Nucleus as an open many-body system, statistical physics, and quantum chaos

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Work in progress

PHYSICS OF ATOMIC NUCLEI IN XXI CENTURY

- *Limits of stability - drip lines, superheavy...*
- *Nucleosynthesis in the Universe; charge asymmetry*
- *Neutron matter; dark matter*
- *Structure of exotic nuclei*
- *Collective effects – superfluidity, shape transformations, ...*
- *Mesoscopic physics – chaos, thermalization, statistics, ...*
  - *random matrix ensembles*
  - *physics of open and marginally stable systems*
  - *enhancement of weak perturbations*
  - *quantum signal transmission*
- *Applied physics – isotopes, isomers, reactor technology, ...*
- *Fundamental physics and violation of symmetries*

Many-body quantum system with no random elements, internally developed chaotic behavior

In physical terms, one may say that quantum thermalization occurs in the Hilbert space rather than phase space
/T. Mori et al. J. Phys. B51, 112001 (2018)./

Classical dynamics – extreme sensitivity to initial conditions, exponential divergence of trajectories (classical limit of path integrals). Quantum many-body dynamics – chaotic mixture of close trajectories in the Hilbert space.

Eigenfunction Thermalization Hypothesis – see Landau and Lifshits, SF

WIKIPEDIA definition:

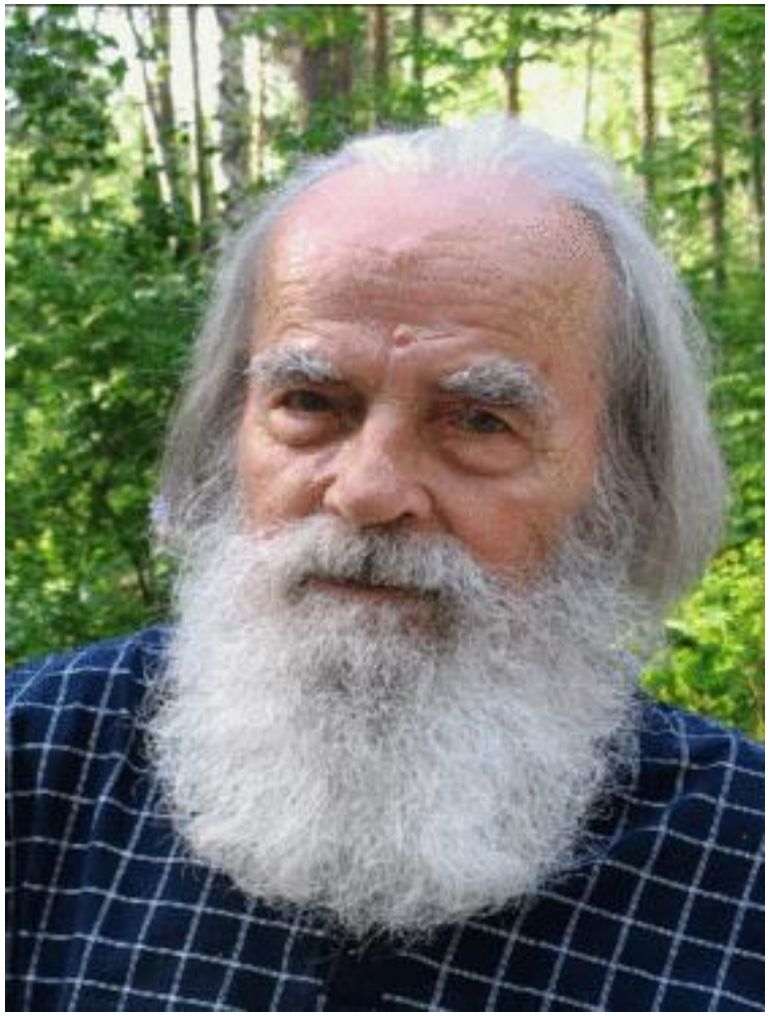
Quantum chaos is a branch of physics which studies how chaotic classical dynamic systems can be described in terms of quantum theory. The primary question that quantum chaos seeks to answer is:

What is the relationship between quantum mechanics and classical chaos?

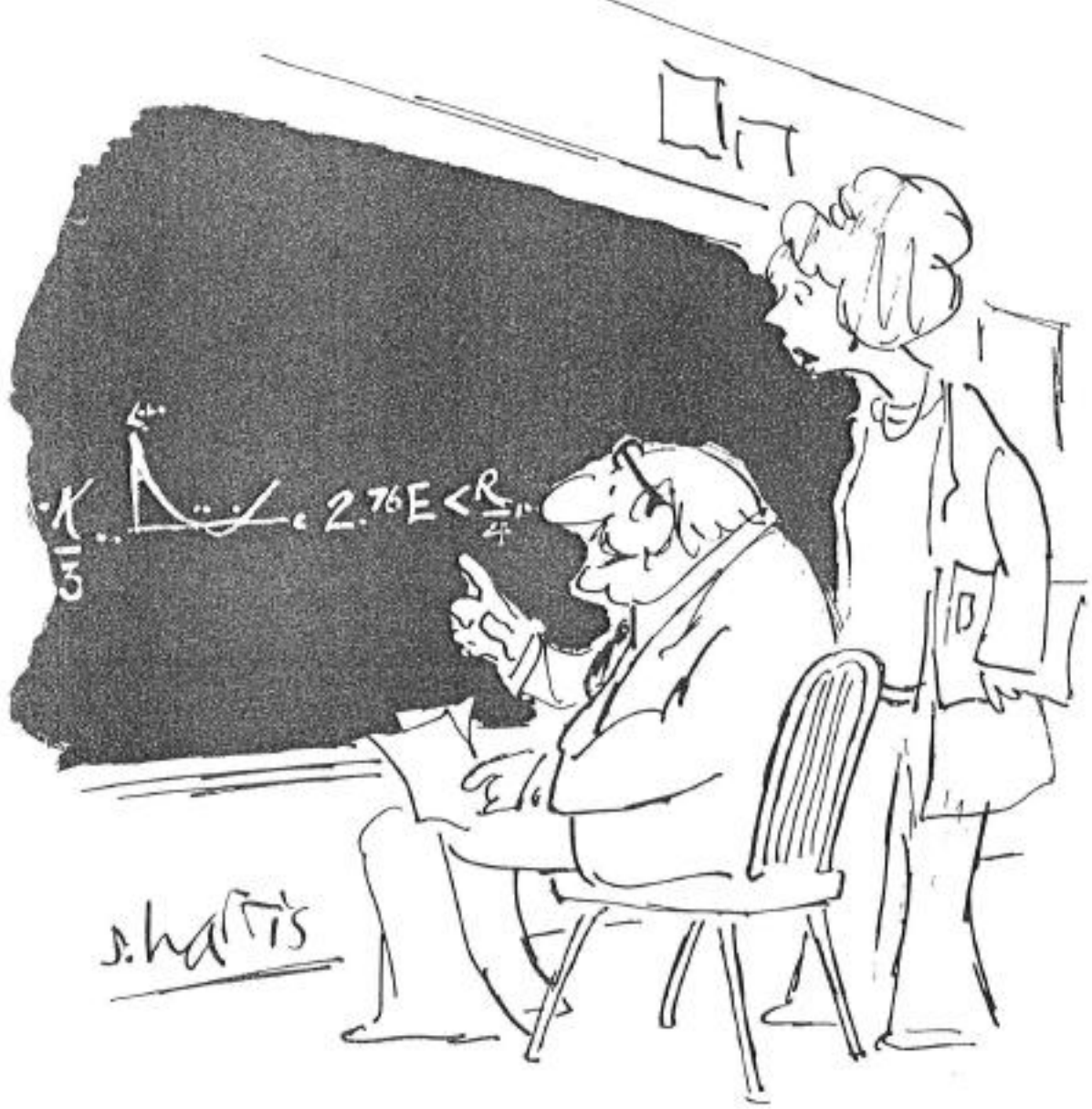
==========

B.V. Chirikov (1992):

Quantum chaos is finite-time statistical relaxation in discrete spectrum



Boris V. CHIRIKOV (1928 – 2008)



"THE BEAUTY OF THIS IS THAT IT IS ONLY OF THEORETICAL IMPORTANCE, AND THERE IS NO WAY IT CAN BE OF ANY PRACTICAL USE WHATSOEVER."

MANY-BODY QUANTUM CHAOS AS AN INSTRUMENT

SPECTRAL STATISTICS – *signature of chaos*

- *missing levels*
- *purity of quantum numbers*
- ***level density without full diagonalization***
- *presence of time-reversal invariance*

EXPERIMENTAL TOOL – *unresolved fine structure*

- *width distribution (more work required)*
- *damping of collective modes*

NEW PHYSICS

- *statistical enhancement of weak perturbations (parity violation in neutron scattering and fission)*
- *mass fluctuations*
- *chaos on the border with continuum*

THEORETICAL CHALLENGES

- order out of chaos
- **chaos and thermalization**
- development of computational tools
- new approximations in many-body problem

INSIDE CHAOS

I. Percival, J. Phys. **B6** (1973) L229

- *DISORDERED* wave functions
- Any *SIMPLE* operator has matrix elements of the same order of magnitude between any two of these eigenfunctions
- All typical wave functions of roughly the same energy *LOOK ROUGHLY THE SAME* being spread over the large region of configuration space

Random matrix canonical ensembles – only as mathematical limit

Chaotic motion in mesoscopic systems

* Mean field (**one-body** chaos) - classical features
* Strong interaction (**many-body** chaos)
* High level density
* Mixing of simple configurations
* Destruction of quantum numbers,
  (in nuclei: conserved only energy; J,M; T,T3; parity)
* Local spectral statistics – **Gaussian Orthogonal Ensemble**
* Correlations between classes of states
* Coexistence with (damped) collective motion
* Thermal equilibrium – **without a heat bath**
* Continuum effects – open system… (**current interest**)
* Superradiance in the limit

STATISTICAL MECHANICS of CLOSED MESOSCOPIC SYSTEMS

* SPECIAL ROLE OF MEAN FIELD BASIS
  (**separation of regular and chaotic motion;**
  mean field out of chaos)

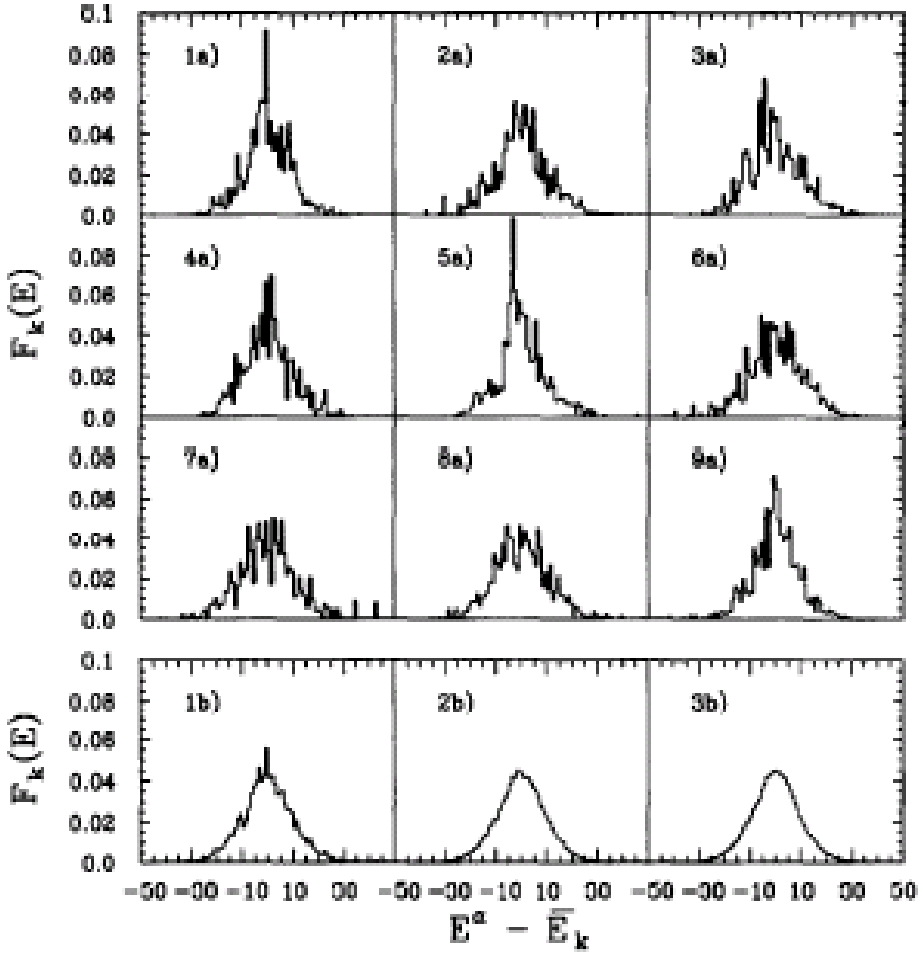
* CHAOTIC INTERACTION as HEAT BATH

* SELF – CONSISTENCY OF
  mean field, interaction and thermometer

* SIMILARITY OF CHAOTIC WAVE FUNCTIONS

* SMEARED PHASE TRANSITIONS

* CONTINUUM EFFECTS (IRREVERSIBLE DECAY)
  new effects when widths are of the order of spacings –
  enhancement of symmetries violation
  super-radiant and trapped states
  conductance fluctuations …



9 INDIVIDUAL STATES

AVERAGE OVER

10, 100, 400 STATES

STRENGTH FUNCTION

$$F_k(E) = \sum_\alpha (C_k^\alpha)^2 \delta(E - E_\alpha)$$

Local density of states in condensed matter physics

POSSIBLE NUCLEAR ENHANCEMENT of weak interactions

* Close levels of opposite parity
  - = near the ground state (accidentally)
  - = at high level density – very weak mixing?
    (statistical = chaotic) enhancement

* Kinematic enhancement

* Coherent mechanisms
  - = deformation
  - = parity doublets
  - = collective modes

* Atomic effects

N - scaling

$$|\alpha\rangle = \sum_l C_l^\alpha |l\rangle, \quad |C_l^\alpha| \sim \frac{1}{\sqrt{N}}$$

N – large number of "simple" components in a typical wave function

Q – "simple" operator $\sim a^\dagger a$

Single – particle matrix element $\langle k|Q|l\rangle \sim q$

Between a simple and a chaotic state

$$\langle k|Q|\alpha\rangle = \sum_l \langle k|Q|l\rangle \, C_l^\alpha \sim \frac{q}{\sqrt{N}}$$

Between two fully chaotic states

$$Q_{\alpha\beta} = \sum_{kl} C_k^\alpha C_l^\beta Q_{kl} \sim \frac{1}{\sqrt{N}} \frac{1}{\sqrt{N}} \sqrt{N}\, q \sim \frac{q}{\sqrt{N}}$$

STATISTICAL ENHANCEMENT

$$\mathcal{P} = \frac{\sigma^+ - \sigma^-}{\sigma^+ + \sigma^-}$$

up to

10%

Parity nonconservation in scattering of slow polarized neutrons

Coherent part of weak interaction (violating parity)

$$H_w = \frac{G}{\sqrt{2}m}(\vec{\sigma}\cdot\mathbf{p})\,\rho(\mathbf{r})$$

Single-particle mixing

$$F_{s-p} \sim \frac{\langle H_w\rangle_{s-p}}{\Delta E_{s-p}} \sim 10^{-(7\div 8)}$$

Chaotic mixing

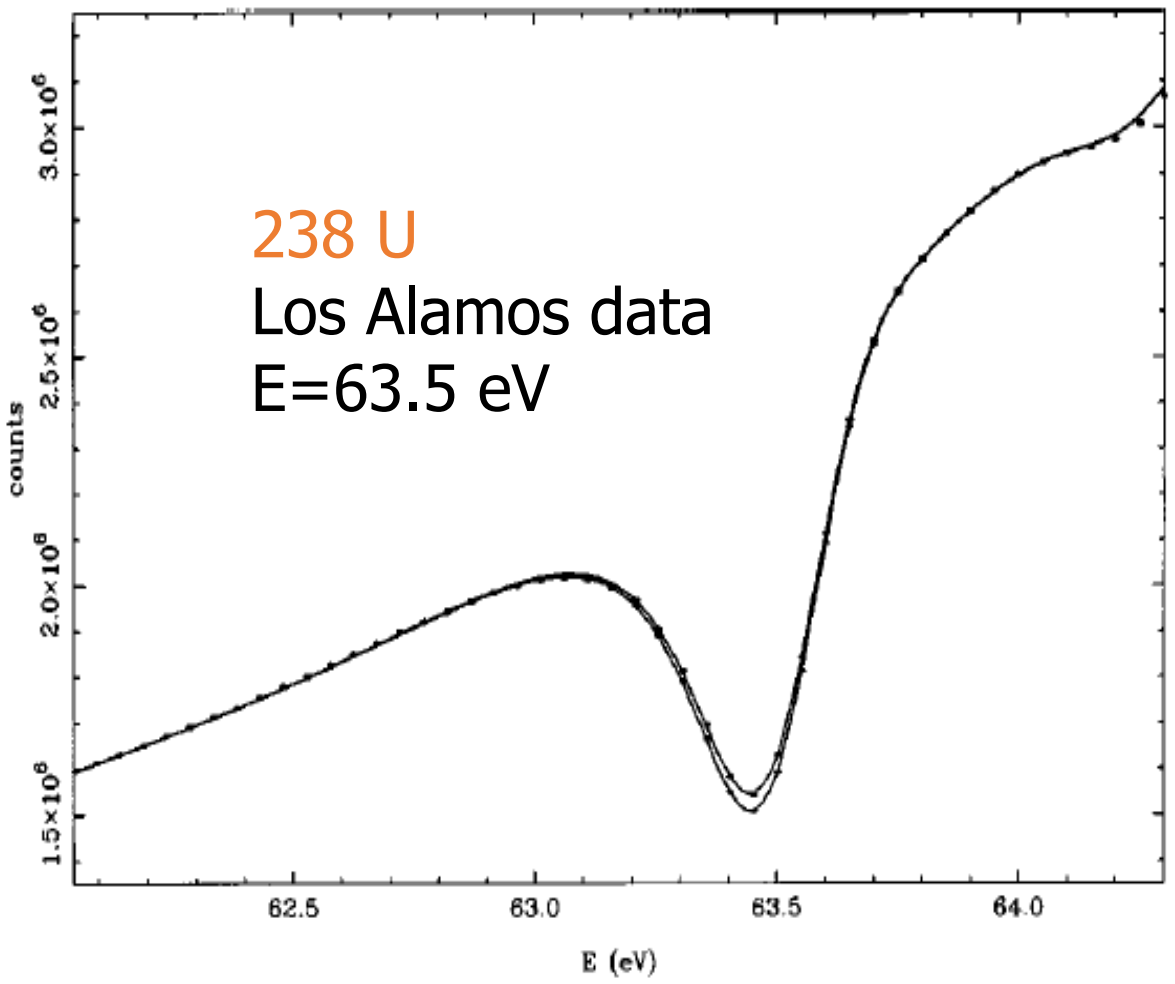
$$F_{ch} \sim \frac{\langle H_w\rangle_{ch}}{\Delta E_{res}} \sim \frac{\langle H_w\rangle_{s-p}}{\sqrt{N}}\frac{N}{\Delta E_{s-p}} \sim F_{s-p}\sqrt{N}$$

Neutron resonances in heavy nuclei

$$N \sim 10^6$$

Additional kinematic enhancement

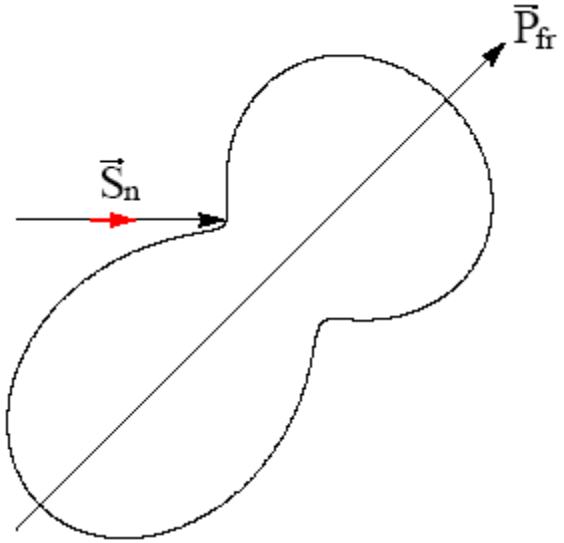
$$\Gamma_s/\Gamma_p \sim 10^3$$



$$\mathcal{P} = \frac{\sigma^+ - \sigma^-}{\sigma^+ + \sigma^-}$$

10.2 eV	-0.16(0.08)%
11.3	0.67(0.37)
63.5	2.63(0.40) *
83.7	1.96(0.86)
89.2	-0.24(0.11)
98.0	-2.8 (1.30)
125.0	1.08(0.86)

Transmission coefficients for two helicity states
(longitudinally polarized neutrons)



Parity nonconservation in fission

Correlation of neutron spin and momentum of fragments

Transfer of elementary asymmetry to ALMOST MACROSCOPIC LEVEL –

What about 2nd law of thermodynamics?

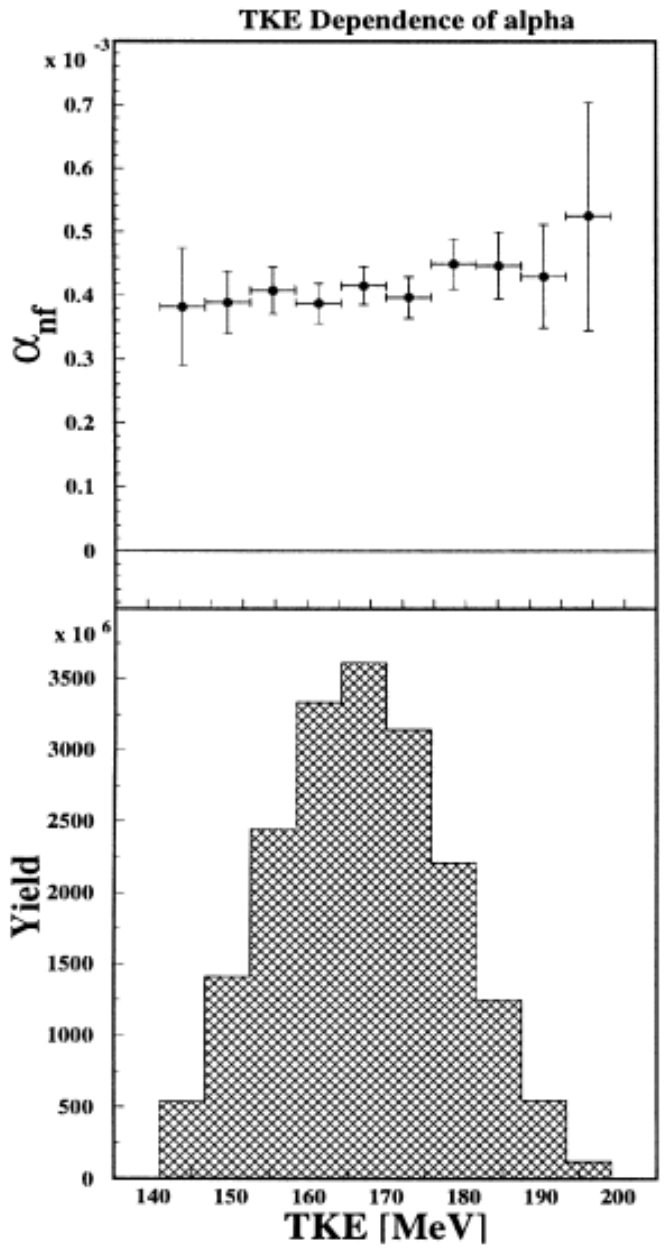
Statistical enhancement – "hot" stage ~ $\sqrt{N} \sim 10^3$

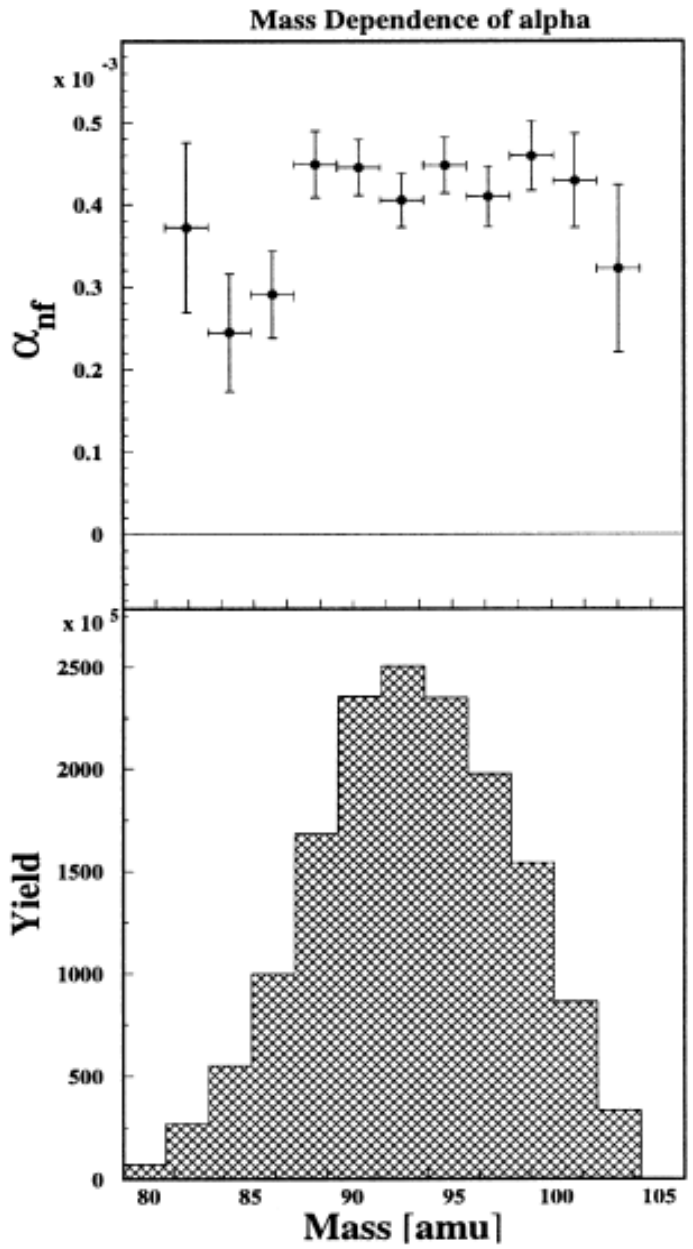
- mixing of parity doublets

Angular asymmetry – "cold" stage,

- fission channels, memory preserved

Complexity refers to the natural basis (mean field)

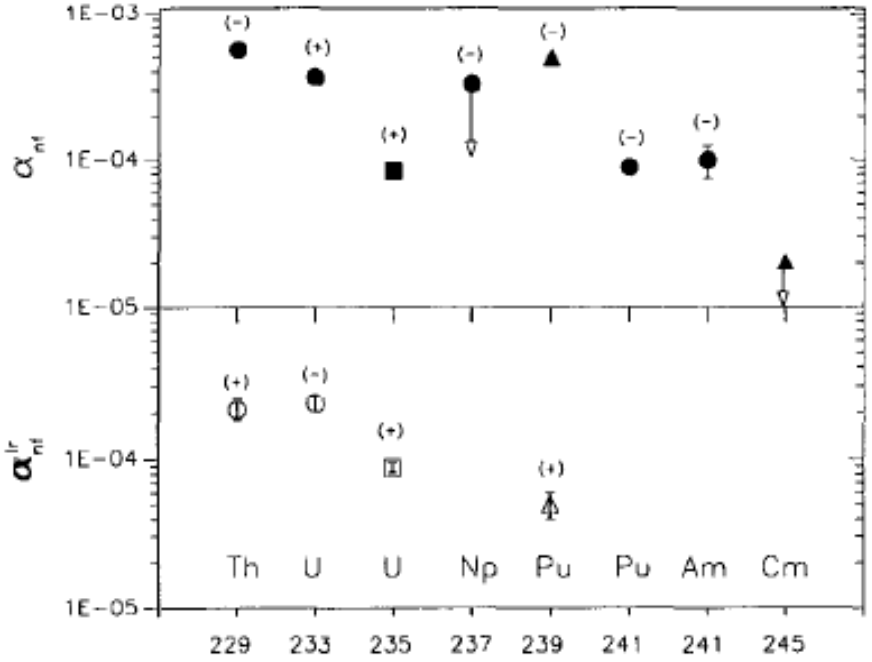




Fission of 233 U by cold polarized neutrons, (Grenoble)

A. Koetzle et al. 2000

Asymmetry determined at the "hot" chaotic stage



$$W(\vartheta) = 1 + \alpha_{nf}(\sigma_n \cdot p_f) = 1 + \alpha_{nf} \cos \vartheta$$

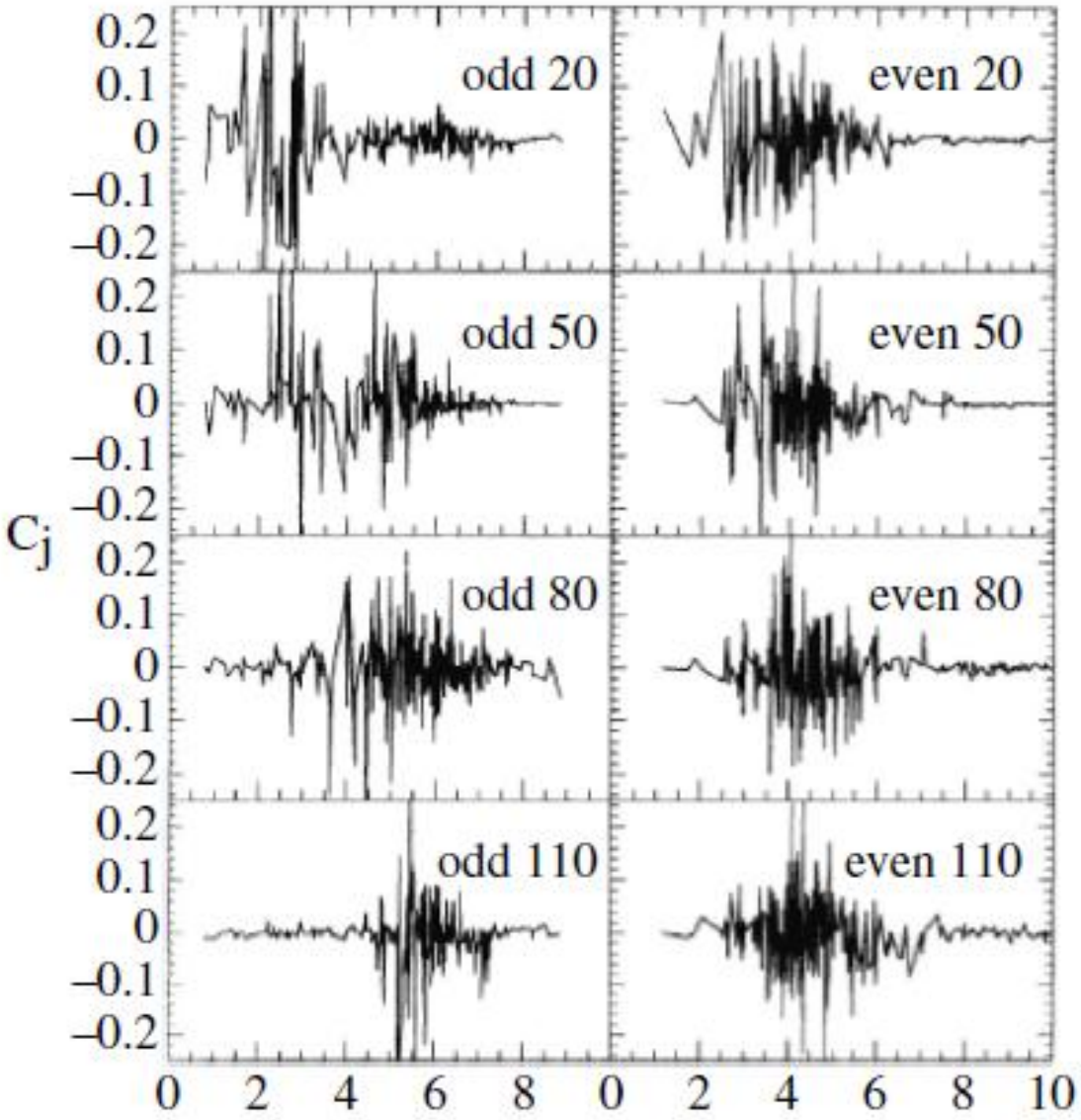
Parity violating asymmetry

$$W(\phi) = 1 + \alpha_{nf}^{lr}(p_f \cdot \langle \sigma_n \times p_n \rangle)$$

Parity preserving asymmetry

[Grenoble]
A. Alexandrovich et al . 1994

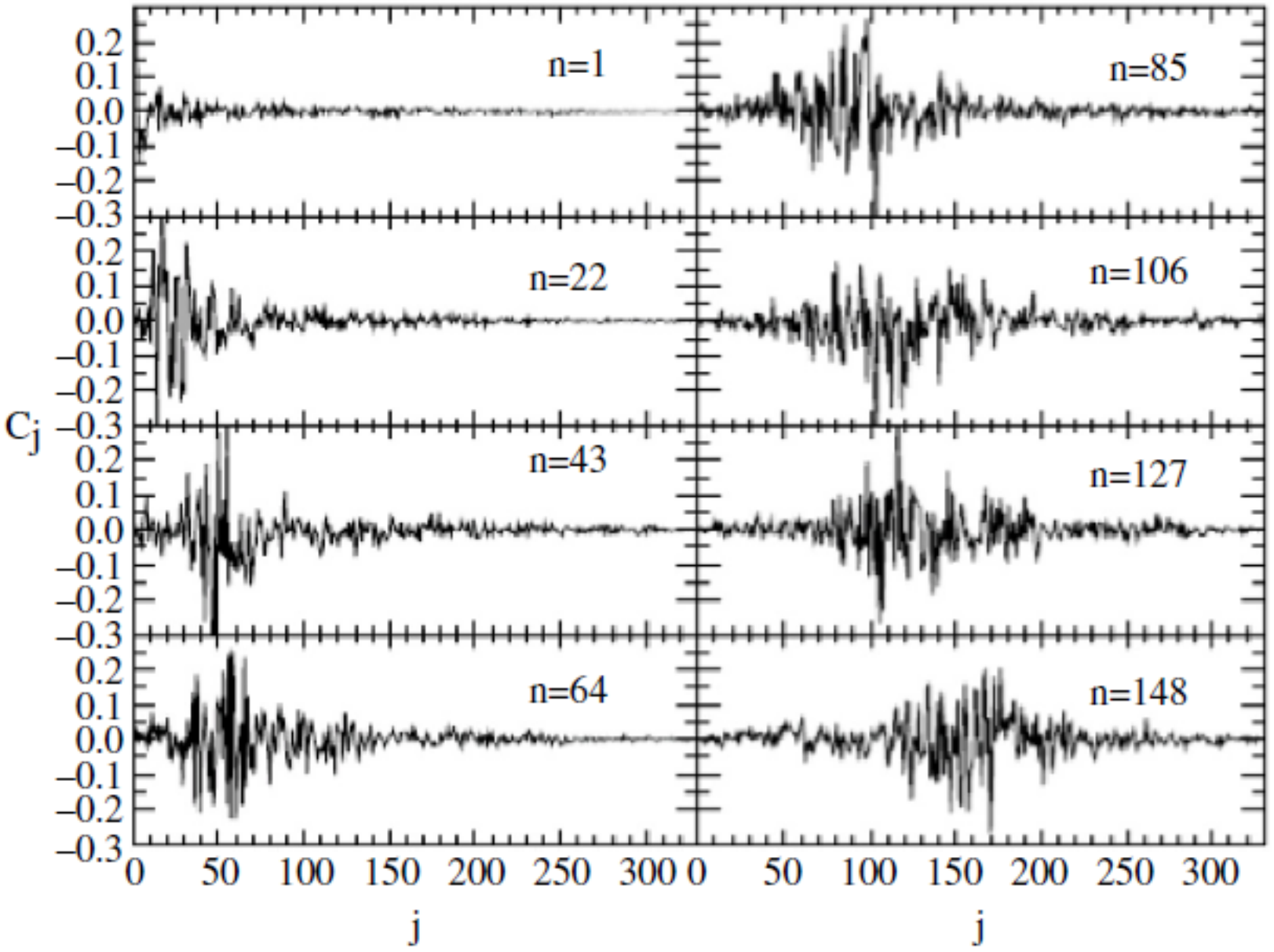
Parity non-conservation in fission by polarized neutrons –
on the level up to 0.001



4- and 4+ states in the model of Cerium atom

/Flambaum et al. 1994, E> 1 eV/

Fixed basis !



4+ excited states in the model of random two-body Interaction

/4 particles, 11 levels/

Fixed basis !

ONE-BODY CHAOS – SHAPE (BOUNDARY CONDITIONS)

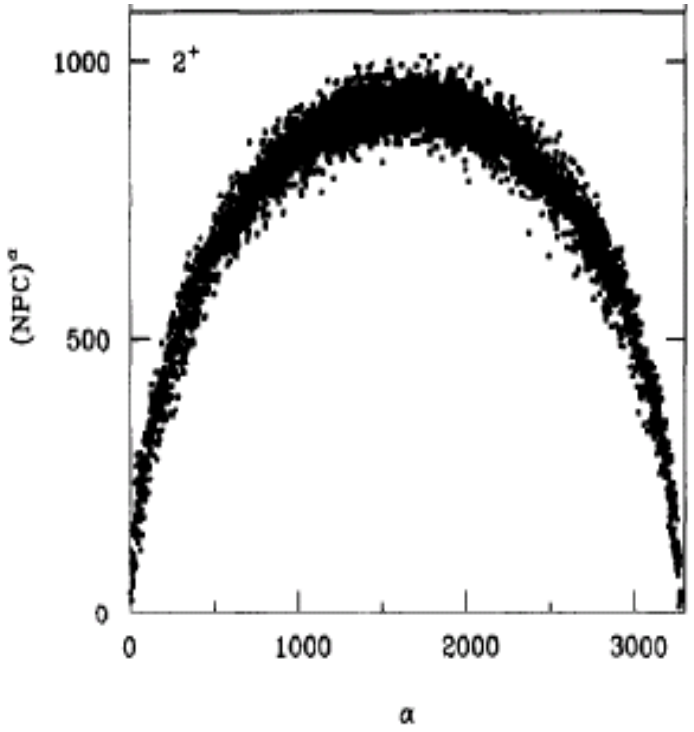
MANY-BODY CHAOS – INTERACTION BETWEEN PARTICLES

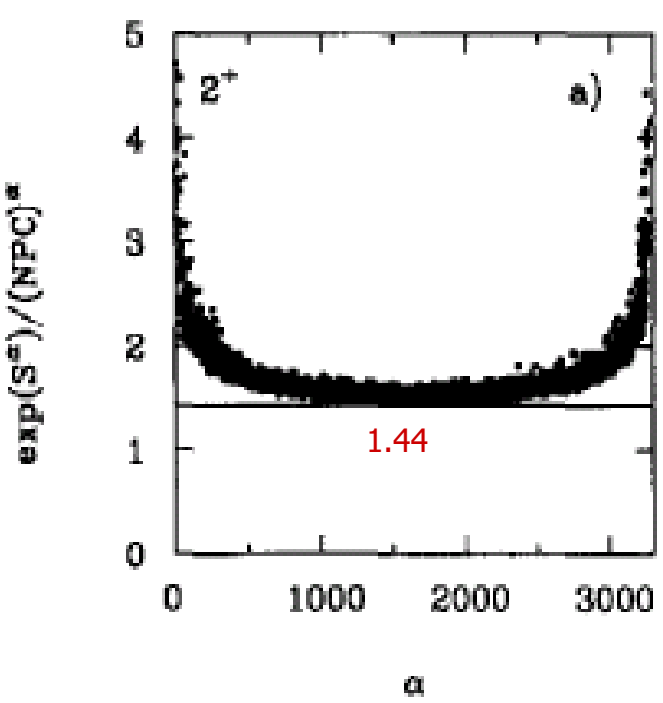
Nuclear Shell Model – realistic testing ground

- Fermi – system with mean field and strong interaction
- Exact solution in finite space
- Good agreement with experiment
- Conservation laws and symmetry classes
- Variable parameters
- Sufficiently large dimensions (statistics)
- Sufficiently low dimensions (easy to analyze) (light nuclei)
- Observables:

energy levels (spectral statistics)
wave functions (complexity)
transitions (correlations)
destruction of symmetries
cross sections (correlations)

Heavy nuclei – dramatic growth of dimensions





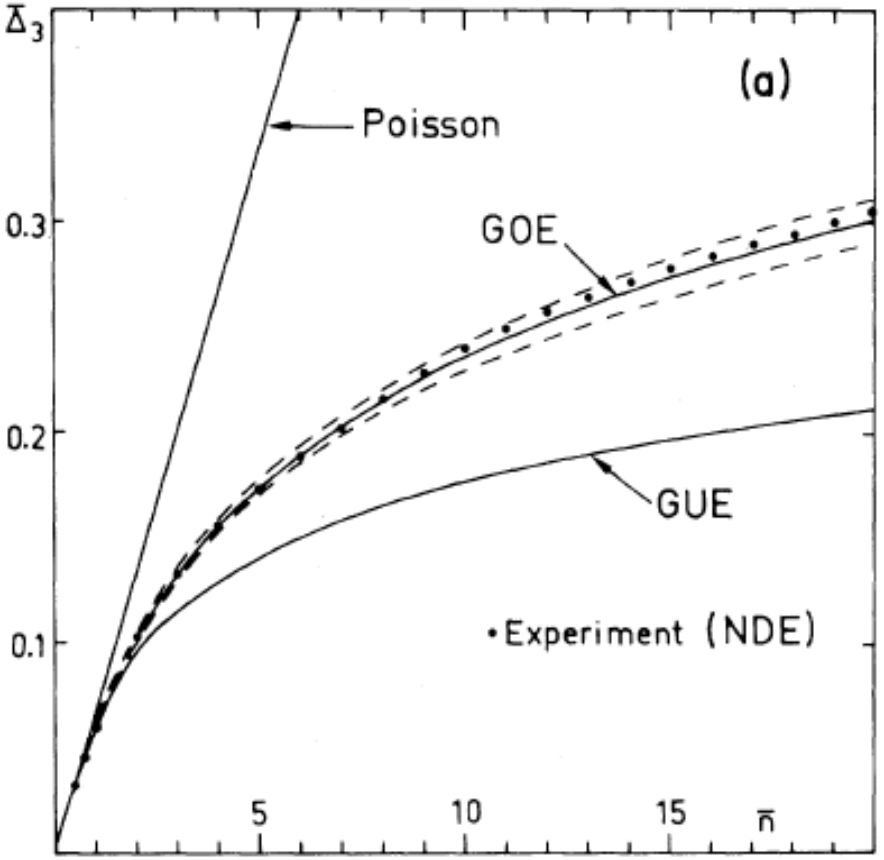
NUMBER of PRINCIPAL COMPONENTS

$$NPC(\alpha) = \left[\Sigma_k (C_k^\alpha)^4\right]^{-1}$$

$$\overline{NPC(GOE)} = \frac{N}{3}$$

Ratio $\exp[S(\alpha)]/\mathrm{NPC}(\alpha)$

GOE value = 1.44



R. Haq et al. 1982

Nuclear Data Ensemble

1407 resonance energies

30 sequences

For 27 nuclei

Neutron resonances
Proton resonances
(n,gamma) reactions

Regular spectra = L/15

(universal for small L)

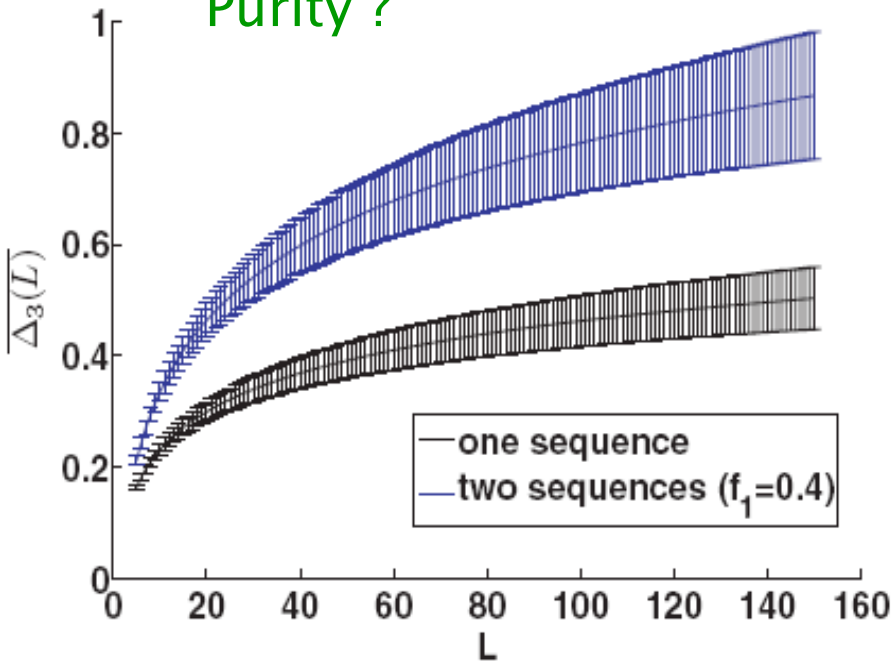
Chaotic spectra

= a log L +b for L>>1

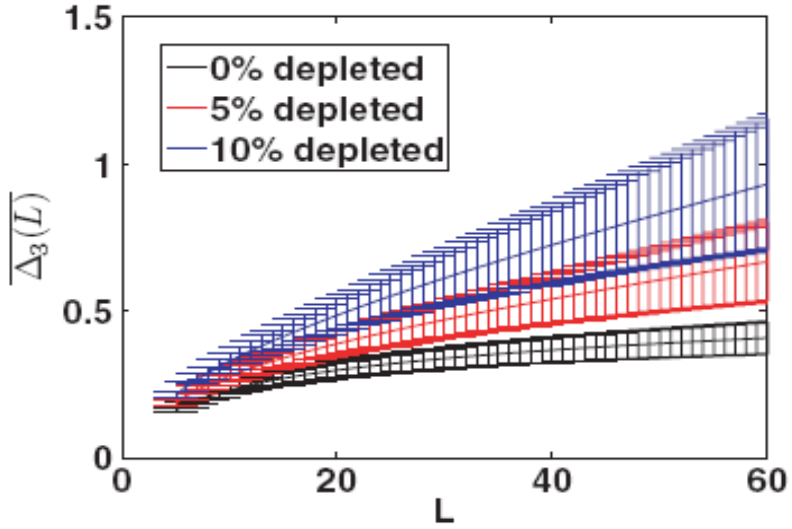
SPECTRAL RIGIDITY

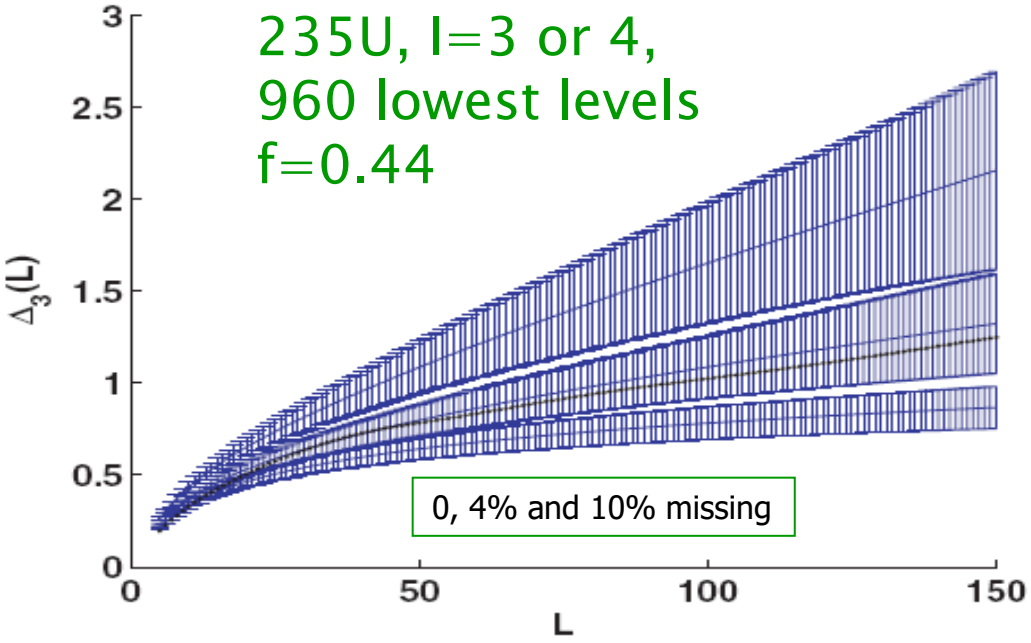
$$\Delta_3(L) = \frac{1}{L} \min_{A,B} \left\langle \int_x^{x+L} d\epsilon [\mathcal{N}(\epsilon) - A\epsilon - B]^2 \right\rangle_x$$

Purity ?



Missing levels ?





Data agree with

f=(7/16)=0.44

and

4% missing levels

D. Mulhall, Z. Huard, V.Z., PRC 76, 064611 (2007).

CHAOS versus THERMALIZATION

L. BOLTZMANN – *Stosszahlansatz* = MOLECULAR CHAOS

N. BOHR - *Compound nucleus* = MANY-BODY CHAOS

N. S. KRYLOV - *Foundations of statistical mechanics*

L. Van HOVE – *Quantum ergodicity*

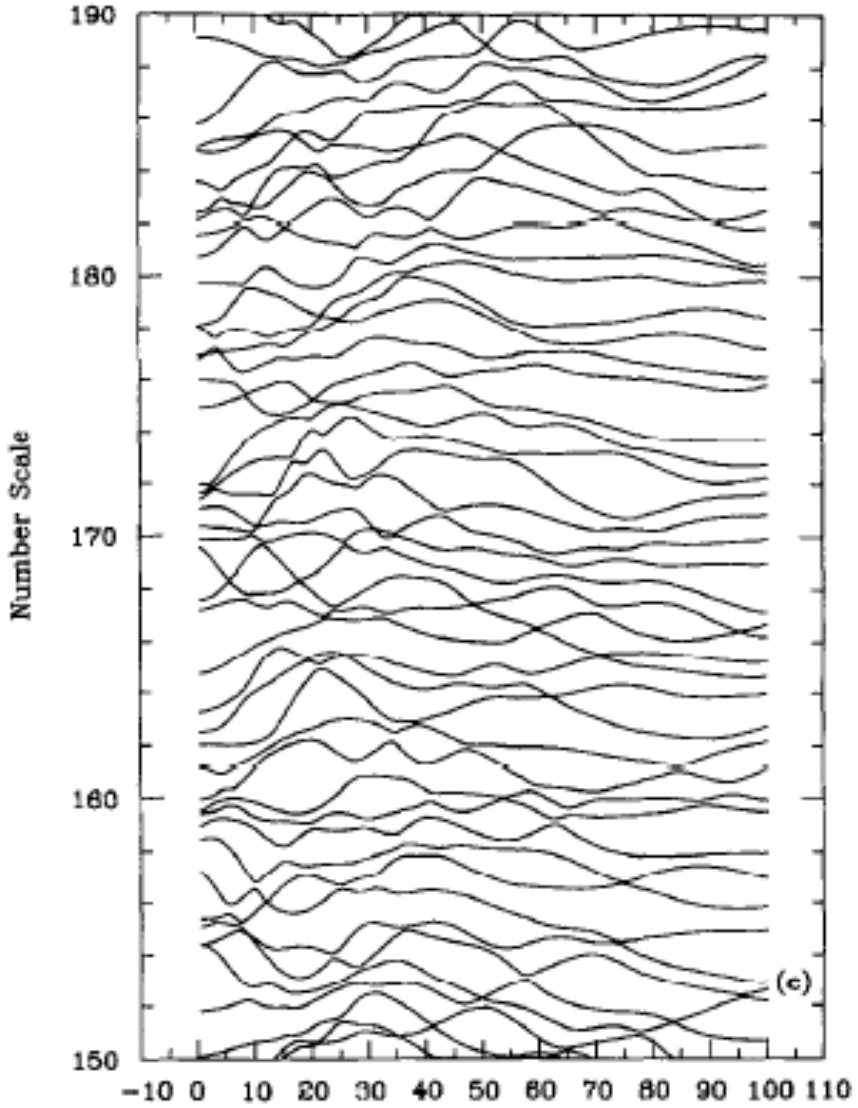
L. D. LANDAU and E. M. LIFSHITZ – "*Statistical Physics*"

Average over the equilibrium ensemble should coincide with the expectation value in a generic individual eigenstate of the same energy – the results of measurements in a closed system do not depend on exact microscopic conditions or phase relationships if the eigenstates at the same energy have similar macroscopic properties

Eigenstate Thermalization Hypothesis

TOOL: MANY-BODY QUANTUM CHAOS

LEVEL DYNAMICS



WAY to CHAOS:

MULTIPLE

AVOIDED

CROSSINGS

as a function

of interaction strength

(shell model of 24Mg
as a typical example)

Fraction (%) of realistic strength

From turbulent to laminar level dynamics

Chaos due to particle interactions at high level density

MEASURING COMPLEXITY

Eigenstate $|\alpha\rangle$ in a shell model basis $|k\rangle$

$|\alpha\rangle = \sum_k C_k^\alpha |k\rangle$

Information entropy

$S^\alpha = -\sum_k |C_k^\alpha|^2 \ln |C_k^\alpha|^2$

No mixing: $S^\alpha \to 0$

"Microcanonical" mixing: $S^\alpha \to \ln N$

GOE: $\overline{S^\alpha} = \ln(0.48N)$

Shannon entropy

Information entropy is basis-dependent
- special role of mean field

FAMILY OF ENTROPIES FOR A MESOSCOPIC SYSTEM

- **THERMODYNAMIC** (*Boltzmann*)

$$\rho(E) \propto \exp(S_{\rm th})$$

- **QUASIPARTICLE** (*Landau Fermi-liquid*)

$$S^{\alpha}_{\rm s.p.} = -\Sigma_i \{ n_i^{\alpha} \ln(n_i^{\alpha}) + (1-n_i^{\alpha}) \ln(1-n_i^{\alpha}) \}$$

- **INFORMATION** (*Shannon*)

$$|\alpha\rangle = \Sigma_k C_k^{\alpha} |k\rangle, \quad S^{\alpha}_{\rm inf} = -\Sigma_k \{ |C_k^{\alpha}|^2 \ln |C_k^{\alpha}|^2 \}$$

$$\langle n_i \rangle_E = [e^{(\epsilon_i - \mu)/T_{\rm s.p.}} + 1]^{-1}$$

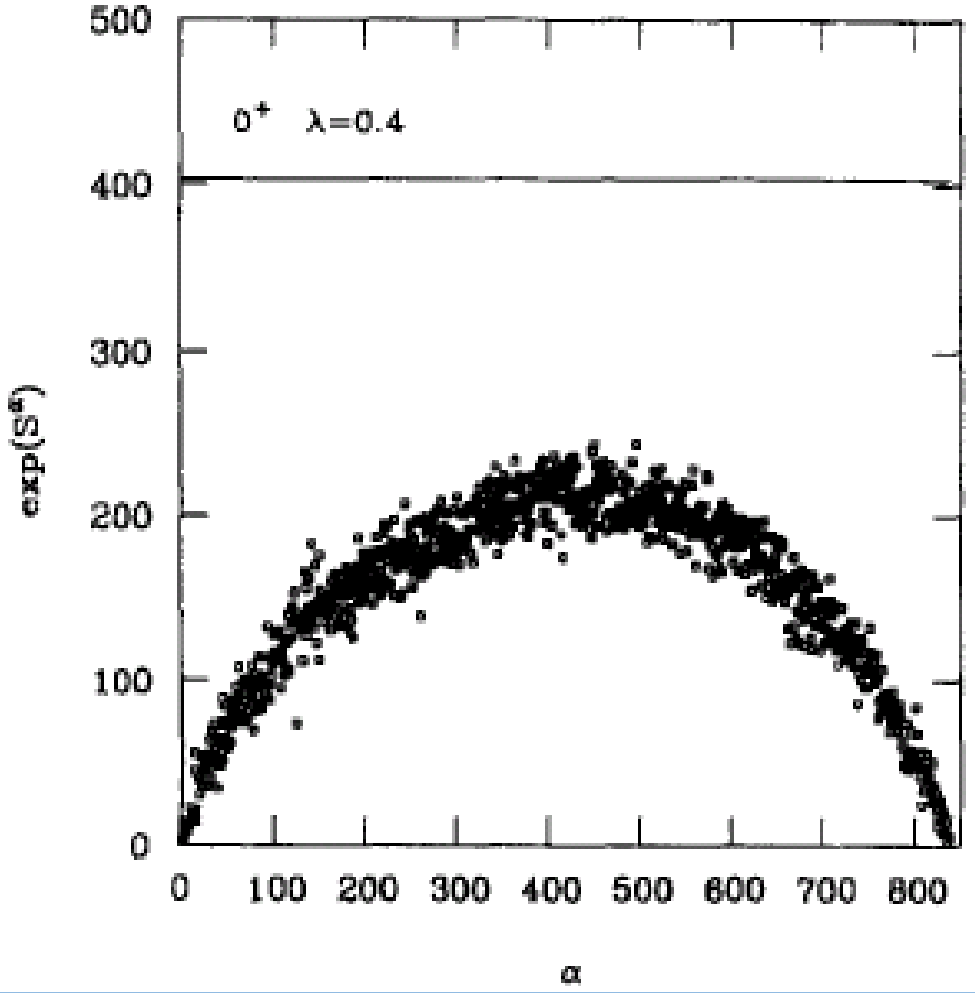
Temperature T(E)

$$T_{\rm th} = \left(\frac{dS_{\rm th}}{dE} \right)^{-1}$$

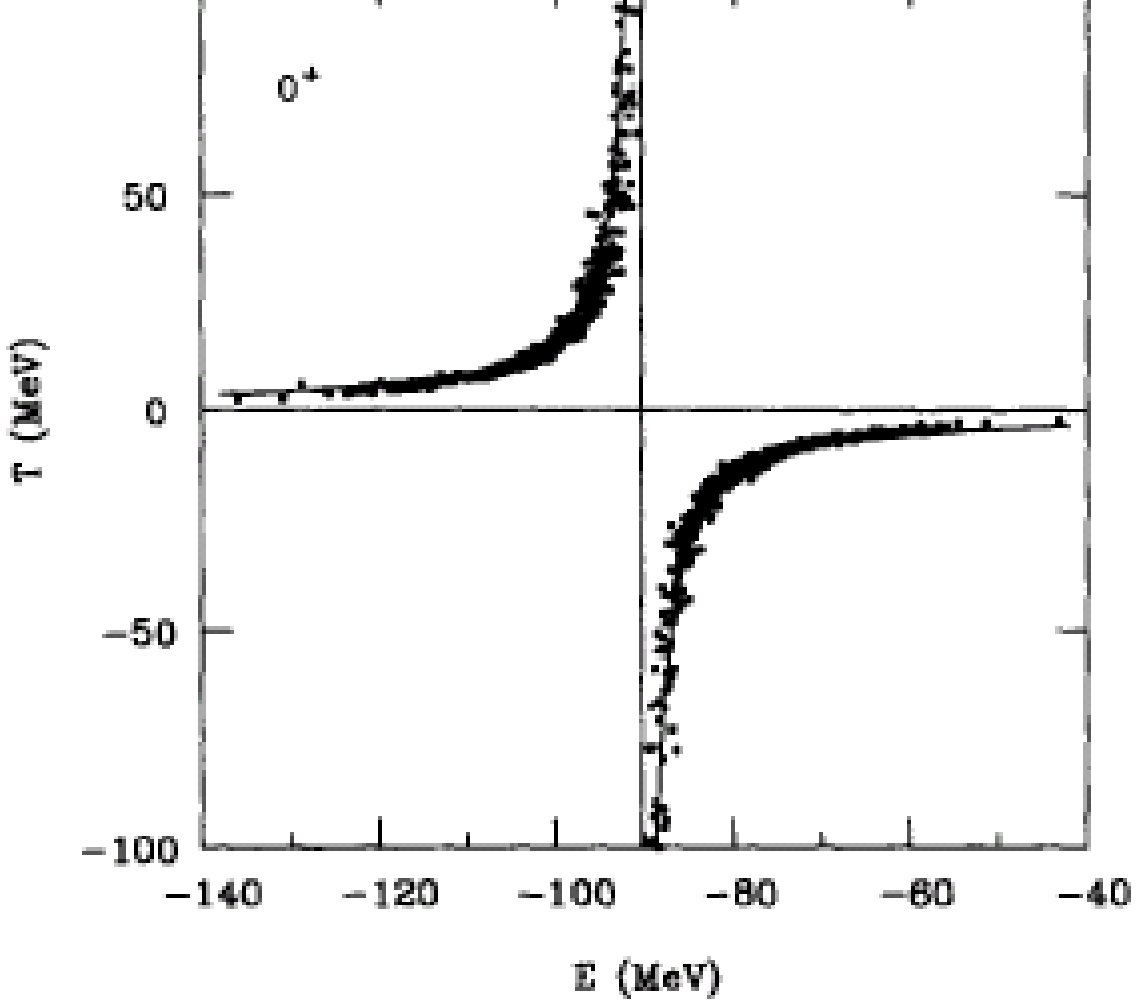
$$T_{\rm inf} = \left(\frac{d\bar{S}_{\rm inf}}{dE} \right)^{-1}$$

T(s.p.) and T(inf) =
for individual states !

28 Si Shell Model (artificially weak interaction)



INFORMATION ENTROPY AT WEAK INTERACTION



Gaussian level density

$$\text{CENTROID}\ E_0$$
$$\text{WIDTH}\ \sigma_E$$

Microcanonical temperature

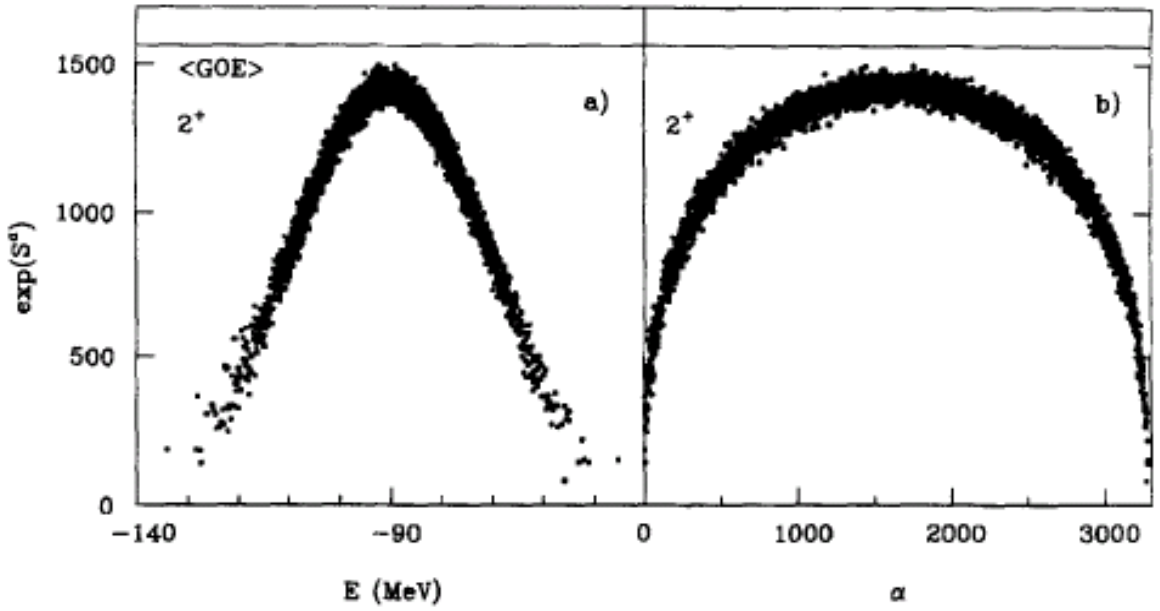
$$T_{\rm th} = \sigma_E^2/(E_0 - E)$$

839 states (28 Si) J=0

EFFECTIVE TEMPERATURE of INDIVIDUAL STATES

From occupation numbers in the shell model solution **(dots)**
From thermodynamic entropy defined by level density **(lines)**

28 Si shell model
Realistic interaction
strength



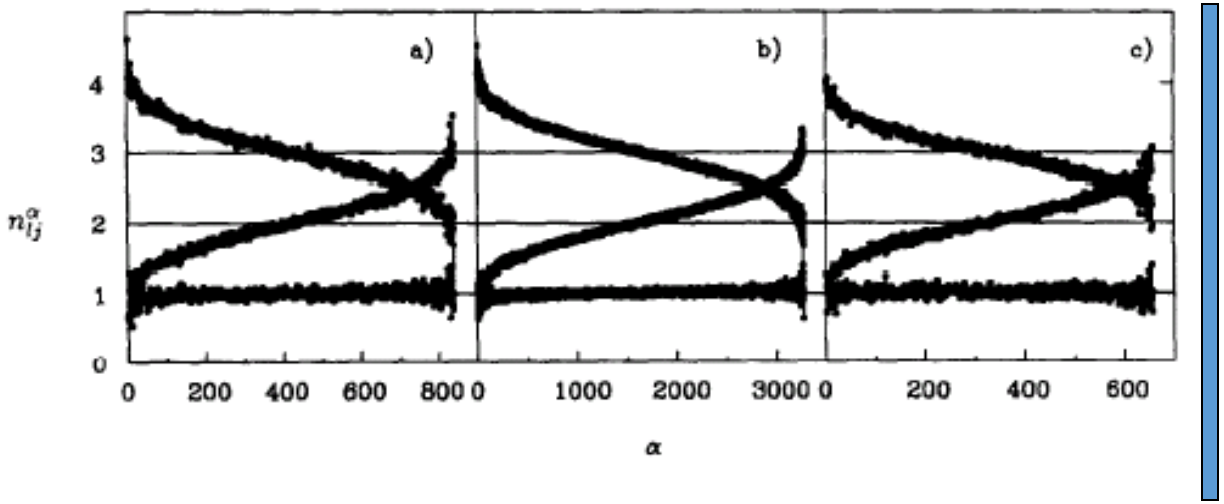
INFORMATION ENTROPY of EIGENSTATES

(a) function of energy; (b) function of ordinal number

ORDERING of EIGENSTATES of GIVEN SYMMETRY

SHANNON ENTROPY AS THERMODYNAMIC VARIABLE

d5/2, d3/2, s1/2



28 Si

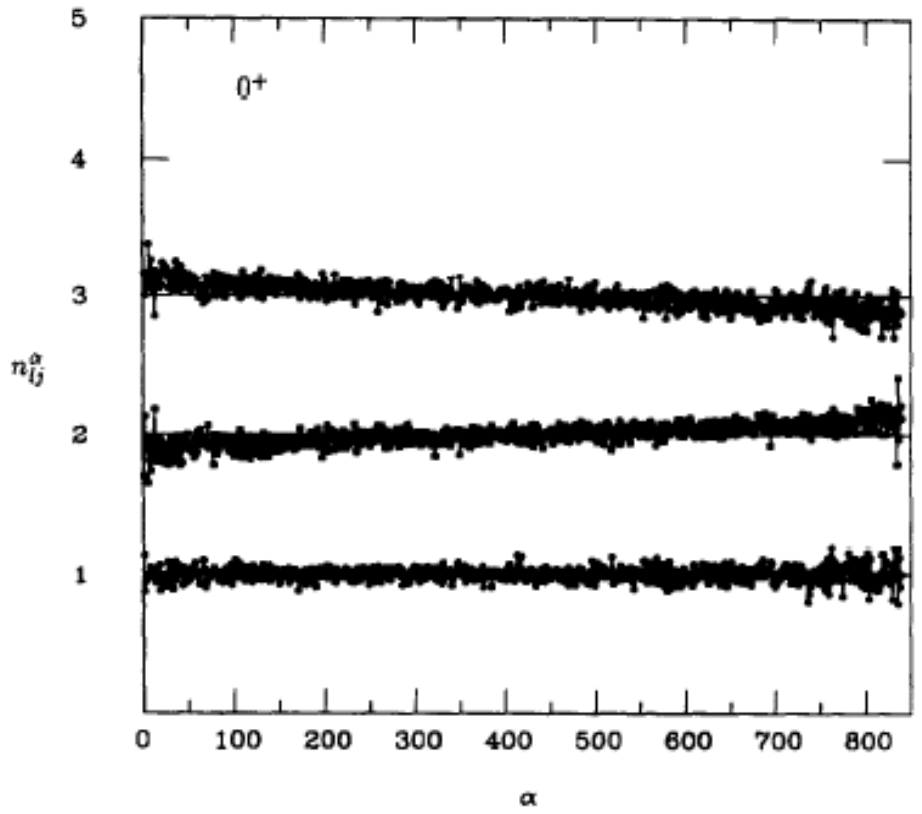
J=0 *J=2* *J=9*

Single – particle occupation numbers

Thermodynamic behavior
identical in all symmetry classes

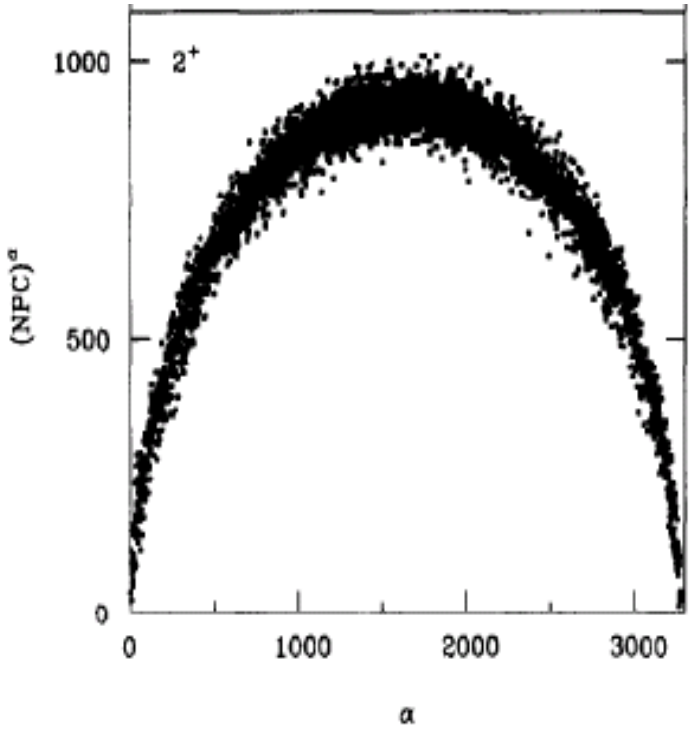
FERMI-LIQUID PICTURE

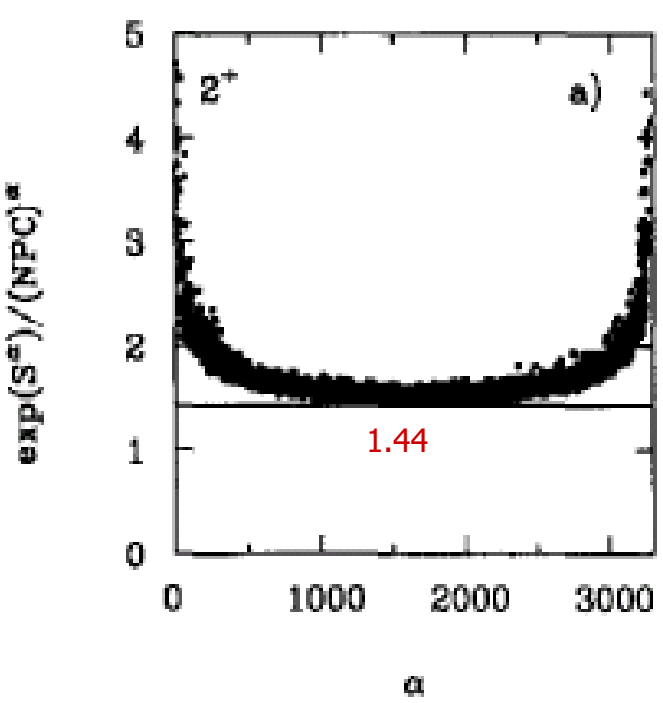
J=0



Artificially strong interaction (factor of 10)

Single-particle thermometer cannot resolve spectral evolution





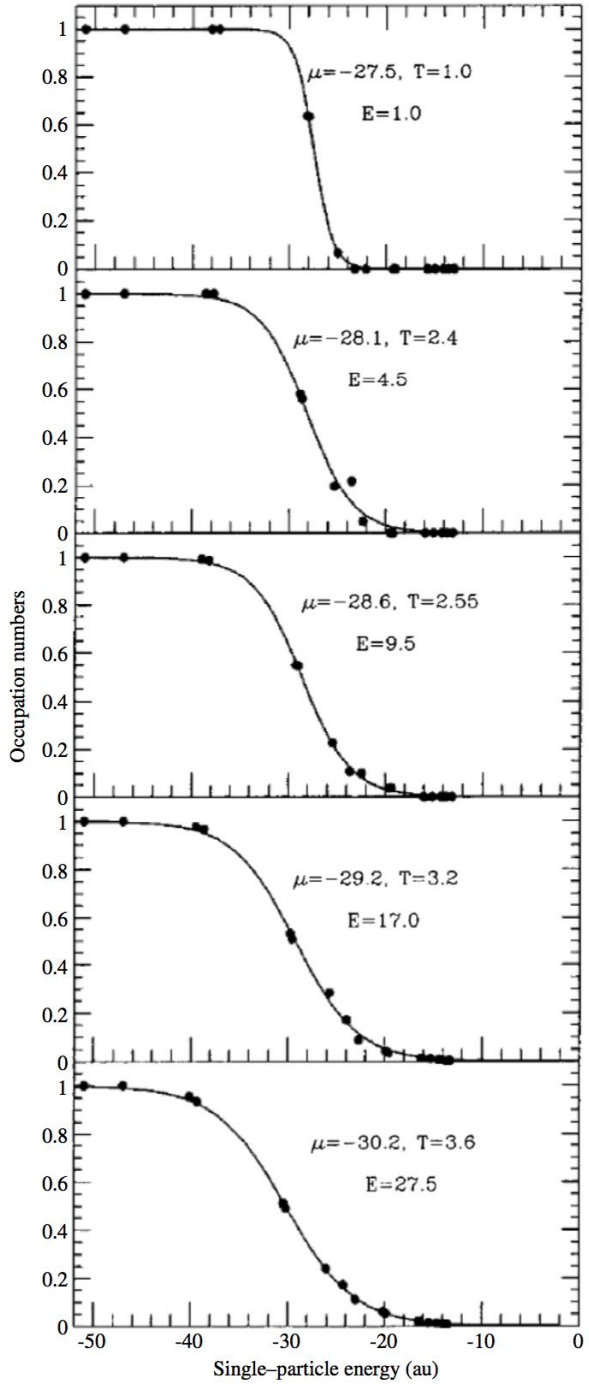
NUMBER of PRINCIPAL COMPONENTS

$$NPC(\alpha) = \left[\Sigma_k (C_k^\alpha)^4\right]^{-1}$$

$$\overline{NPC(GOE)} = \frac{N}{3}$$

Ratio $\exp[S(\alpha)]/\mathrm{NPC}(\alpha)$

GOE value = 1.44



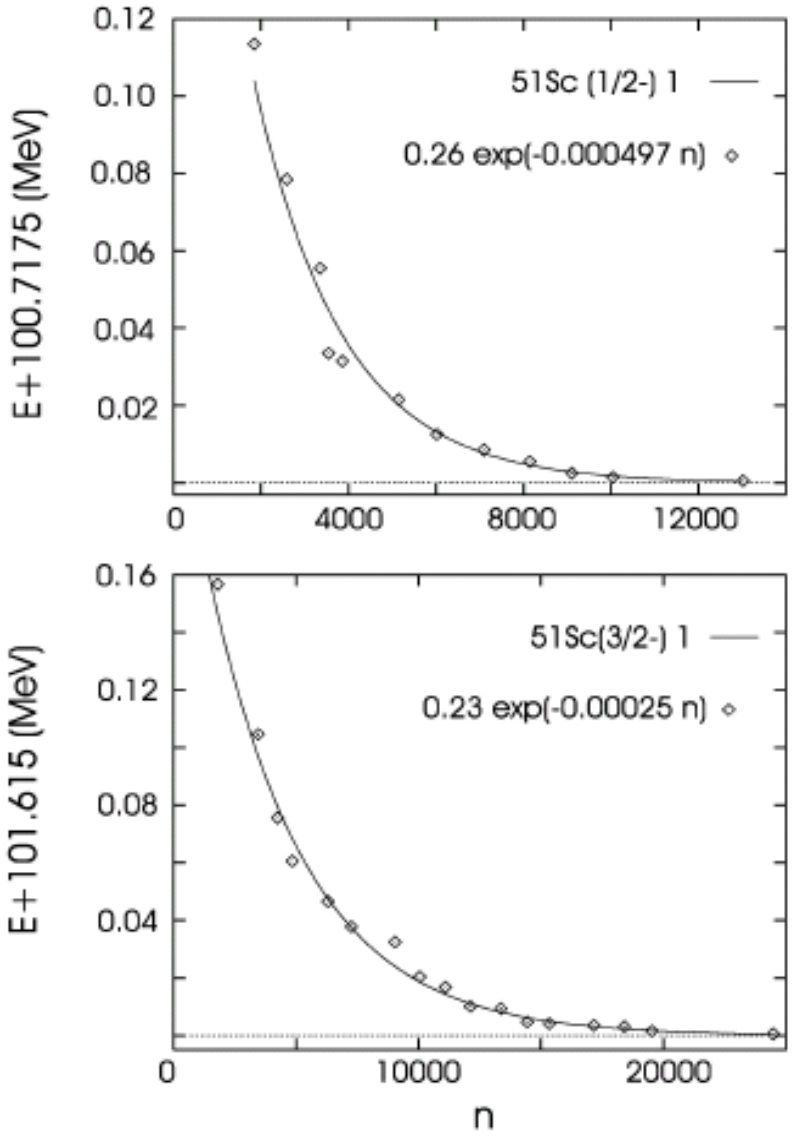
Occupation numbers in multicharged ions Au25+

(recombination as analog of neutron resonances in nuclei)

$$n_s^\alpha = \langle\alpha|\,\hat{n}_s\,|\alpha\rangle = \sum_k \left|C_k^\alpha\right|^2 \langle k|\,\hat{n}_s\,|k\rangle$$

/G. Gribakin, A. Gribakina, V. Flambaum/

Average over individual states is equivalent to a thermal ensemble



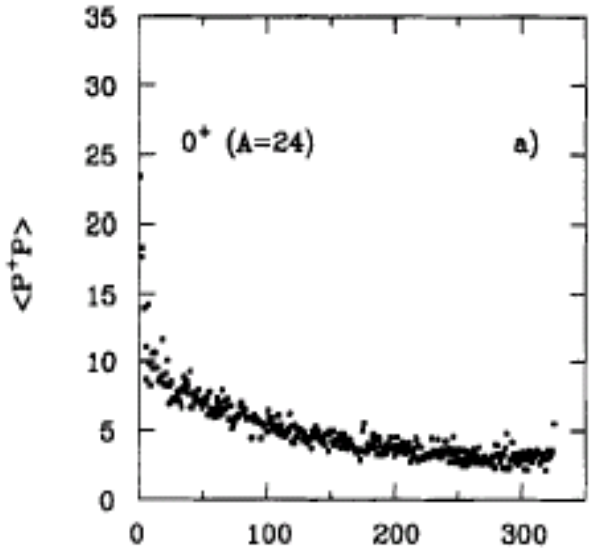
REALISTIC
SHELL
MODEL

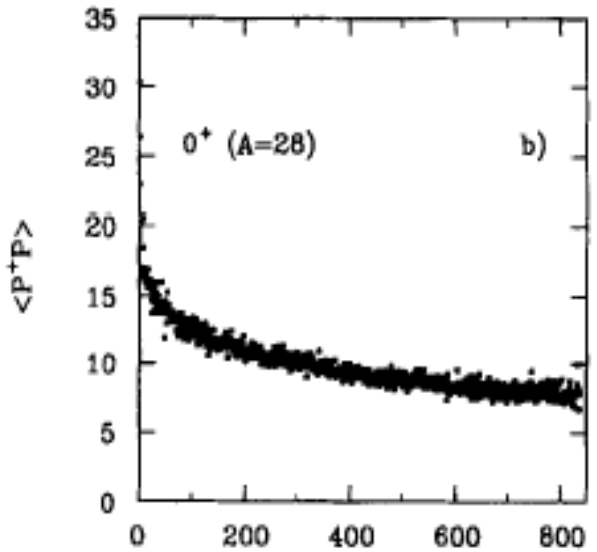
EXCITED STATES
51Sc

1/2-, 3/2-

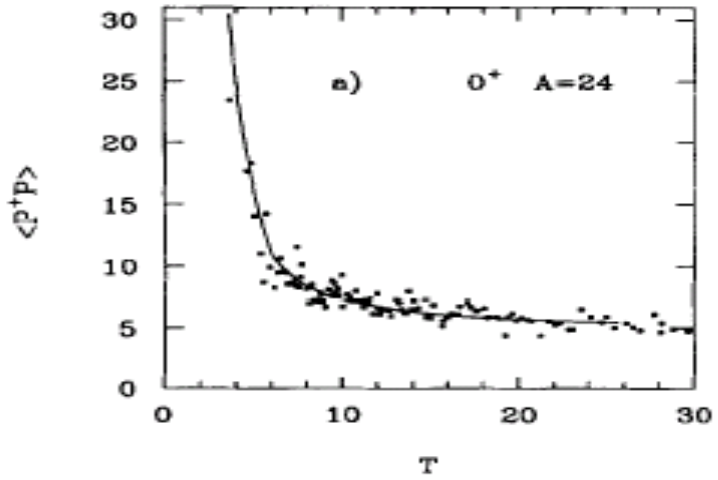
Faster convergence:
$E(n) = E + \exp(-an)$
$a \sim 6/N$

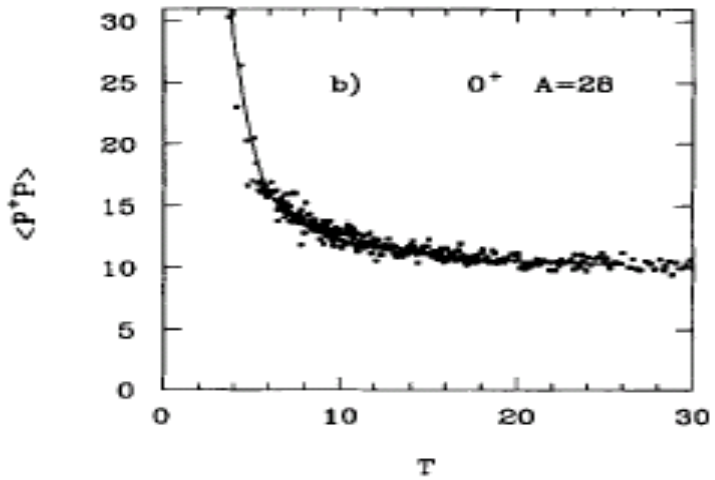
Exponential convergence



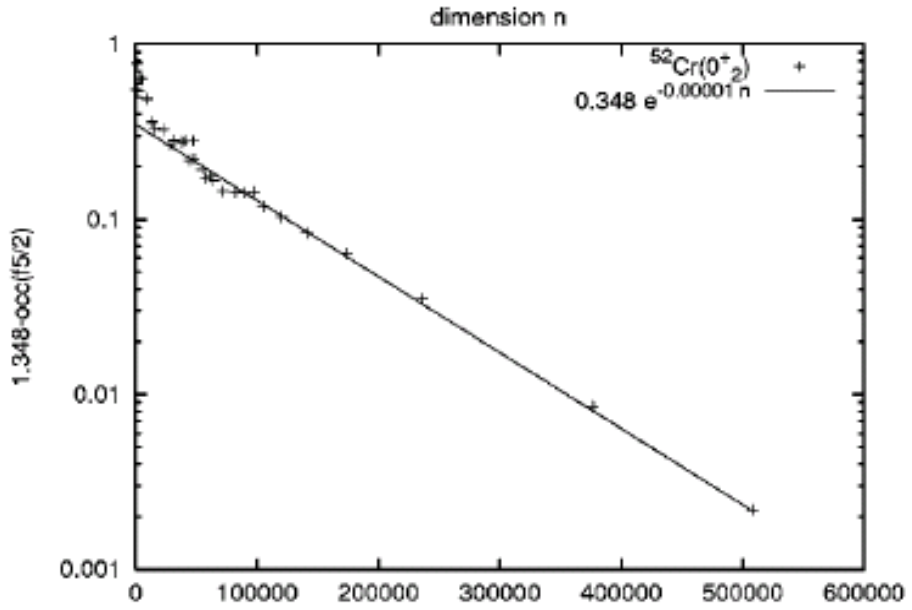


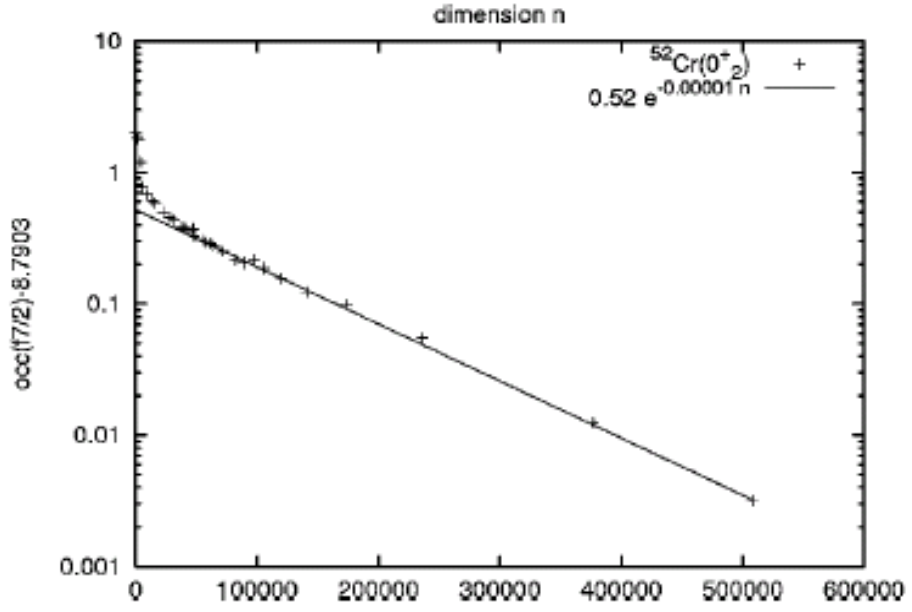
PAIRING
PHASE
TRANSITION





PAIR CORRELATOR as a THERMODYNAMIC FUNCTION



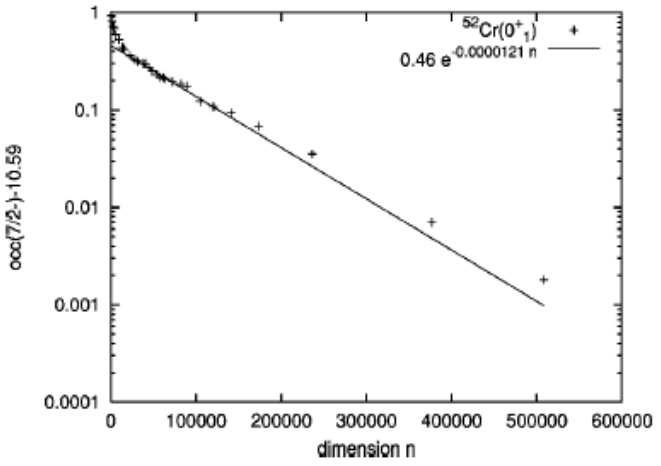


EXPONENTIAL CONVERGENCE OF SINGLE-PARTICLE OCCUPANCIES

(first excited state J=0)

52

Cr



Fit with $\gamma' = \gamma$

Analytical results for tridiagonal matrices

$$H = \begin{pmatrix} \epsilon_1 & V_2 & 0 & 0 & 0 & 0 \\ V_2 & \epsilon_2 & V_3 & 0 & 0 & 0 \\ 0 & V_3 & \epsilon_3 & V_4 & 0 & 0 \\ 0 & 0 & \dots & \dots & \dots & 0 \\ 0 & 0 & 0 & \dots & \dots & V_n \\ 0 & 0 & 0 & 0 & V_n & \epsilon_n \end{pmatrix}$$

Recurrence relation for determinants

$$D_n(E) = (\epsilon_n - E)D_{n-1}(E) - V_n^2 D_{n-2}(E)$$

Convergence is determined by

$$\lambda_n^2 = V_n^2/(\epsilon_n \epsilon_{n-1})$$

Assume

existence

of the limit

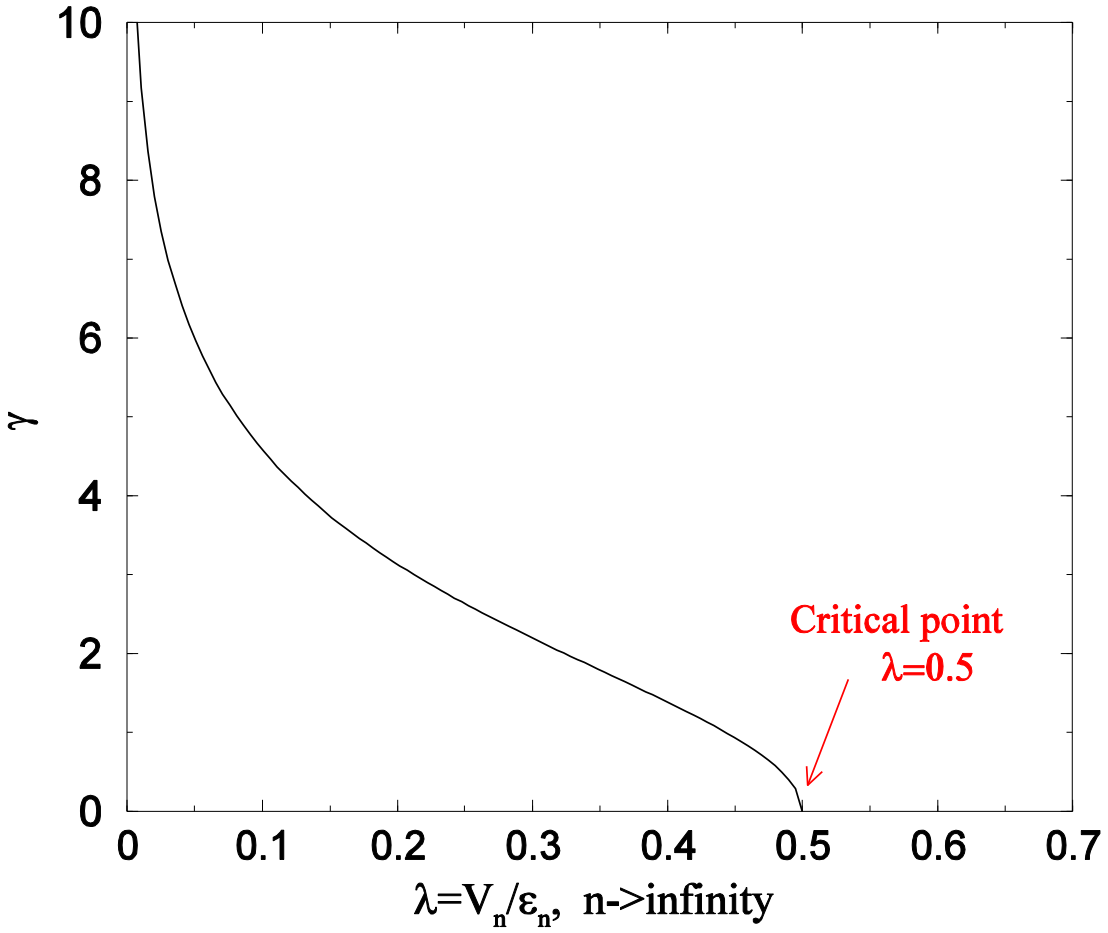
$$\lambda_n \Rightarrow \lambda$$

at

$$n \Rightarrow \infty$$

CONVERGENCE REGIMES

- $\lambda = 0$ — *Fast convergence*
- $0 < \lambda < 1/2$ — *Exponential convergence*
- $\lambda = 1/2$ — Power law
- $\lambda > 1/2$ — *Divergence*



Exponential convergence

$\sim \exp(-\gamma n)$

$$\gamma = -\ln\left\{\frac{1}{2\lambda^2}\left(1 - 2\lambda^2 - \sqrt{1 - 4\lambda^2}\right)\right\}$$

PRACTICAL ALGORITHM

(Ground state energy problem)

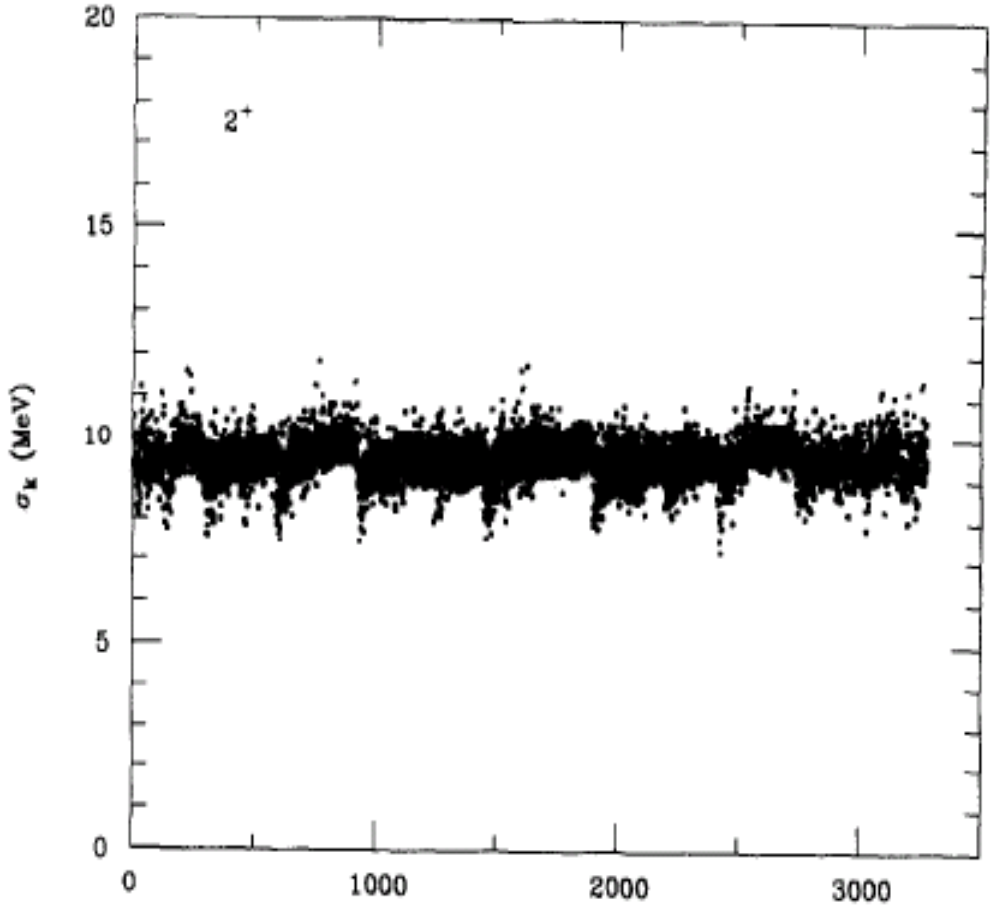
- Generate the set of partitions in given orbital space
  Example ^{56}Ni in full pf-space:
  1 087 455 228 m-scheme states,
  2 581 576 $J^\pi T = 0^+0$ states, 475 partitions
- Calculate partition centroids
  (Hamiltonian traces/ dimension)
- Order partitions by their centroids
- Truncate shell model matrices including consecutively partitions in their entirety
- Calculate the lowest states (Lanczos) with progressive truncation
- Identify onset of the exponential regime
- Obtain the exact energy value

GLOBAL PROBLEMS

1. New approach to many-body theory for mesoscopic systems –
   instead of blunt diagonalization -
   mean field out of chaos,
   coherent modes plus
   thermalized chaotic background

2. Chaos-free scalable quantum computing
   (internal and external chaos)

28
Si



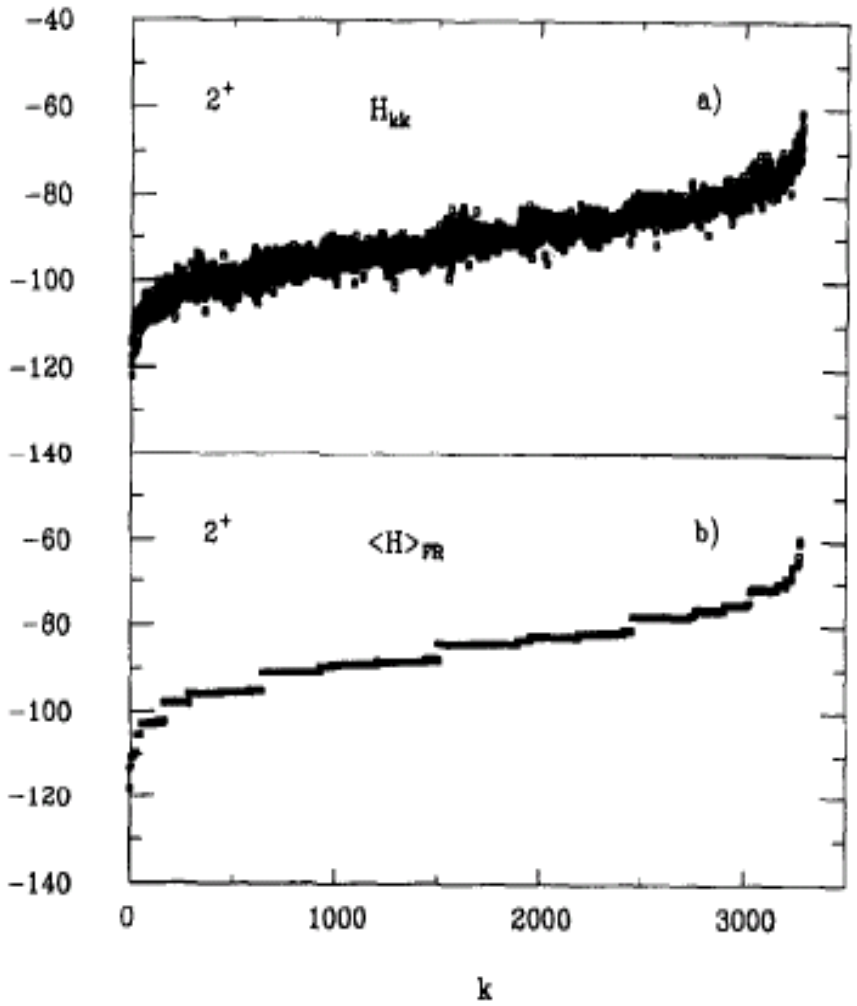
Energy dispersion for individual states is nearly constant
(result of **geometric chaoticity**!)
Also in multiconfigurational method (hybrid of shell model and density functional)

$$\sigma_k^2 = \langle k|(H - H_{kk})^2|k\rangle = \sum_{l\neq k} H_{kl}^2 \ ,$$

Widths add in quadratures

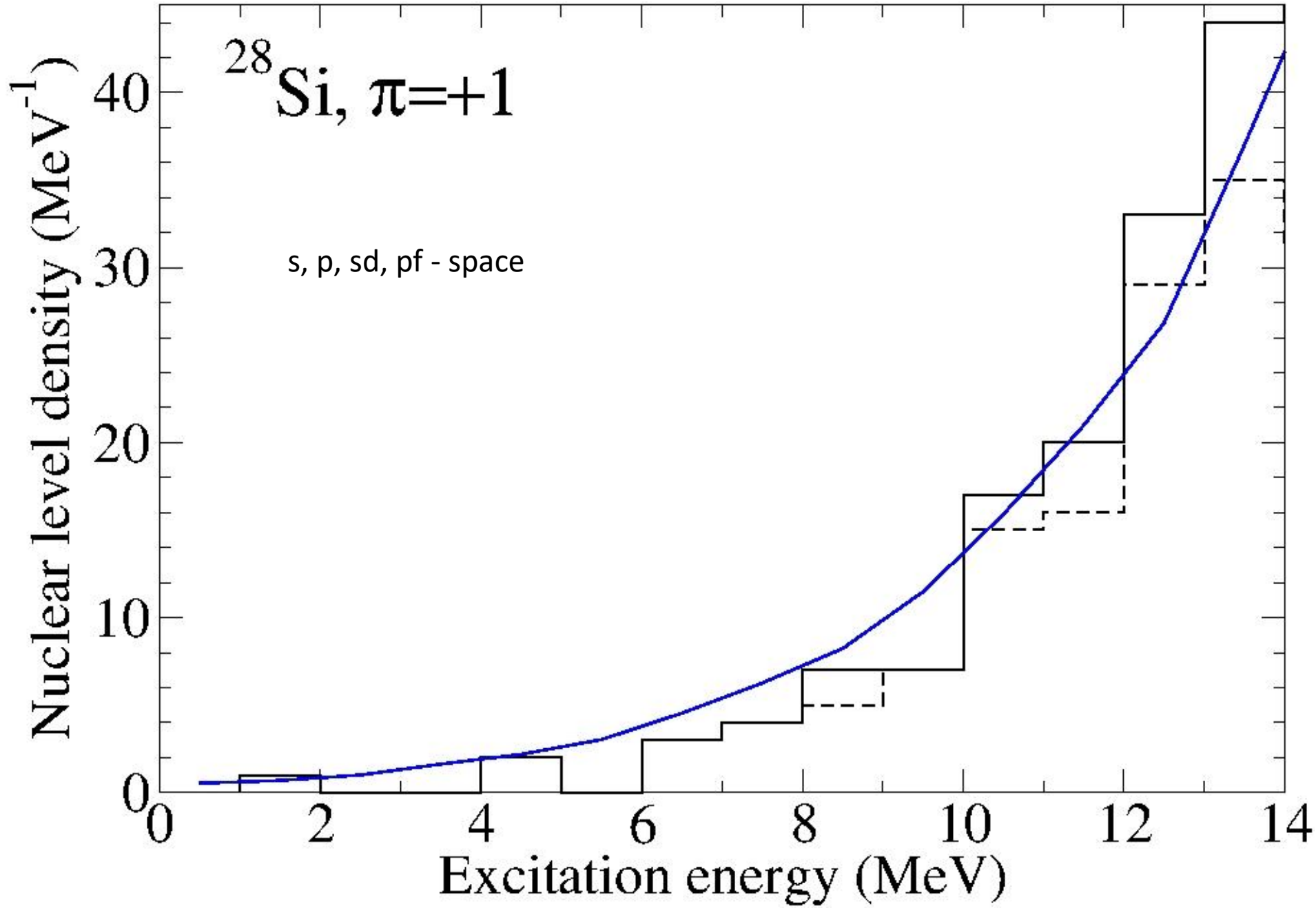
^{28}Si

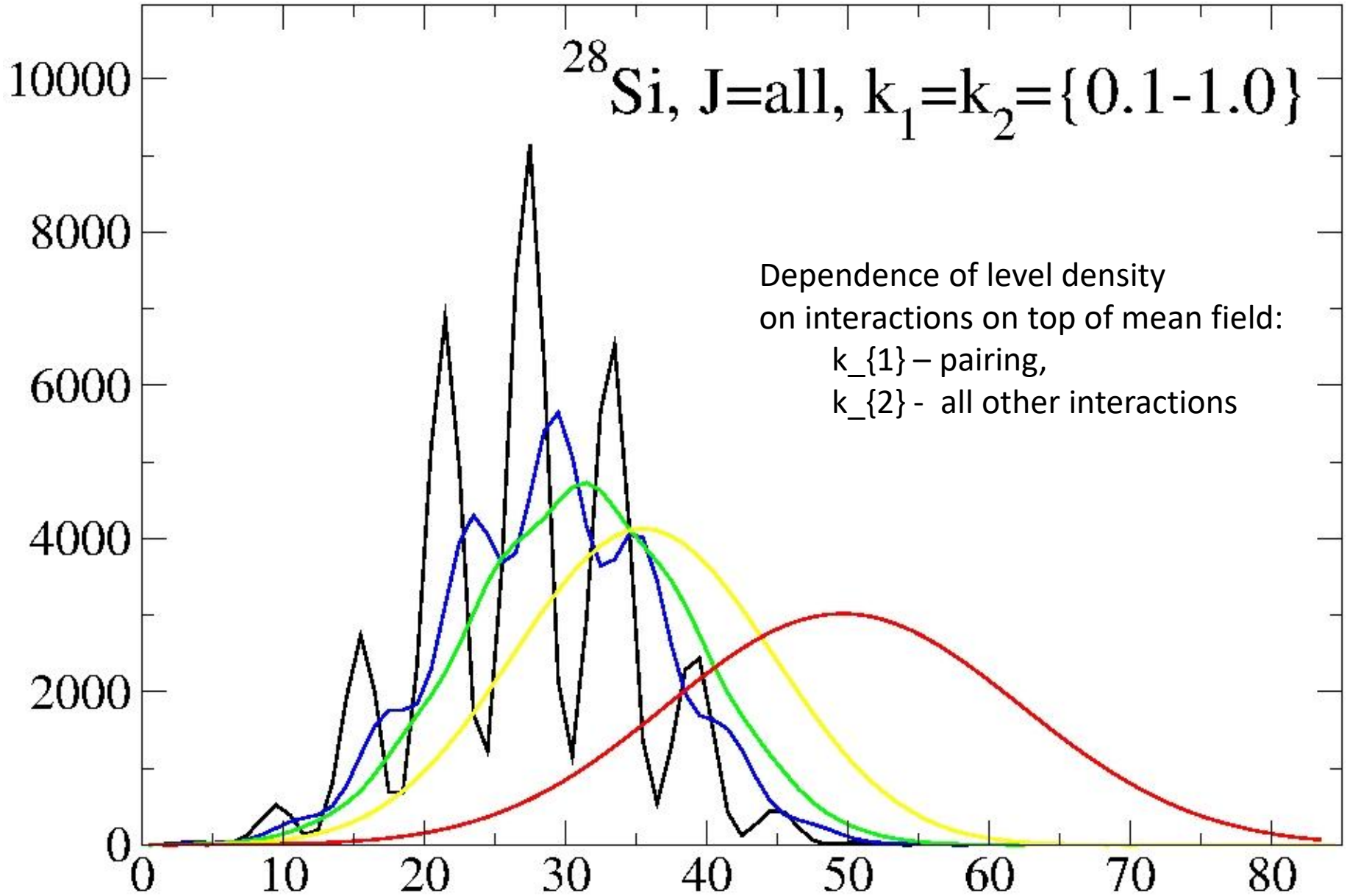
Diagonal matrix elements of the Hamiltonian in the mean-field representation

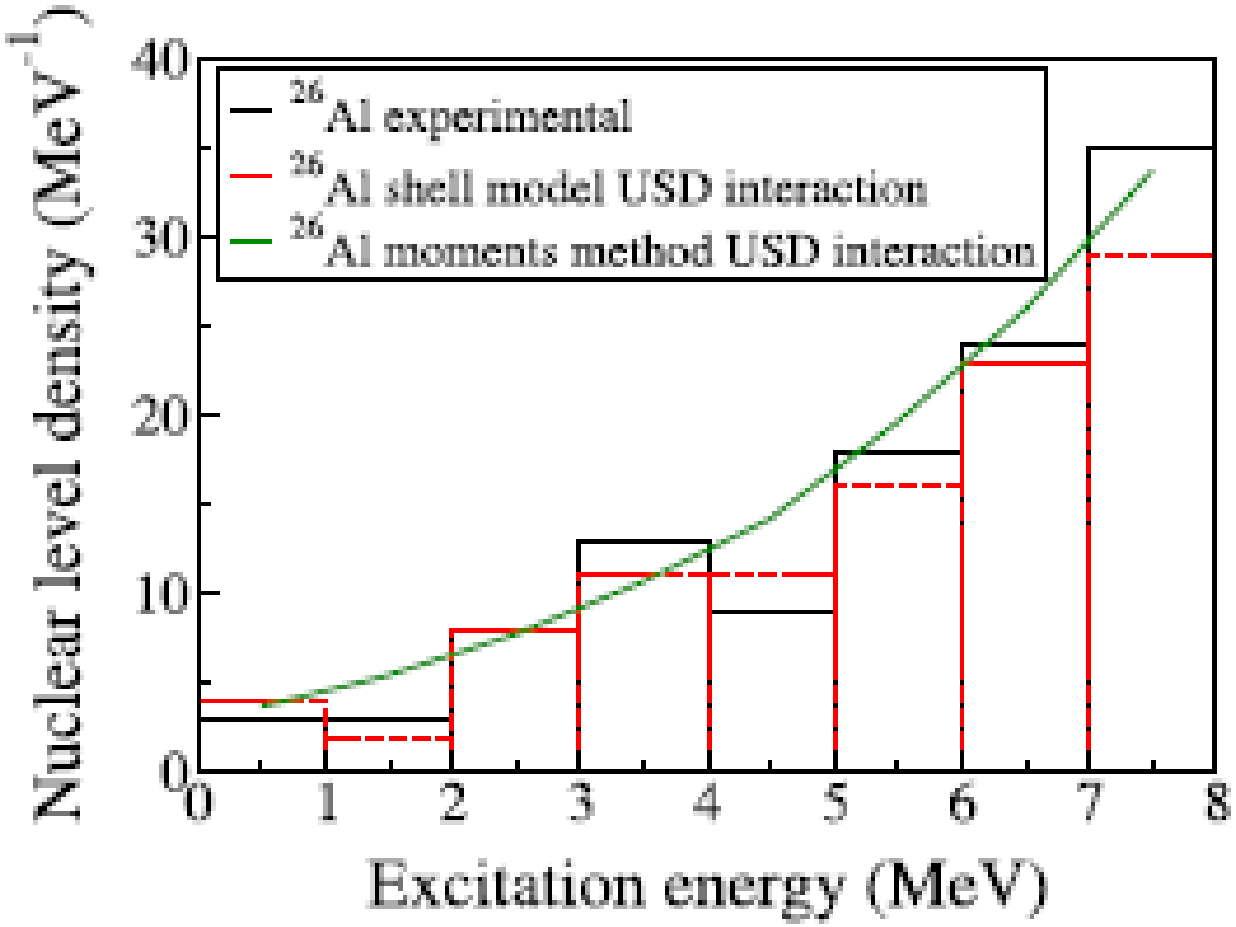


Partition structure in the shell model

(a) *All 3276 states* ; (b) *energy centroids*


^{28}Si, π=+1
s, p, sd, pf - space
Nuclear level density (MeV^{-1})
Excitation energy (MeV)
0
10
20
30
40
0
2
4
6
8
10
12
14


^{28}Si, J=all, $k_1=k_2=\{0.1\text{-}1.0\}$
Dependence of level density
on interactions on top of mean field:
k_{1} – pairing,
k_{2} - all other interactions
10000
8000
6000
4000
2000
0
0
10
20
30
40
50
60
70
80



S. Karampagia, V.Z.
Nucl. Phys. A962 (2017)

J = 0 – 7, positive parity level density

Moments method

No diagonalization required

$$\rho(E,\alpha) = \sum_{\kappa} D_{\alpha\kappa} \cdot G_{\alpha\kappa}(E)$$

$$\alpha = \{n, J, T_z, \pi\}$$

Quantum numbers

Exact quantum numbers

$$\kappa = \{n_1, n_2, \ldots, n_q\}$$

Partitions

[Wong]

$$G_{\alpha\kappa}(E) = G(E + E_{g.s.} - E_{\alpha\kappa}, \sigma_{\alpha\kappa})$$

$$G(x,\sigma) = C \cdot \begin{cases} \exp\left(-x^2/2\sigma^2\right) & , \ |x| \le \eta \cdot \sigma \\ 0 & , \ |x| > \eta \cdot \sigma \end{cases}$$

Finite range Gaussian

$D_{\alpha\kappa}$

Many-body dimension

$$E_{\alpha\kappa} = \langle H \rangle_{\alpha\kappa};$$

$$\sigma_{\alpha\kappa} = \sqrt{\langle H^2 \rangle_{\alpha\kappa} - \langle H \rangle^2_{\alpha\kappa}}$$

$$\mathrm{Tr}^{(J)}[\cdots] = \mathrm{Tr}^{(J_z)}[\cdots]_{J_z=J} - \mathrm{Tr}^{(J_z)}[\cdots]_{J_z=J+1}$$

$$\langle H \rangle_{\alpha\kappa} = \mathrm{Tr}^{(\alpha\kappa)}[H]/D_{\alpha\kappa},$$

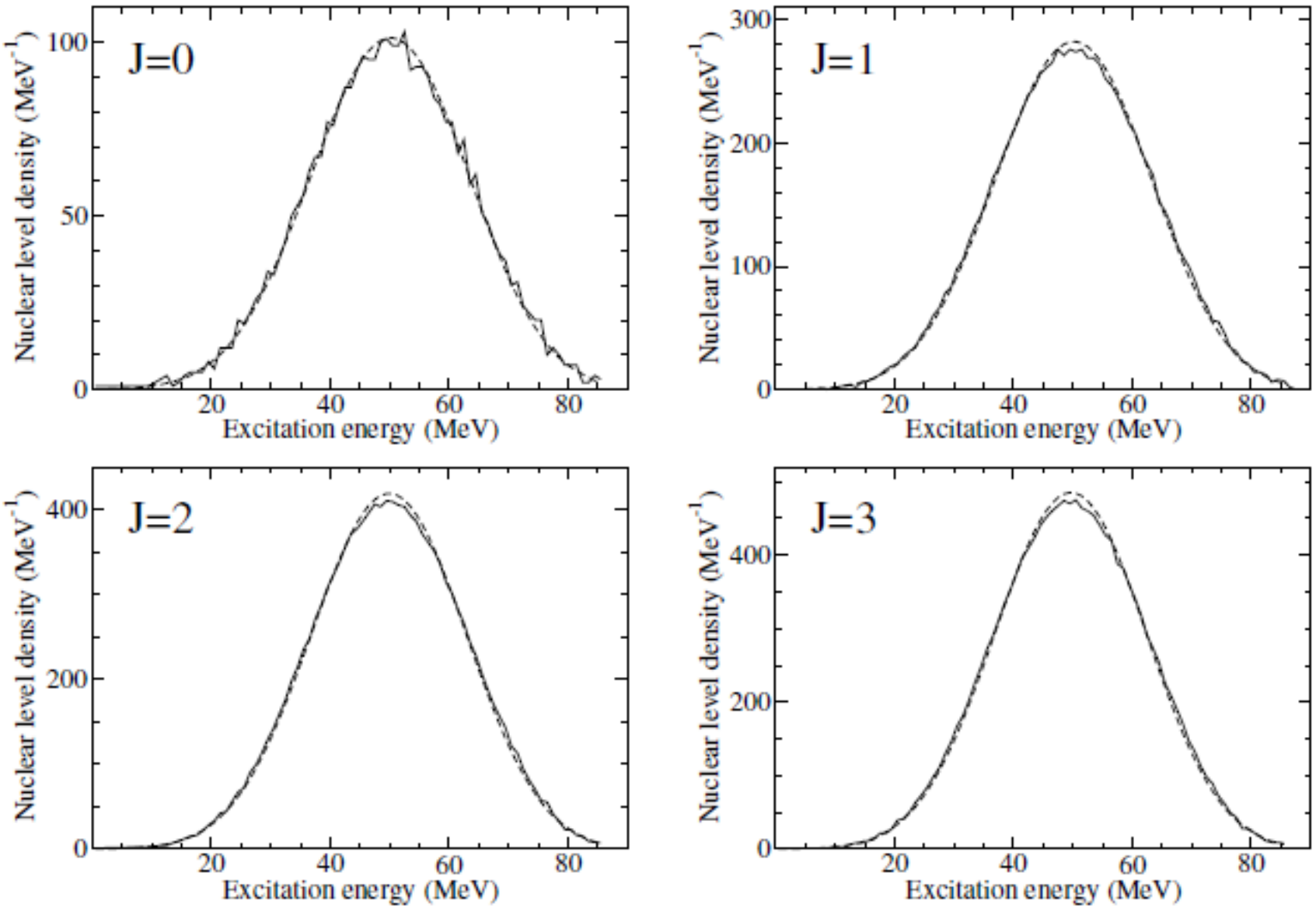
Centroids – first moment

$$\langle H^2 \rangle_{\alpha\kappa} = \mathrm{Tr}^{(\alpha\kappa)}[H^2]/D_{\alpha\kappa}$$

Widths - second moment

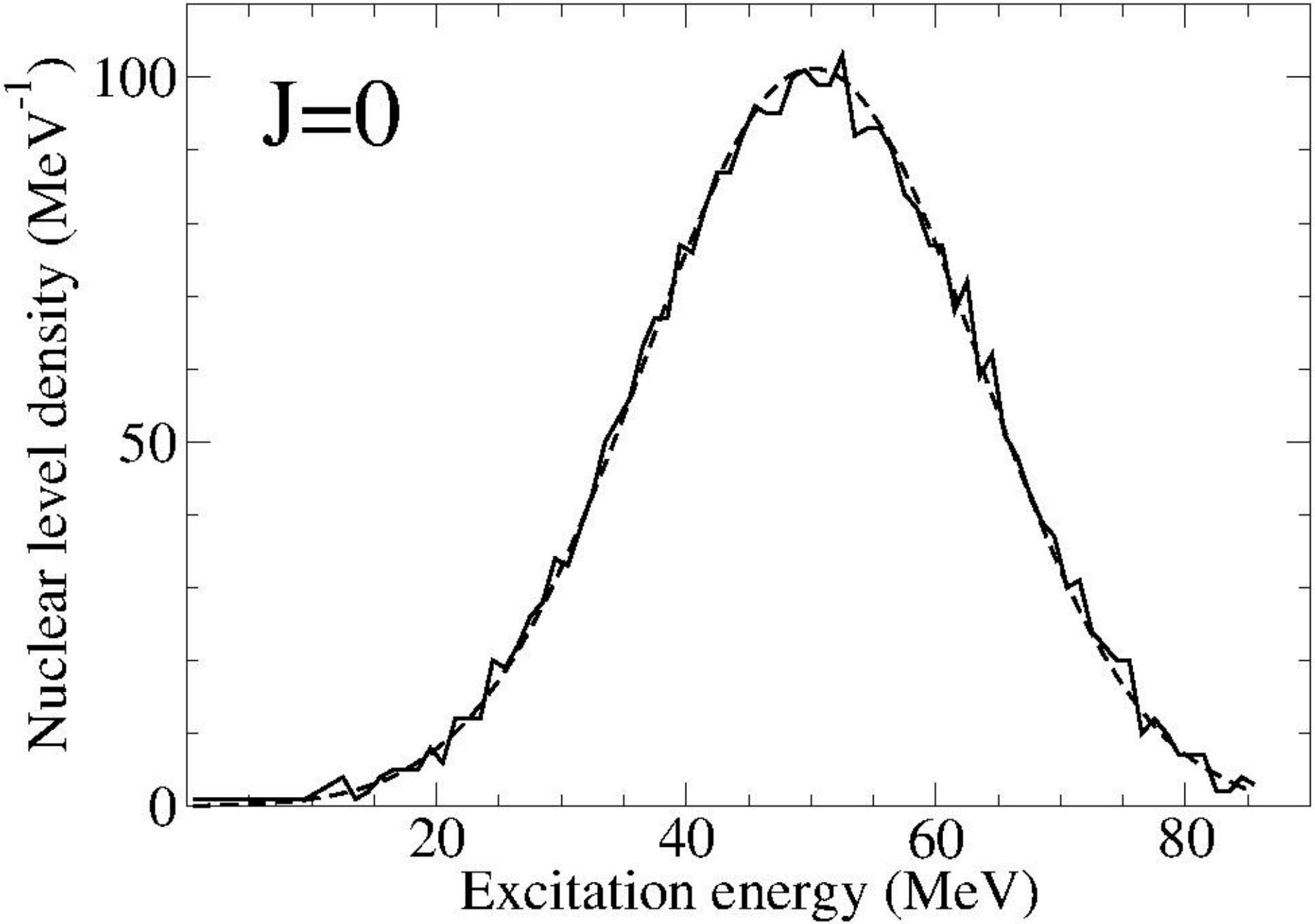
^{28}Si, parity=+1, some J, *sd*-shell

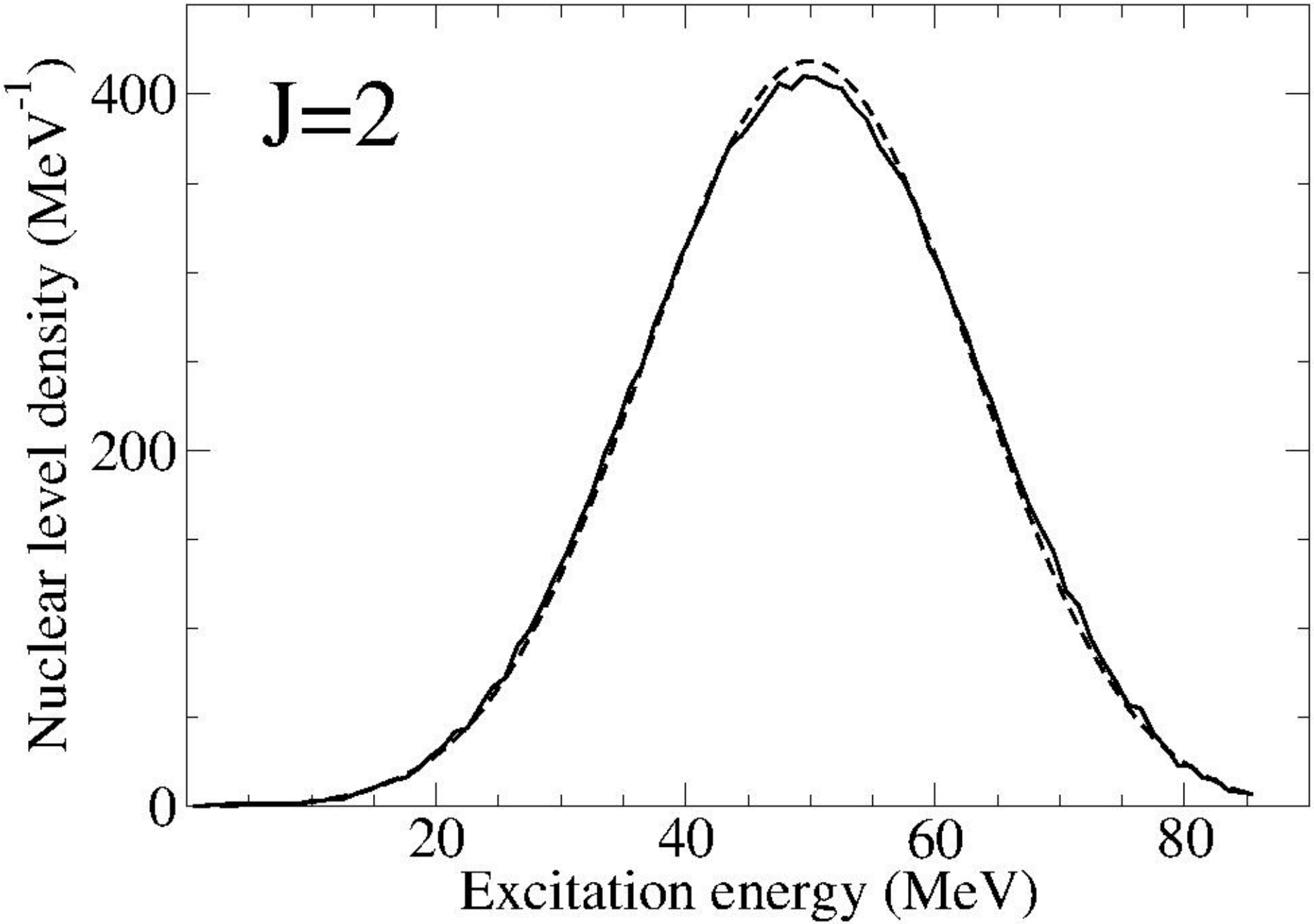
Shell Model (solid line) vs. Moments Method (dashed line).

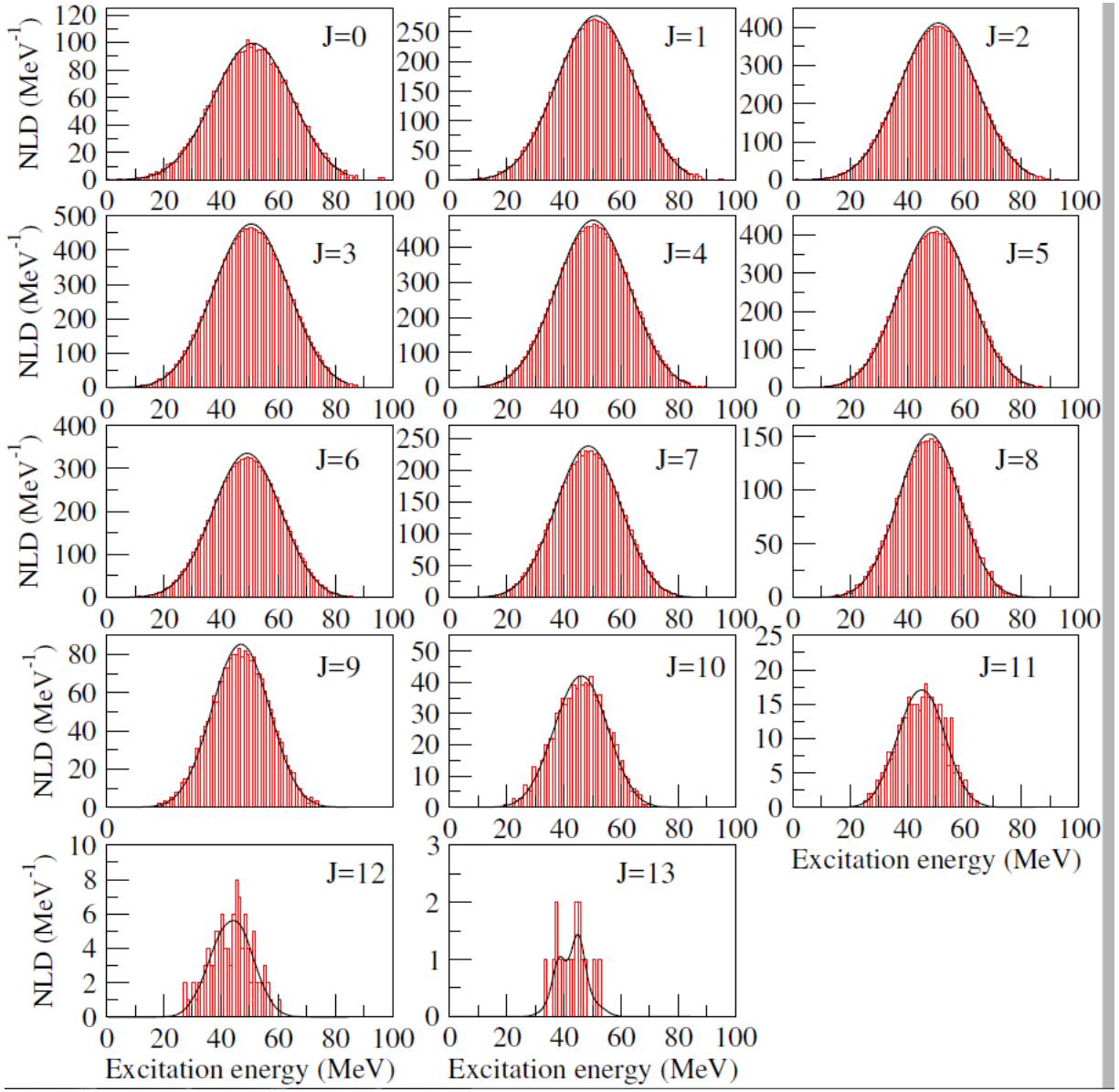


Shell-model level density.

Moments method (no diagonalization)


J=0
Nuclear level density (MeV^{-1})
100
50
0
20
40
60
80
Excitation energy (MeV)


J=2
Nuclear level density (MeV^{-1})
0
200
400
20
40
60
80
Excitation energy (MeV)

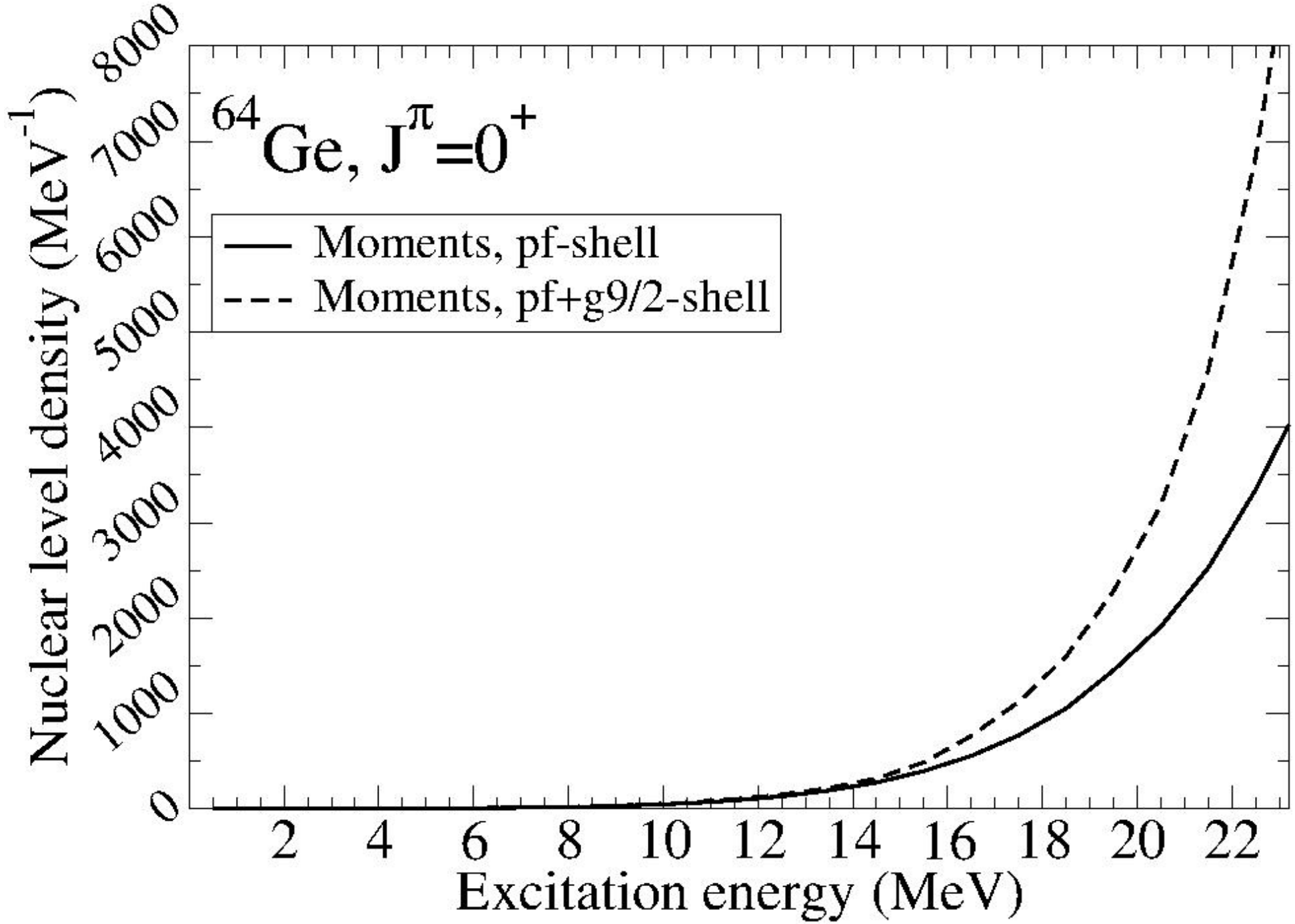


Generic shape (Gaussian)

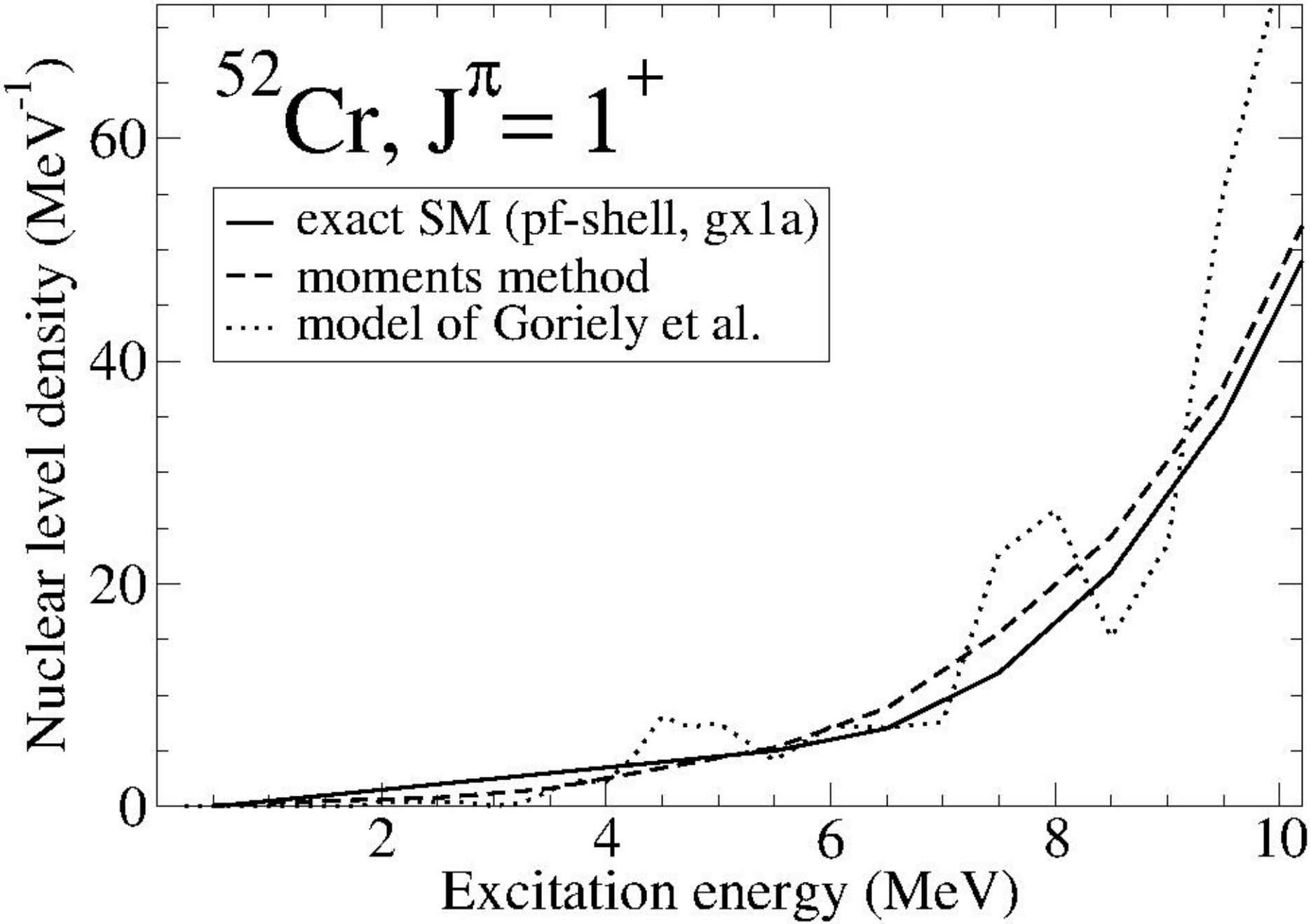
Level density for different classes of states in 28Si

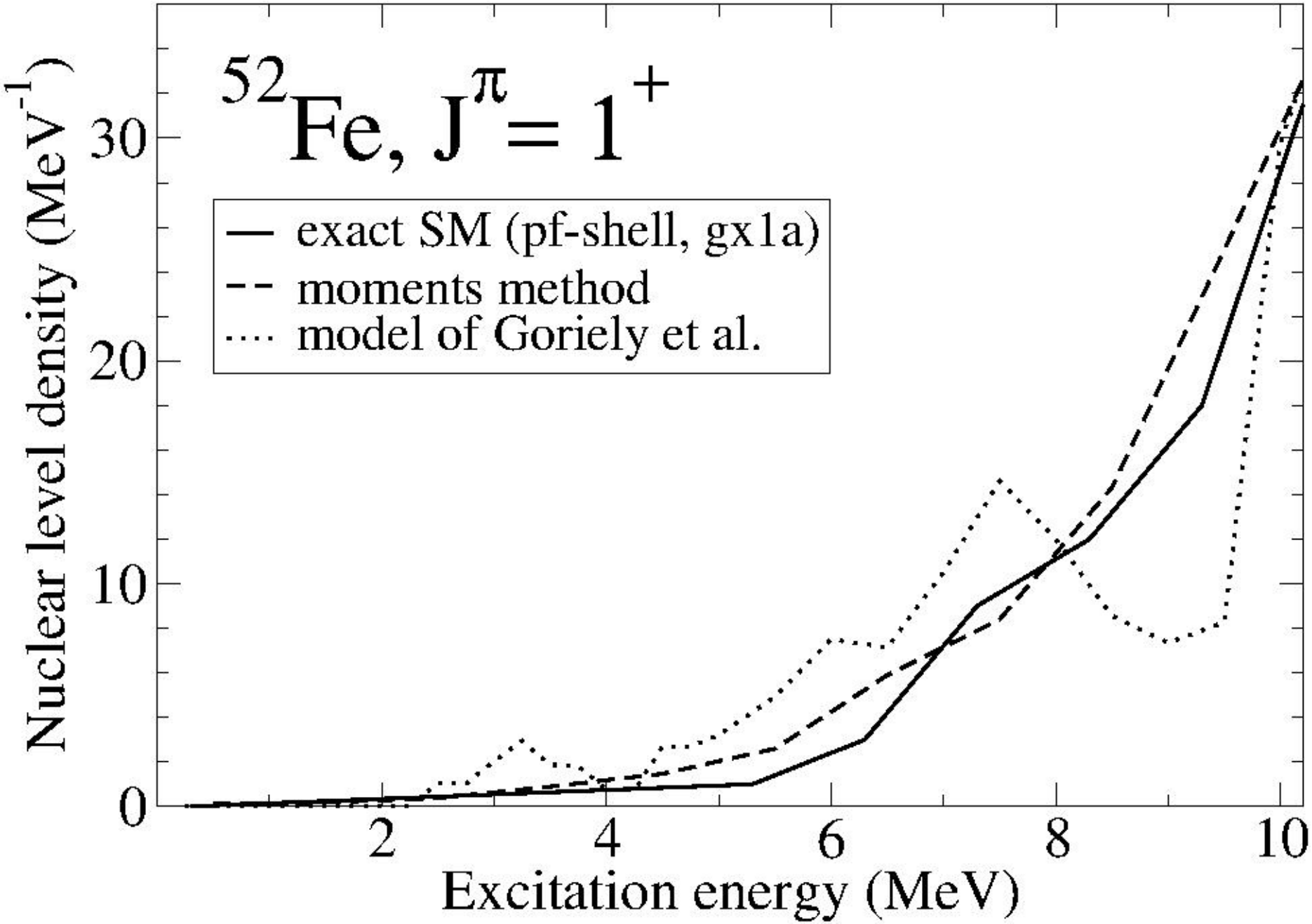
Full agreement between exact shell model and moments method

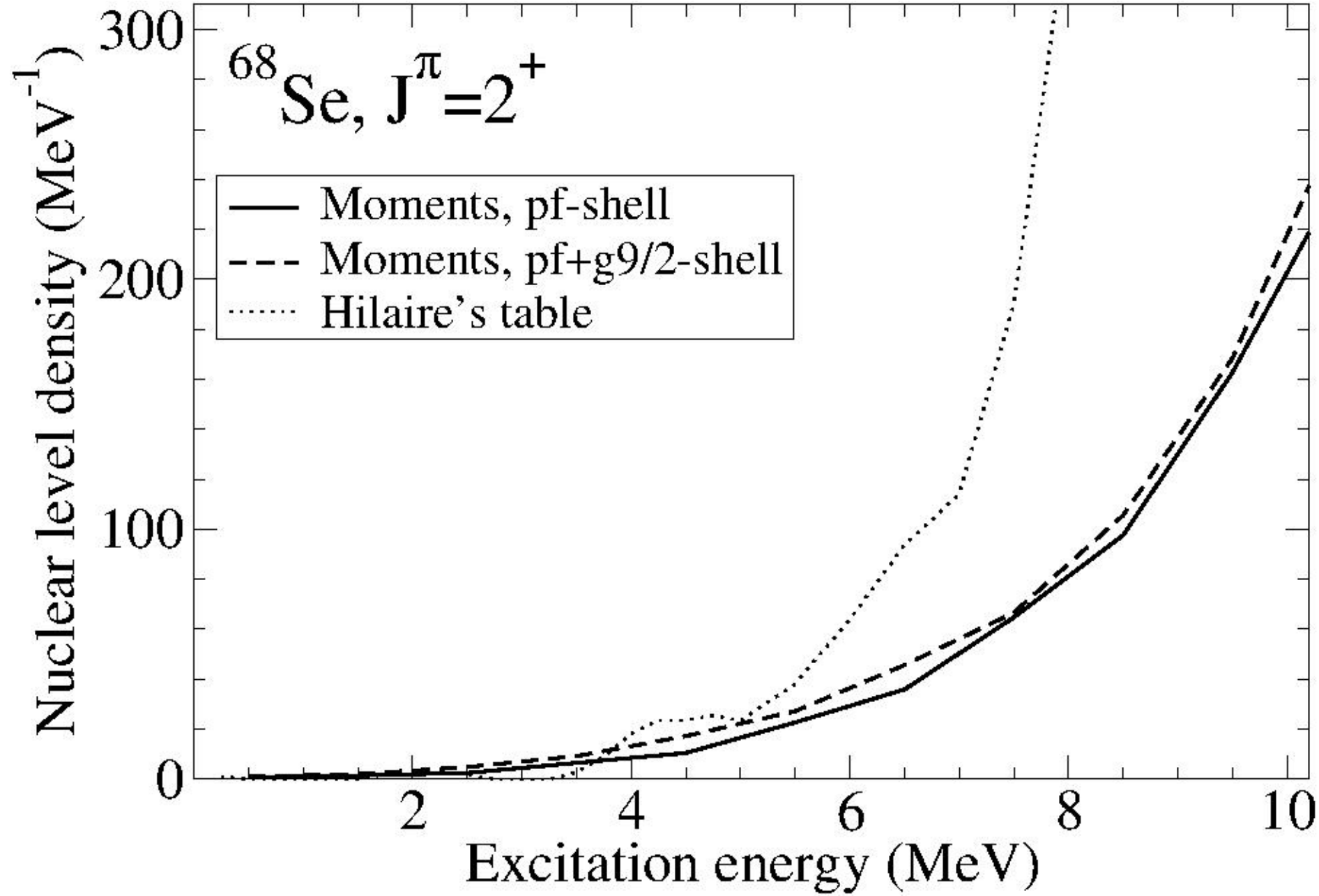
Problems: truncated orbital space, only positive parity in sd-model, …

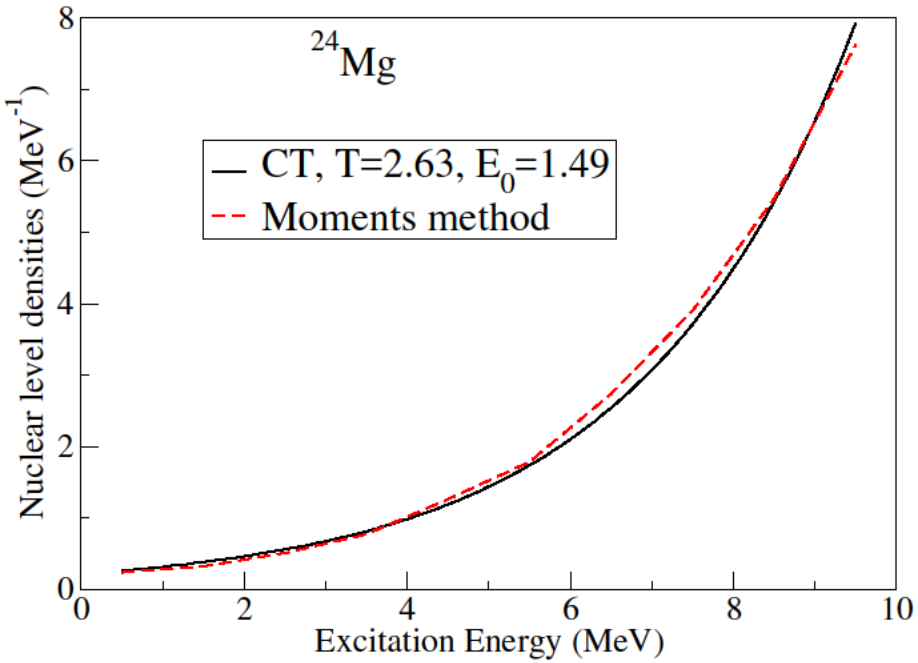


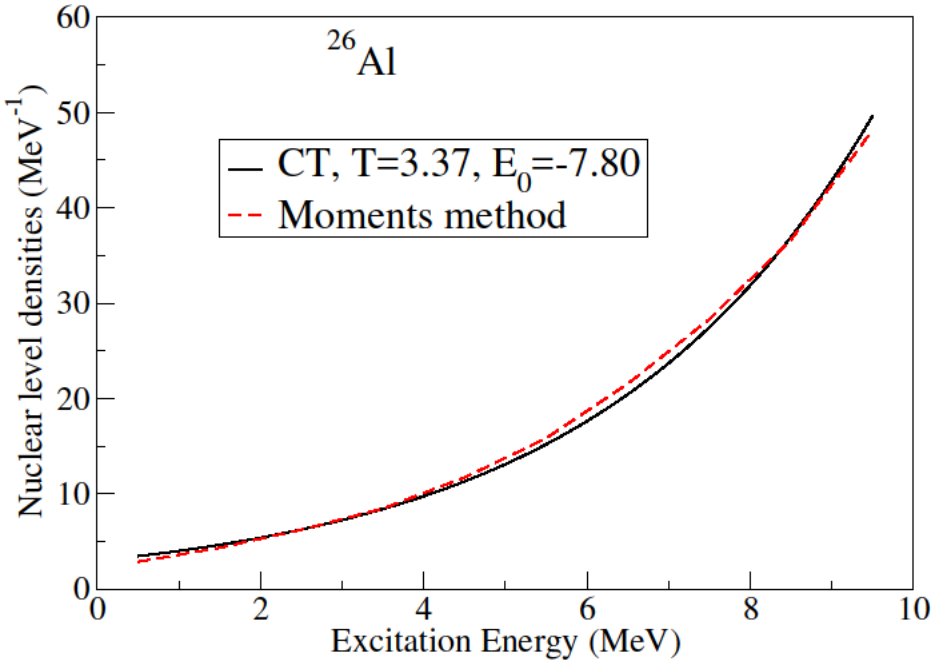
R.Sen'kov, V.Z.
PRC 93 (2016)


^{52}Cr, J$^{\pi}$= 1^{+}
exact SM (pf-shell, gx1a)
moments method
model of Goriely et al.
Nuclear level density (MeV^{-1})
Excitation energy (MeV)
0
20
40
60
2
4
6
8
10


^{52}Fe, $J^{\pi} = 1^{+}$
exact SM (pf-shell, gx1a)
moments method
model of Goriely et al.
Nuclear level density (MeV^{-1})
Excitation energy (MeV)
0
10
20
30
2
4
6
8
10


^{68}Se, $J^{\pi}=2^{+}$
Moments, pf-shell
Moments, pf+g9/2-shell
Hilaire's table
Nuclear level density (MeV^{-1})
Excitation energy (MeV)
0
100
200
300
2
4
6
8
10



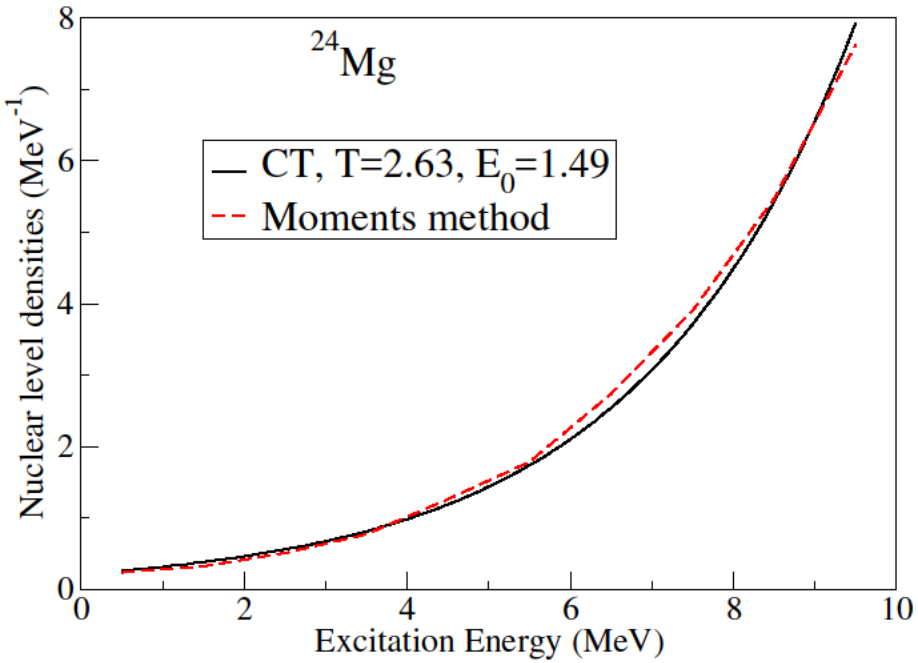


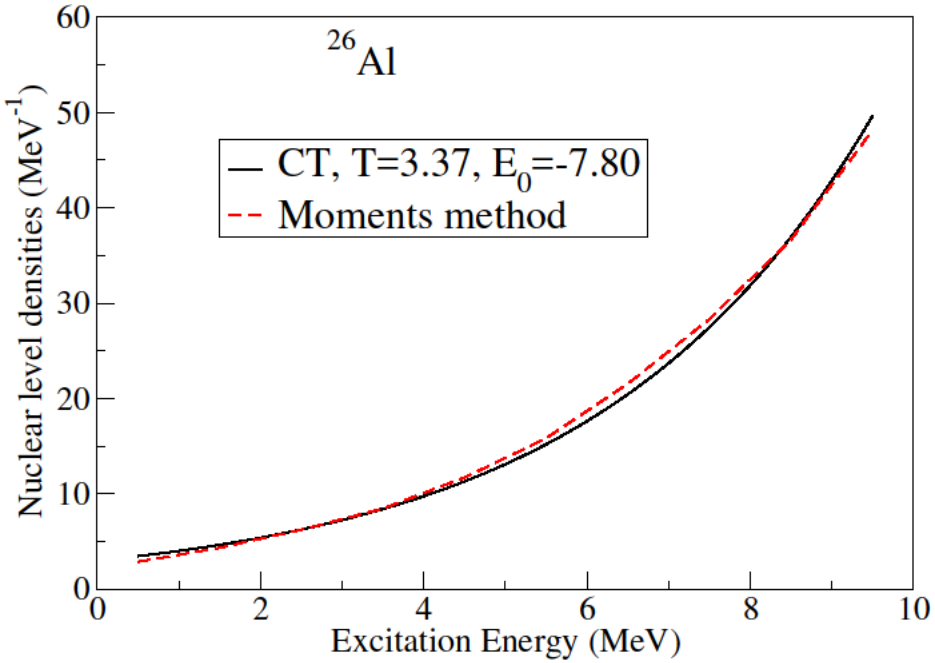
CONSTANT TEMPERATURE PHENOMENOLOGY

Level density (const) exp(E/T)

$$T_{\mathrm{t-d}} = \left(\frac{\partial S}{\partial E}\right)^{-1} = T\left(1 - e^{-E/T}\right)$$

Partition function = **Trace{exp[-H/T(t-d)]}** **diverges** at **T > T(t-d), this is limiting temperature – speed of chaotization**





CONSTANT TEMPERATURE PHENOMENOLOGY

LEVEL DENSITY (E) = (const) exp (E/T)

Ericson (1962), Gilbert and Cameron (1965)

Moretto (1975) – pairing phase transition

T – "effective constant temperature"

1/T – rate of increase of level density

Cumulative level number

N(E) = exp(S),

Entropy **S(E)= ln(N)**

Thermodynamic temperature

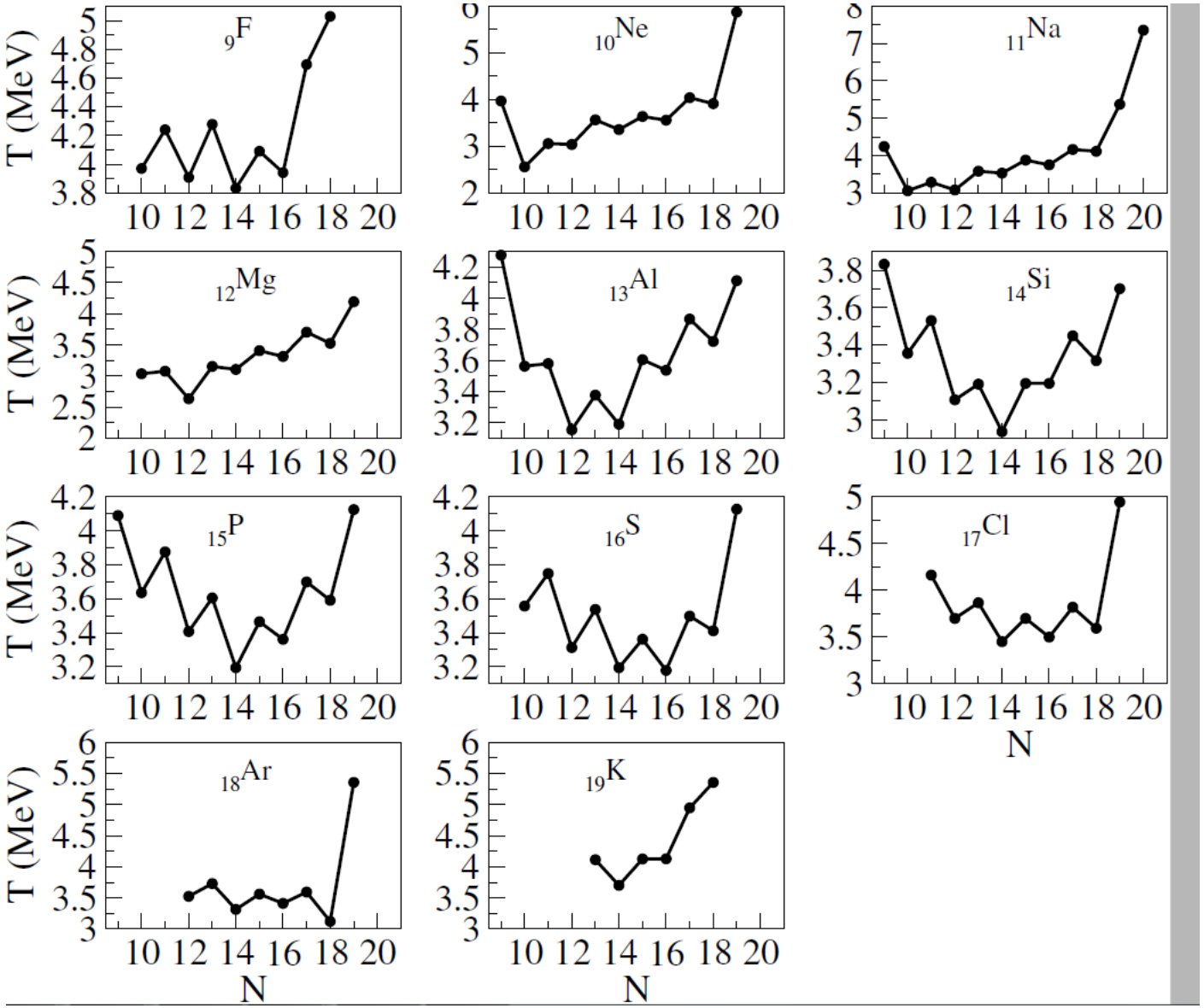
T(t-d) = dS/dE = T[1 – exp(- E/T)]

Parameter T is ***limiting temperature***

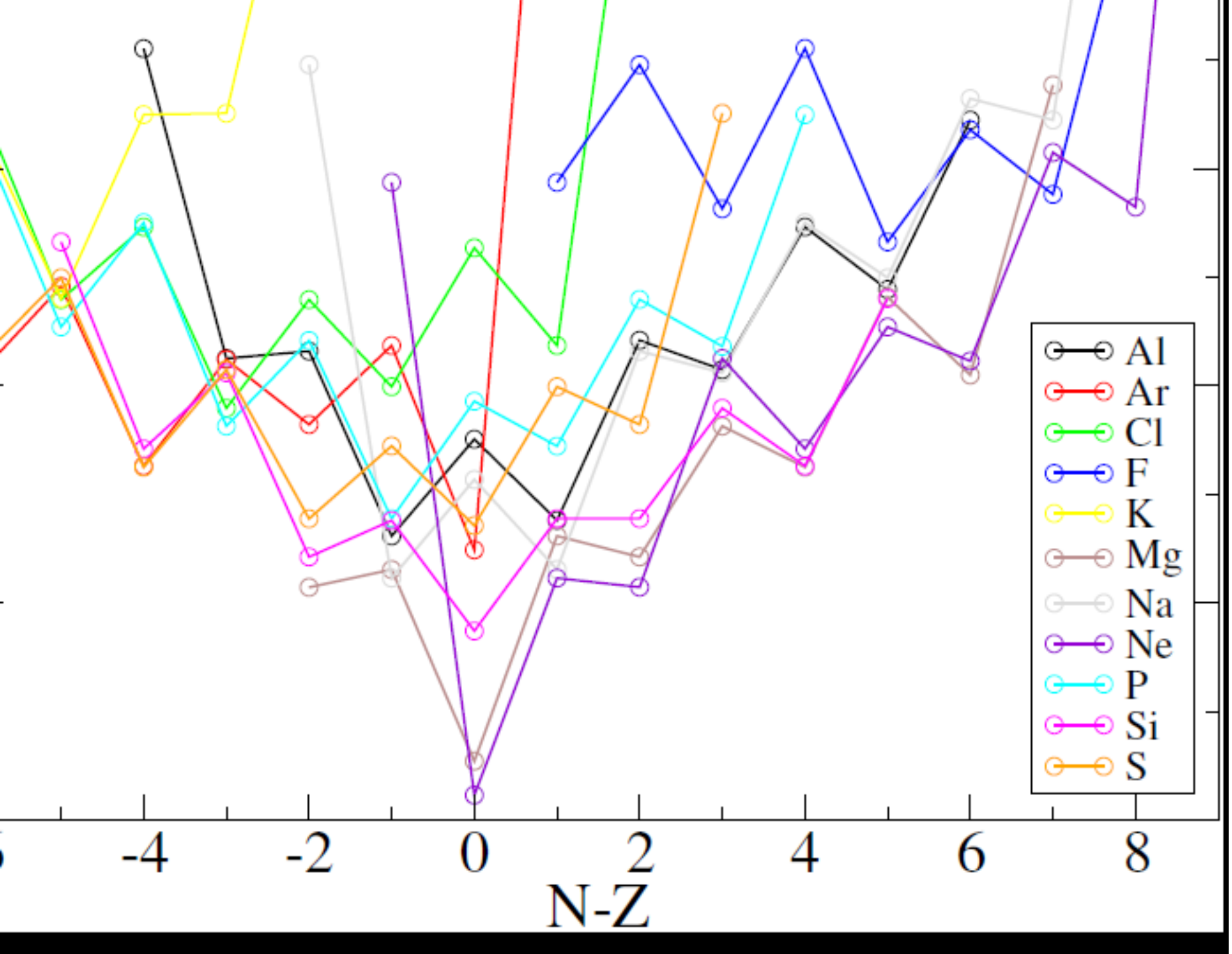
(***Hagedorn temperature*** in particle physics)

Pairing phase transition? (Moretto) - Chaotization

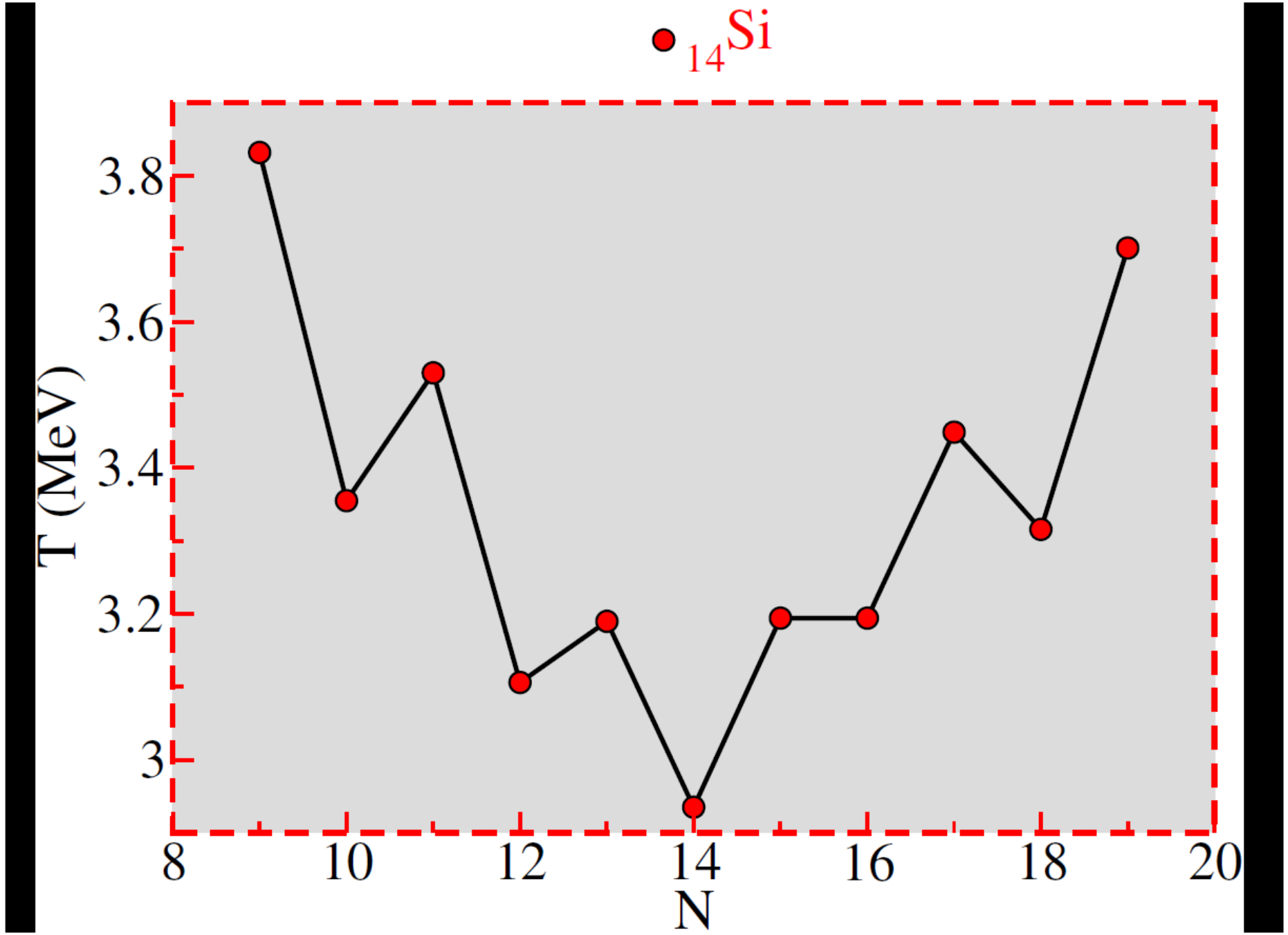
1/T – rate of increase of the level density



Effective temperature **T** for **(sd)** – nuclei, tabulated for all classes of spin (ADNDT, 2018)



Effective temperature for the level density at low energy (up to 6 – 8 Mev)
Even-odd staggering
Clear minima in the vicinity of N=Z



FASTEST CHAOTIZATION FOR N=Z (ALL INTERACTIONS AT WORK)

$$\rho^{(0)}(E,J,0) = \rho(E,J,0)$$ $N\hbar\omega$ classification

Pure **Total** **(N=0)**

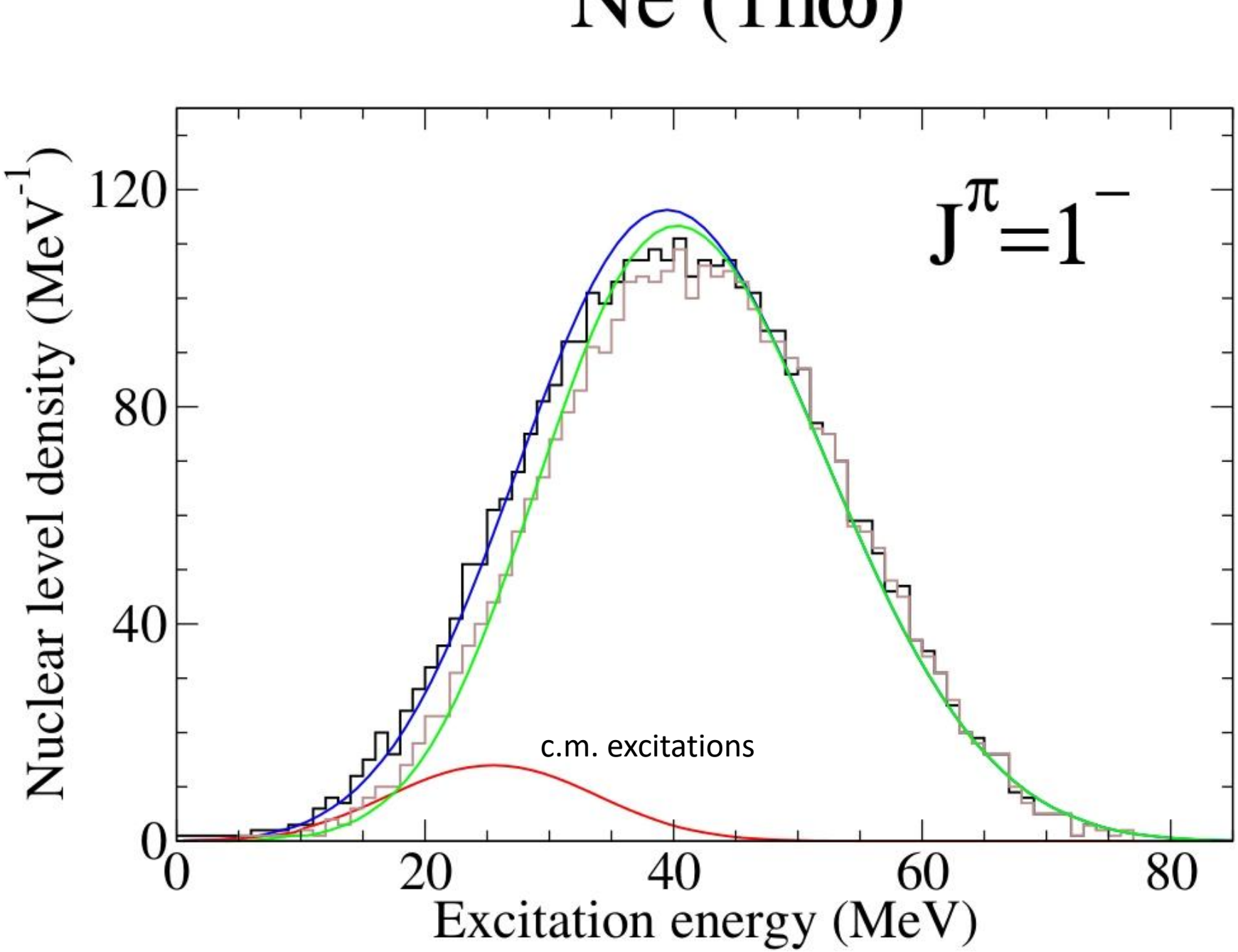
$$\rho^{(0)}(E,J,1) = \rho(E,J,1) - \sum_{J'=|J-1|}^{J+1} \rho(E,J',0)$$ **(N=1)**

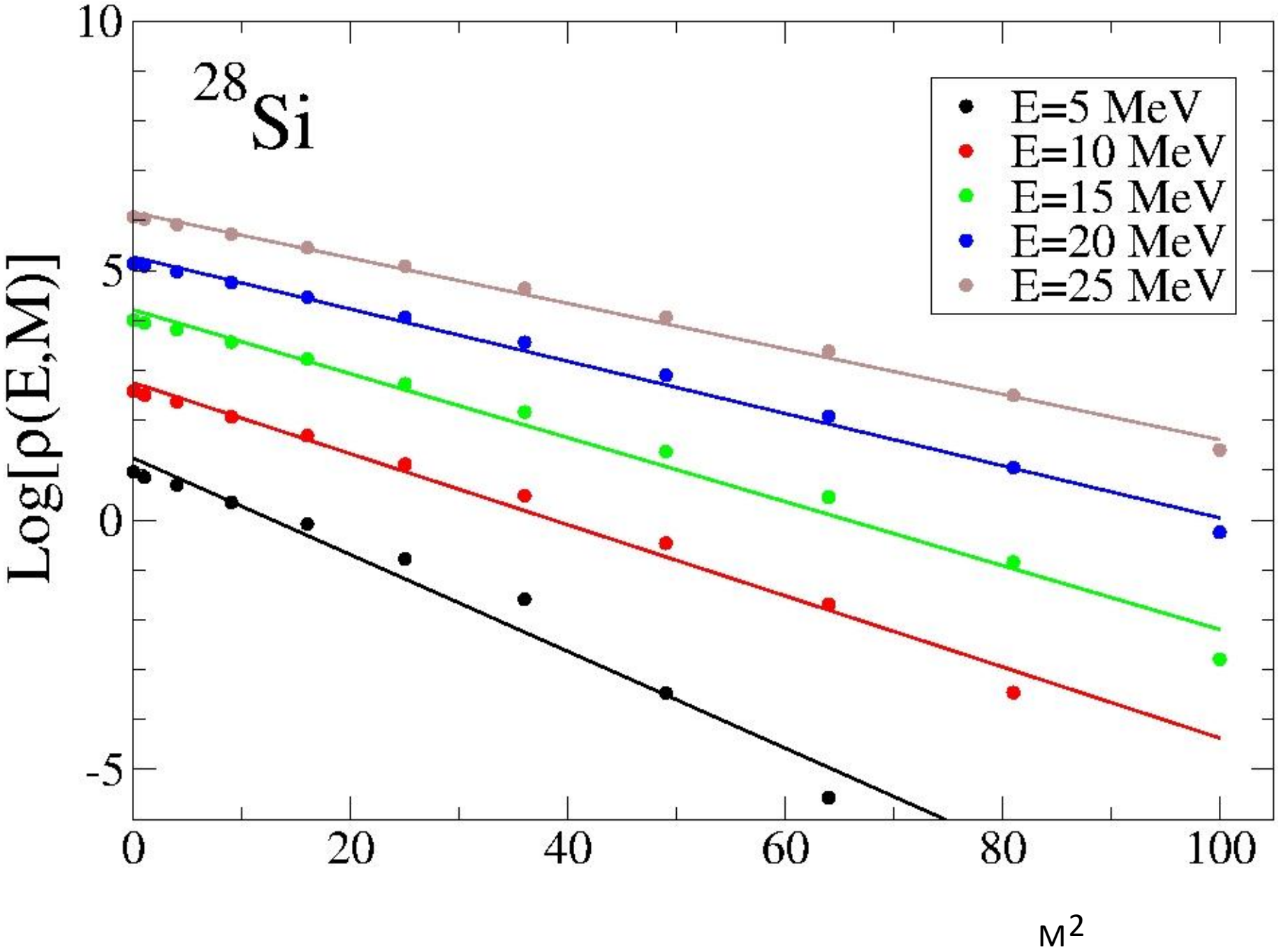
$$\rho^{(0)}(E,J,N) = \rho(E,J,N) -$$

$$-\sum_{K=1}^{N} \sum_{J_K=J_{\min}}^{N,\text{step }2} \sum_{J'=|J-J_K|}^{J+J_K} \rho^{(0)}(E,J',(N-K))$$

Recursive relation

Exclusion of c.m. states in the shell model

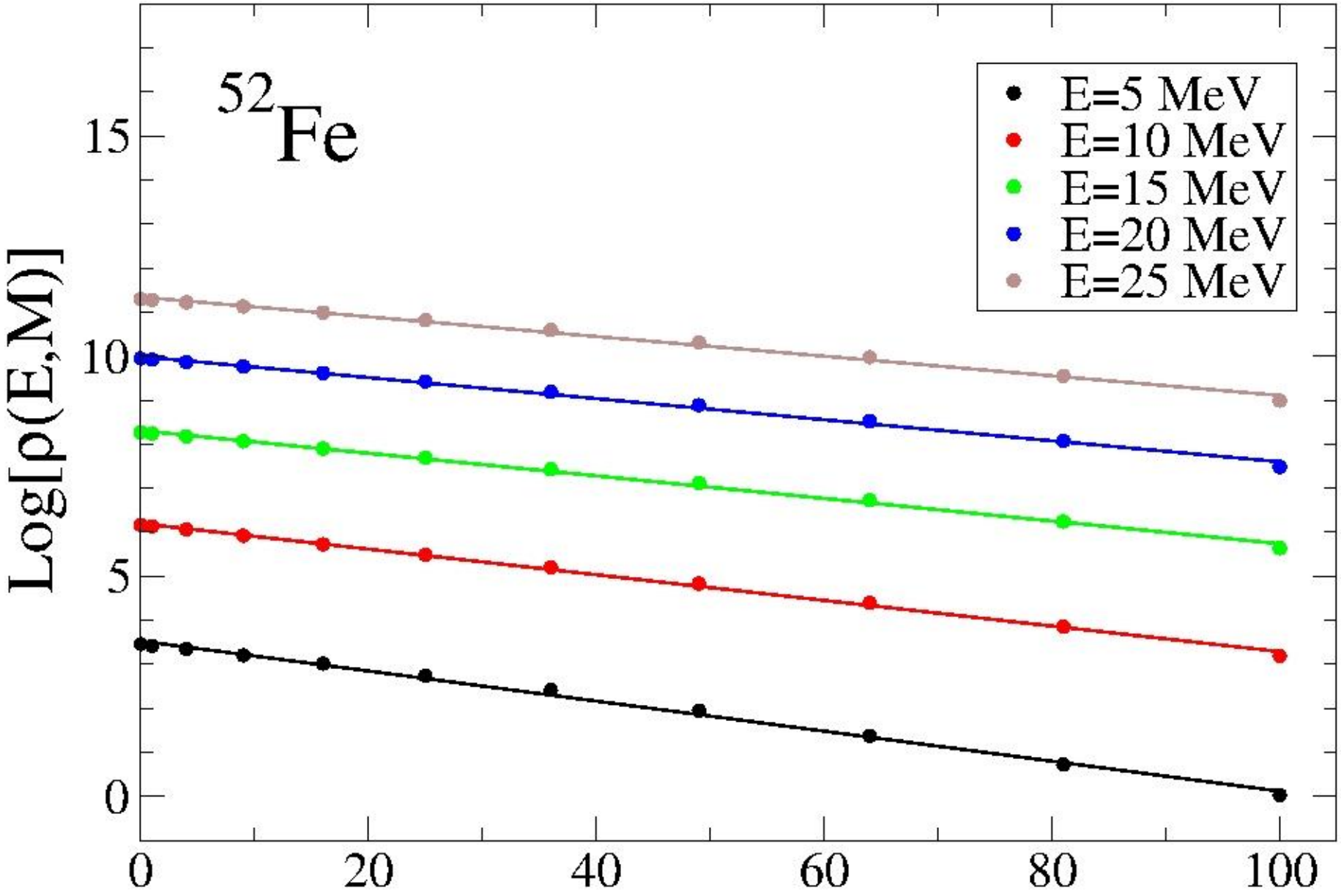

Ne (1ħω)
$J^{\pi}=1^{-}$
c.m. excitations
Nuclear level density (MeV^{-1})
Excitation energy (MeV)
0
40
80
120
0
20
40
60
80



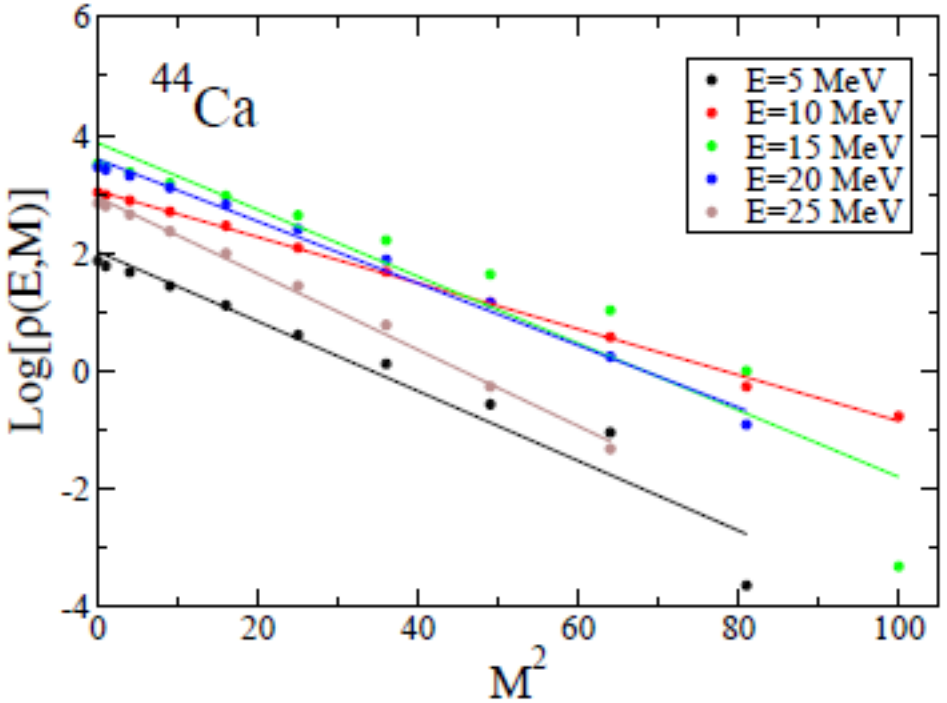
"Spin cut-off" parameter

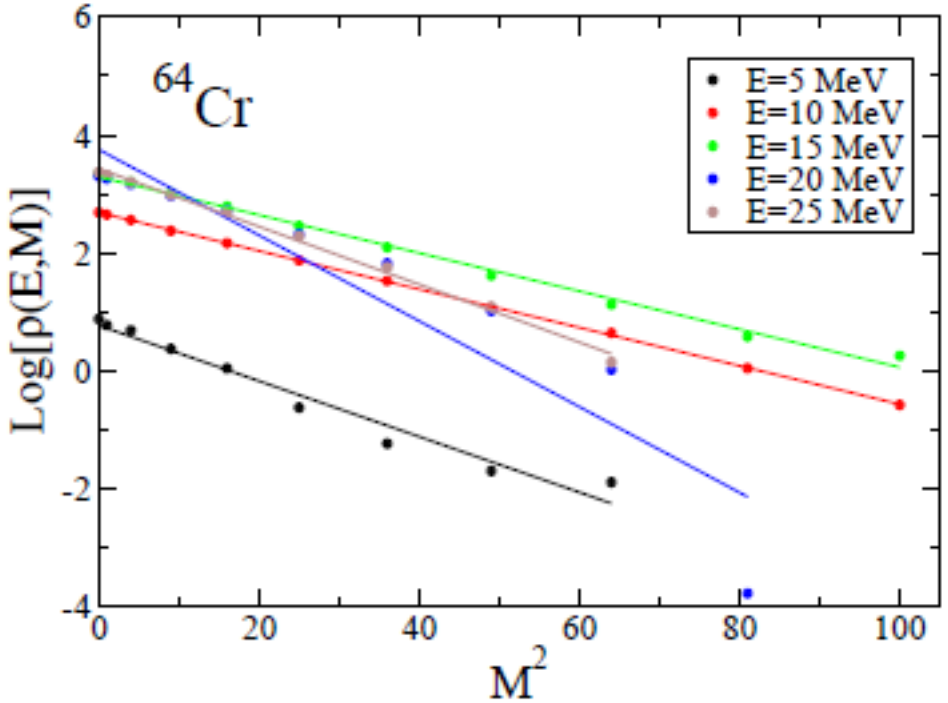
$$\frac{\rho(E,M)}{\rho(E)} = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-M^2/2\sigma^2}$$

Markovian
random process
of angular momentum
coupling



$$\frac{\rho(E,M)}{\rho(E)} = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-M^2/2\sigma^2}$$

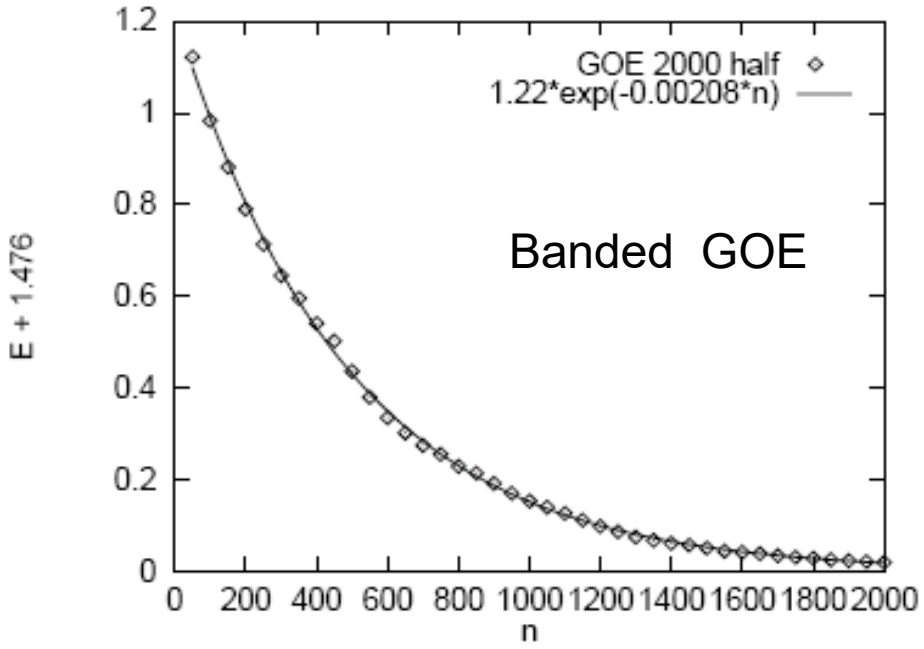


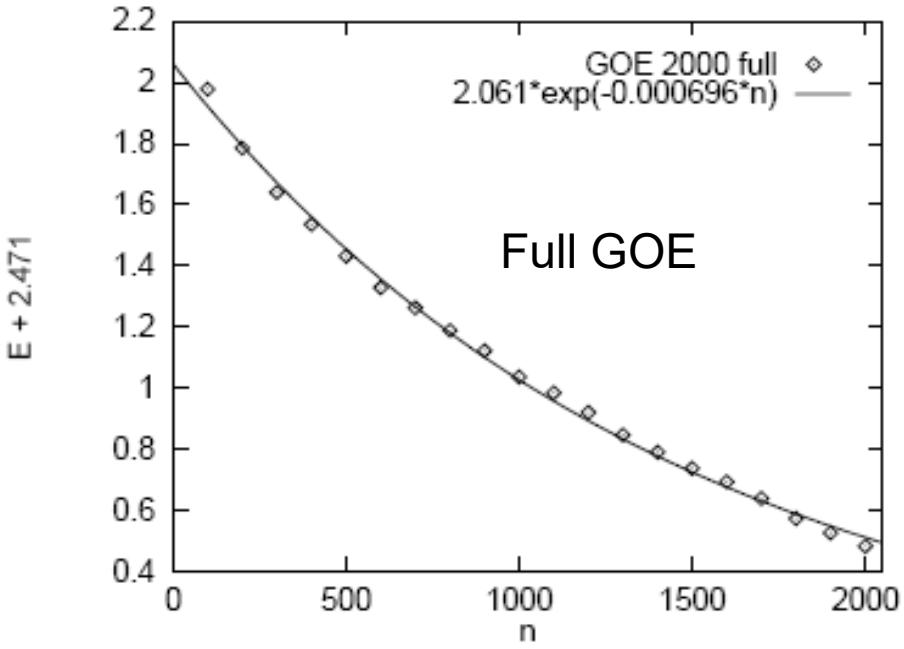


4 valence neutrons

4 proton holes

Space – only T=2,
Two-body interaction through T=1 channel





GROUND STATE ENERGY OF RANDOM MATRICES

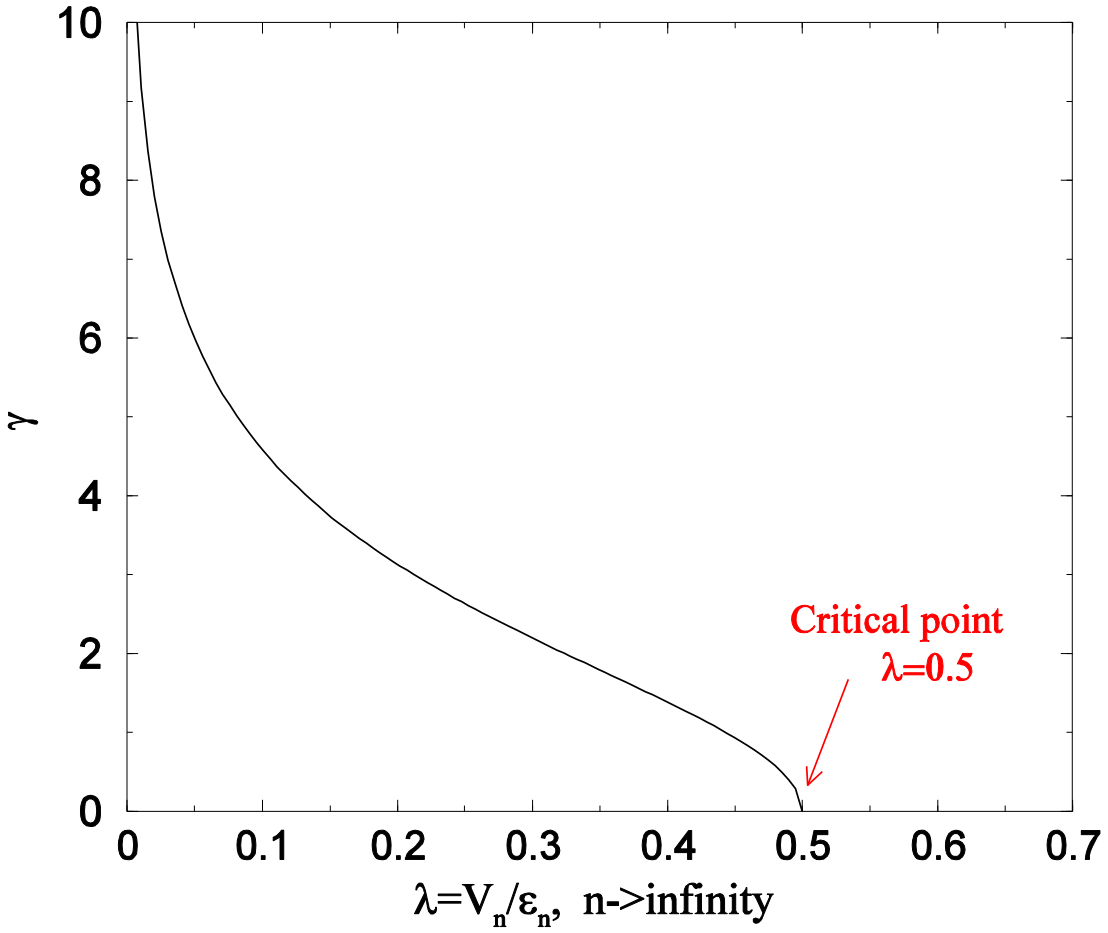
EXPONENTIAL CONVERGENCE

SPECIFIC PROPERTY of RANDOM MATRICES ?

/The proof based on the Lanczos algorithm/

CONVERGENCE REGIMES

- $\lambda = 0$ — *Fast convergence*
- $0 < \lambda < 1/2$ — *Exponential convergence*
- $\lambda = 1/2$ — Power law
- $\lambda > 1/2$ — *Divergence*



Exponential convergence

$\sim \exp(-\gamma n)$

$$\gamma = -\ln\left\{\frac{1}{2\lambda^2}\left(1 - 2\lambda^2 - \sqrt{1 - 4\lambda^2}\right)\right\}$$

New method for shell-model level density /B.A. Brown, 2018/

Analytical results for tridiagonal matrices

$$H = \begin{pmatrix} \epsilon_1 & V_2 & 0 & 0 & 0 & 0 \\ V_2 & \epsilon_2 & V_3 & 0 & 0 & 0 \\ 0 & V_3 & \epsilon_3 & V_4 & 0 & 0 \\ 0 & 0 & \dots & \dots & \dots & 0 \\ 0 & 0 & 0 & \dots & \dots & V_n \\ 0 & 0 & 0 & 0 & V_n & \epsilon_n \end{pmatrix}$$

Recurrence relation for determinants

$$D_n(E) = (\epsilon_n - E)D_{n-1}(E) - V_n^2 D_{n-2}(E)$$

Convergence is determined by

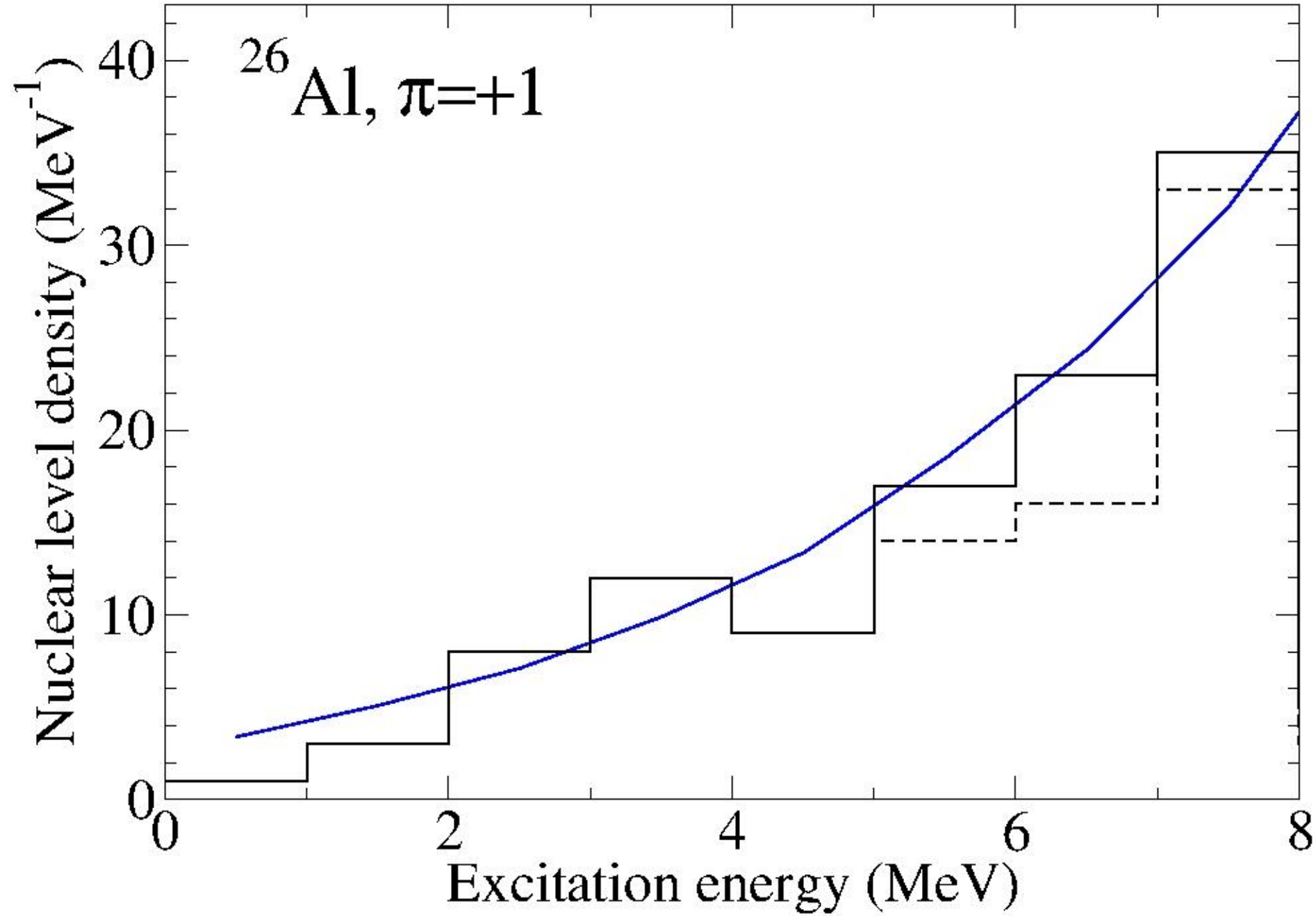
$$\lambda_n^2 = V_n^2/(\epsilon_n \epsilon_{n-1})$$

Assume existence of the limit

$$\lambda_n \Rightarrow \lambda$$

at

$$n \Rightarrow \infty$$


^{26}Al, π=+1
Nuclear level density (MeV^{-1})
Excitation energy (MeV)
0
10
20
30
40
0
2
4
6
8

TYPICAL COMPUTATIONAL PROBLEM

DIAGONALIZATION OF HUGE MATRICES

(*dimensions dramatically grow with the particle number*)

Practically we need not more than few dozens –
is the rest just useless garbage?

Process of progressive truncation –

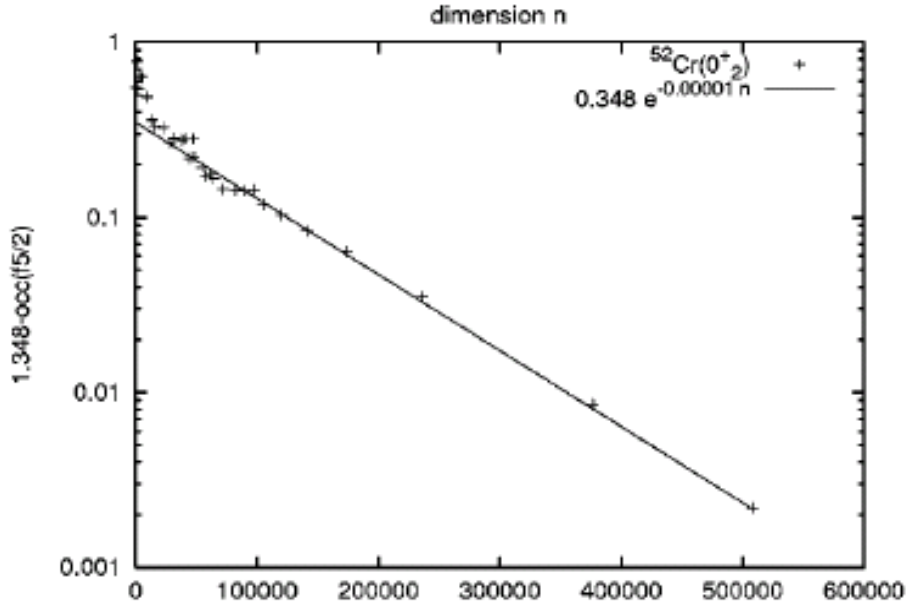
* how to order?

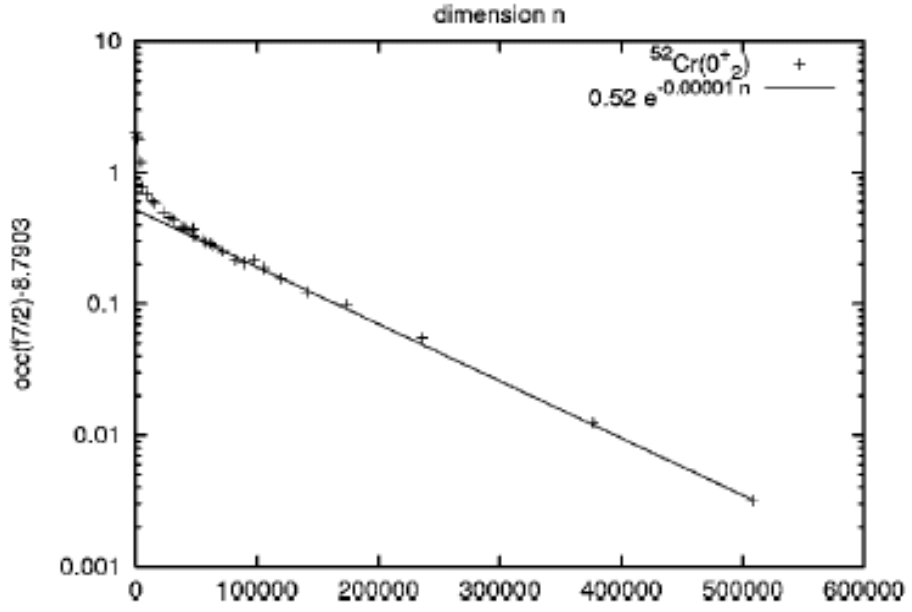
* is it convergent?

* how rapidly?

* in what basis?

* which observables?



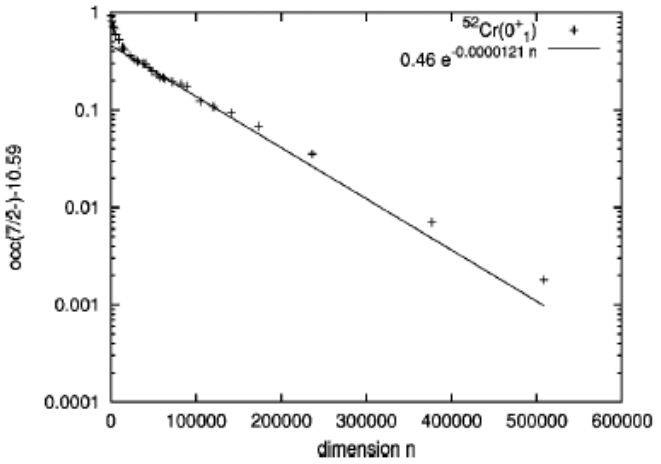


EXPONENTIAL CONVERGENCE OF SINGLE-PARTICLE OCCUPANCIES

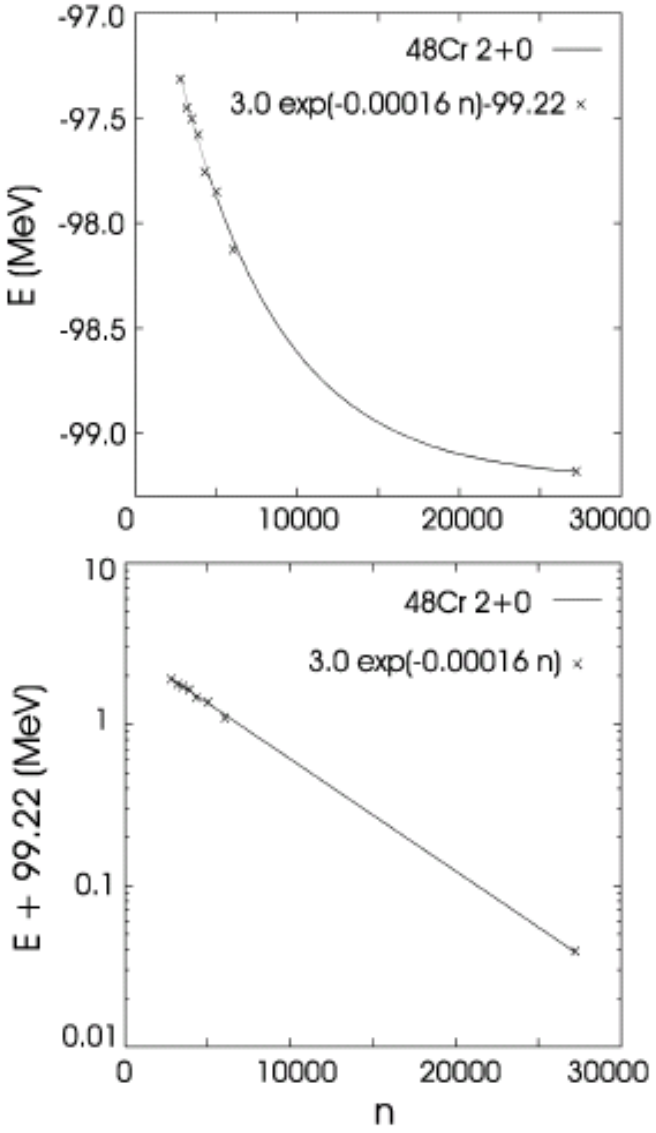
(first excited state J=0)

52

Cr



Fit with $\gamma' = \gamma$



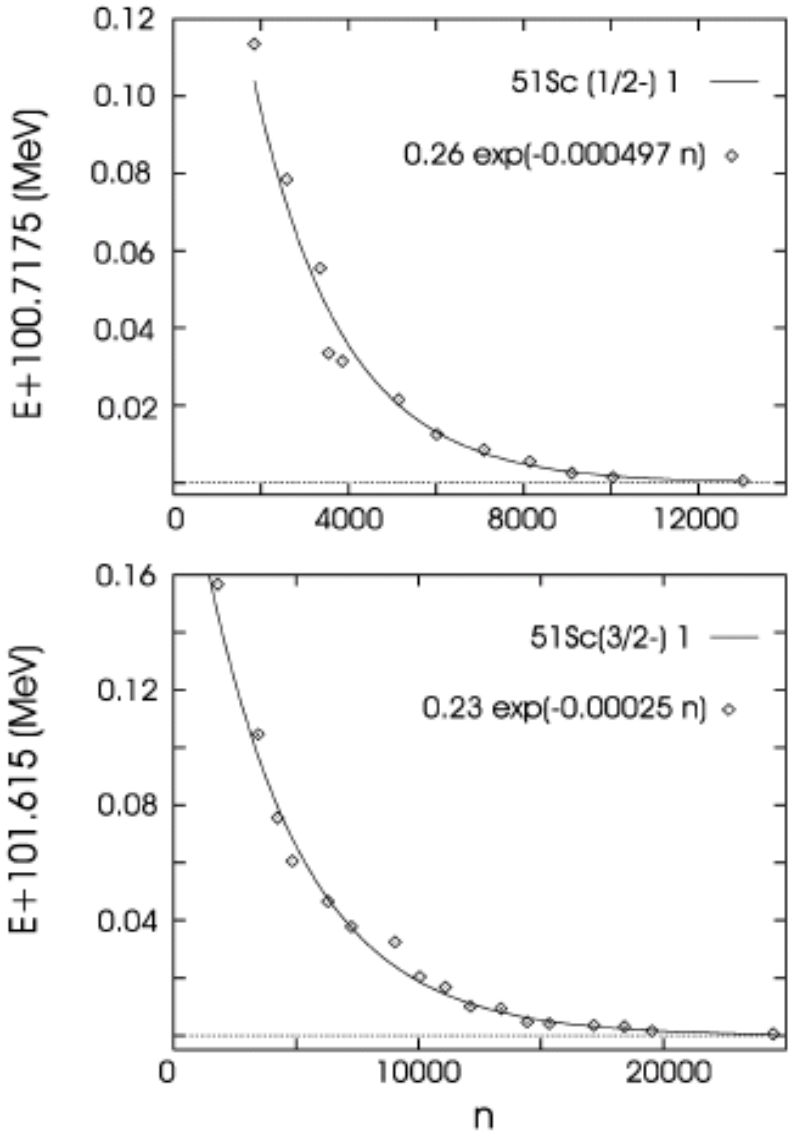
REALISTIC SHELL MODEL 48 Cr

Excited state
J=2, T=0

EXPONENTIAL CONVERGENCE !

$E(n) = E + \exp(-an)$

$a \sim 4/N$



REALISTIC
SHELL
MODEL

EXCITED STATES
51Sc

1/2-, 3/2-

Faster convergence:
$E(n) = E + \exp(-an)$
$a \sim 6/N$

What next?

* Tables for **pf-shell** – and further?
* Comparison of phenomenological descriptions with “Constant temperature” model
* New methods - Lanczos algorithm
  - hybrid methods
  - **random interactions – quite promising**
* **Mesoscopic applications (disordered solids)**
* **Can we analytically derive CTM?**
* Computational progress
* **Continuum effects, width distribution, overlapping resonances**
* Application to reactions
*
*

Dynamics in a complex quantum system

$$\hat{H}|n\rangle = E_n|n\rangle \qquad |\Psi_a\rangle = \sum_n a_n|n\rangle$$

$$Q_a(t) = \langle\Psi_a(t)|\hat{Q}|\Psi_a(t)\rangle$$

$$\boxed{f_{a'a}(t) = \langle a'|e^{-i\hat{H}t}|a\rangle} \qquad f_{aa'}(t) = f_{a'a}(-t) \qquad f_{aa'}(0) = \delta_{aa'}$$

Chaotic limit

$$\overline{\langle a|n\rangle} = 0 \qquad \overline{\langle a|n\rangle\langle n'|a'\rangle} = \frac{1}{\mathcal{N}}\delta_{aa'}\delta_{nn'}$$

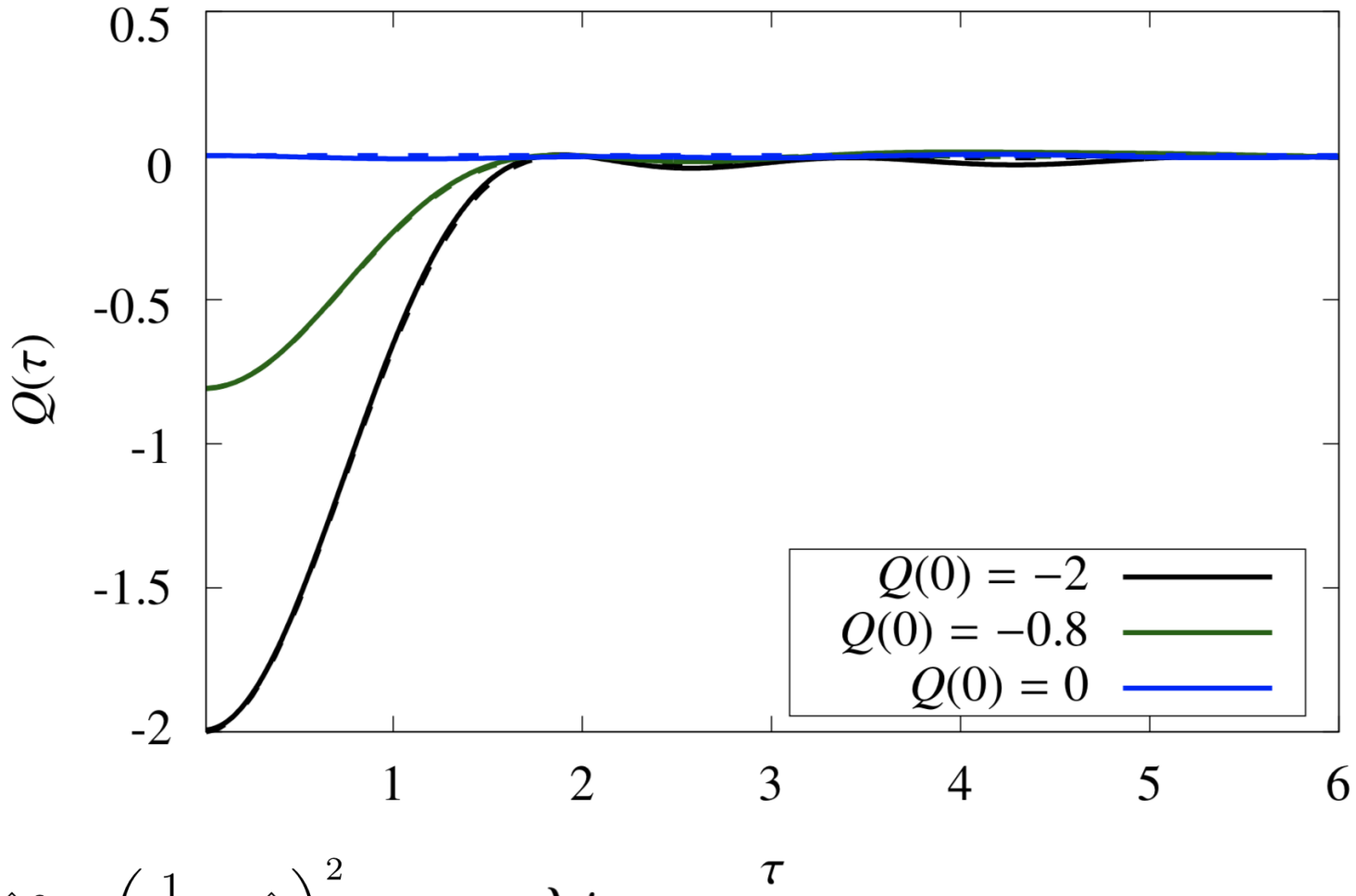
$$\overline{f_{aa'}(t)} = \sum_n e^{-iE_nt}\overline{\langle a|n\rangle\langle n|a'\rangle} = \delta_{aa'}f(t)$$

$$Q_a(t) = \sum_{a'} Q_{a'}|f_{a'a}(t)|^2 = Q_a|f(t)|^2 + \frac{1-|f(t)|^2}{\mathcal{N}-1}\sum_{a'\neq a} Q_{a'}$$

$$\boxed{Q(t) = Q(0)|f(t)|^2}$$

A. Volya and V. Zelevinsky, J. Phys. Complex., vol. 1, no. 2, p. 025007, 2020.

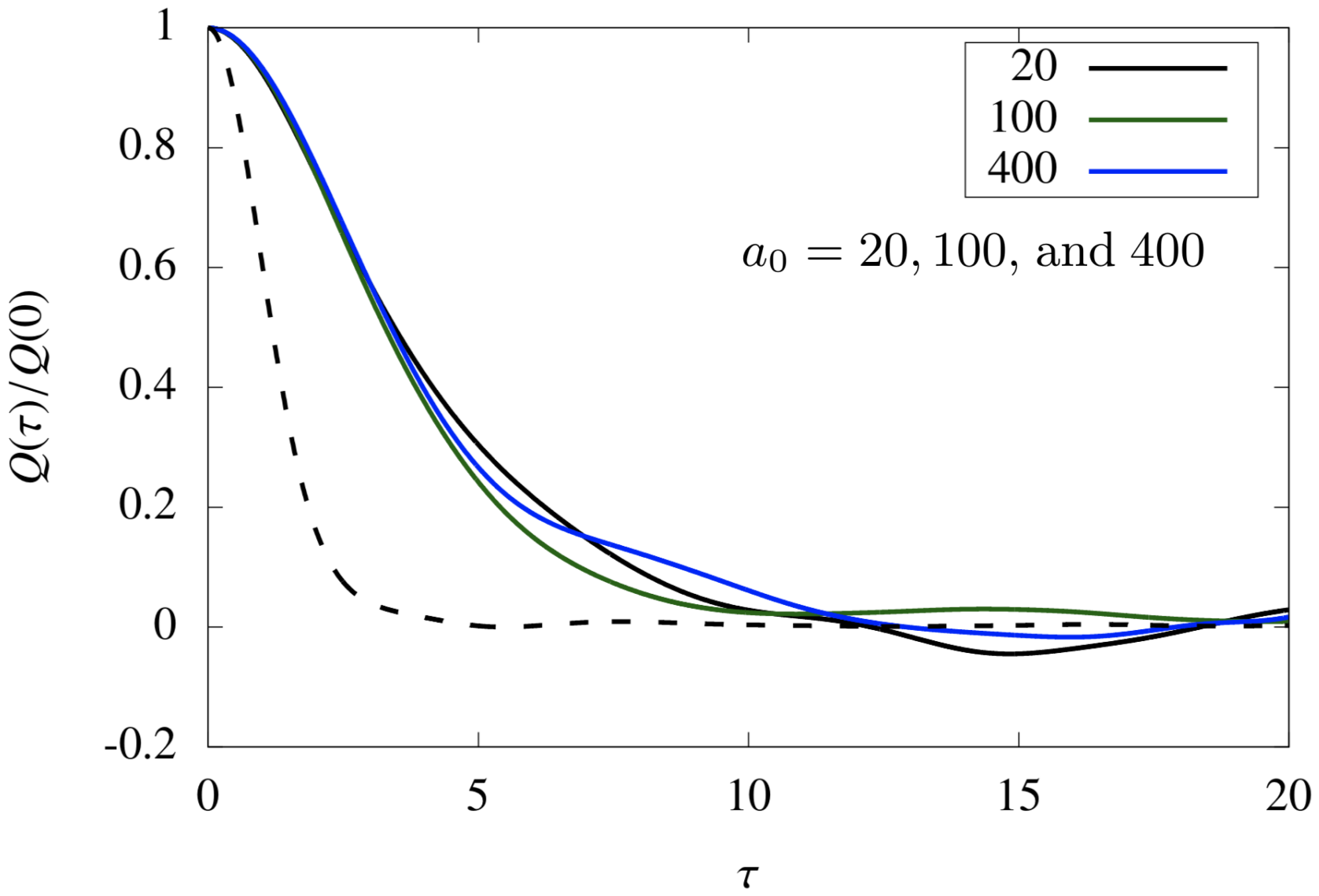
GOE limit



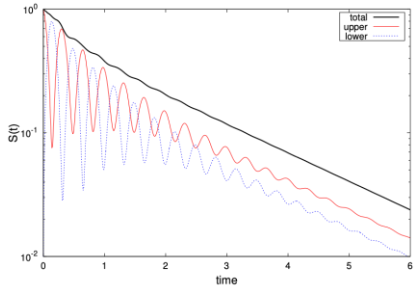
$$\lambda^2 = \frac{1}{\mathcal{N}}\mathrm{Tr}\hat{H}^2 - \left(\frac{1}{\mathcal{N}}\mathrm{Tr}\hat{H}\right)^2 \quad \tau = \lambda t$$

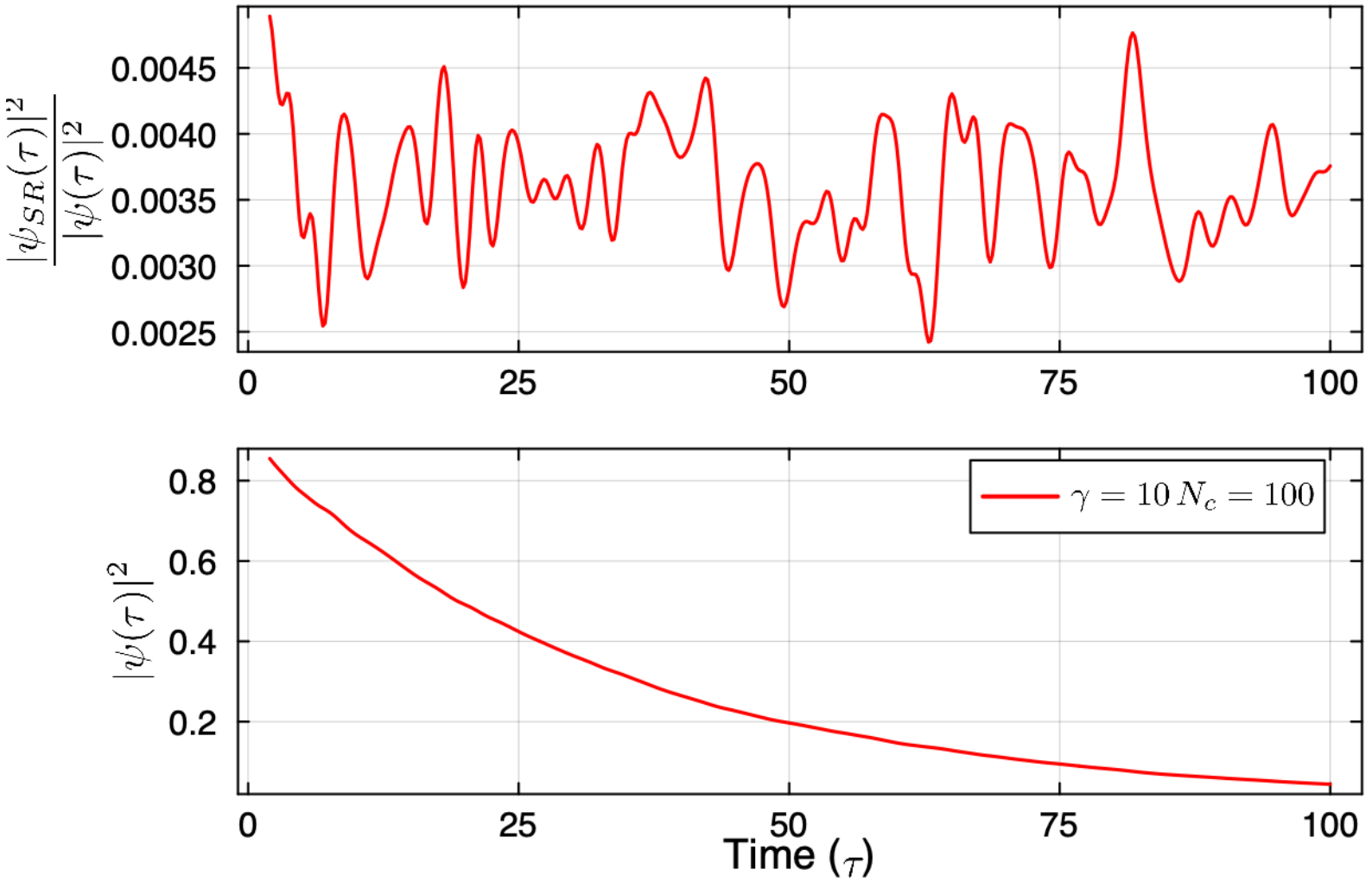
$$f(\tau) = \frac{1}{2\pi}\int_{-2}^{2} d\epsilon\, \sqrt{4-\epsilon^2}\, e^{-i\epsilon\tau} = \frac{1}{\tau}J_1(2\tau)$$

Evolution of quadrupole moments in ^{24}Mg



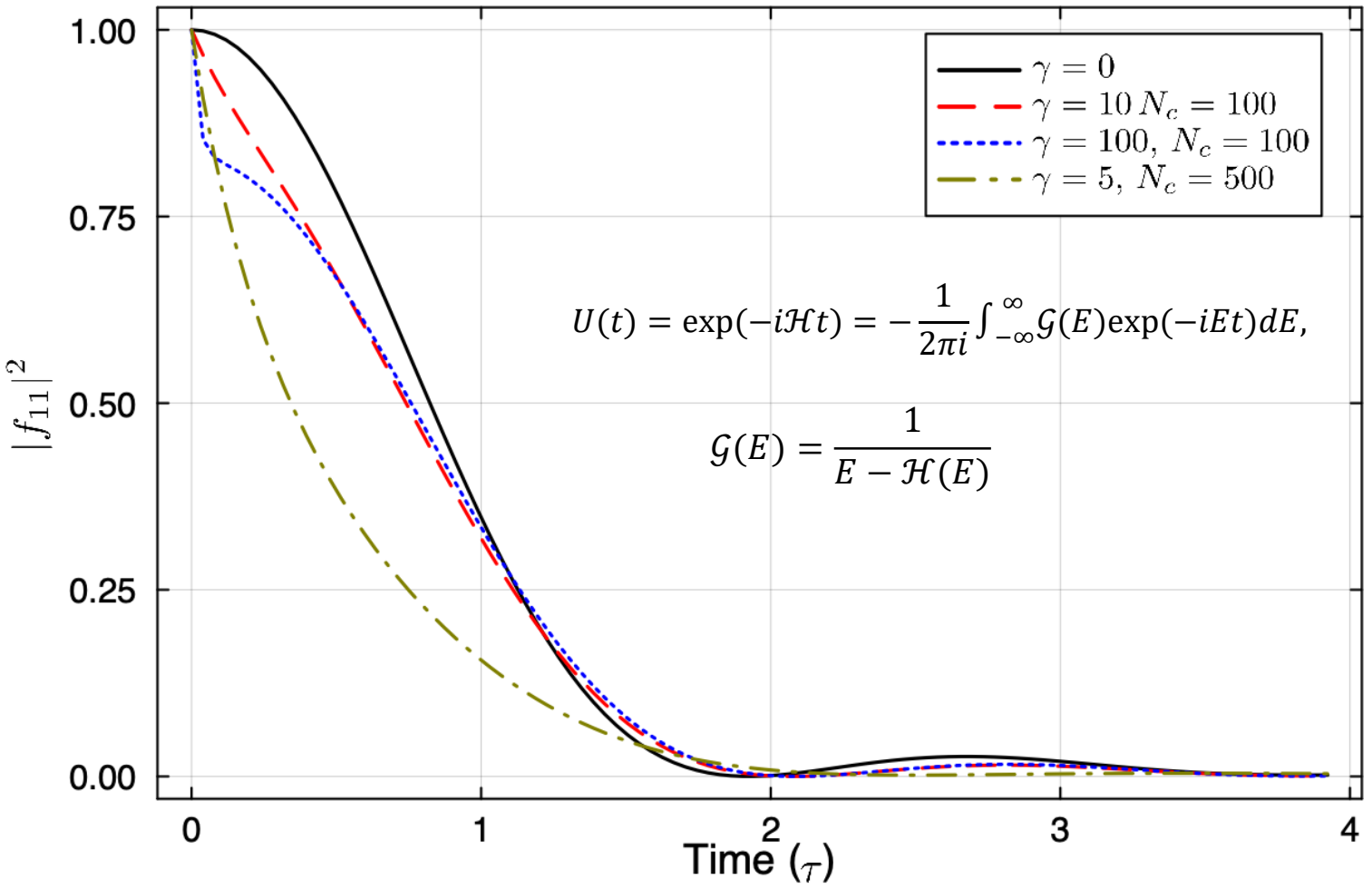
Long time dynamics

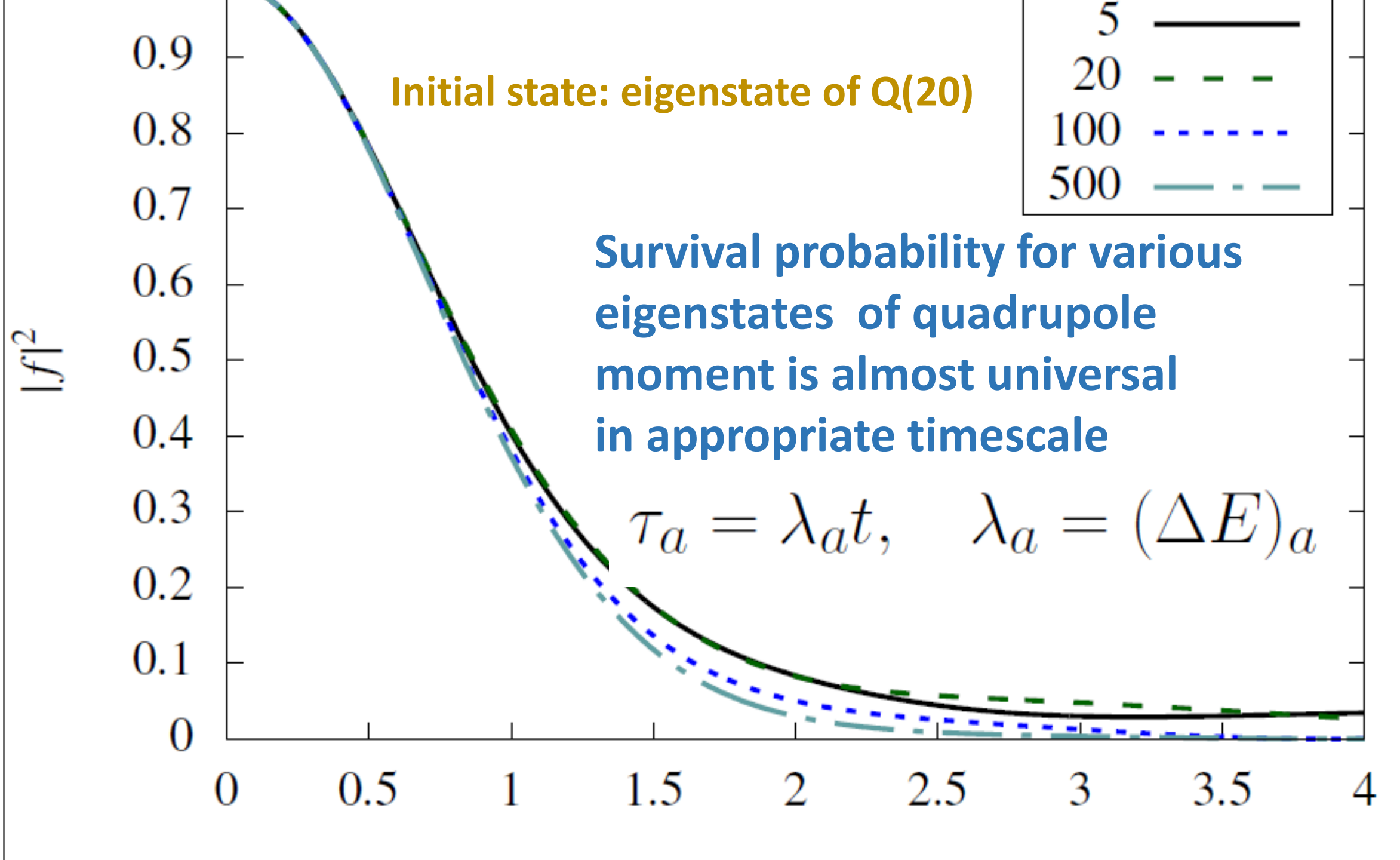




Statistical equilibrated fluctiations

Survival probability in decaying system




Initial state: eigenstate of Q(20)
5
20
100
500
Survival probability for various eigenstates of quadrupole moment is almost universal in appropriate timescale
$\tau_a = \lambda_a t, \quad \lambda_a = (\Delta E)_a$
$|f|^2$
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
0
0.5
1
1.5
2
2.5
3
3.5
4

Interplay of collectivities

Definitions

n - labels particle-hole state

ε_ν □ εξχιτατιον ενεργψ οφ στατε ν

d_n - dipole operator

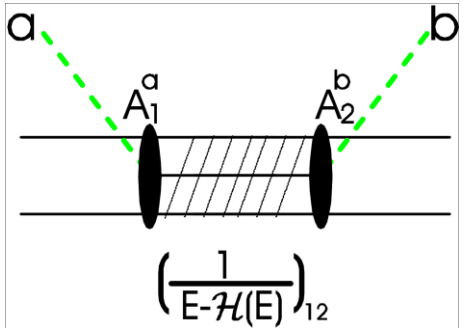
A_n – decay amplitude of n

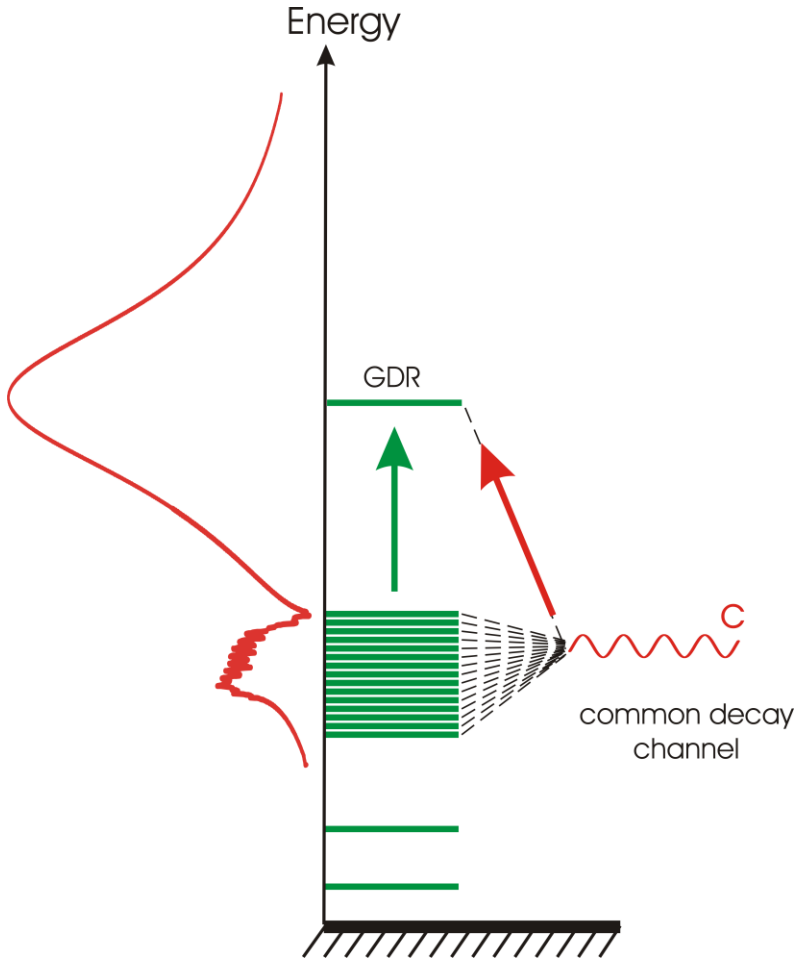
Model Hamiltonian

$$\mathcal{H}_{nn'} = \epsilon_n \delta_{nn'} + \lambda d_n d_{n'} - \frac{i}{2} A_n A_{n'},$$

Driving GDR externally

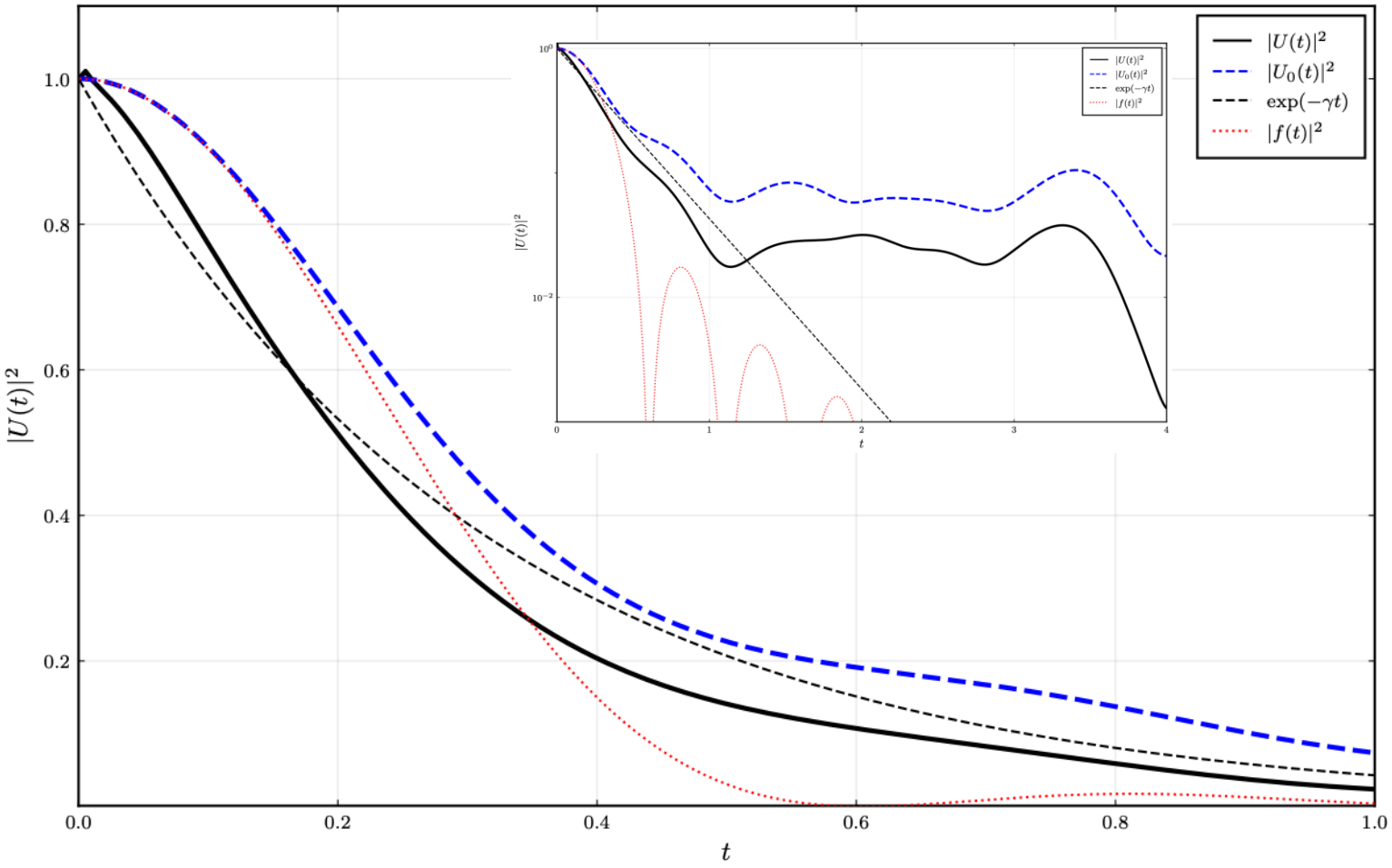
(doing scattering)





Everything depends on
angle between multi dimensional vectors
A and d

Survival probability of GR state



Survival probability of GR state as a function of time in a decaying system with particle decay threshold -1, comparison with non-decaying GR, exponential decay (closed system), and chaotic relaxation.

$$\epsilon_{\mathrm{th}} = -1$$

EXAMPLES of CURRENT PROBLEMS

- ***Level density***: **comparison with phenomenological model**
  microscopic calculation of parameters
  higher order moments
  application to reactions

- ***Resonance width distribution***: **from Porter-Thomas to … ?**

- ***Statistical reactions***: **Hauser-Feshbach prescription**
  Ericson fluctuations

- ***Phase transitions***: **invariant correlational entropy**