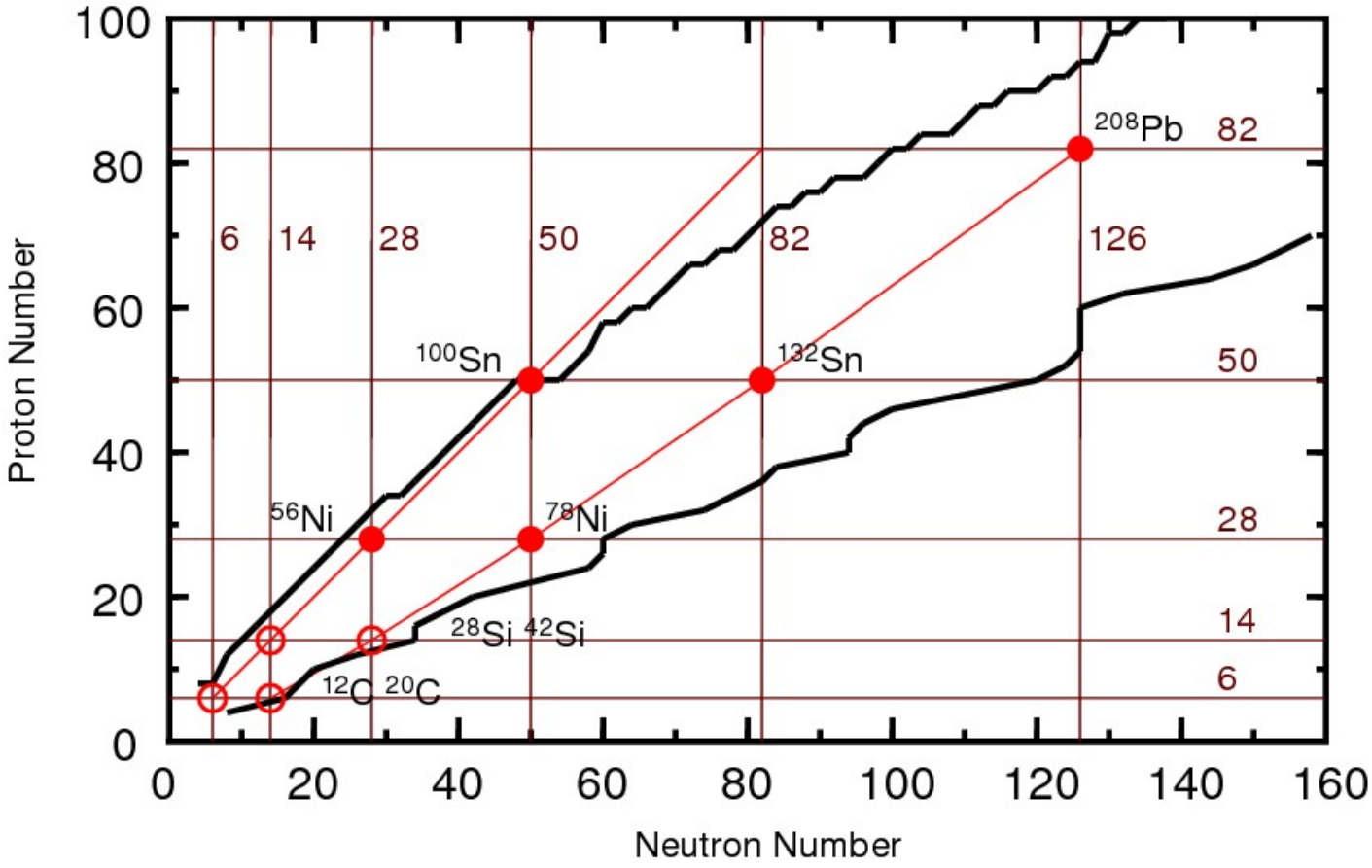
E2 maps of nuclei, from order to chaos

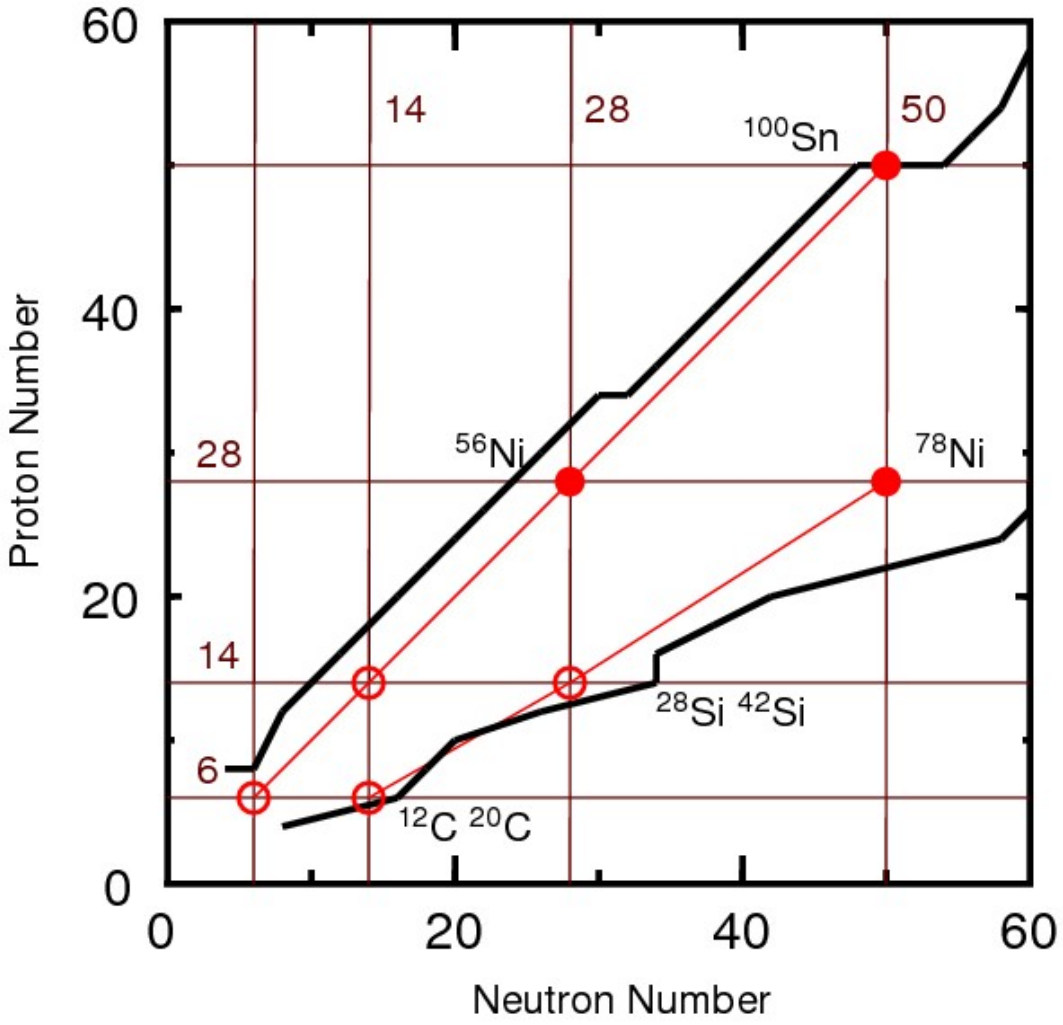
- Alex Brown

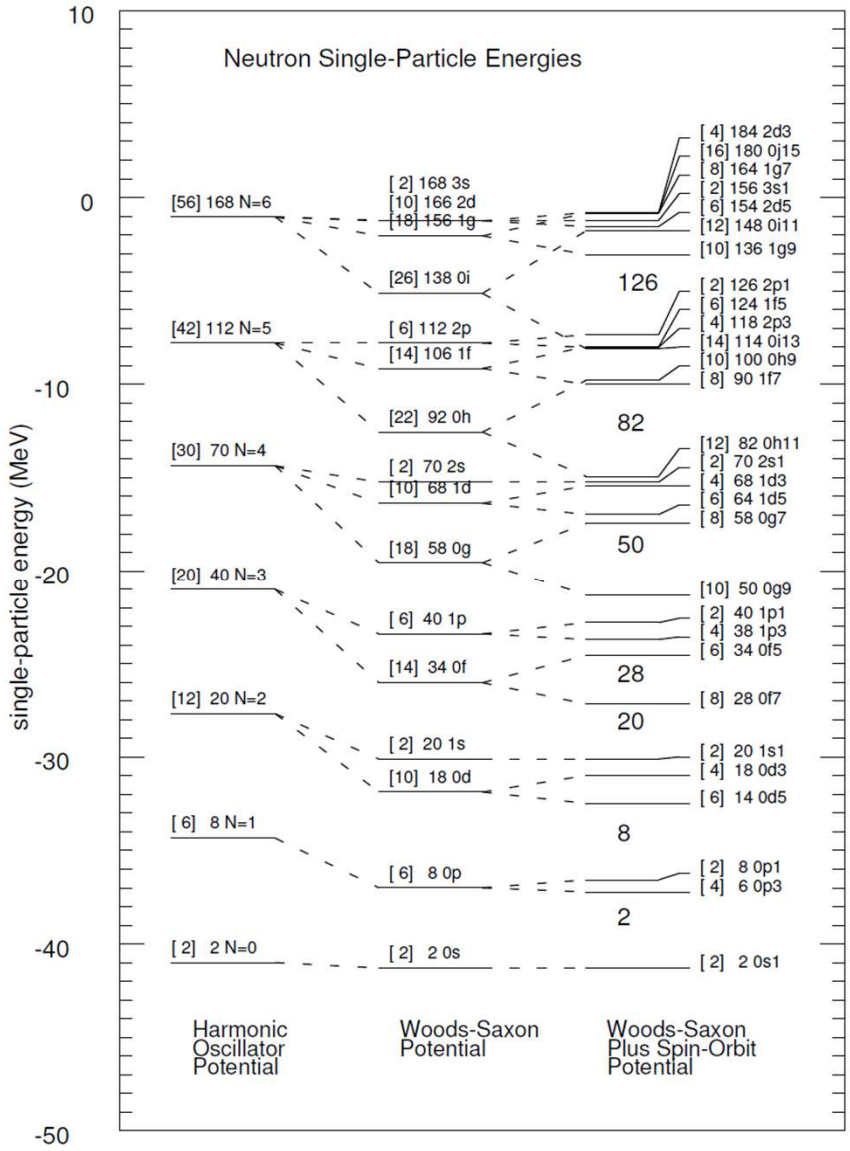
- Presentation given on July 7, 2026 at the

- FRIB-TA Topical Program: Statistical Nuclear Properties in the AI/ML Era: Advances, Challenges, and Synergies for FRIB Science

JJ-magic numbers




60
40
20
0
0
20
40
60
Proton Number
Neutron Number
14
28
50
28
14
6
^{100}Sn
^{56}Ni
^{78}Ni
^{28}Si ^{42}Si
^{12}C ^{20}C

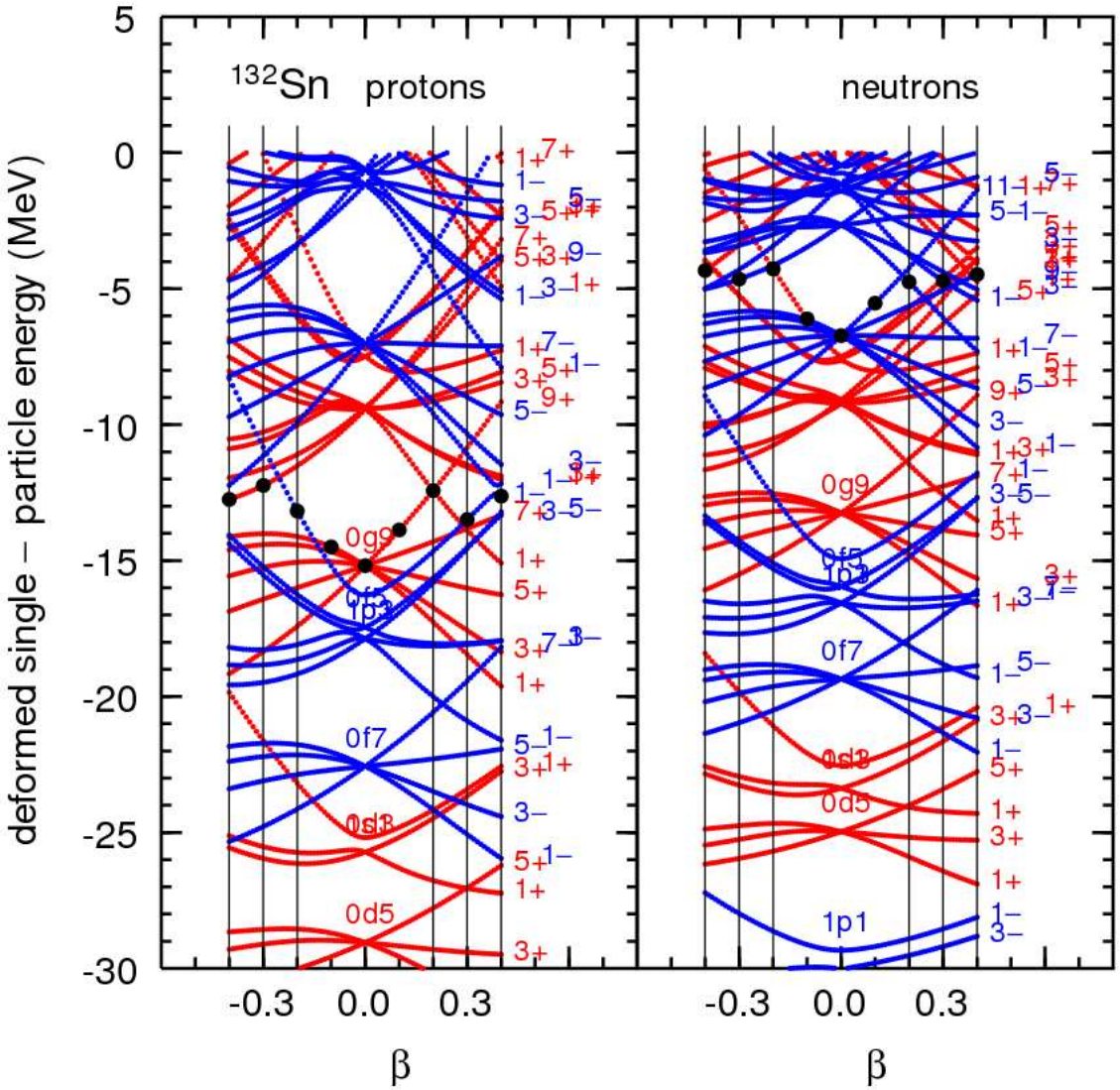


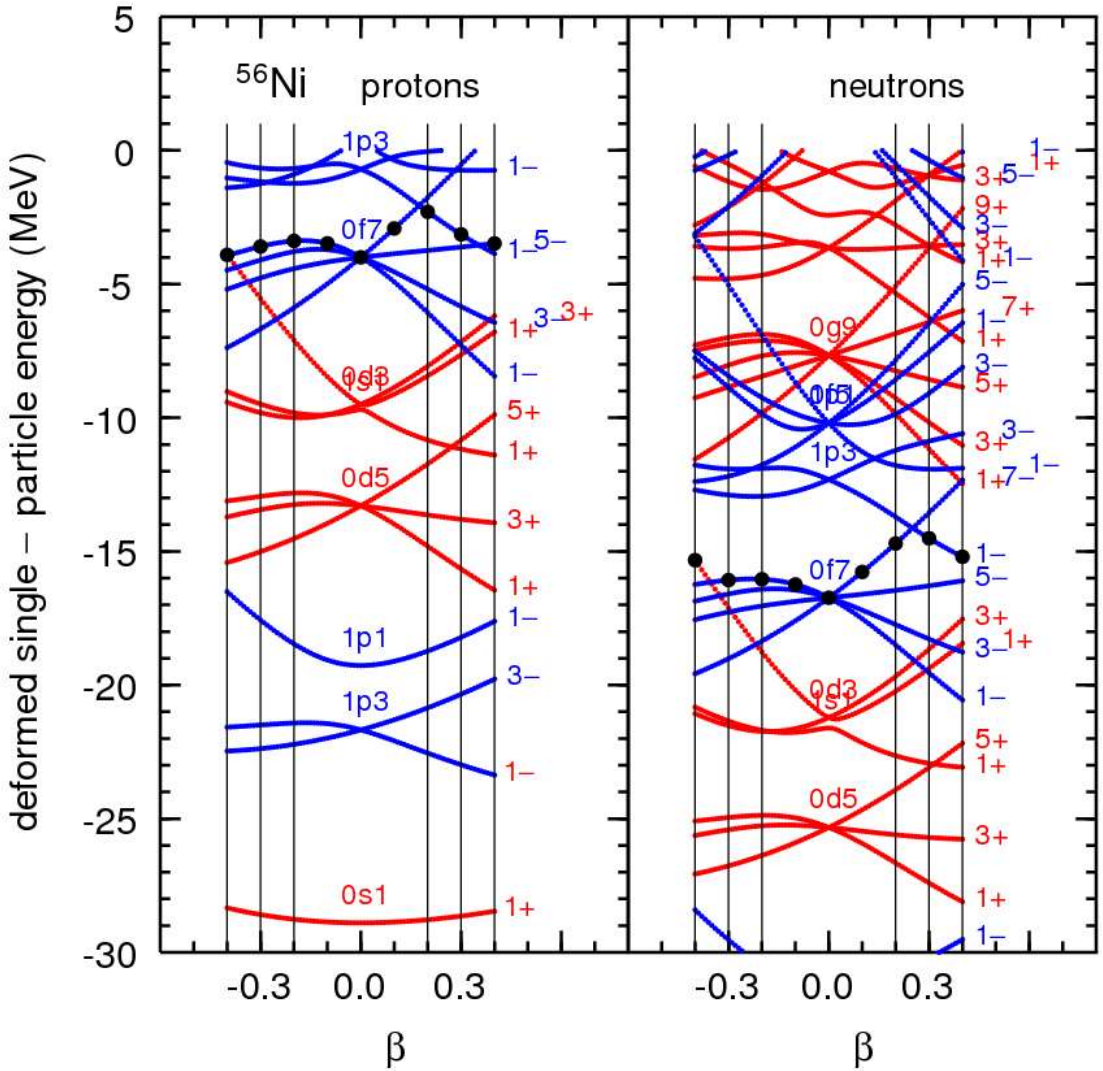
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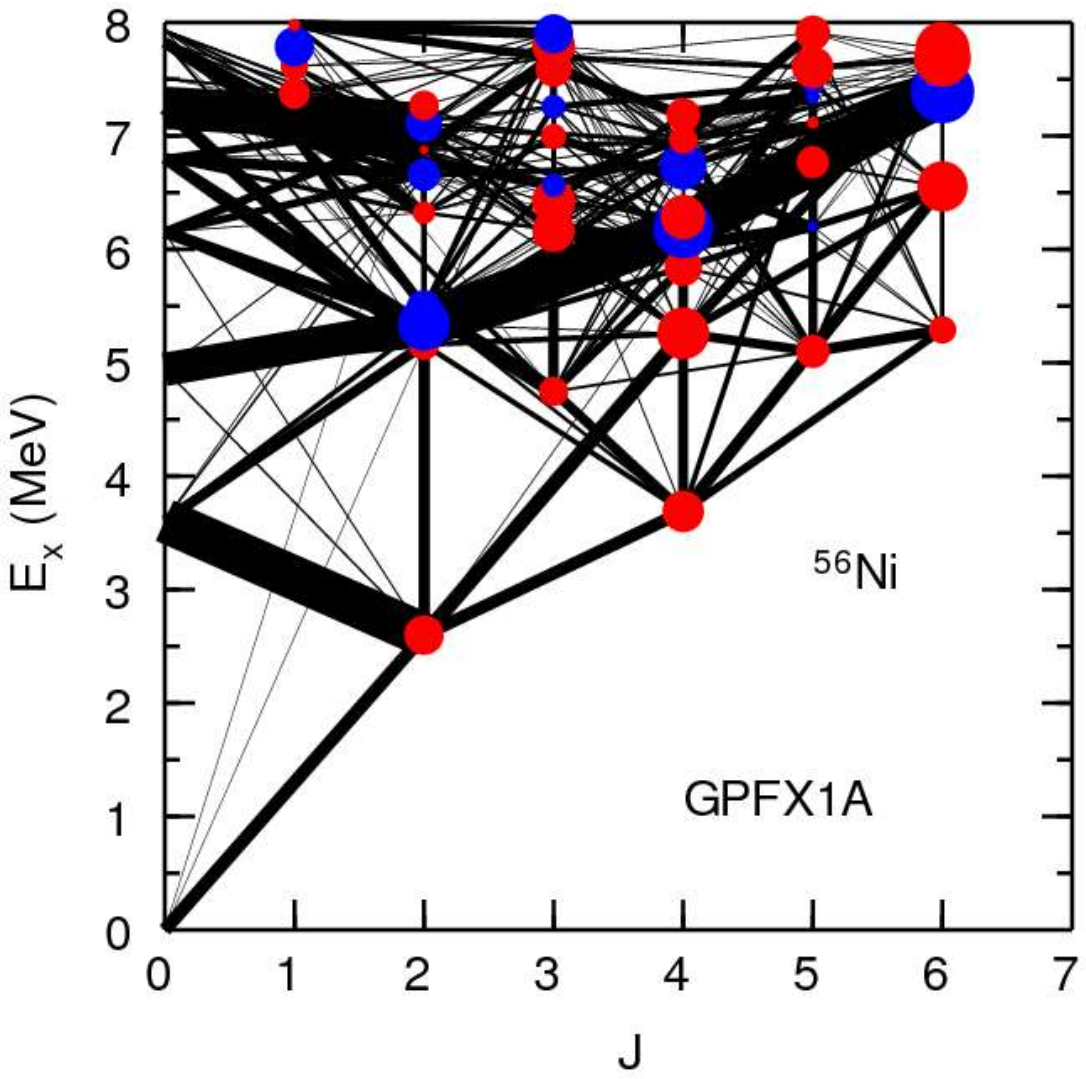
6

5

4 j4 model space


^{132}Sn protons
neutrons
deformed single – particle energy (MeV)
β
β
0g9
0f7
1p1
0d5
0g9
0f7
1p1
0d5

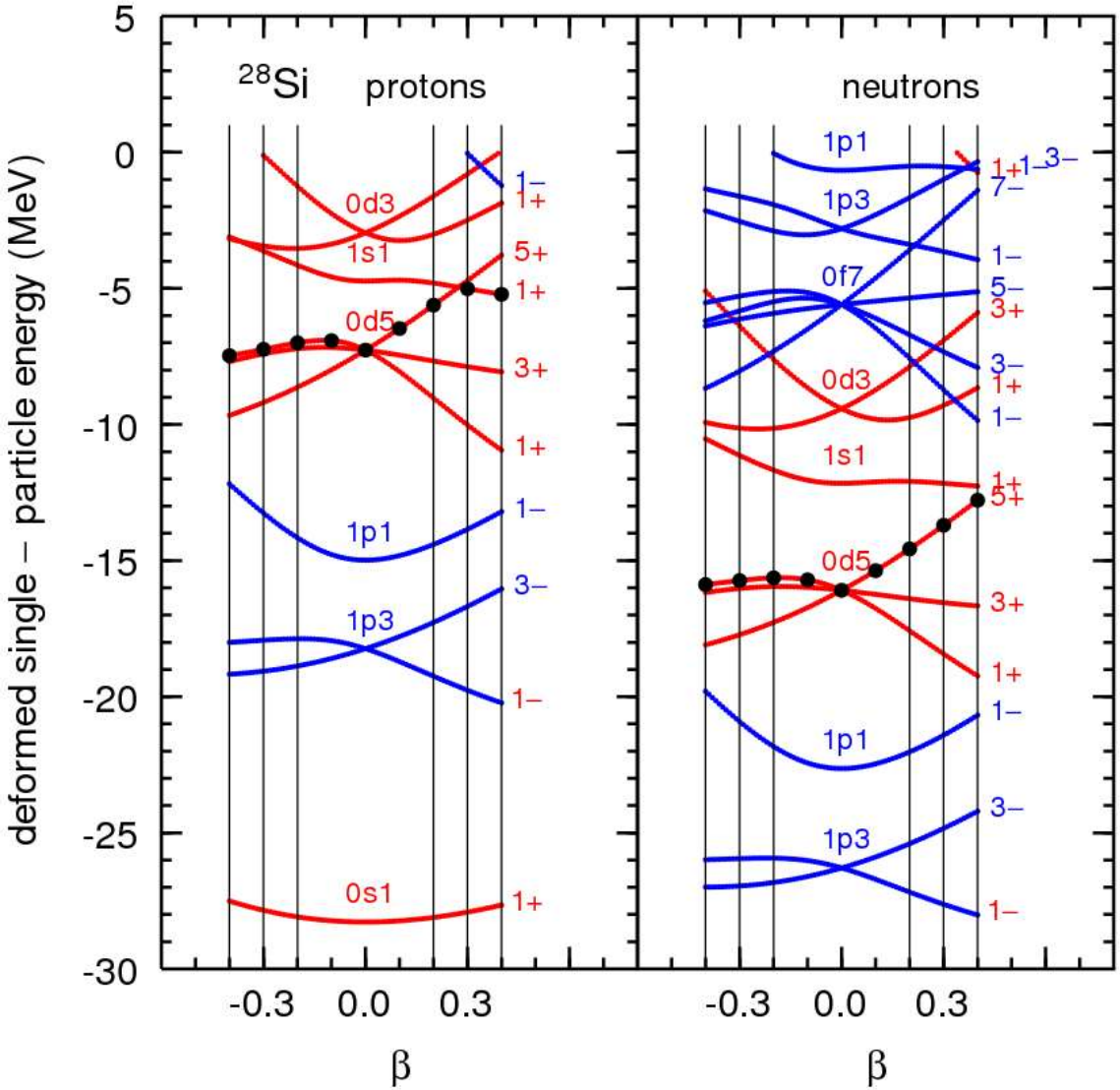

^{56}Ni
protons
neutrons
deformed single – particle energy (MeV)
β
β
1p3
0f7
0d3
0s1
0d5
1p1
0g9
5
0
-5
-10
-15
-20
-25
-30
-0.3
0.0
0.3

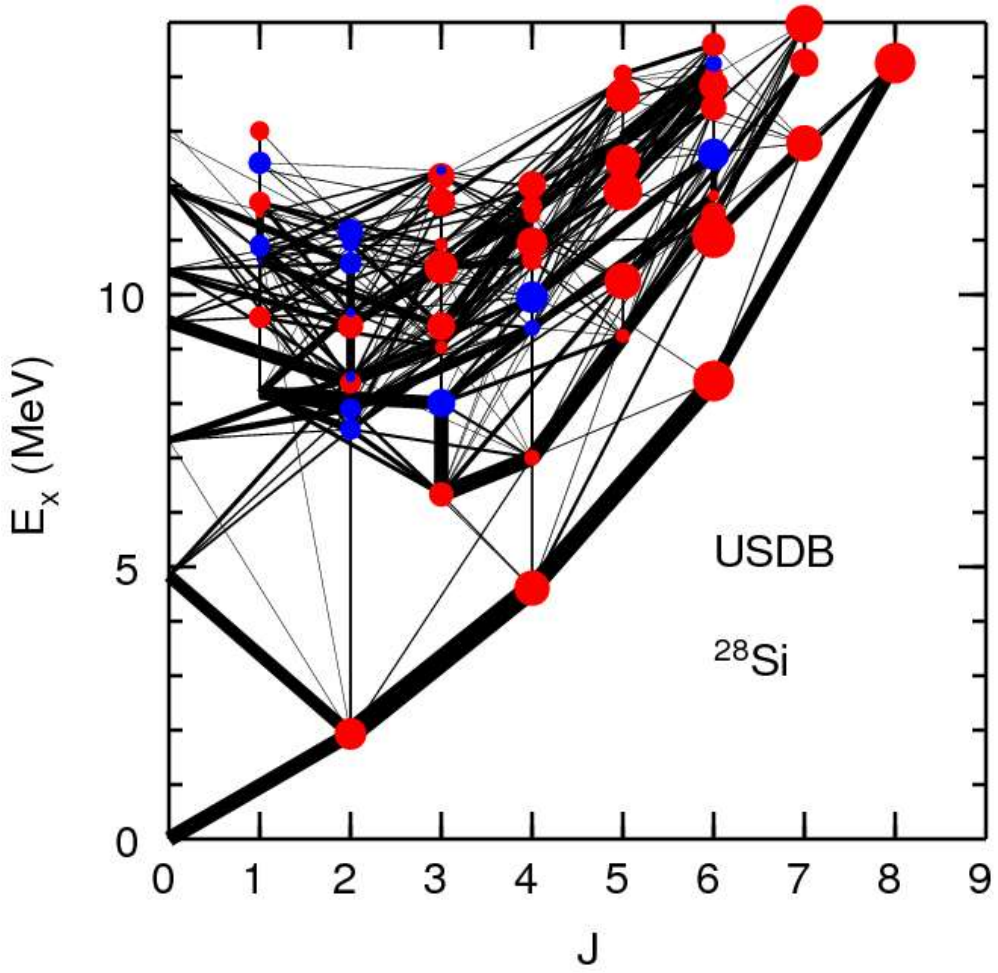


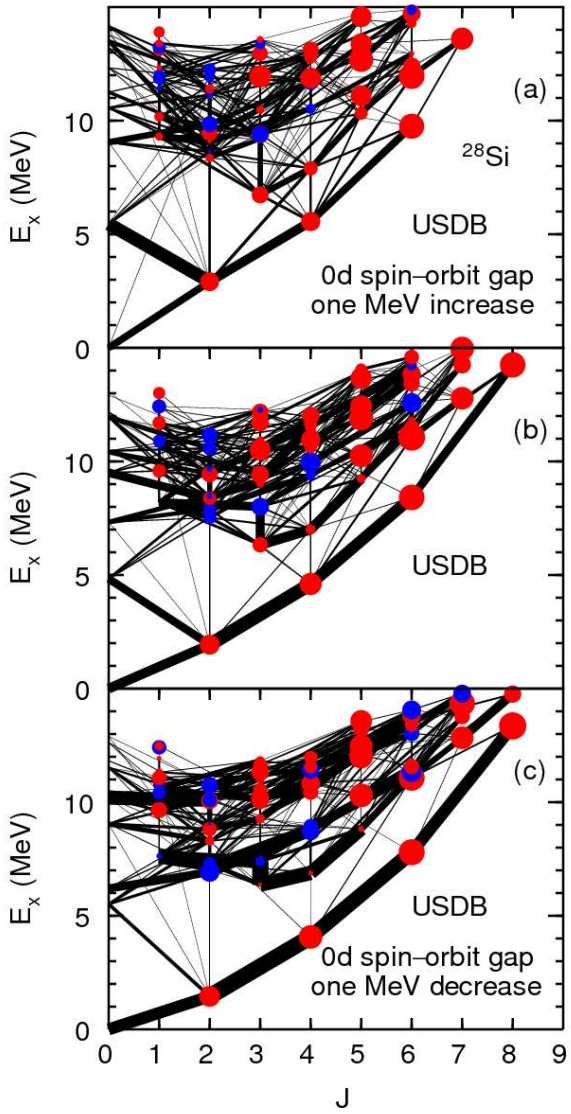
$$\langle Q \rangle_{JK} = \frac{3K^2 - J(J+1)}{(J+1)(2J+3)} (Q_0)_{\rm sp},$$

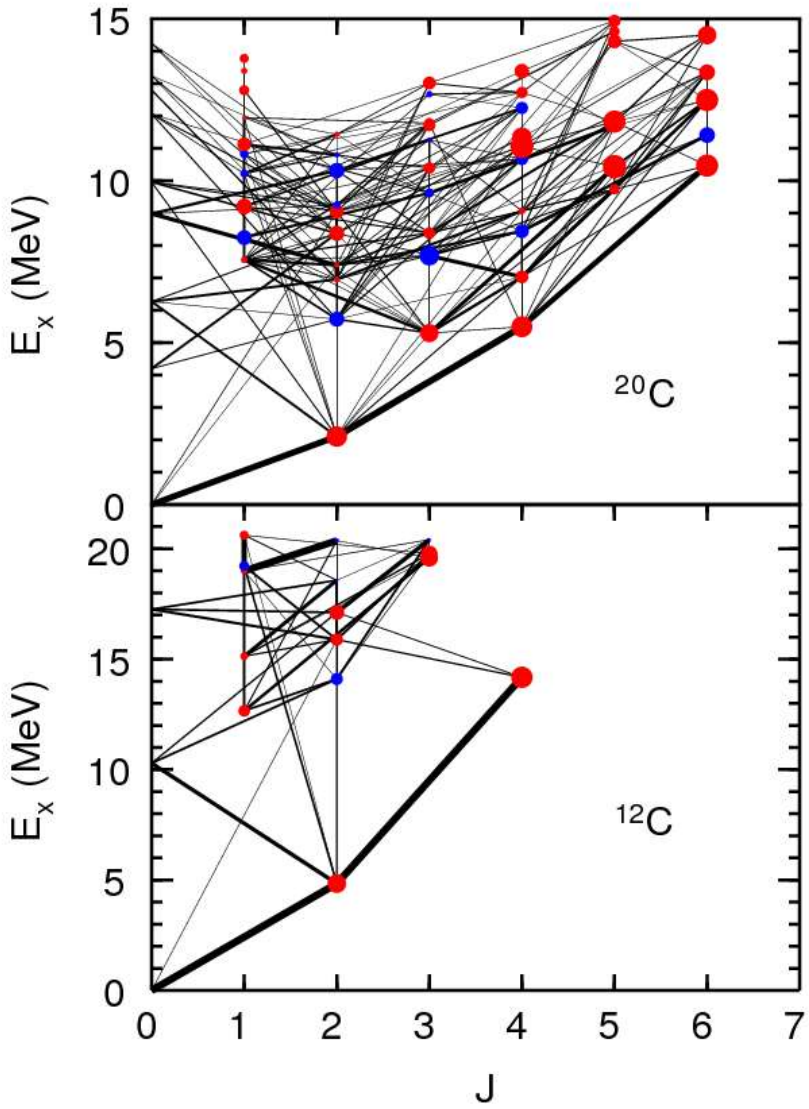
K=0 for even-even gs

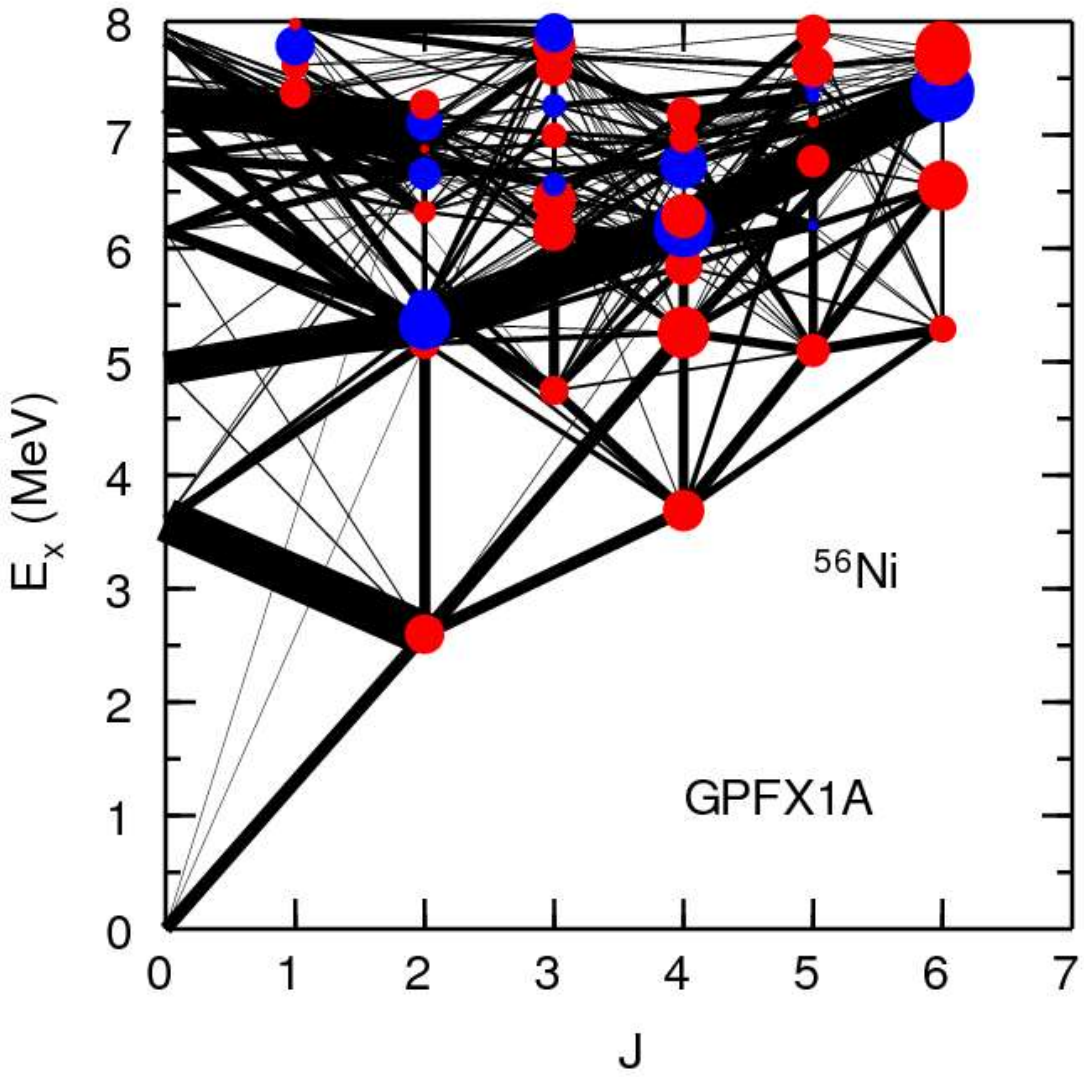
So gs band has oblate intrinsic Q


^{28}Si protons
neutrons
deformed single – particle energy (MeV)
β
β
5
0
-5
-10
-15
-20
-25
-30
-0.3
0.0
0.3
0d3
1s1
0d5
1p1
1p3
0s1
0f7
1–
1+
5+
3+
3–
7–
5–


E$_x$ (MeV)
0
5
10
0
1
2
3
4
5
6
7
8
9
J
USDB
^{28}Si


(a)
^{28}Si
USDB
0d spin–orbit gap
one MeV increase
(b)
USDB
(c)
USDB
0d spin–orbit gap
one MeV decrease
E$_x$ (MeV)
J
0
5
10
0 1 2 3 4 5 6 7 8 9


15
10
5
0
E_x (MeV)
20C
20
15
10
5
0
E_x (MeV)
12C
0
1
2
3
4
5
6
7
J


8
7
6
5
4
3
2
1
0
E_x (MeV)
0
1
2
3
4
5
6
7
J
^{56}Ni
GPFX1A

PHYSICAL REVIEW C **73**, 061305(R) (2006)

Shell model analysis of the ^{56}Ni spectrum in the full *pf* model space

M. Horoi,[1] B. A. Brown,[2,3] T. Otsuka,[4,5] M. Honma,[6] and T. Mizusaki[7]

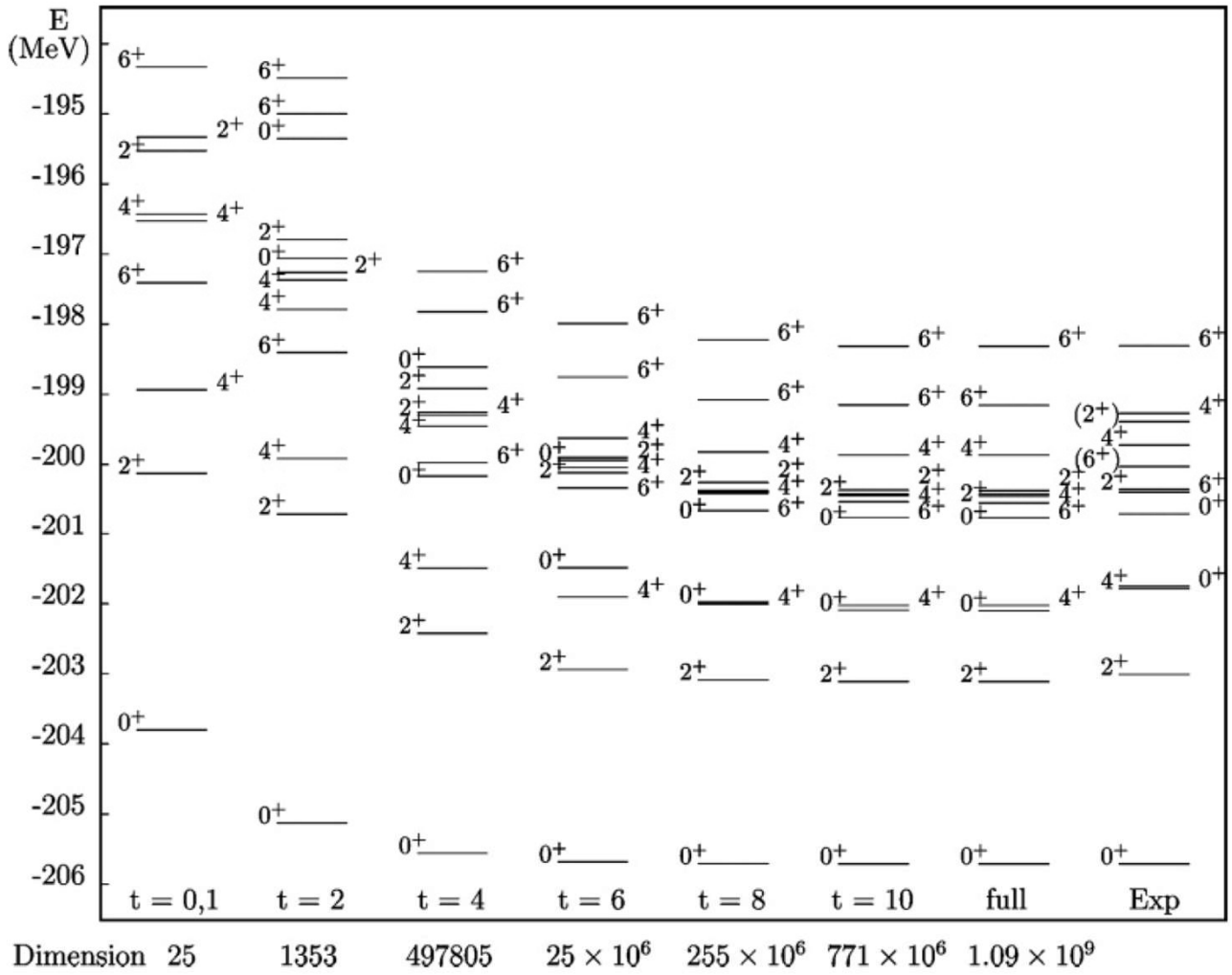
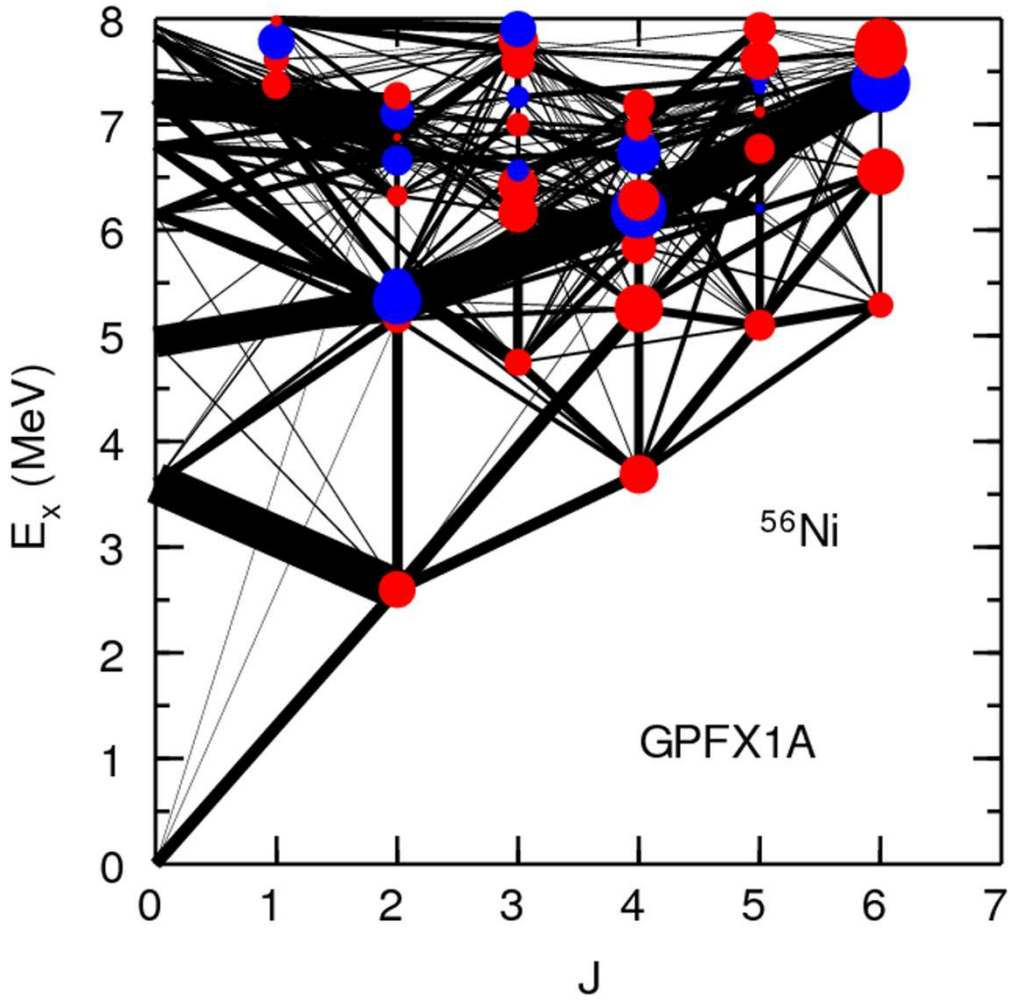


FIG. 1. The evolution of the first three 0^+, 2^+, 4^+, and 6^+ states as a function of the truncation level t.



$$M_{ij} = \frac{M_{sp}}{\sqrt{N}}$$

What is N?

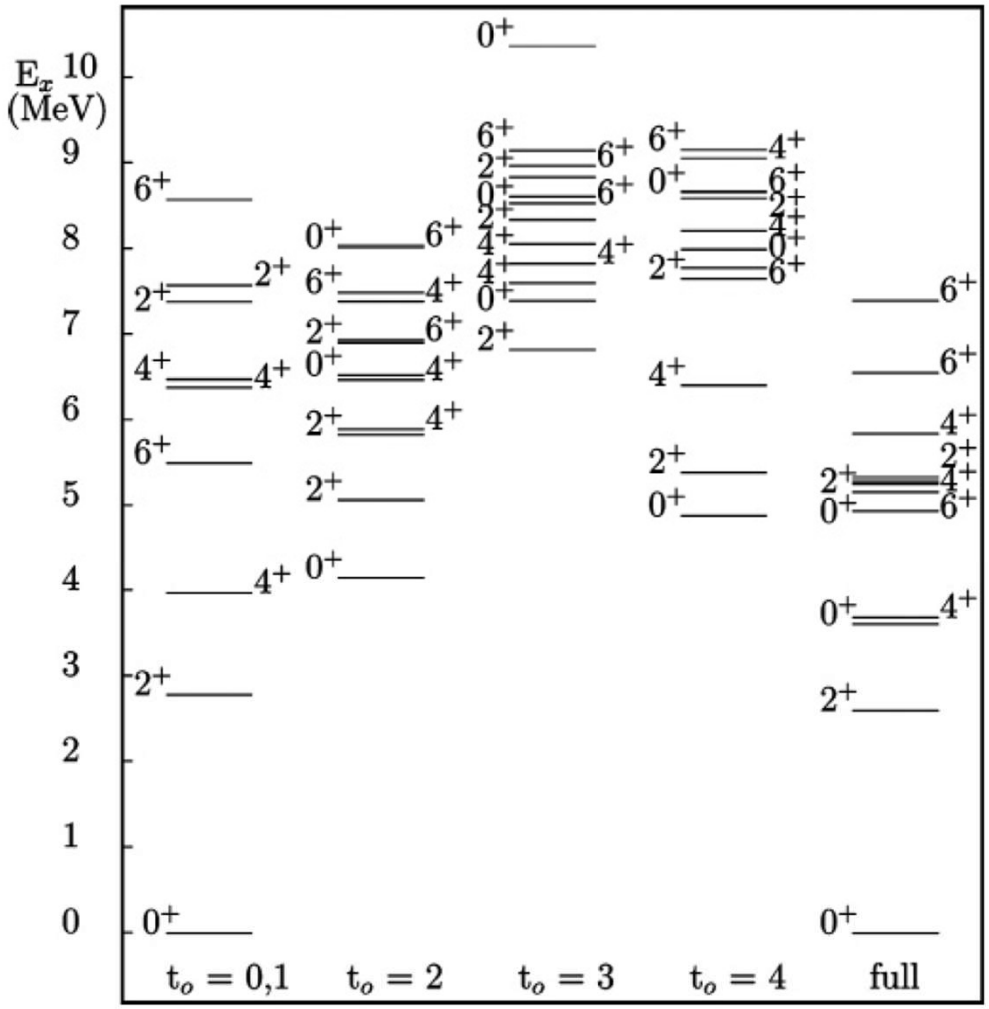


FIG. 2. The first three 0^+, 2^+, 4^+, and 6^+ states for the pure t_o configurations.

Eur. Phys. J. A **27**, 157–165 (2006)
DOI 10.1140/epja/i2005-10259-y

THE EUROPEAN
PHYSICAL JOURNAL A

Gamma-ray spectroscopy of the doubly magic nucleus ^{56}Ni

E.K. Johansson[1,a], D. Rudolph[1], J. Ekman[1,b], C. Fahlander[1], C. Andreoiu[1,c], M.A. Bentley[2,d], M.P. Carpenter[3], R.J. Charity[4], R.M. Clark[5], P. Fallon[5], R.V.F. Janssens[3], F.G. Kondev[3], T.L. Khoo[3], T. Lauritsen[3], A.O. Macchiavelli[5], W. Reviol[4], D.G. Sarantites[4], D. Seweryniak[3], C.E. Svensson[5,c], and S.J. Williams[2,e]

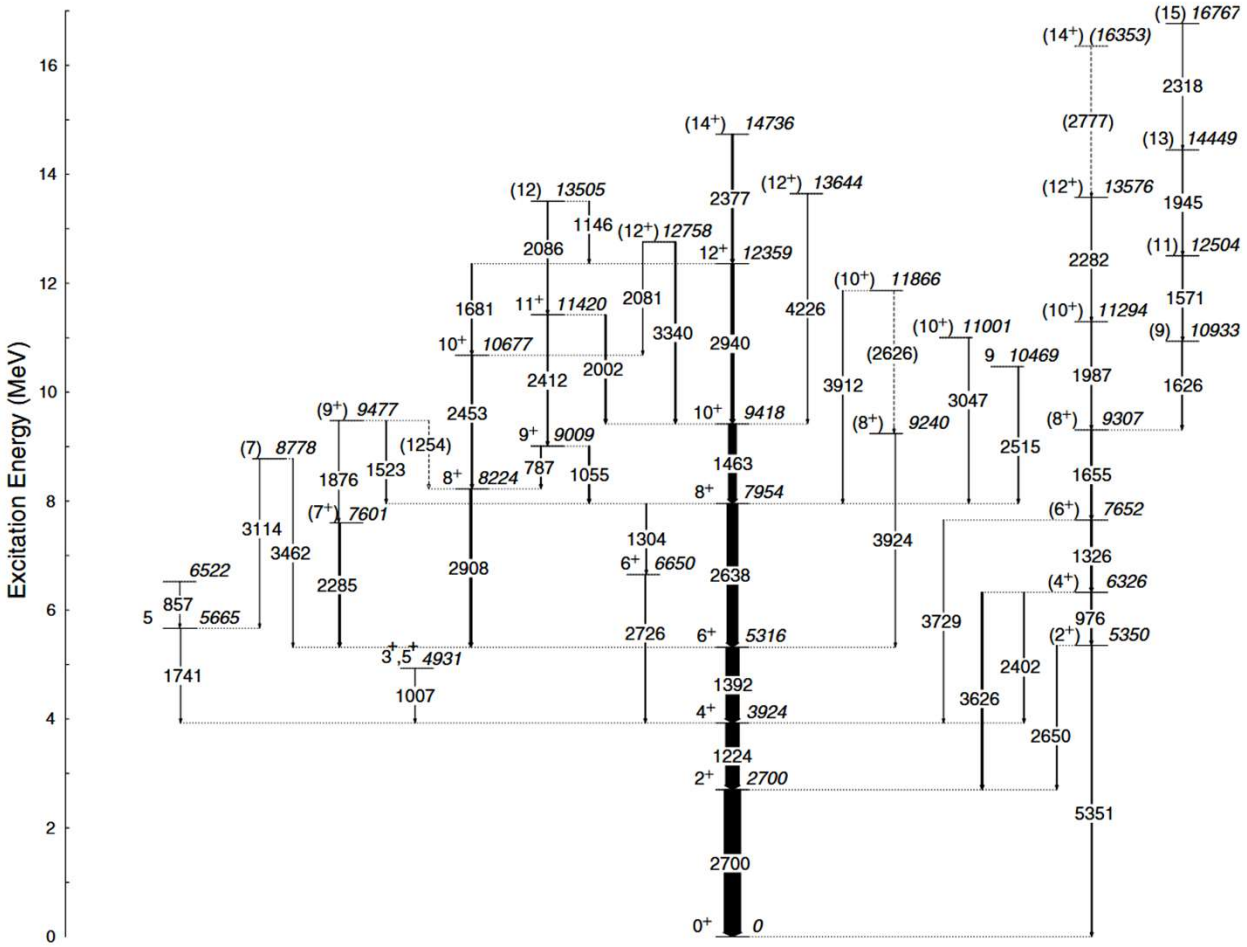


Fig. 1. The proposed experimental level scheme of ^{56}Ni from the present work, featuring some 20 new transitions. The energy labels are given in keV. The widths of the arrows are proportional to the relative intensities of the γ-rays. Dashed lines indicate tentative transitions and states.

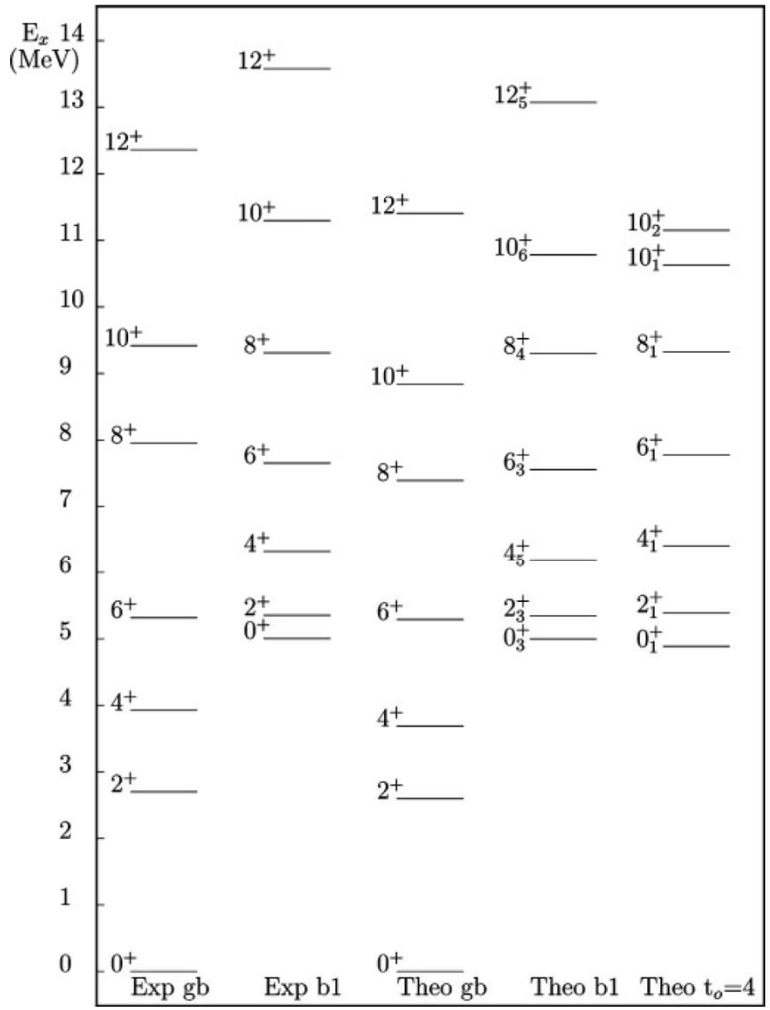


FIG. 3. Experimental and theoretical spectra for the ground band (gb) and the first rotational band (b1). The experimental energies are taken from Ref. [4].

In the collective model with $K = 0$

$$B(E2; J \to J-2) = \frac{15}{32\pi}\frac{J(J+1)}{(2J+1)(2J-1)}(Q_o)^2_{tr}$$

$$Q = \frac{-J(J+1)}{(J+1)(2J+3)}(Q_o)^2_{sp}$$

TABLE I. Electromagnetic properties of the states in the deformed band obtained in the $t = 10$ model space. The units are e fm^2 for the electric quadrupole moments and e^2 fm^4 for the $J \to J-2$ $B(E2)$ values.

J_n	E_x (MeV)	$\langle Q \rangle_{J0}$	$(Q_0)_{\rm sp}$	$B(E2)$	$\lvert Q_0 \rvert_{\rm tr}$
2_3	5.342	−41.6	+145.3	413.2	144.1
4_5	6.027	−55.2	+151.9	598.0	143.8
6_3	7.556	−56.2	+140.6	609.3	139.5
8_4	9.300	−47.2	+112.2	558.4	130.5
10_6	10.782	−63.9	+147.0	591.1	132.5
12_5	13.071	−62.7	+141.1	612.3	133.7

PHYSICAL REVIEW C VOLUME 10, NUMBER 5 NOVEMBER 1974

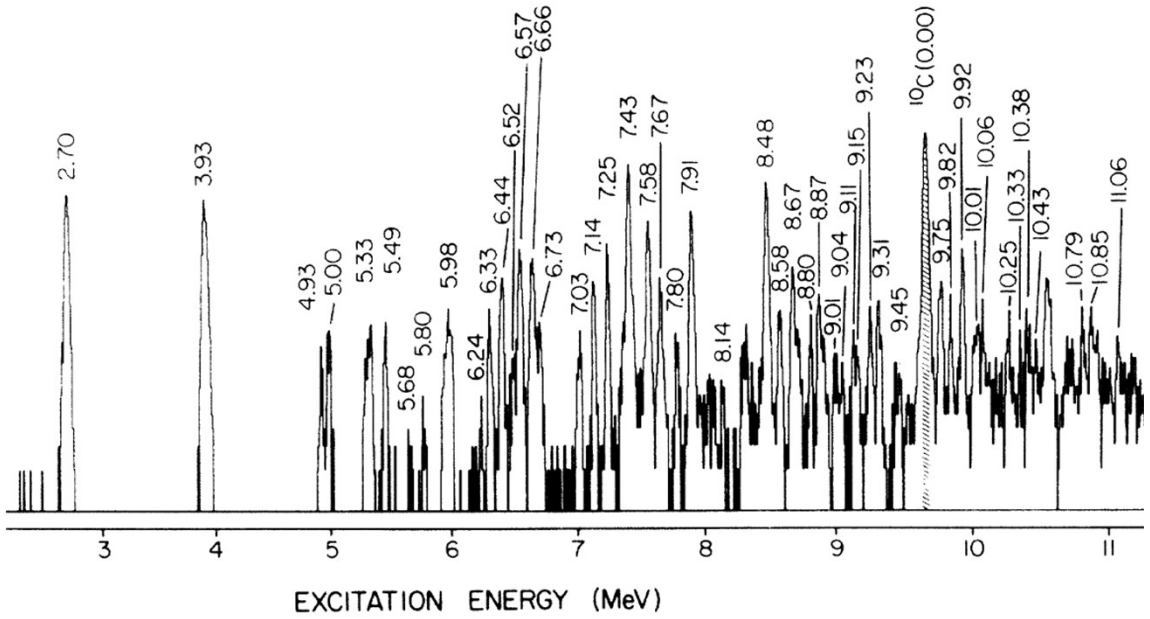
Levels of ^{56}Ni†

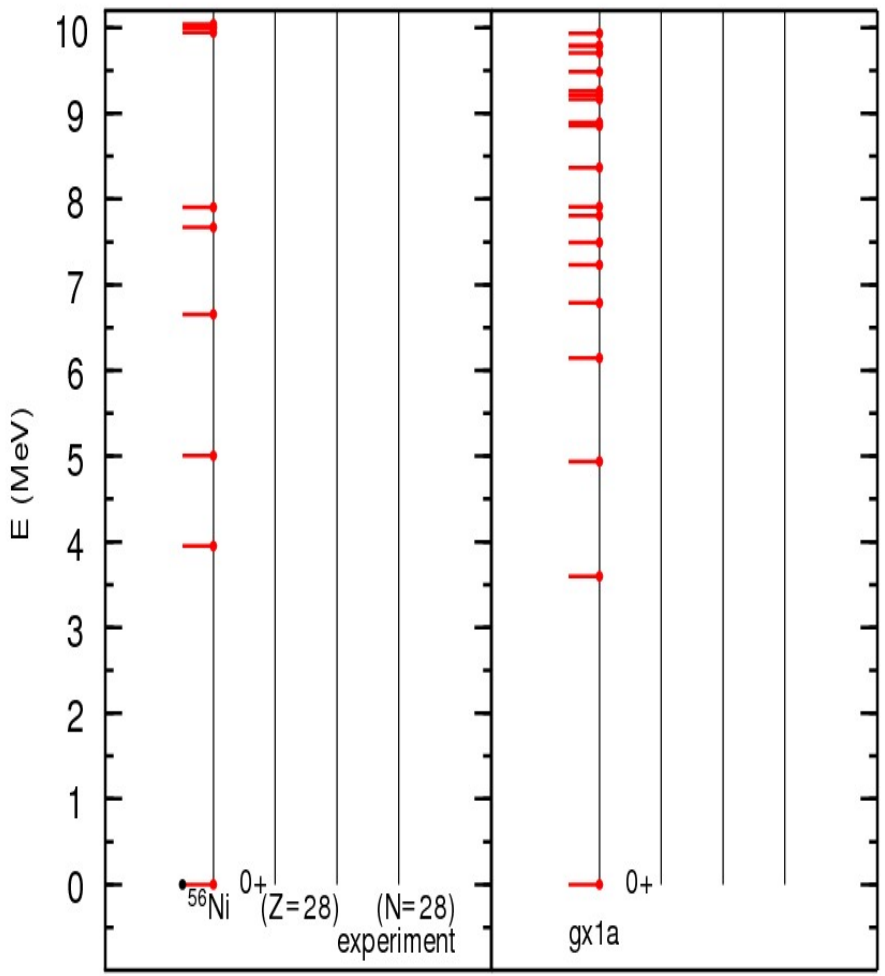
H. Nann* and W. Benenson

Cyclotron Laboratory and Department of Physics, Michigan State University, East Lansing, Michigan 48824

(Received 5 August 1974)

The ^{58}Ni$(p, t)^{56}$Ni reaction was studied at 40 and 45 MeV beam energy. An energy resolution of 10–25 keV permitted observation of 60 levels with excitation energy up to 10.5 MeV. Spin and parity are assigned to levels which were excited with characteristic angular distributions. These include 0^+ states at 3.95, 5.00, 6.44, 7.91, 9.92, 9.99, and 10.02 MeV.




E (MeV)
10
9
8
7
6
5
4
3
2
1
0
^{56}Ni
0+
(Z=28)
(N=28)
experiment
0+
gx1a

Special case for the second part of my talk

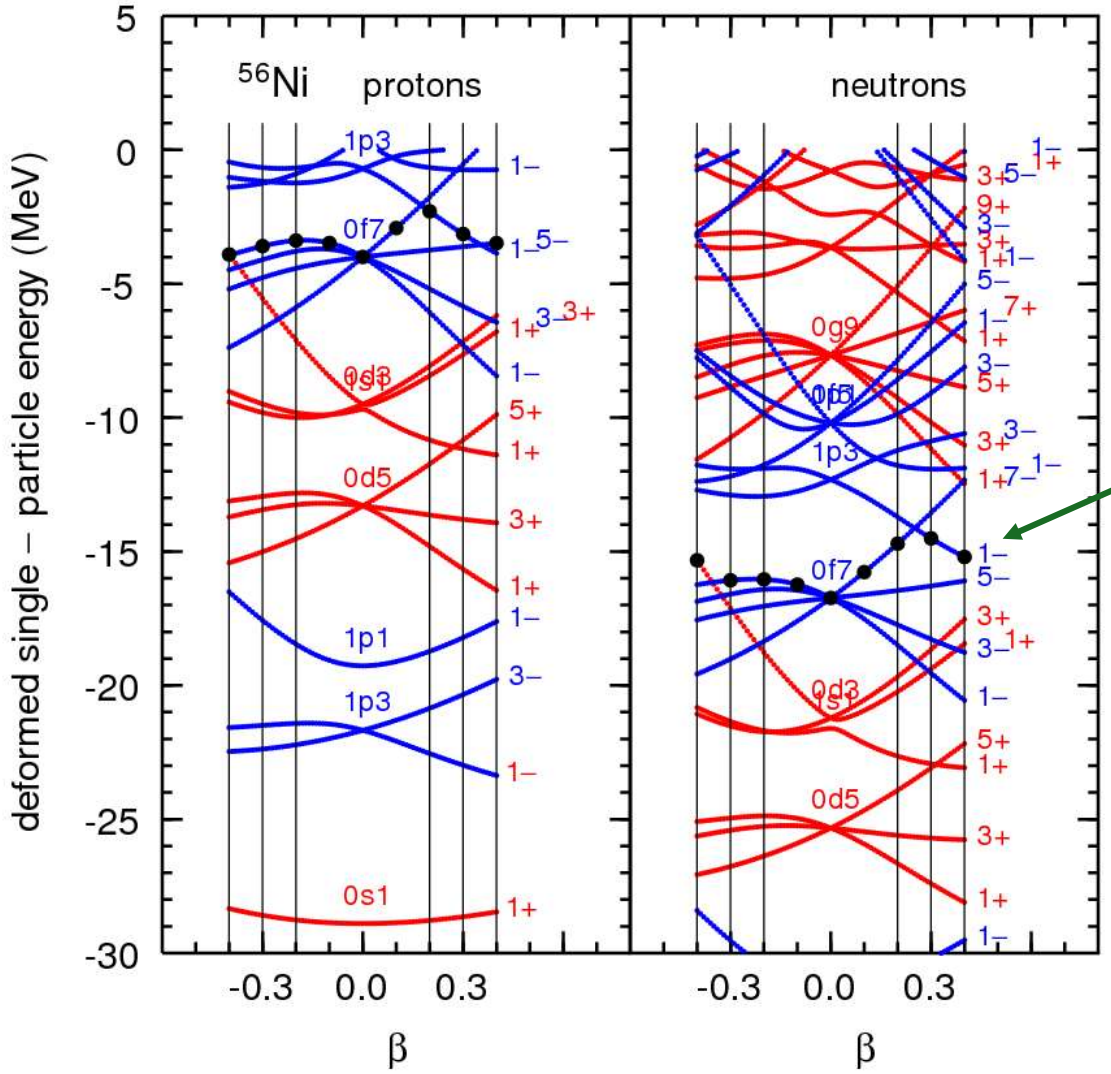
$B = B(E2; 0^+ \rightarrow 2^+) = \frac{5}{16\pi} Q_o^2$

$Q = -\frac{2}{7} \cos(3\gamma) Q_o$

$q \equiv \frac{Q}{\sqrt{aB}} = -\cos(3\gamma)$

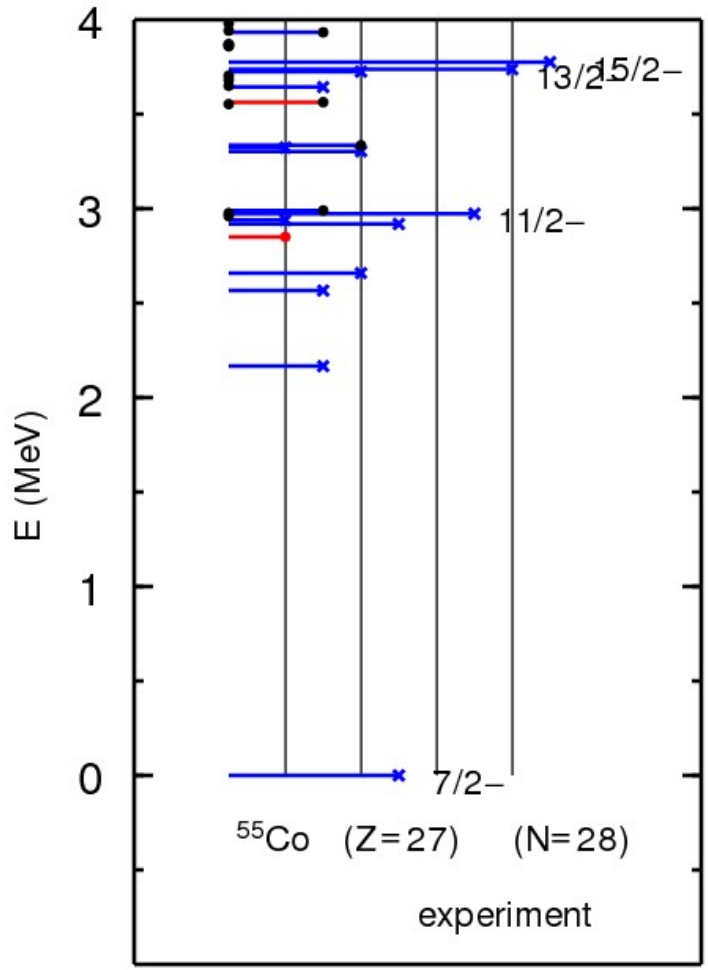
$a = \frac{64\pi}{245} = 0.82$

q goes between -1 (prolate, γ=0) and +1 (oblate, γ=60^o)



4p-4h in Nilsson

Deformation narrows the gap between positive negative parity states


4
3
2
1
0
E (MeV)
15/2−
13/2−
11/2−
7/2−
55Co (Z=27) (N=28)
experiment

PHYSICAL REVIEW C **102**, 014315 (2020)

Microscopic calculations of nuclear level densities with the Lanczos method

W. E. Ormand [1,2,*] and B. A. Brown [2]

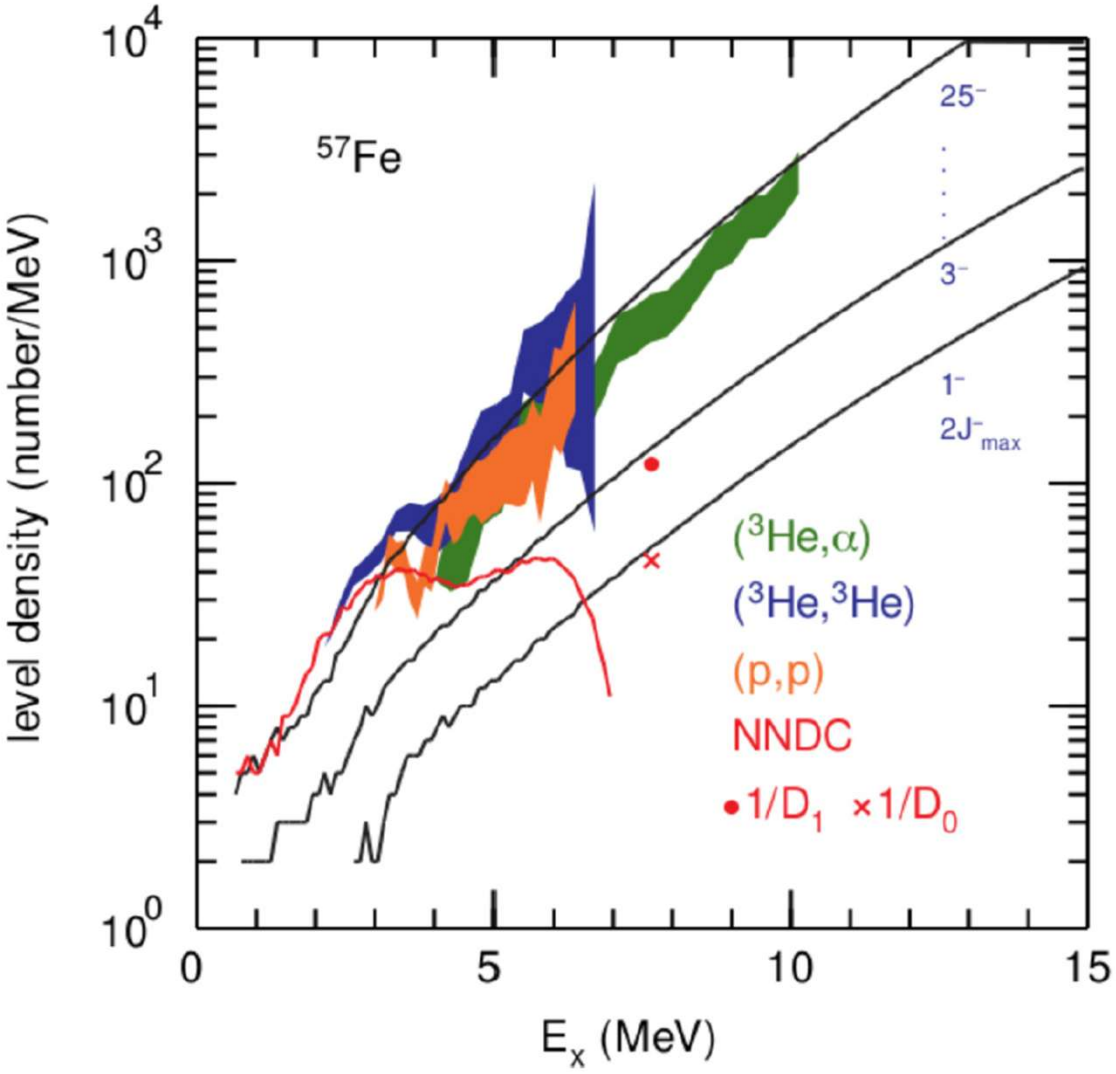
[1] *Lawrence Livermore National Laboratory, P.O. Box 808, L-414, Livermore, California 94551, USA*

[2] *Department of Physics and the National Superconducting Cyclotron Laboratory, Michigan State University, East Lansing, Michigan 42284-1321, USA*

(Received 4 March 2018; revised 19 December 2019; accepted 17 June 2020; published 23 July 2020)

A new method for computing the density of states in nuclei making use of an extrapolated form of the tridiagonal matrix obtained from the Lanczos method is presented. It will be shown that the global, average properties of the entire Lanczos matrix can be predicted from just four Lanczos iterations. The extrapolated Lanczos matrix (ELM) approach provides for an accurate computation of the density of states described within the configuration space, which, in some cases, is sufficient to accurately calculate the density of states at, or near, the neutron separation energy. Comparisons between theory and experiment are shown for ^{57}Fe, ^{74}Ge, and ^{76}Ge. In addition, we show results for the J-dependence of moments and the level density for these three nuclei.

DOI: 10.1103/PhysRevC.102.014315



fp shell calculation for the level density

At 8 MeV the level density for + and – are about the same.

But the fp shell calculations has no + parity

PHYSICAL REVIEW C **109**, 054318 (2024)

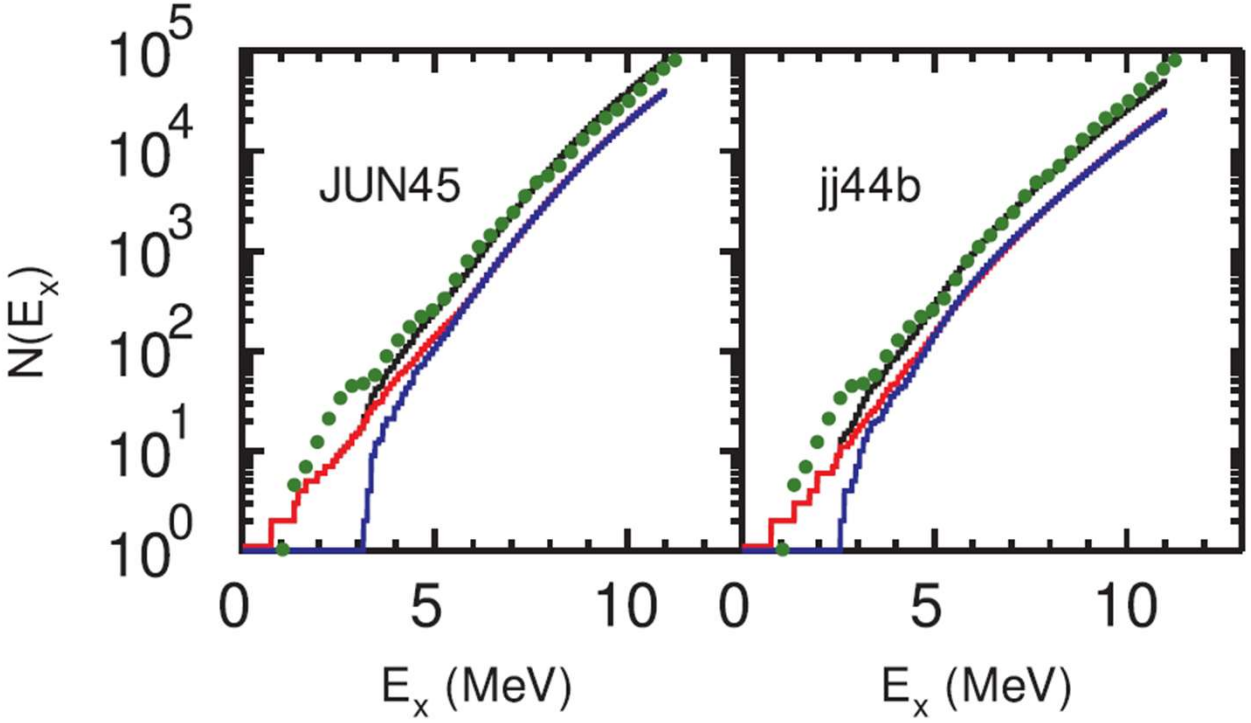
Nuclear structure of ^{74}Ge from inelastic neutron scattering measurements and shell-model calculations

E. E. Peters,[1,*] B. A. Brown,[2,3] S. Mukhopadhyay,[1,4,†] A. P. D. Ramirez,[1,4,‡] and S. W. Yates[1,4,§]

PHYSICAL REVIEW C **99**, 054609 (2019)

Level densities of ^{74,76}Ge from compound nuclear reactions

A. V. Voinov,[1,*] T. Renstrøm,[2] D. L. Bleuel,[3] S. M. Grimes,[1] M. Guttormsen,[2] A. C. Larsen,[2] S. N. Liddick,[4,5] G. Perdikakis,[5,6] A. Spyrou,[5,7] S. Akhtar,[1] N. Alanazi,[1,†] K. Brandenburg,[1] C. R. Brune,[1] T. W. Danley,[1] S. Dhakal,[1,‡] P. Gastis,[6] R. Giri,[1] T. N. Massey,[1] Z. Meisel,[1] S. Nikas,[5,§] S. N. Paneru,[1] C. E. Parker,[1,‖] and A. L. Richard[1,¶]



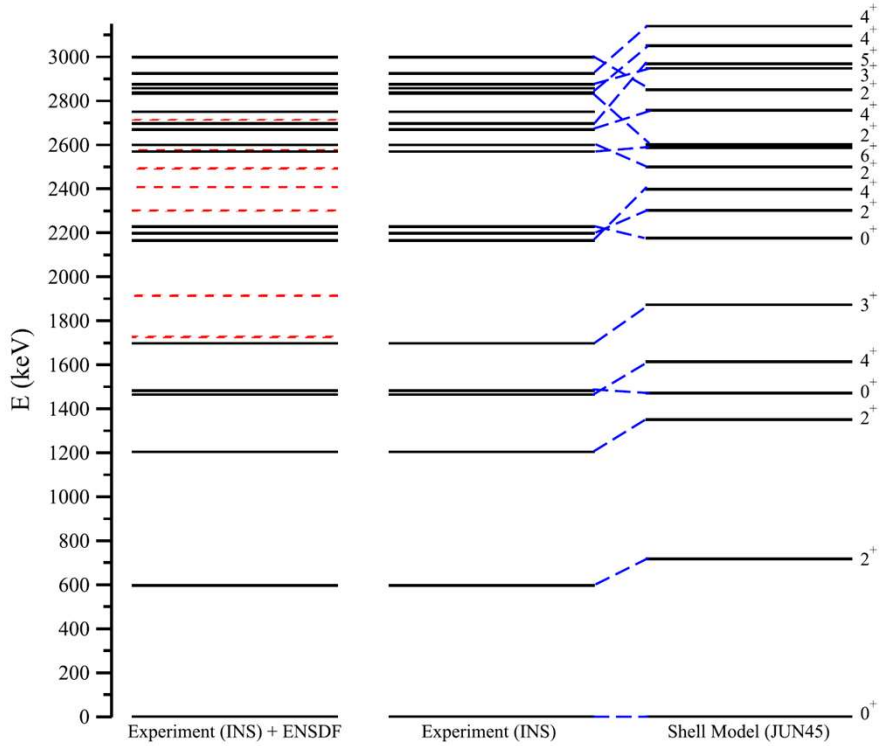
jj44 shell calculation for the level density of 74Ge

Voinov results in green

Above 6 MeV the level density for + (red) and – (blue) are about the same in the calculation

There are ½+ states but no $s_{1/2}$ strength

What is the peak in exp around 3 MeV from NNDC?


E (keV)
3000
2800
2600
2400
2200
2000
1800
1600
1400
1200
1000
800
600
400
200
0
4+
4+
5+
3+
2+
4+
2+
6+
2+
4+
2+
0+
3+
4+
0+
2+
2+
0+
Experiment (INS) + ENSDF
Experiment (INS)
Shell Model (JUN45)

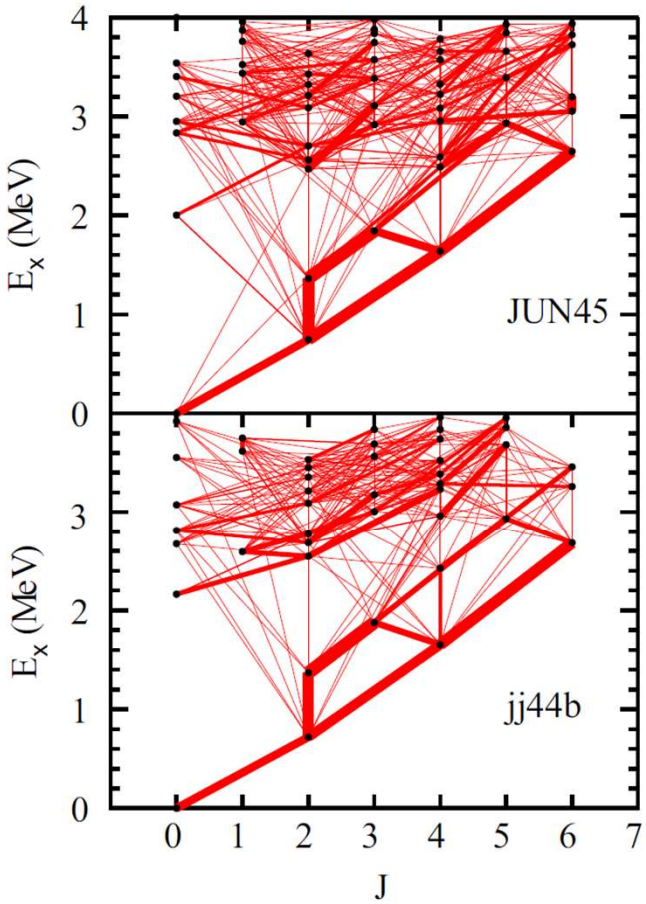
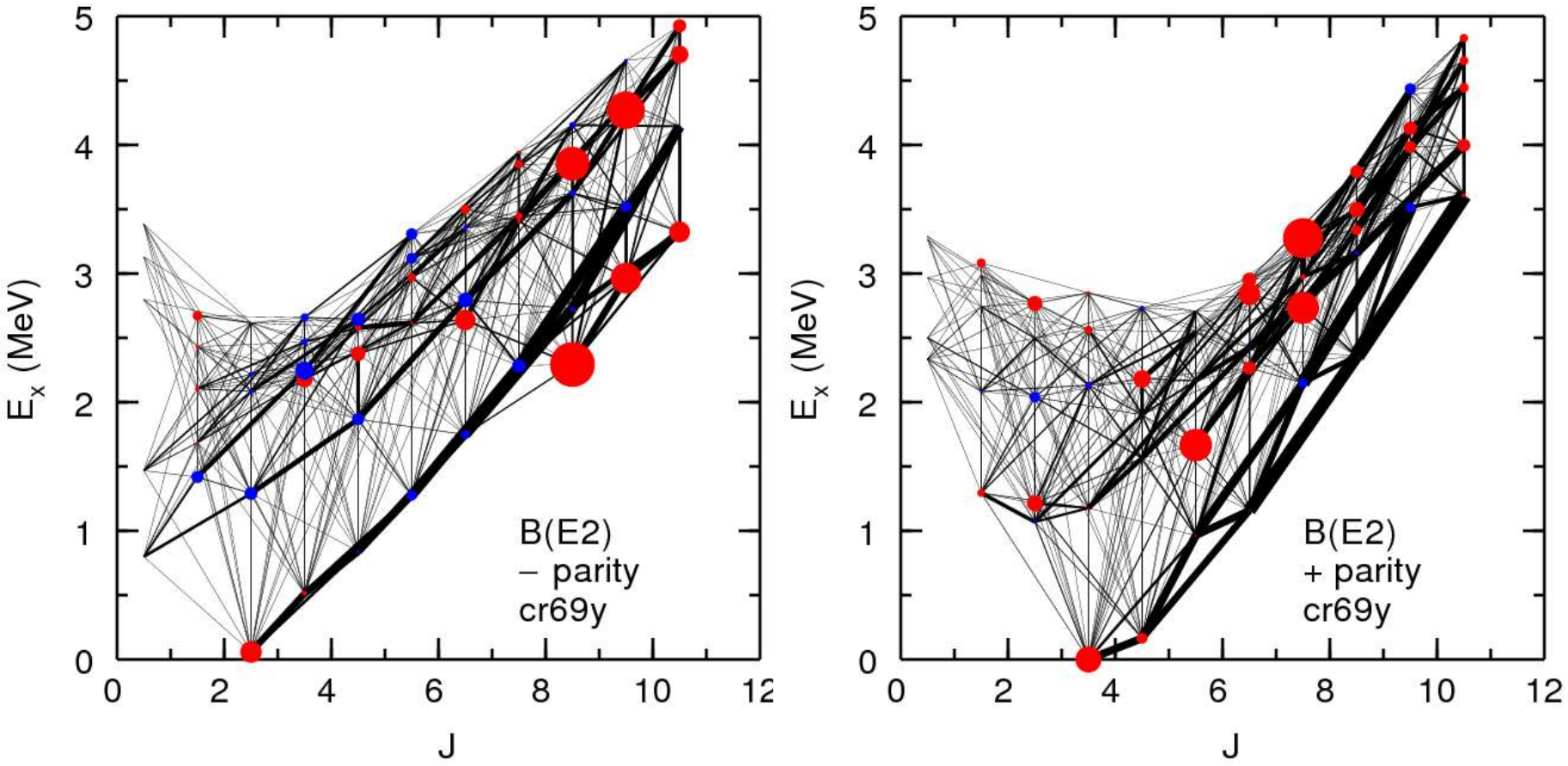
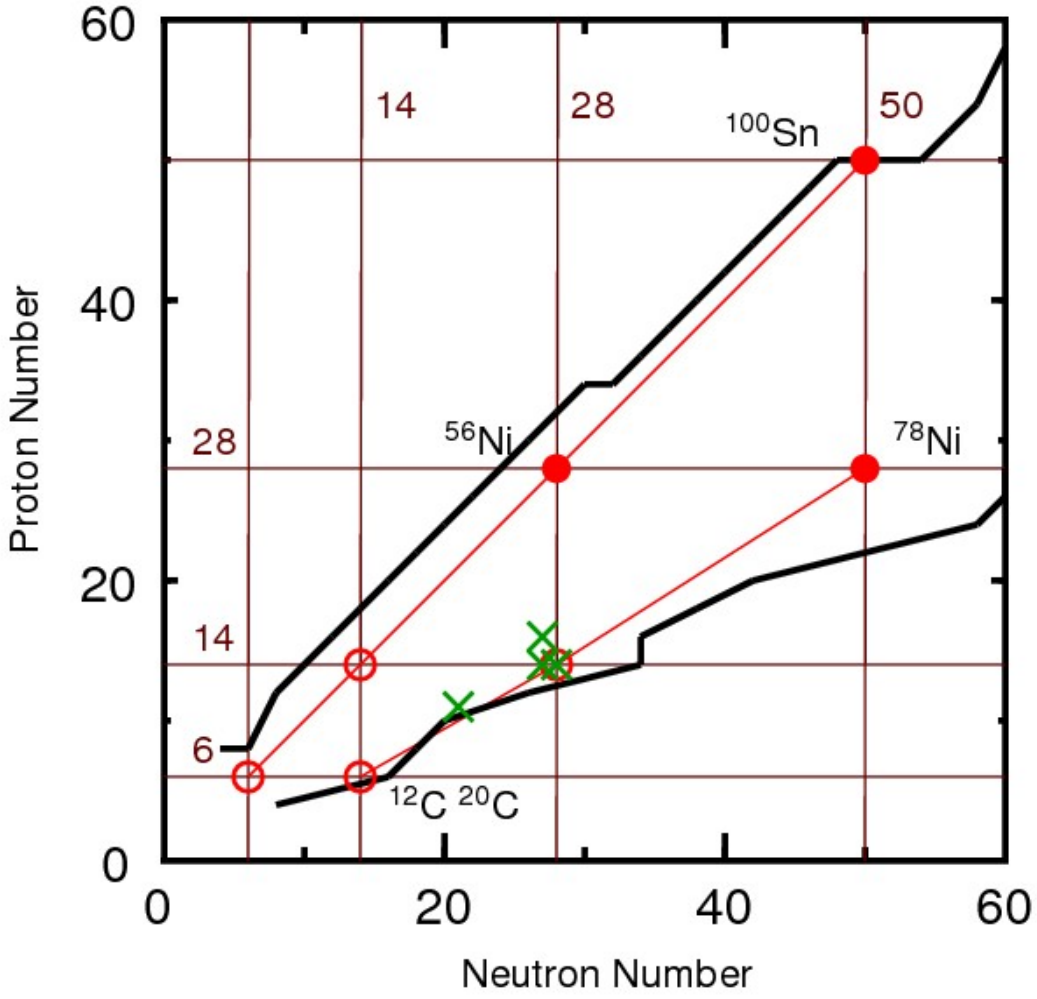


FIG. 6. Levels in ^{76}Ge connected by bars whose widths are proportional to the $B(E2)$ values obtained with experiment and the shell model calculations. Only those transitions which decay with $B(E2)$ values larger than one W.u. are depicted in the figure.

^{76}Ge



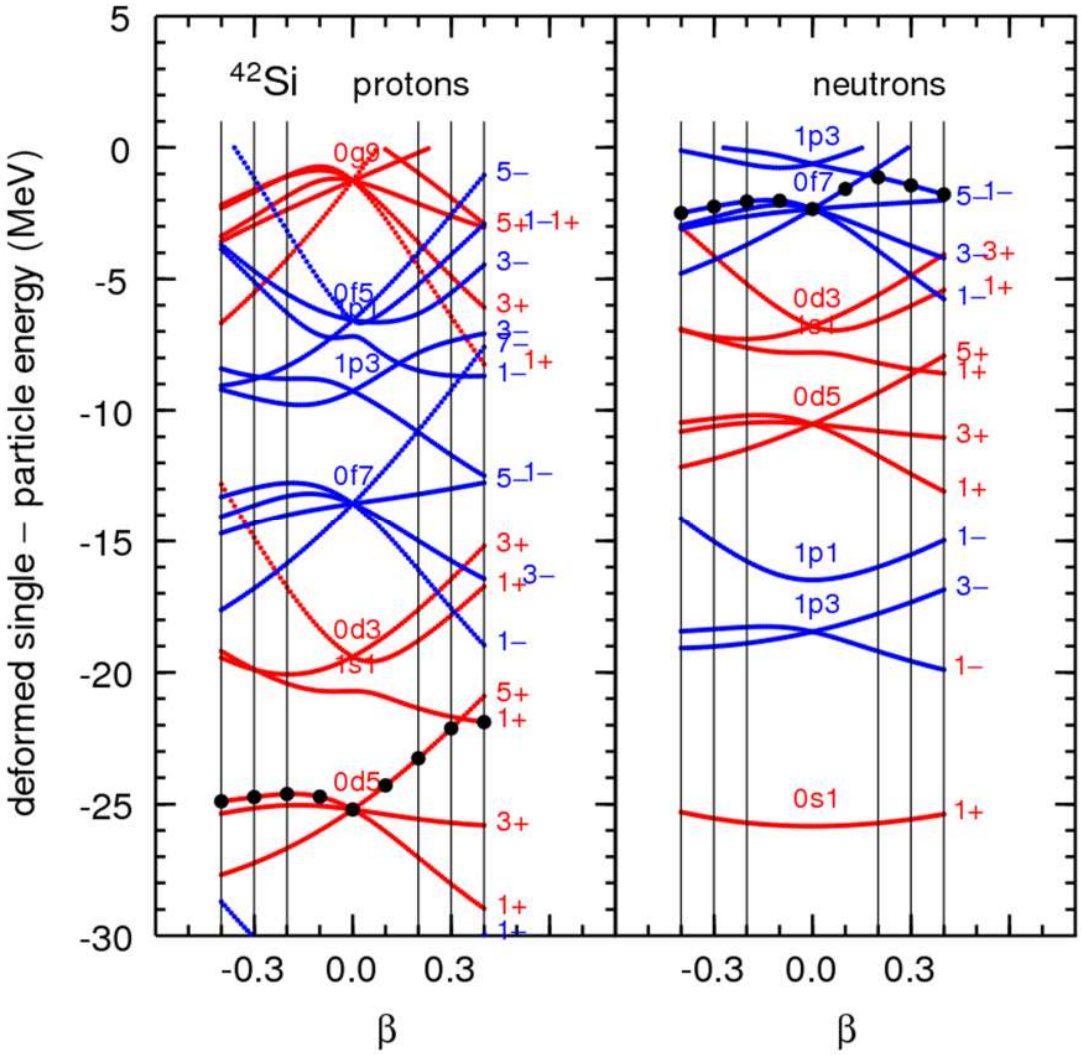
^{69}Cr

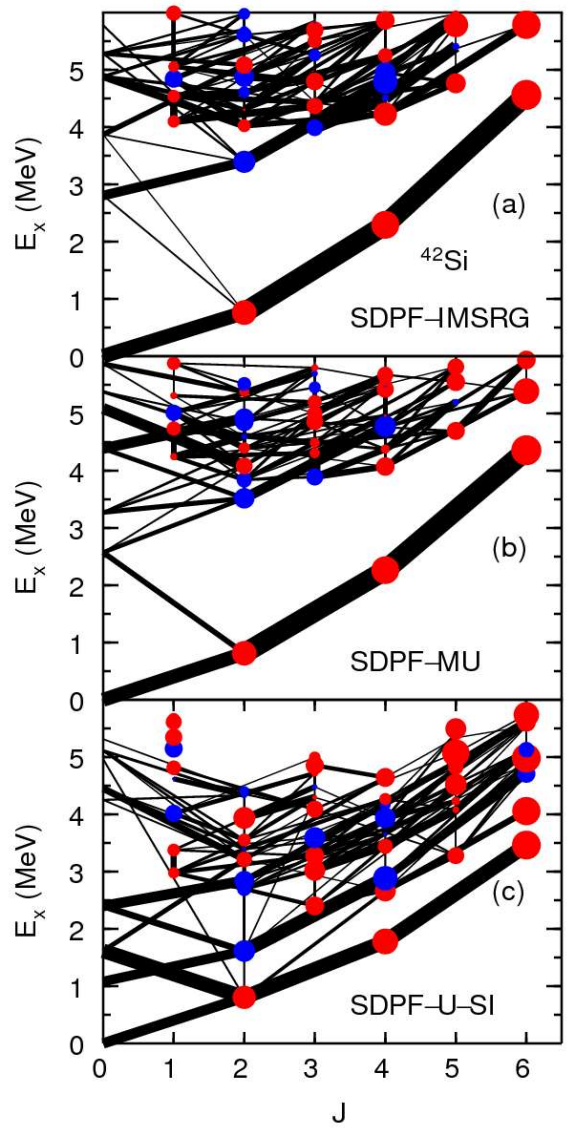


^{43}S

^{41}Si ^{42}Si

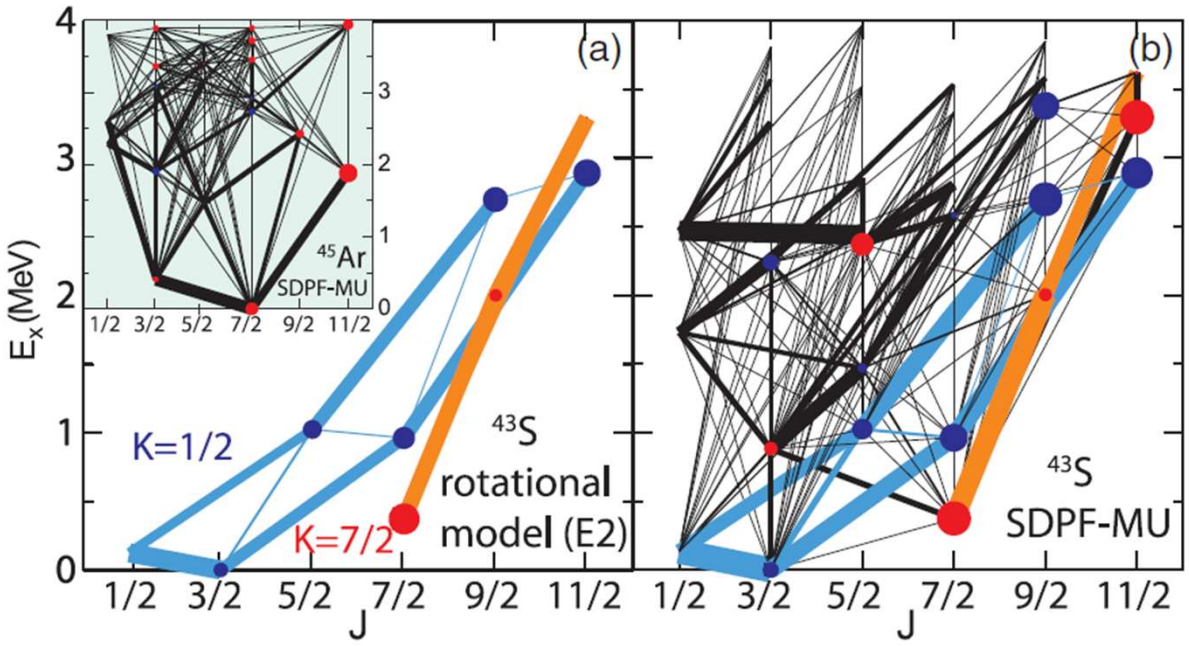
^{32}Na


^{42}Si
protons
neutrons
deformed single – particle energy (MeV)
β
β
0g9
0f5
1p3
0f7
0d3
1s1
0d5
1p1
0s1
1p3
0f7
0d3
1s1
0d5
1p1
1p3
0s1


E_x (MeV)
(a)
42Si
SDPF–IMSRG
(b)
SDPF–MU
(c)
SDPF–U–SI
J

Shape Changes in the $N = 28$ Island of Inversion: Collective Structures Built on Configuration-Coexisting States in ^{43}S

B. Longfellow,[1,2,3] D. Weisshaar,[1] A. Gade,[1,2] B. A. Brown,[1,2] D. Bazin,[1,2] K. W. Brown,[1,4] B. Elman,[1,2] J. Pereira,[1] D. Rhodes,[1,2] and M. Spieker[1,*]



Rapid structural evolution of neutron-rich silicon isotopes toward $N = 28$

G. L. Zimba,[1,2,*] H. Iwasaki,[1,3] B. A. Brown,[1,3] Y. Utsuno,[4,5] N. Shimizu,[6] N. Aoi,[5,7] M. Basson,[1,3] T. Beck,[1,†] J. Chen,[1] J. Chung-Jung,[1,3] A. Douglas,[1,3] A. Ertoprak,[8] P. Farris,[1,3] C. Fransen,[9] A. Gade,[1,3] S. A. Gillespie,[1] A. Hill,[1,3] K. Kolos,[10] D. Lempke,[1,3] I. Lihtar,[1,11] T. Mijatović,[11] S. Neupane,[10] S. Noji,[1] T. Parry,[1] A. Revel,[1,3] R. Salinas,[1,3] A. Sanchez,[1,3] E. Schieb,[1,3] D. Weisshaar,[1] and Y. Yamamoto[5,7]

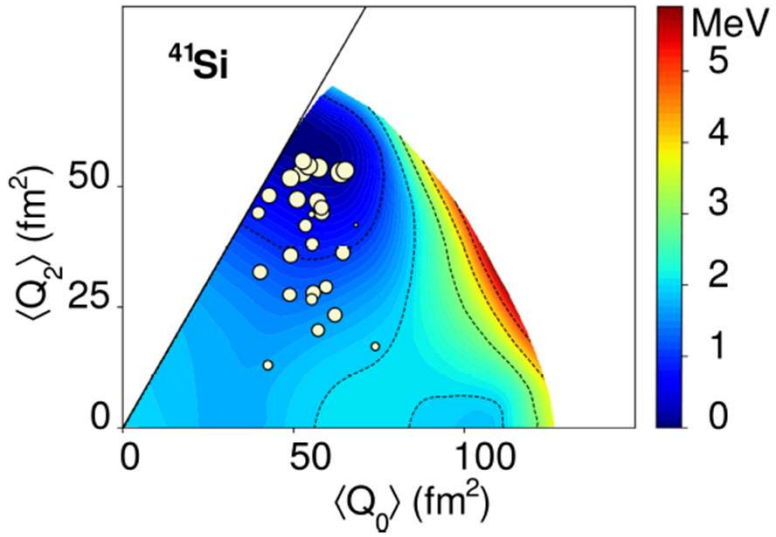
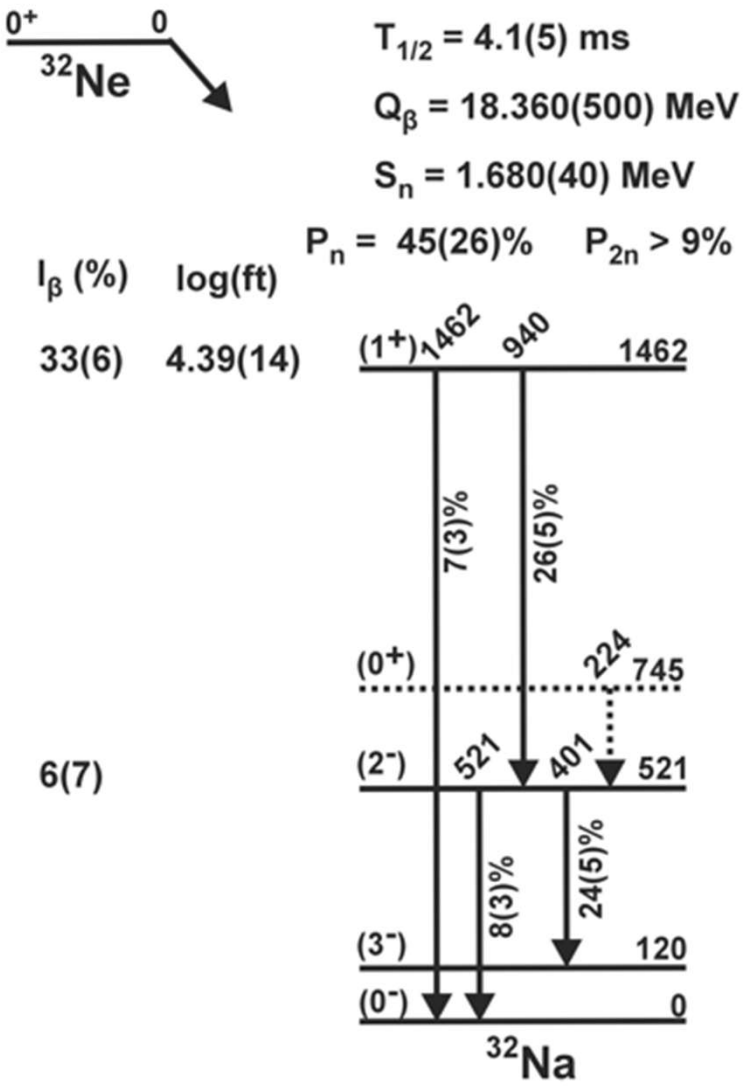
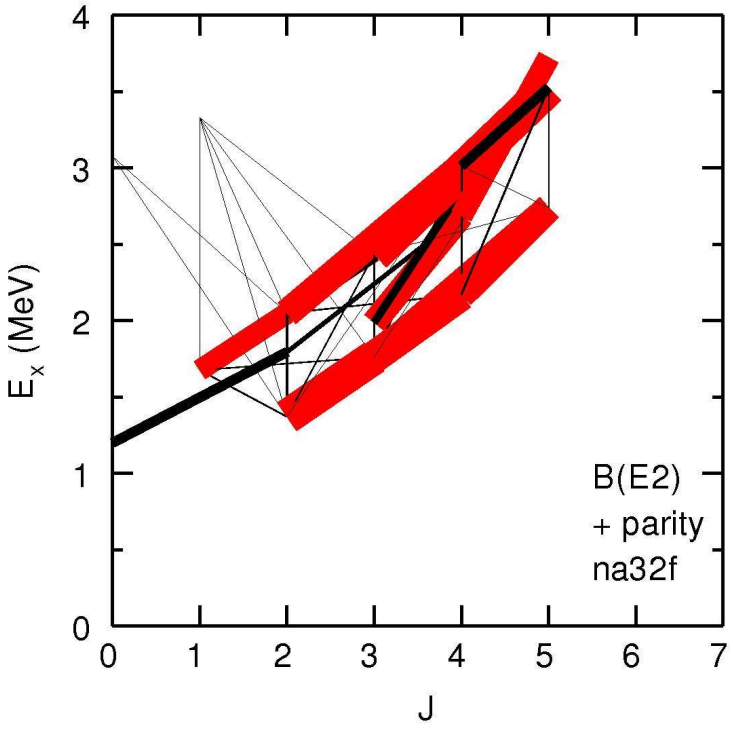
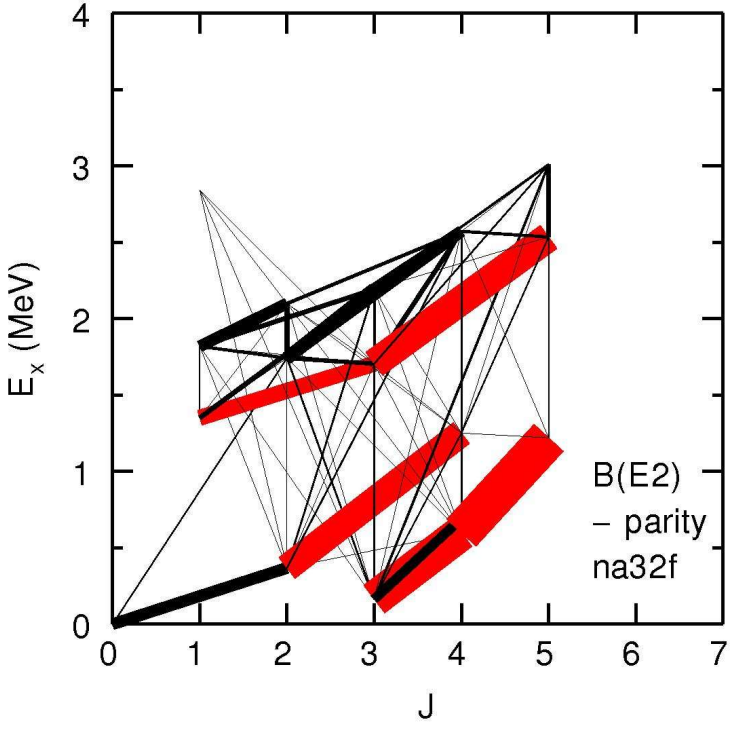


FIG. 3. Potential-energy surfaces (PES) for the ground state of ^{41}Si as a function of quadrupole moments. The energy relative to the minimum is shown by contour plots. Circles on the PES represent shapes of MCSM basis vectors (see the text).

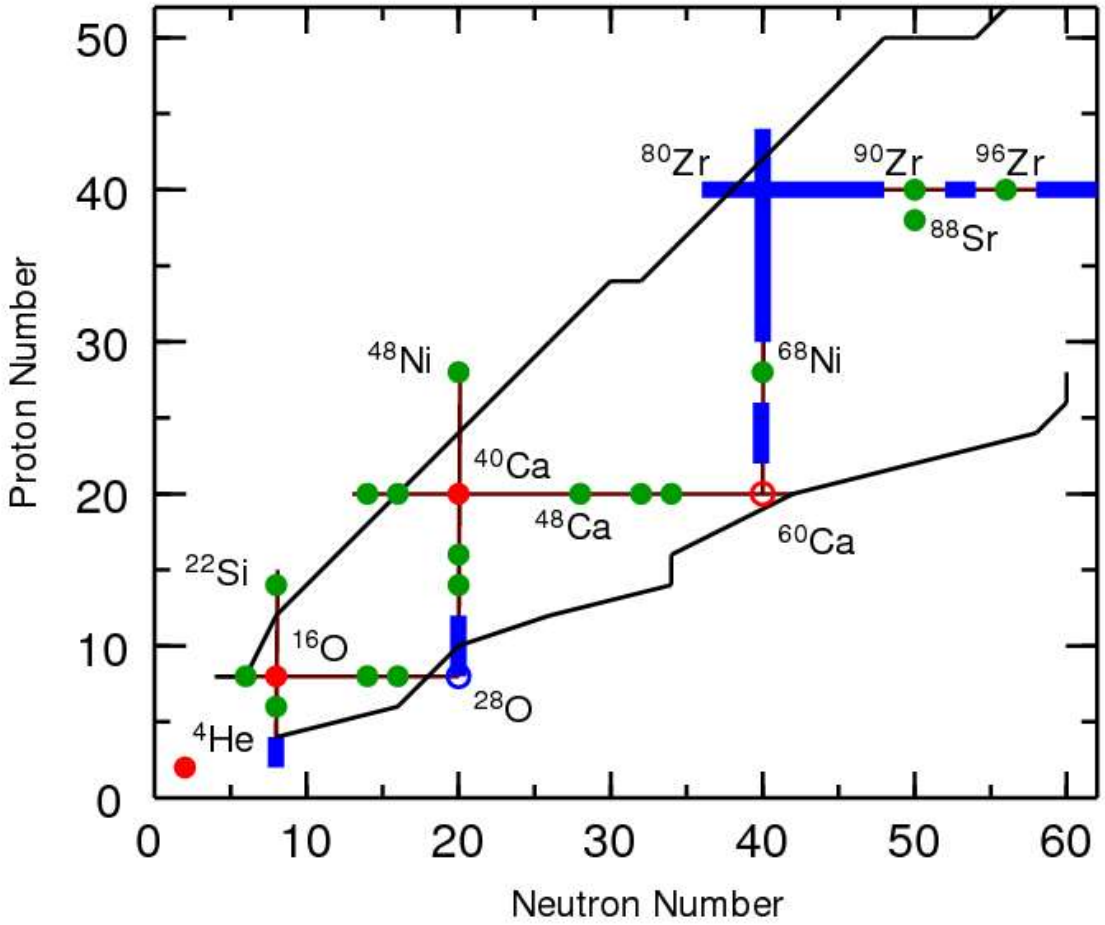


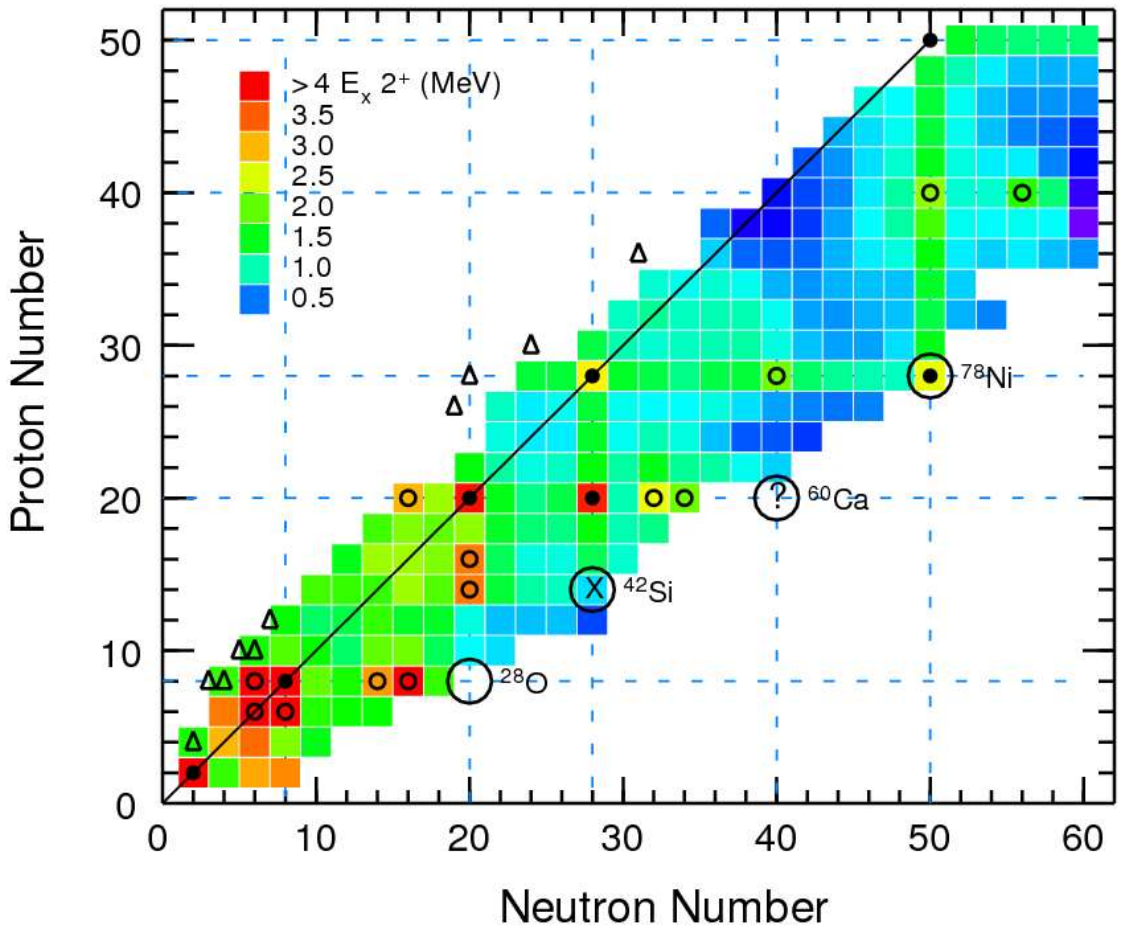
Amelia Doetsch (thesis)


E$_x$ (MeV)
J
B(E2)
+ parity
na32f


E$_x$ (MeV)
J
B(E2)
– parity
na32f

LS-magic numbers - oscillator filled shells 2, 8, 20, 40



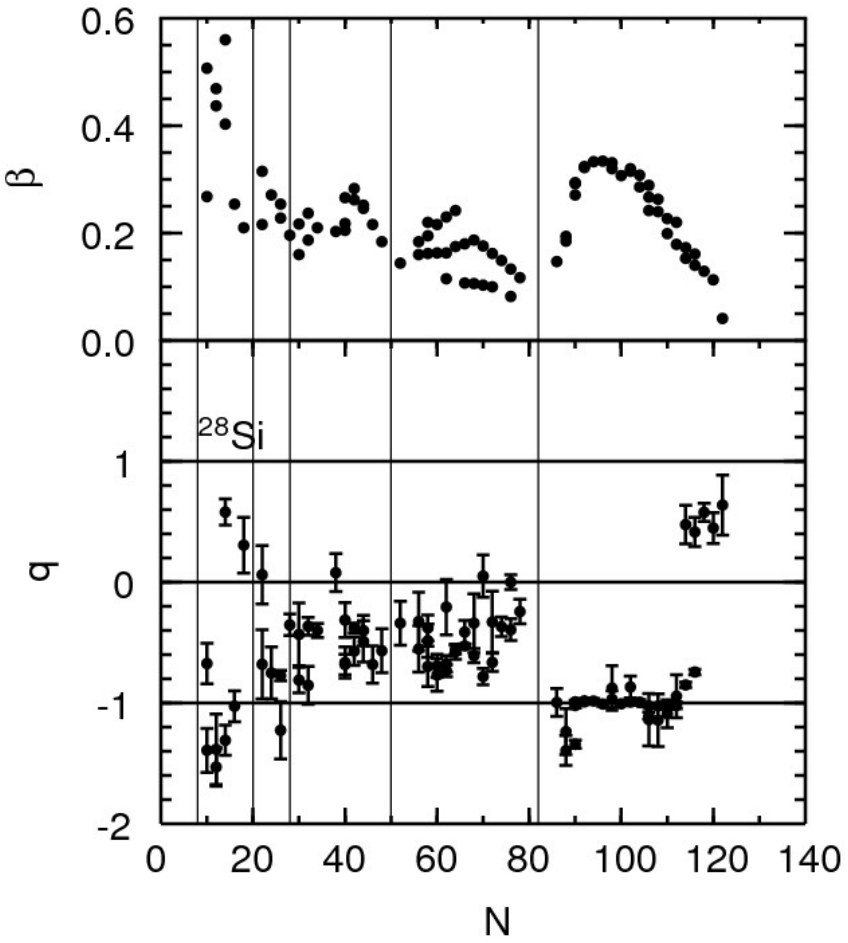

Proton Number
Neutron Number
0
10
20
30
40
50
60
>4 E_x 2^+ (MeV)
3.5
3.0
2.5
2.0
1.5
1.0
0.5
^{78}Ni
^{60}Ca
^{42}Si
^{28}O

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Systematics of the ratio of electric quadrupole moments $Q(2_1^+)$ to the square root of the reduced transition probabilities $B(E2; 0_1^+ \rightarrow 2_1^+)$ in even–even nuclei

Y.Y. Sharon [a], N. Benczer-Koller [a,*], G.J. Kumbartzki [a], L. Zamick [a], R.F. Casten [b,c]

$$q = Q/[a\ B(E2,up)]^{1/2}$$
$$a = (64\pi/245) = 0.82$$

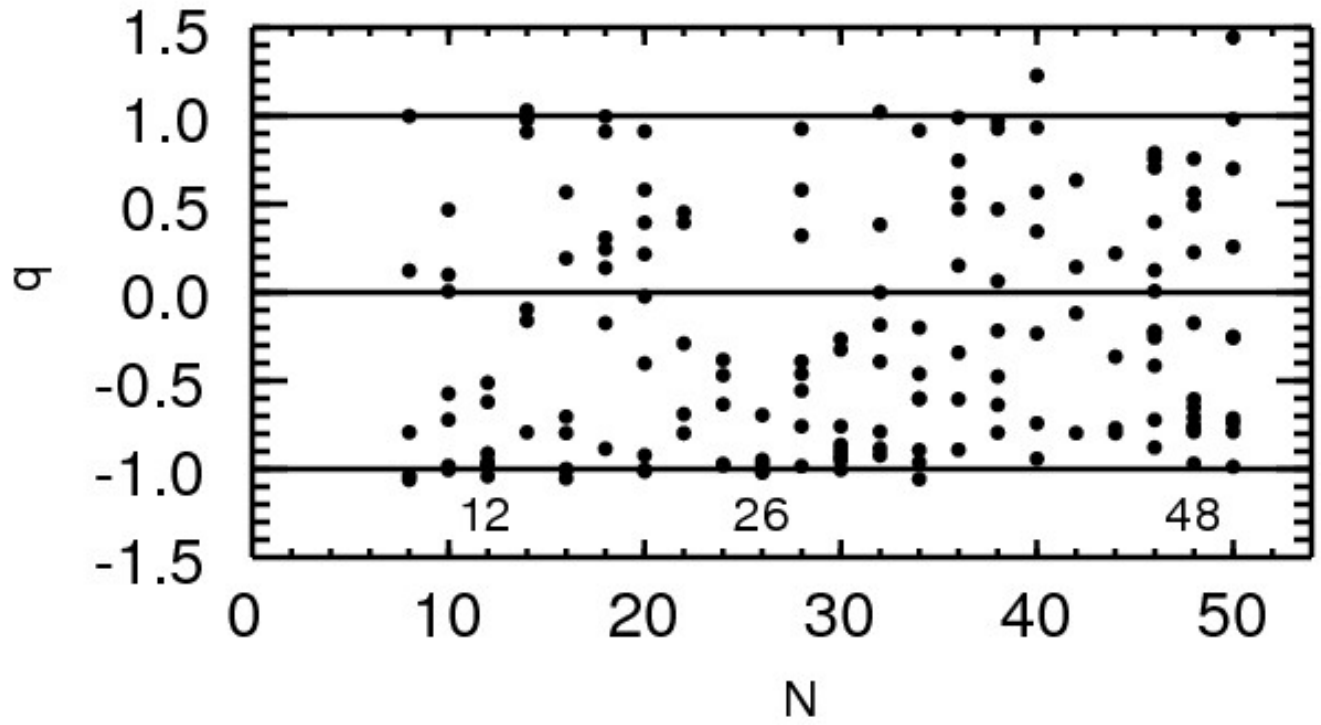


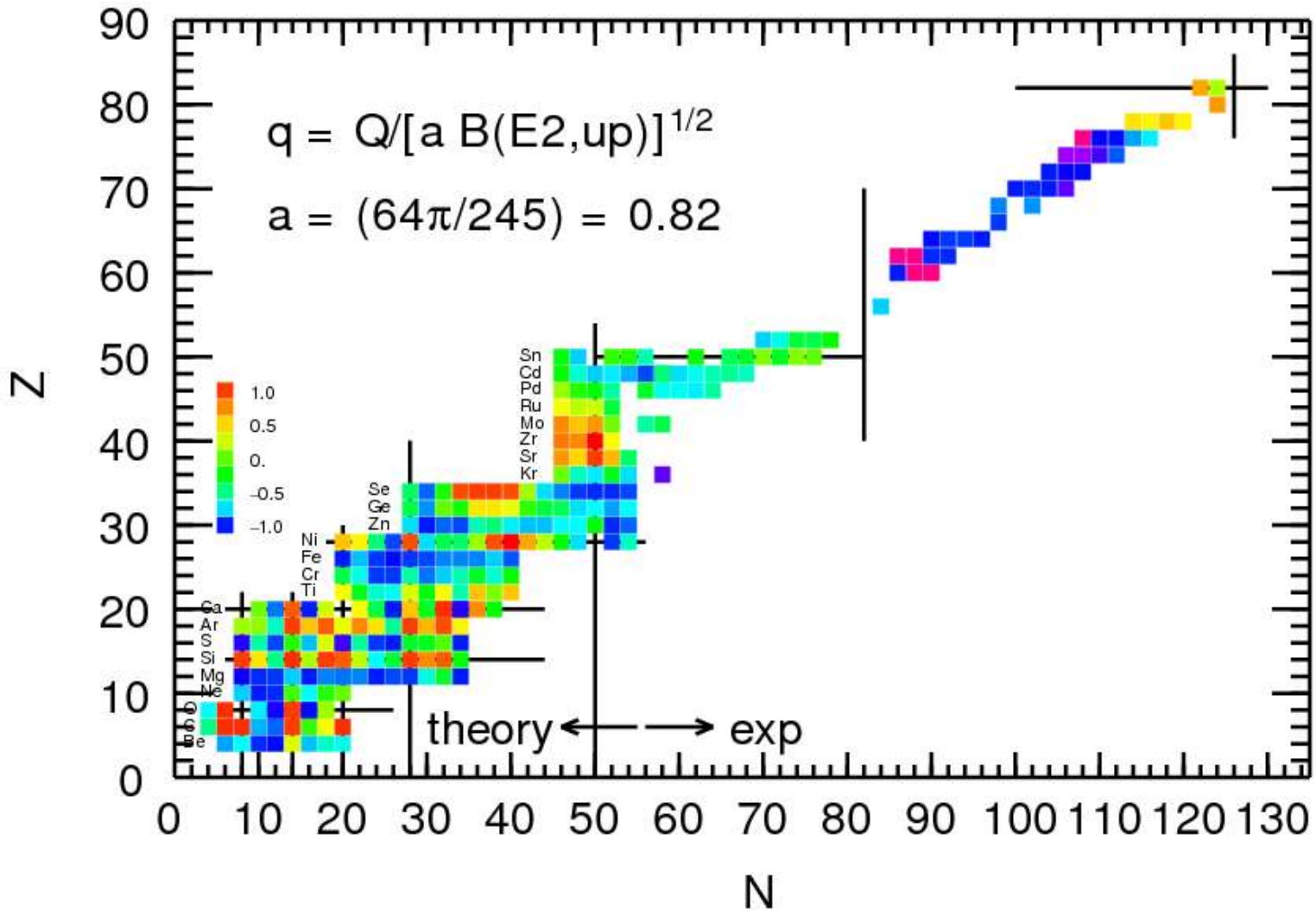
PHYSICAL REVIEW C **79**, 034317 (2009)

Further examination of prolate-shape dominance in nuclear deformation

Ikuko Hamamoto[1,2] and Ben R. Mottelson[2,3]
[1]*Division of Mathematical Physics, Lund Institute of Technology at the University of Lund, Lund, Sweden*
[2]*The Niels Bohr Institute, Blegdamsvej 17, Copenhagen Ø, DK-2100, Denmark*
[3]*NORDITA, Blegdamsvej 17, Copenhagen Ø, DK-2100, Denmark*
(Received 31 January 2009; published 26 March 2009)

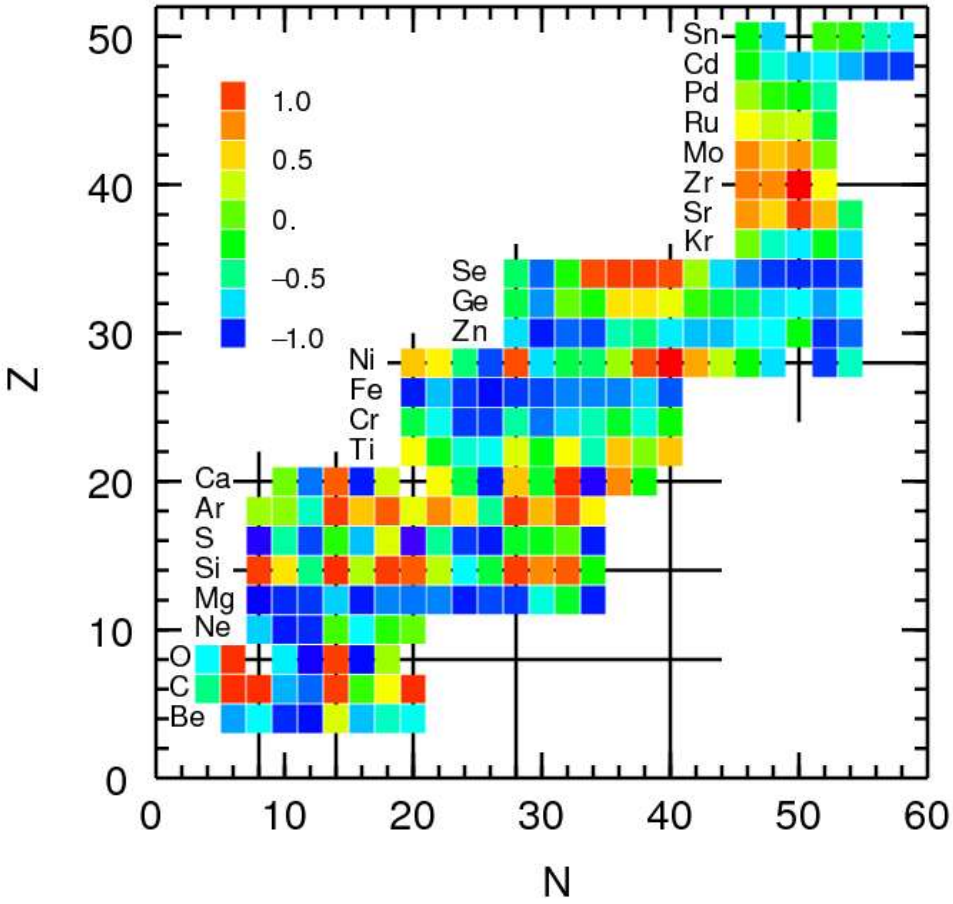
The observed almost complete dominance of prolate over oblate deformations in the ground states of deformed even-even nuclei is related to the splitting of high ℓ "surface" orbits in the Nilsson diagram: on the oblate side the occurrence of numerous strongly avoided crossings which reduce the fanning out of the low Λ orbits, while on the prolate side the same interactions increase the fanning out. It is further demonstrated that the prolate dominance is rather special for the restricted particle number of available nuclei and is not generic for finite systems with mean-field potentials resembling those in atomic nuclei.

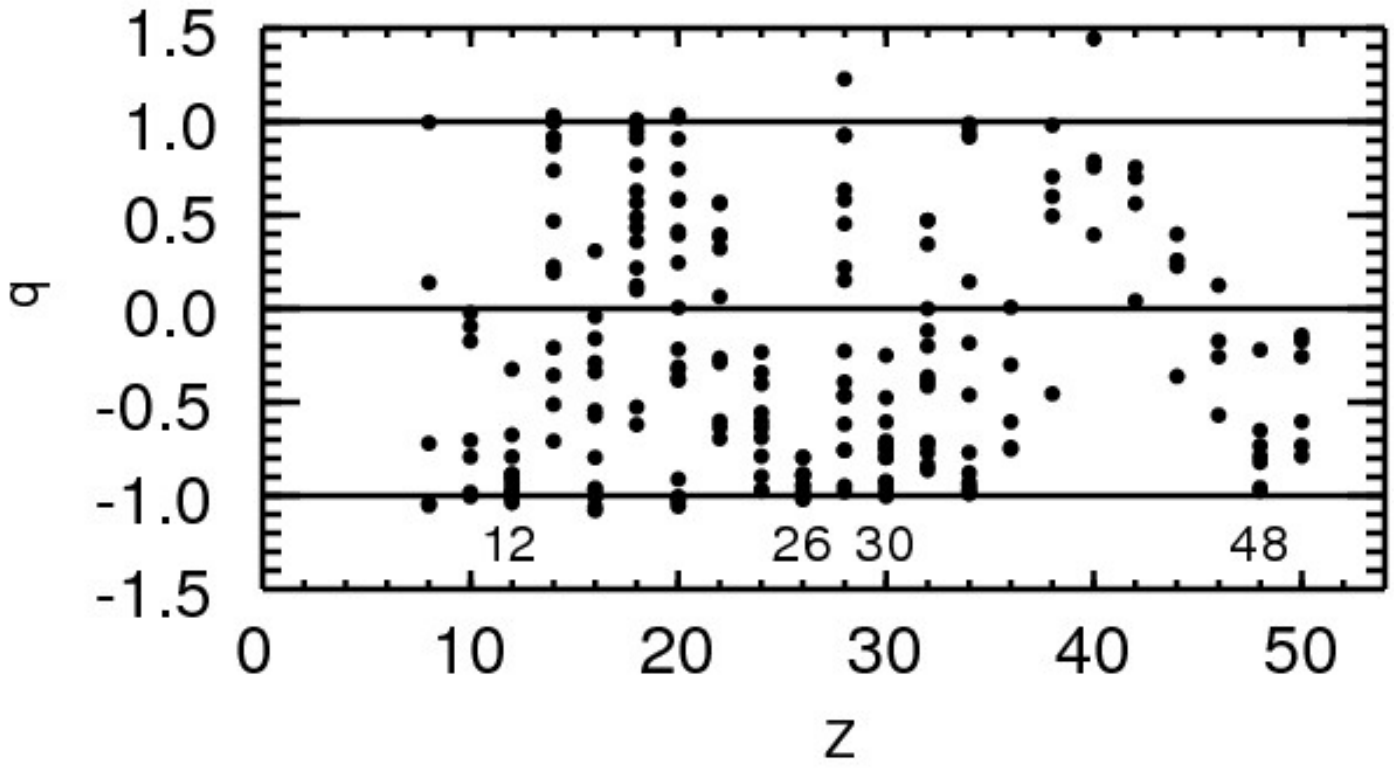

1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
q
12
26
48
0
10
20
30
40
50
N


q = Q/[a B(E2,up)]^1/2
a = (64π/245) = 0.82
1.0
0.5
0.
−0.5
−1.0
Sn
Cd
Pd
Ru
Mo
Zr
Sr
Kr
Se
Ge
Zn
Ni
Fe
Cr
Ti
Ca
Ar
S
Si
Mg
Ne
O
C
Be
theory
exp
Z
N
0
10
20
30
40
50
60
70
80
90
100
110
120
130

$q = Q/[a\ B(E2,up)]^{1/2}$

$a = (64\pi/245) = 0.82$

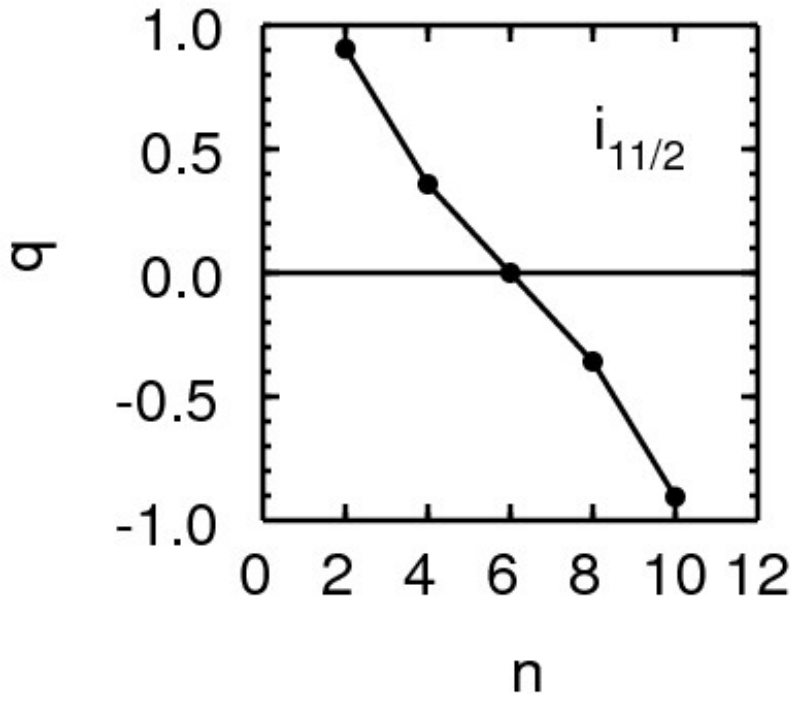


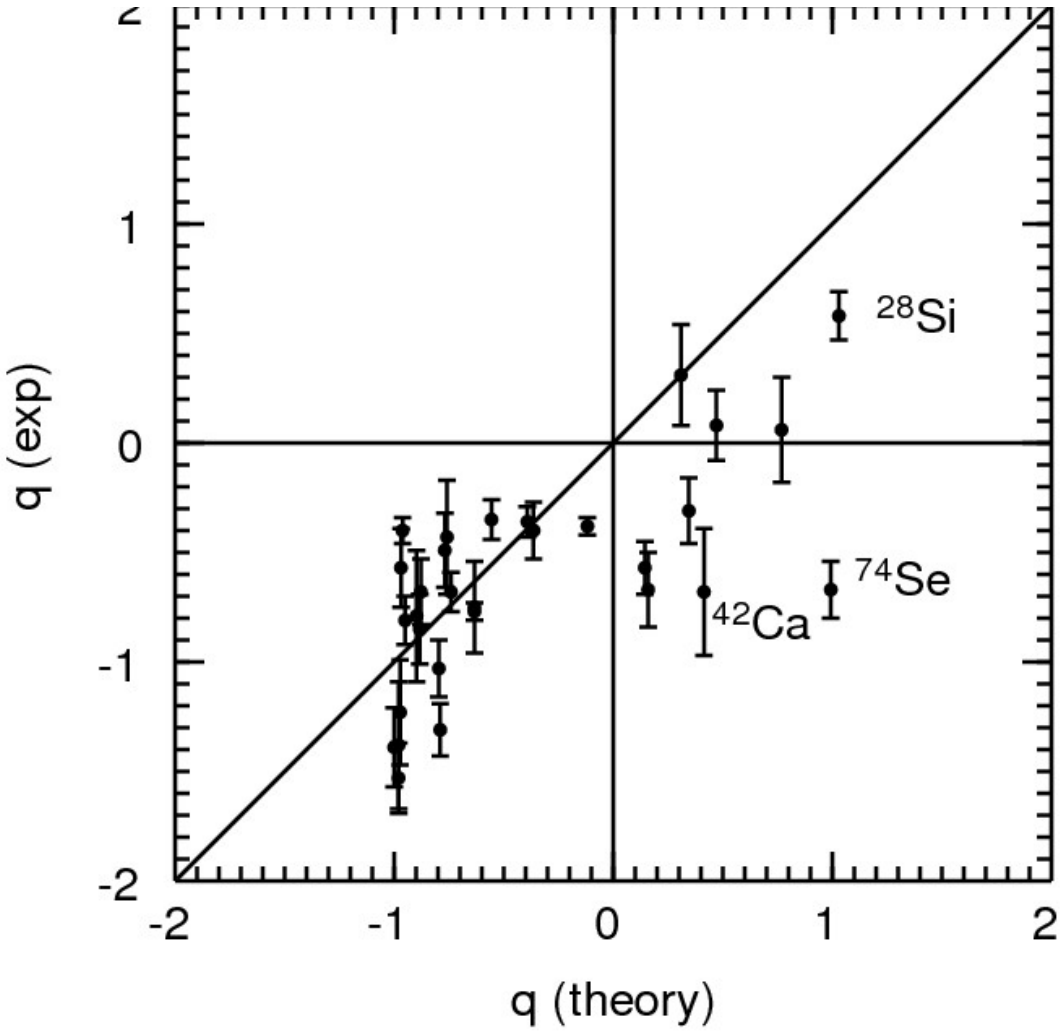


12 (two holes in d5/2)

26 (two holes in f7/2)

48 (twp holes in g9/2)

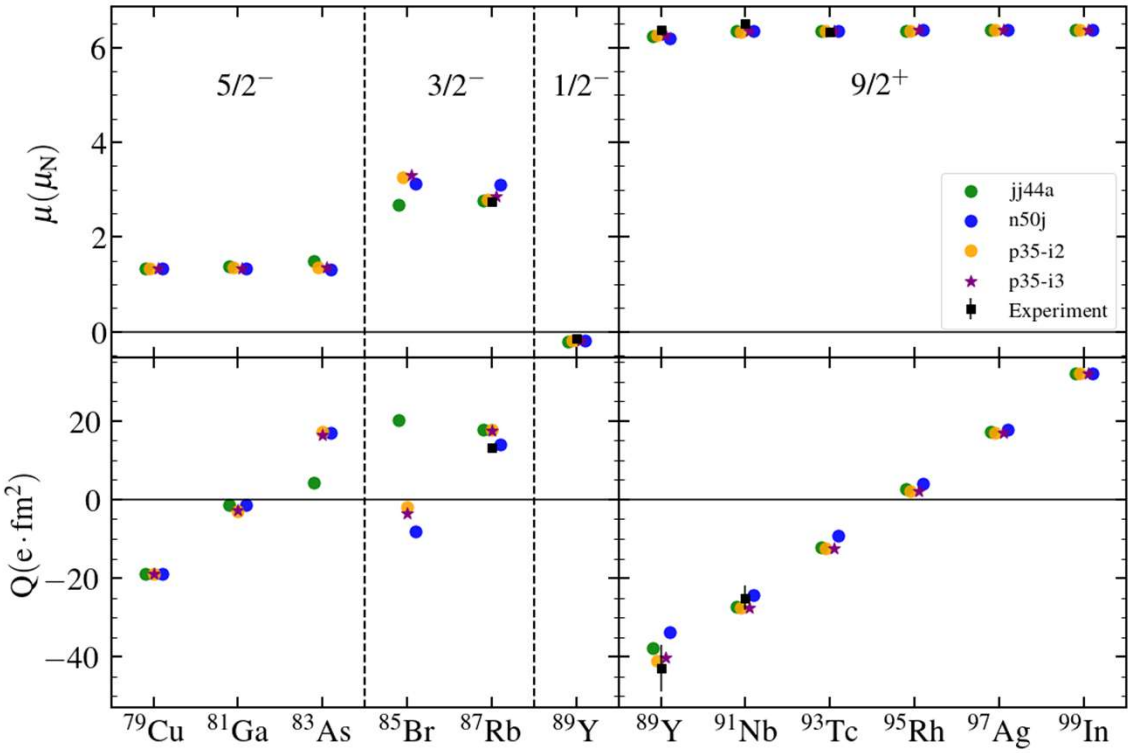

1.0
0.5
0.0
-0.5
-1.0
q
0
2
4
6
8
10
12
n
$i_{11/2}$

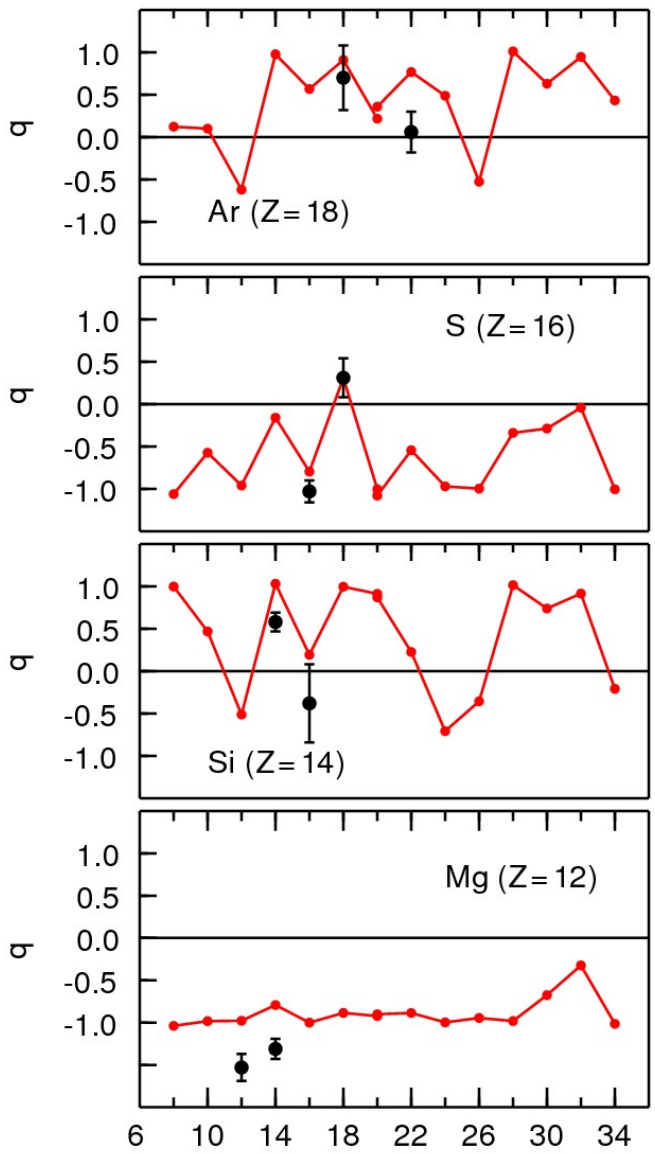

q (exp)
q (theory)
2
1
0
-1
-2
-2
-1
0
1
2
28Si
74Se
42Ca

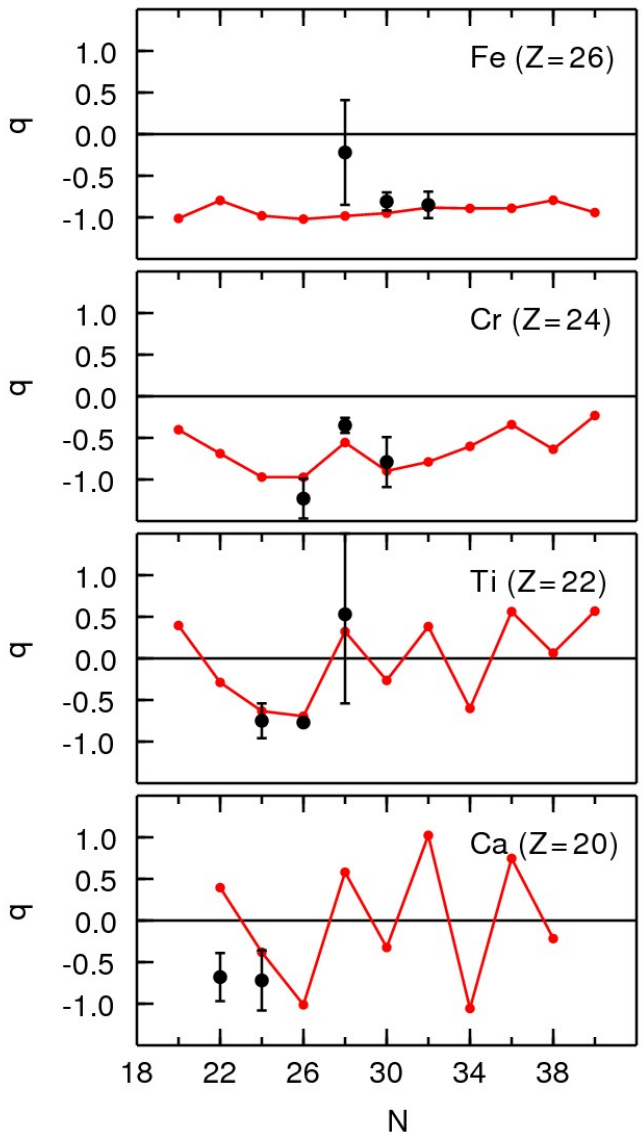
Electromagnetic Properties of the N=50 Isotones with the p35-i3 Hamiltonian

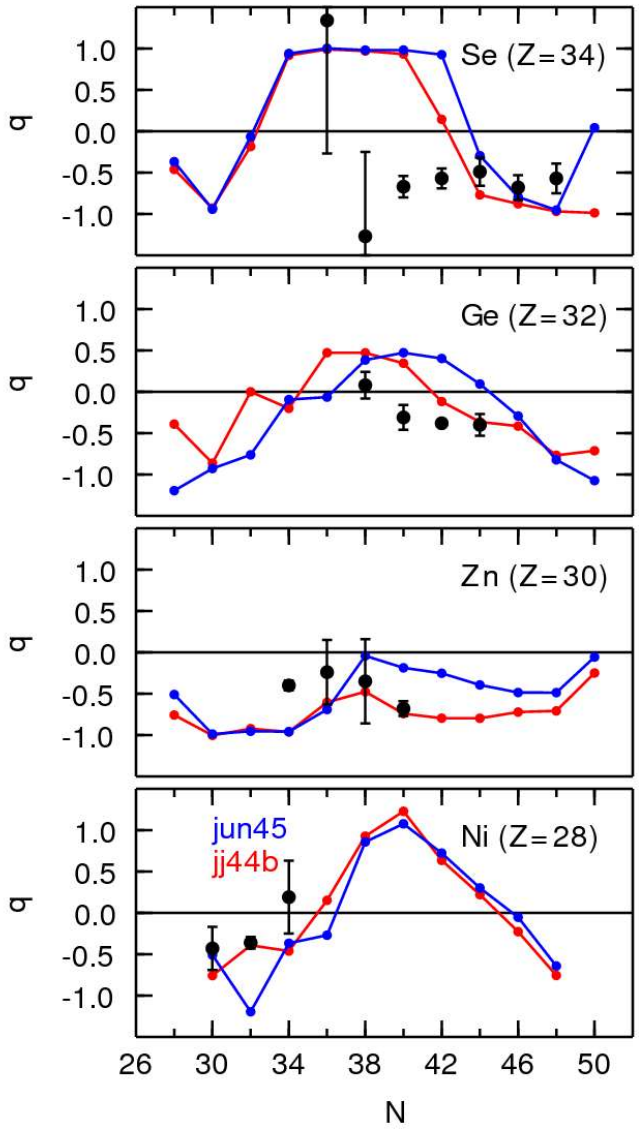
J. A. Purcell and B. A. Brown[1]

[1] *Department of Physics and Astronomy, Michigan State University, East Lansing, Michigan 48824-1321, USA and Facility for Rare Isotope Beams, Michigan State University, East Lansing, Michigan 48824-1321, USA*




q
1.0
0.5
0.0
-0.5
-1.0
Ar (Z = 18)
S (Z = 16)
Si (Z = 14)
Mg (Z = 12)
6
10
14
18
22
26
30
34


Fe (Z=26)
Cr (Z=24)
Ti (Z=22)
Ca (Z=20)
q
1.0
0.5
0.0
-0.5
-1.0
18
22
26
30
34
38
N


Se (Z=34)
Ge (Z=32)
Zn (Z=30)
Ni (Z=28)
jun45
jj44b
q
1.0
0.5
0.0
-0.5
-1.0
26
30
34
38
42
46
50
N

Summary

1960-70 Most structure interpreted by deformed HF models

1980- Configuration Interactions (CI) models take over for light nuclei
- provide complete spectra up to about 5 MeV

CI contains the collective structures plus much more

This provides a rich interpretation of what we see in exp.

Can we use CI plus collective models to calculate level densities to higher energy and encompass both light and heavy nuclei?

The angular momentum dependence of the level density is key to understanding many reactions. A commonly used form comes from the original work of Bethe [35], where the level density for a given J is

with

$$P(J) = \frac{(2J+1)}{2\sigma^2} \exp[-(J+1/2)^2/2\sigma^2], \qquad (47)$$

and σ^2 being the so-called spin cutoff parameter, which is energy dependent. The spin cutoff parameter can be determined at a fixed excitation energy via

$$\sigma^2 = \langle (J+1/2)^2 \rangle / 2. \qquad (48)$$

The calculated spin cutoff parameters for ^{57}Fe, ^{74}Ge, and ^{76}Ge as a function of excitation energy are shown in Figs. 19 and 20. In Fig. 20, both the positive- and negative-parity spin cutoff parameters are shown.

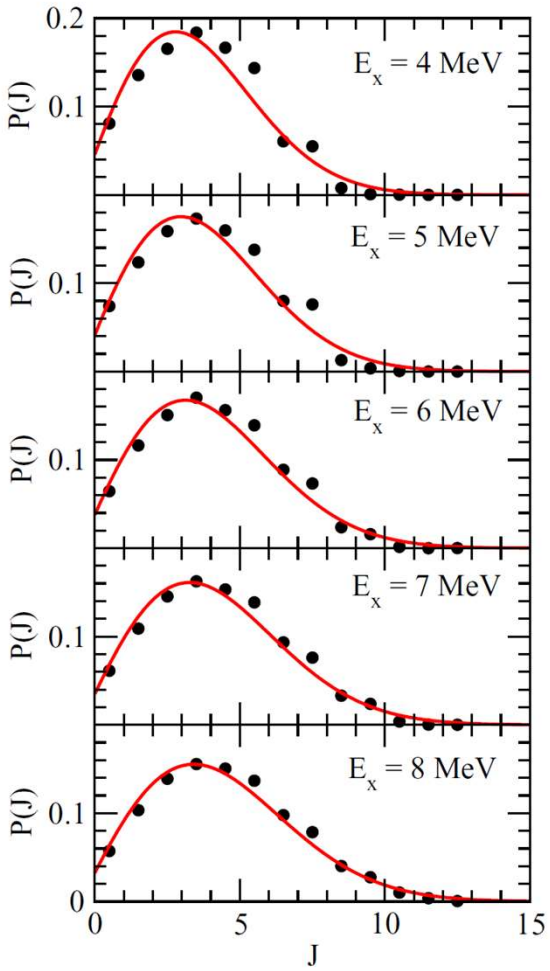


FIG. 21. Angular momenta probabilities for ^{57}Fe are shown for five excitation-energy slices across the density of states. The red lines show the results obtained from Eq. (47) with the spin cutoff parameter shown in Fig. 19.

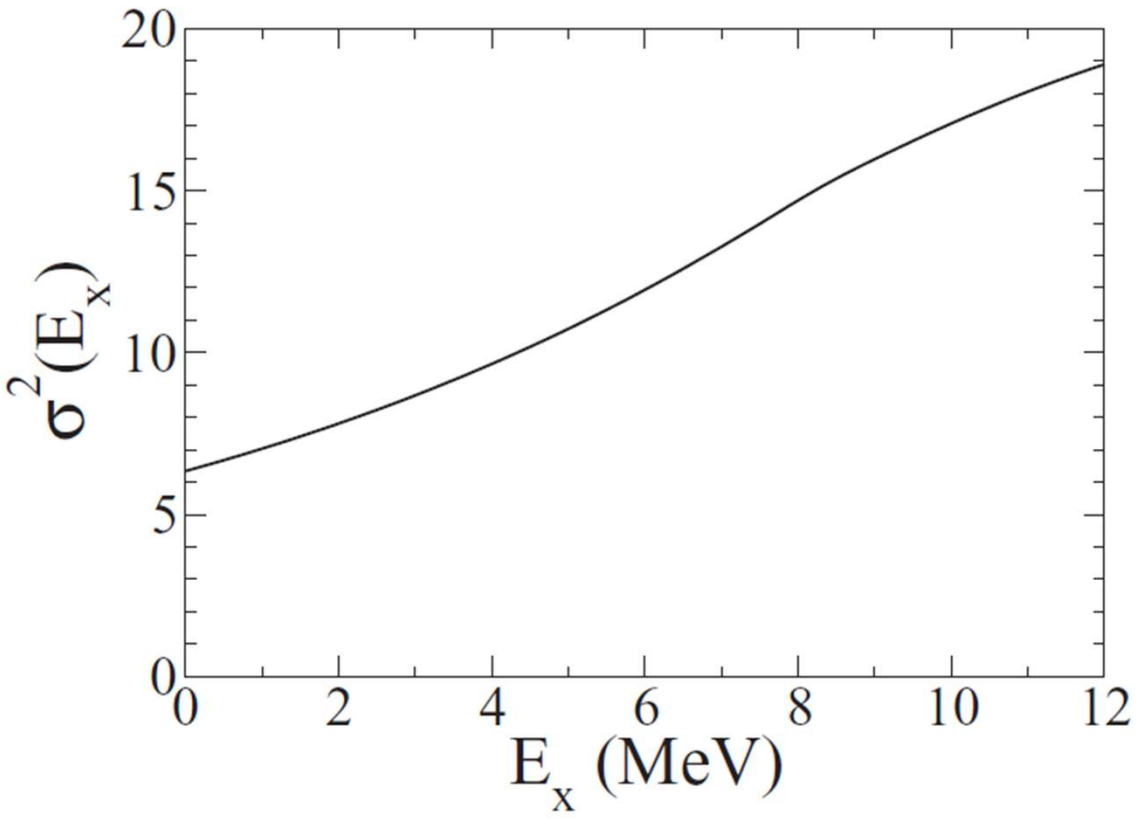
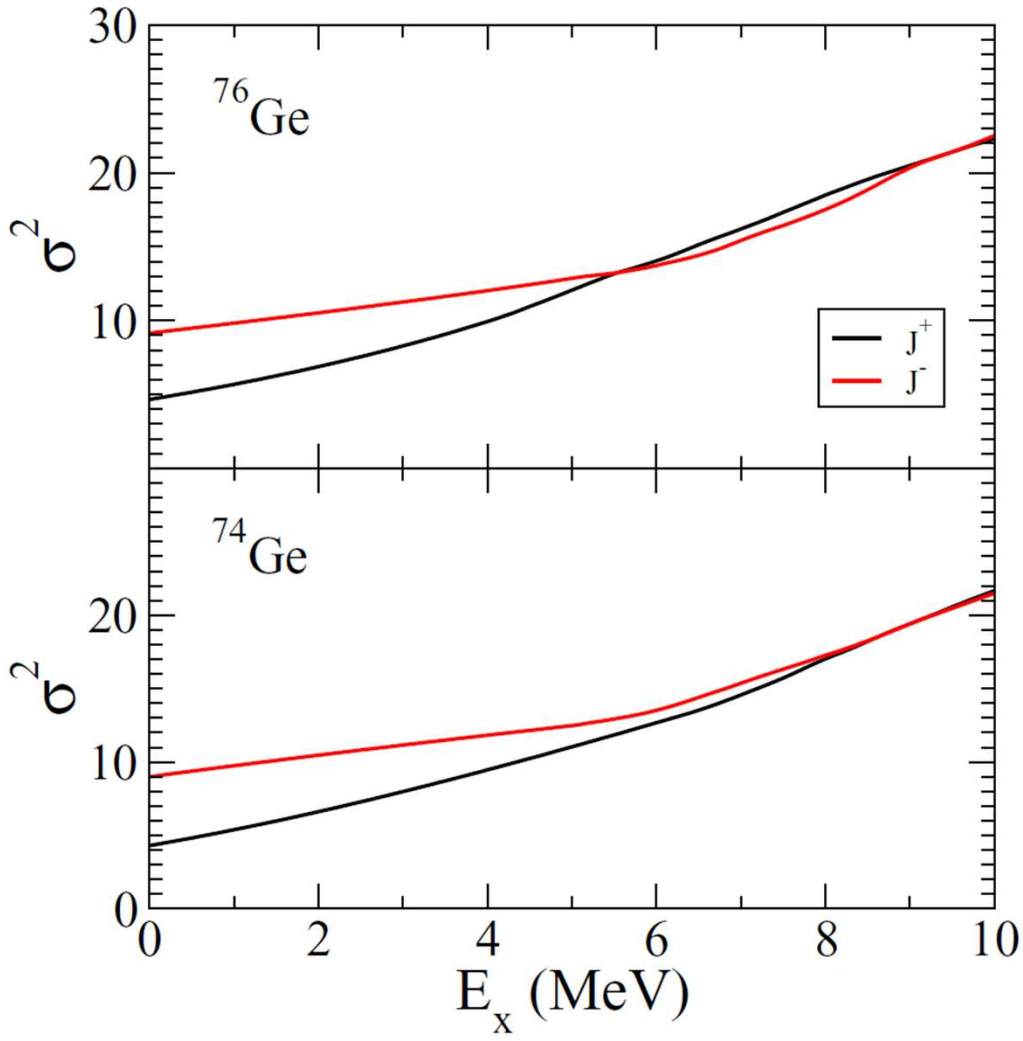


FIG. 19. Calculated spin cutoff parameter for ^{57}Fe as a function of excitation energy.


30
20
10
σ^2
^{76}Ge
J^+
J^-
^{74}Ge
20
10
0
σ^2
0
2
4
6
8
10
E_x (MeV)