

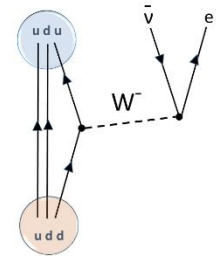
Isospin-symmetry breaking and nuclear beta decay (Theory under experimental constraints)

Nadezda A. Smirnova¹, Latsamy Xayavong²

1 – Laboratory of Physics of Two Infinities Bordeaux
(LP2IB, CNRS/IN2P3 – University of Bordeaux), France

2 - Department of Physics, Yonsei University, Seoul, South Korea

Topic program: Future directions in **Nuclear Beta Decay at FRIB**
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Isospin-symmetry breaking and nuclear beta decay

I.

- Background: Isospin Symmetry and Its Breaking in Nuclear Structure
- Shell-Model Formalism for **isospin-symmetry breaking**: state-of-the-art and application to nuclear structure and nuclear astrophysics

- Isospin-symmetry breaking correction to Superaligned Fermi **beta decay** δ_C within shell model + WS or Hartree-Fock wave functions

L. Xayavong, N. Smirnova, Phys. Rev. C97, 024324 (2018)

L. Xayavong, N. Smirnova, Phys. Rev. C105, 044308 (2022)

II.

- Higher-order terms in δ_C

L. Xayavong, N. Smirnova, Phys. Rev. C109, 014317 (2024)

- Towards *ab-initio* No-core shell-model calculation of isospin-symmetry breaking correction δ_C

L. Xayavong, Y. Lim, N. Smirnova, C. Johnson, submitted.

- Conclusions and perspectives

I. Isospin-symmetry breaking and shell model description

Isospin Symmetry

W. Heisenberg (1932)

$SU_T(2)$

$$\psi_n(\vec{r}) = \psi(\vec{r}) \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \psi_p(\vec{r}) = \psi(\vec{r}) \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$
$$t = 1/2 \qquad t = 1/2$$
$$t_z = +1/2 \qquad t_z = -1/2$$

$$[\hat{t}_j, \hat{t}_k] = i\epsilon_{jkl}\hat{t}_l, \quad [\hat{t}^2, \hat{t}_j] = 0$$

$$\hat{t}_x = \frac{1}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \hat{t}_y = \frac{1}{2} \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}, \quad \hat{t}_z = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Electric Charge operator

$$\hat{q} = \left(\frac{1}{2} - \hat{t}_z \right) e$$



$$\hat{q}\psi_n(\vec{r}) = 0, \quad \hat{q}\psi_p(\vec{r}) = e\psi_p(\vec{r})$$

Operators useful for Beta decay

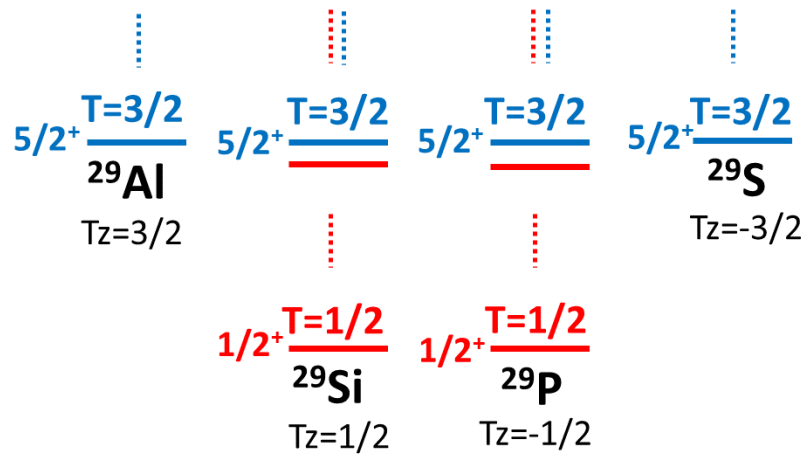
$$\hat{t}_{\pm} = \hat{t}_x \pm i\hat{t}_y.$$



$$\begin{aligned} \hat{t}_+\psi_p(\vec{r}) &= \psi_n(\vec{r}), & \hat{t}_+\psi_n(\vec{r}) &= 0 \\ \hat{t}_-\psi_n(\vec{r}) &= \psi_p(\vec{r}), & \hat{t}_-\psi_p(\vec{r}) &= 0 \end{aligned}$$

Isospin Symmetry in a Many-Body System

E.P. Wigner (1937)



$$\hat{T} = \sum_{k=1}^A \hat{t}(k), \quad \hat{T}_{\pm} = \sum_{k=1}^A \hat{t}_{\pm}(k), \quad \hat{T}_z = \sum_{k=1}^A \hat{t}_z(k)$$

A charge-independent Hamiltonian commutes with isospin operators

$$[\hat{H}_{nucl}, \hat{T}_{\pm}] = [\hat{H}_{nucl}, \hat{T}_z] = 0. \quad \longrightarrow \quad |\eta T T_z\rangle$$

Consequences of isospin symmetry:

- Degenerate isobaric multiplets
- Isospin selection rules for transition operators

$$T_z = \frac{N - Z}{2},$$

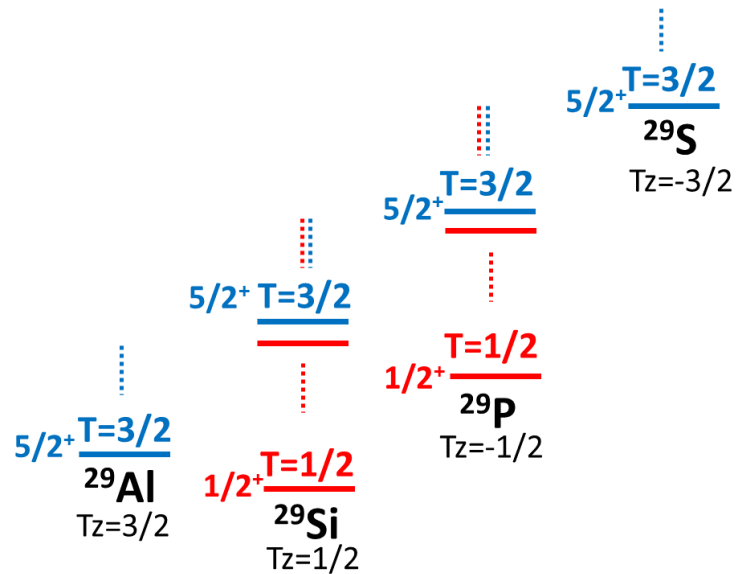
$$T = T_z, T_z + 1, \dots, A/2$$

Dynamical breaking of Isospin Symmetry

E.P. Wigner (1957)

$$\hat{V}_{Coul} = \sum_{i < k}^A \left(\frac{1}{2} - \hat{t}_z(i) \right) \left(\frac{1}{2} - \hat{t}_z(k) \right) \frac{e^2}{|\vec{r}(i) - \vec{r}(k)|}$$

$$= \sum_{i < k}^A \left\{ \underbrace{\left[\frac{1}{4} + \frac{1}{3} \hat{\mathbf{t}}(i) \hat{\mathbf{t}}(k) \right]}_{\text{isoscalar}} - \underbrace{\frac{1}{2} (\hat{t}_z(i) + \hat{t}_z(k))}_{\text{isovector}} + \underbrace{\left[\hat{t}_z(i) \hat{t}_z(k) - \frac{1}{3} \hat{\mathbf{t}}(i) \hat{\mathbf{t}}(k) \right]}_{\text{isotensor}} \right\} \frac{e^2}{|\vec{r}(i) - \vec{r}(k)|}$$



$$\langle \eta T T_z | \hat{V}_{Coul} | \eta T T_z \rangle = E^{(0)}(\eta, T) + E^{(1)}(\eta, T) T_z + E^{(2)}(\eta, T) [3T_z^2 - T(T+1)]$$

$$\mathcal{M}(\eta, T, T_z) = a(\eta, T) + b(\eta, T) T_z + c(\eta, T) T_z^2$$

$$SU_T(2) \supset SO_T(2)$$

Quantum numbers are conserved!

Selection rules (still hold under dynamical breaking): examples

Fermi and Gamow-Teller beta decay operators:

$$\hat{O}_F(\beta^\pm) = \sum_{k=1}^A \hat{t}_\pm(k), \quad \hat{O}_{GT}(\beta^\pm) = \sum_{k=1}^A \hat{\sigma}(k) \hat{t}_\pm(k).$$



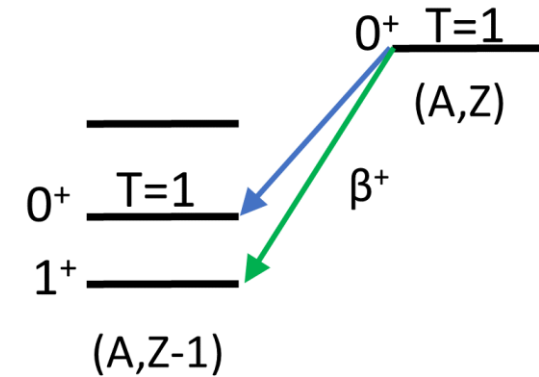
$$\Delta J = 0, \Delta T = 0, \Delta \pi = 0$$

$$|M_F| = \sqrt{T(T+1) - T_{iz}T_{fz}}$$



$$\Delta J = 0, 1, \Delta T = 0, 1, \Delta \pi = 0$$

$$\text{no } J = 0 \rightarrow J = 0$$

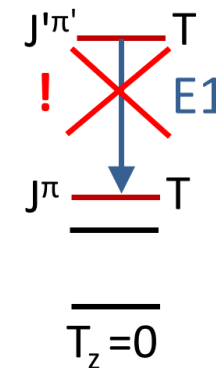


Electric dipole (E1) operator is purely isovector (one-body):

$$\hat{O}(E1) = \sum_{k=1}^A e(k) \vec{r}(k) = \sum_{k=1}^A \left(\frac{1}{2} + \hat{t}_z(k) \right) e \vec{r}(k)$$

$$(T010|T0) = 0$$

Transitions between states with the same T
In N=Z nuclei are isospin-forbidden!



Selection rules for Electromagnetic operators

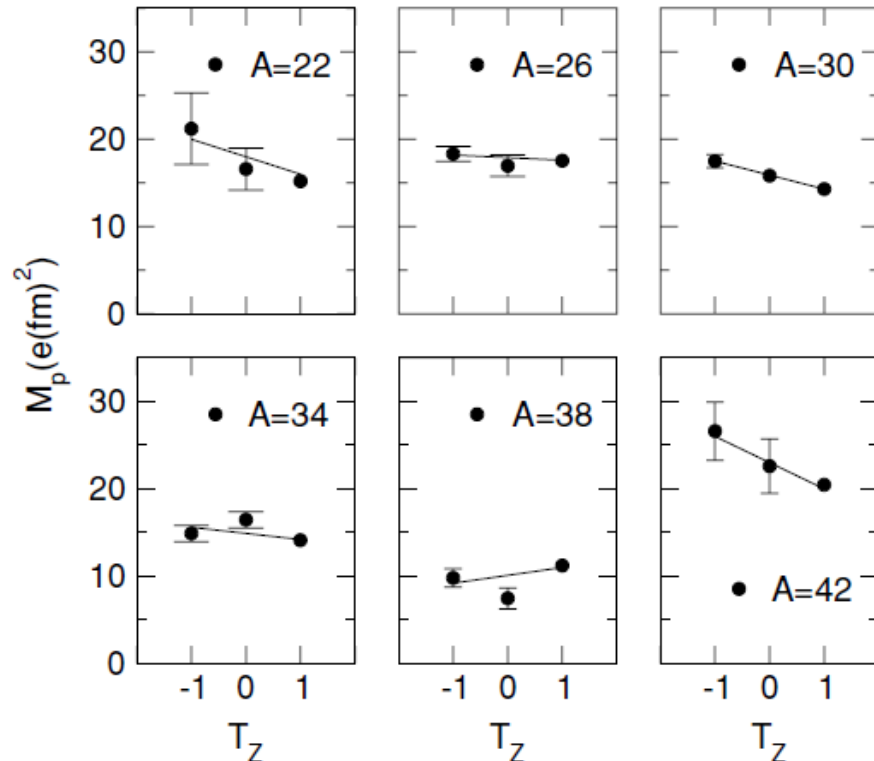
One-body transition operator (except for E1):

Warburton, Wesener (1968)

$$\hat{O}_{LM} = \hat{O}_{LM}^{(0)} + \hat{O}_{LM}^{(1)}$$

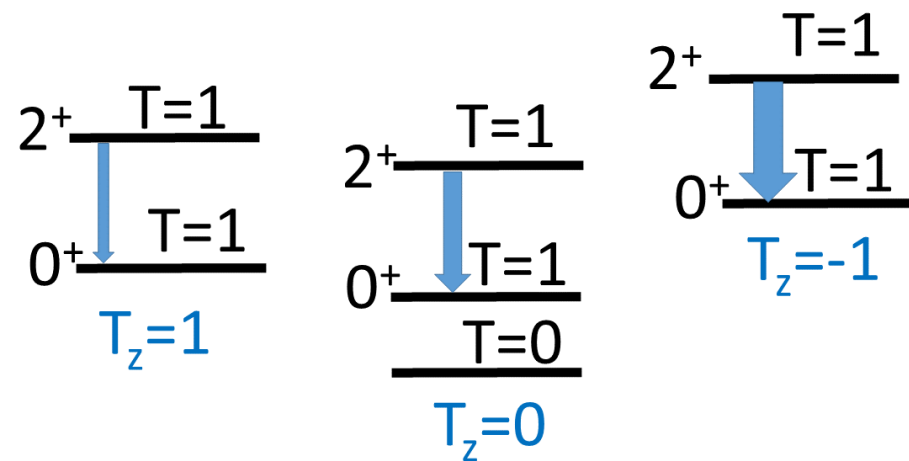
$$\langle J_f M_f; T T_z | \hat{O}_{LM} | J_i M_i; T T_z \rangle = \langle J_f M_f | \hat{O}_{LM}^{(0)} | J_i M_i \rangle + \frac{T_z}{\sqrt{T(T+1)(2T+1)}} \langle J_f M_f; T || \hat{O}_{LM}^{(1)} || J_i M_i; T \rangle.$$

$$\langle J_f M_f T T_z | \hat{O} | J_i M_i T T_z \rangle = A + B T_z$$



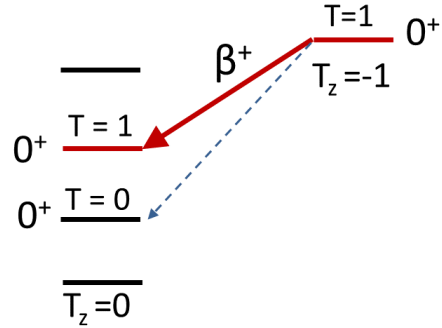
Prados-Estevéz et al, PRC75 (2007)

Example of E2 transitions in an Isobaric triplet:

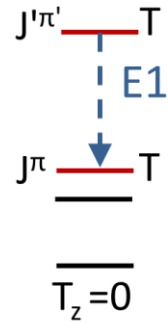


Explicit breaking of isospin symmetry

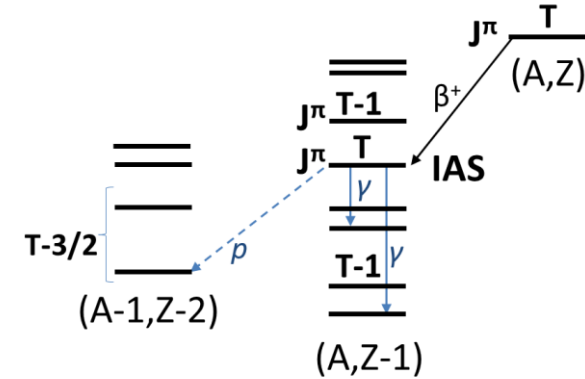
Observation of (typically weak) processes forbidden by isospin symmetry :



Forbidden Fermi beta decay



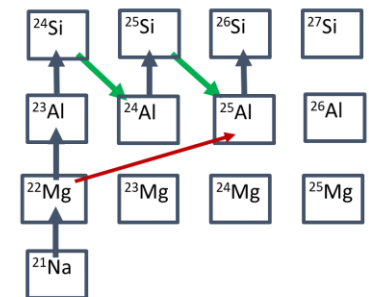
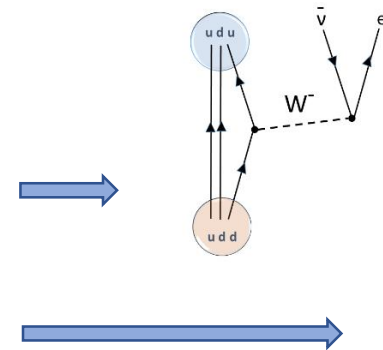
Isospin-forbidden
E1 transitions



Isospin-forbidden nucleon(s) emission

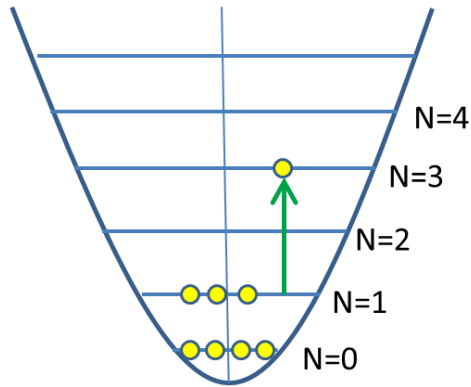
These are small effects, but important for

- Structure and decay modes of proton-rich nuclei
- Fundamental interaction studies (theoretical corrections to nuclear matrix elements)
- Astrophysics applications : proton/alpha capture/emission on proton-rich nuclei (p, γ), (p, α), (α ,p), ...



No-Core Shell model - configuration-interaction approach

$$H = T_{rel} + V_{NN}$$



$$H|\Psi_n\rangle = E_n|\Psi_n\rangle$$

Harmonic Oscillator Basis

$$|\Psi_n\rangle = \sum_k c_{kn} |\Phi_k\rangle$$

$$\langle\Phi_k|\Phi_l\rangle = \delta_{kl}$$

$$\sum_{k=1}^d \langle\Phi_l|H|\Phi_k\rangle c_{kn} = E_n c_{ln}$$

$$\begin{pmatrix} H_{11} & H_{12} & \dots & H_{1d} \\ H_{21} & H_{22} & \dots & H_{2d} \\ \vdots & \vdots & \ddots & \vdots \\ H_{d1} & H_{d2} & \dots & H_{dd} \end{pmatrix} \Rightarrow \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array} J^\pi$$

Ab-initio No-Core Shell Model : sufficiently large model space so that the results for A nucleons do not depend on the basis parameters ($\hbar\omega$ and $N_{\max}$)

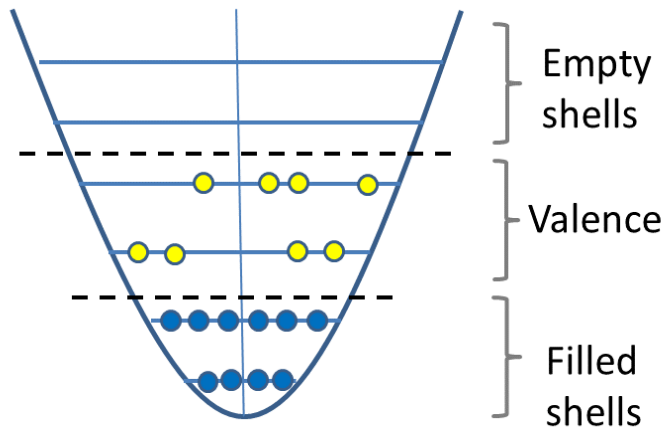
Conservation of symmetries of the Hamiltonian, detailed information on low-energy states and transitions

Valence-space shell model for heavier nuclei

Restricted model space

Effective operators

$$H|\Psi_n\rangle = E_n|\Psi_n\rangle \quad \longrightarrow \quad H_{eff}|\Psi_n^M\rangle = E_n|\Psi_n^M\rangle$$



$$H_{eff} = T + V_{eff} = \underbrace{(T + U)}_{\text{exactly solvable}} + \underbrace{(V_{eff} - U)}_{\text{residual interaction}} = H_0 + V_{res}$$

$$H_{eff} = \sum_{\alpha} \epsilon_{\alpha} a_{\alpha}^{\dagger} a_{\alpha} + \frac{1}{4} \sum_{\alpha\beta\gamma\delta} \langle \alpha\beta | V_{res} | \delta\gamma \rangle a_{\alpha}^{\dagger} a_{\beta}^{\dagger} a_{\gamma} a_{\delta}$$

Empirical
Microscopic derived from NN potential
Phenomenological (constrained by the data)

Current status :

- ❑ Excellent description with empirical (phenomenological) interactions
- ❑ Microscopic interactions -> recent progress and challenges

Isospin-symmetry breaking: NN interaction and light nuclei

Sources of isospin-symmetry breaking : $m_u \neq m_d$ and electromagnetic interactions between quarks

- Coulomb interaction between protons
 - $M_p - M_n \approx 1.3 \text{ MeV}$ and charge-dependent forces of nuclear origin
- } Nucleon level

Experimental evidence on charge-dependent components of NN forces

1) NN scattering length in the $^1S_0$ channel:

$$a_{nn} - a_{pp} = 1.65 \pm 0.60 \text{ fm}$$

⇒ Charge-symmetry breaking (CSB)

$$V_{pp} - V_{nn} \neq 0$$

$$\frac{1}{2}(a_{nn} + a_{pp}) - a_{np} = 5.6 \pm 0.6 \text{ fm}$$

⇒ Charge-independence breaking (CIB)

$$V_{pn}^{T=1} - \frac{1}{2}(V_{pp} + V_{nn}) \neq 0$$

2) Nolen-Schiffer anomaly (1969): mirror binding energy differences ($^3\text{H} - ^3\text{He}$, ...)

Modern high-precision potentials are charge-dependent
(CD-Bonn, Argonne V18, chiral EFT, etc)

⇒ Ab-initio approaches (NCSM, GFMC, ...)

Light nuclei in *ab-initio* approaches :
 $A = 3, 7, 8$: GFMC with CD interactions
Wiringa, Pastore, Pieper, Miller, PRC88 (2013)

G.A. Miller et al PR194, 1 (1990); R. Machleidt, χ EFT : U. van Kolck, J.L. Friar, R. Machleidt, E. Epelbaum, ...

Valence-space isospin non-conserving (INC) Shell Model

We include Coulomb + effective charge-dependent components perturbatively

$$H_{INC} = H_0^{pn} + V_{res} + \Delta V_{CD} \quad \text{with} \quad \Delta V_{CD} = V_C + V_{CSB}^{(1)} + V_{CIB}^{(2)}$$

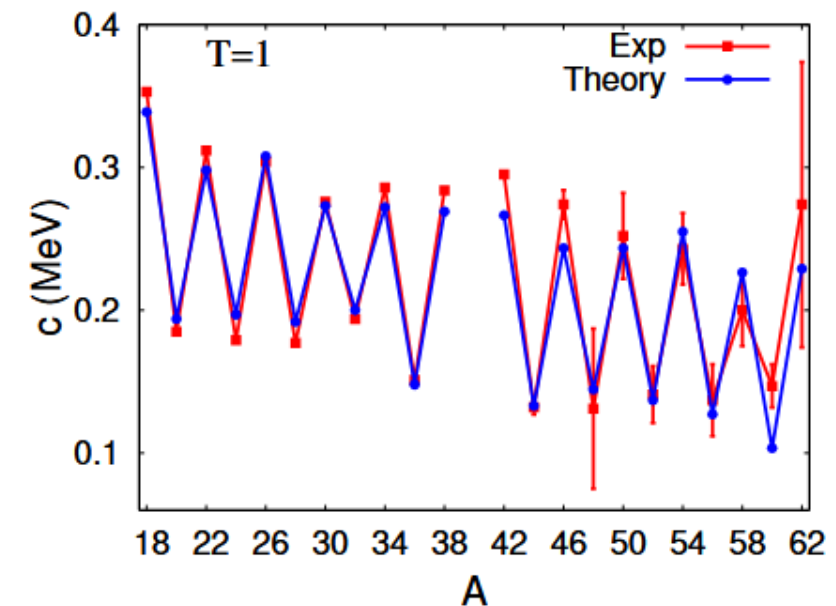
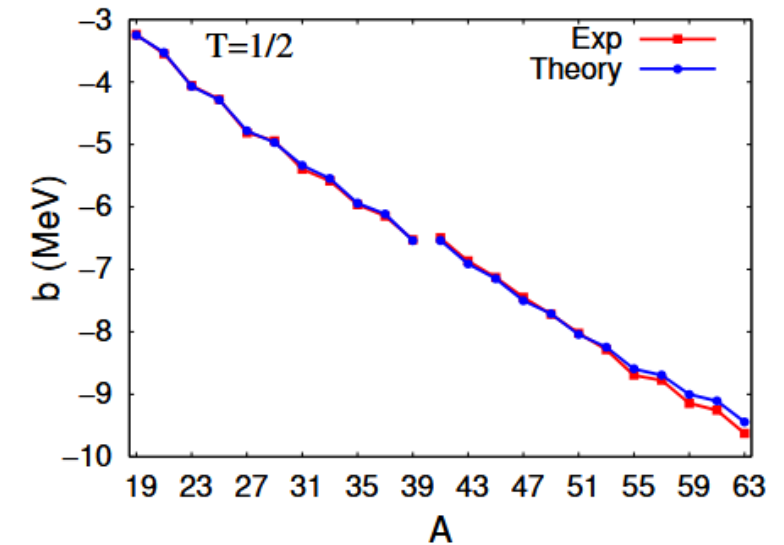
$$\langle \eta T T_z | H_{INC} | \eta T T_z \rangle = a^{th}(\eta, T) + b^{th}(\eta, T) T_z + c^{th}(\eta, T) T_z^2$$

- **Phenomenological Hamiltonians** (still more accurate)
 - fit of isovector s.p.e's and V_{CD} (3 parameters) to experimental b and c coefficients

*Ormand, Brown, NPA491 (1989); Lam, Smirnova, Caurier, PRC87 (2013);
Magiligan, Brown (2020),...*

RMS errors (theory & experiment) :

	b (keV)	c (keV)	N data (b)	N data (c)
<i>sd</i> shell	32	11	81 (all T)	51 (T=1,3/2,2)
<i>pf</i> shell	50	20	g.s. (T=1/2,1)	g.s. (T=1)

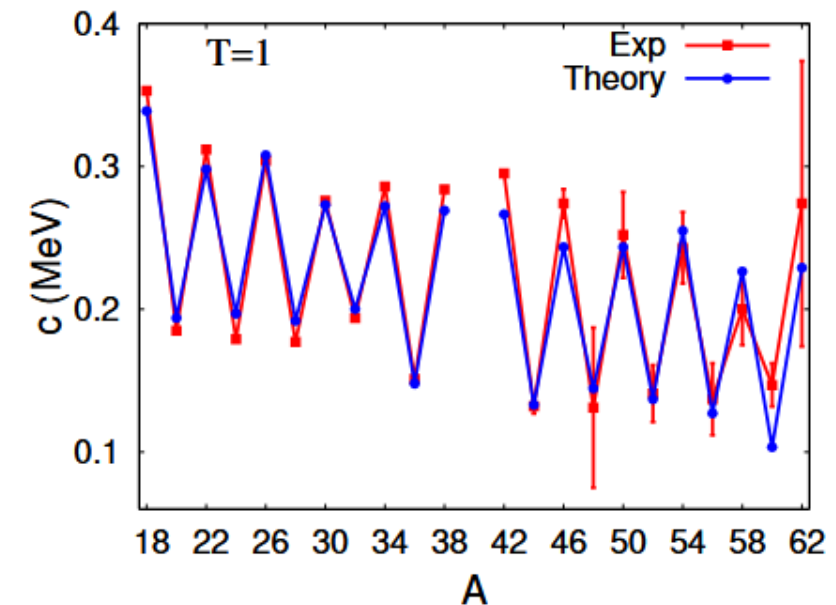
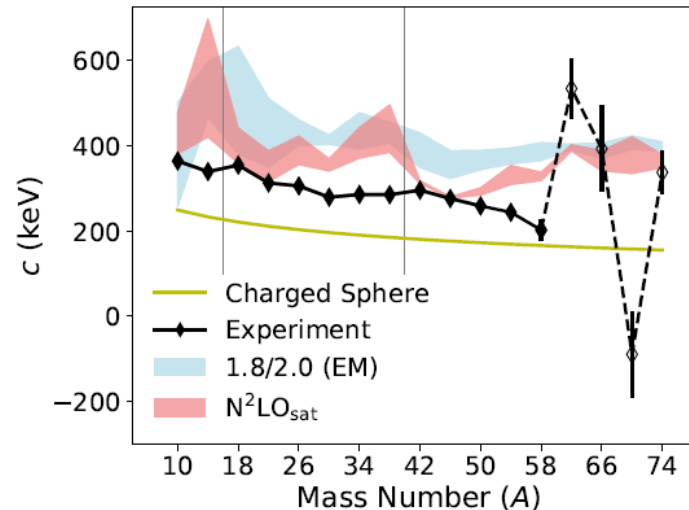
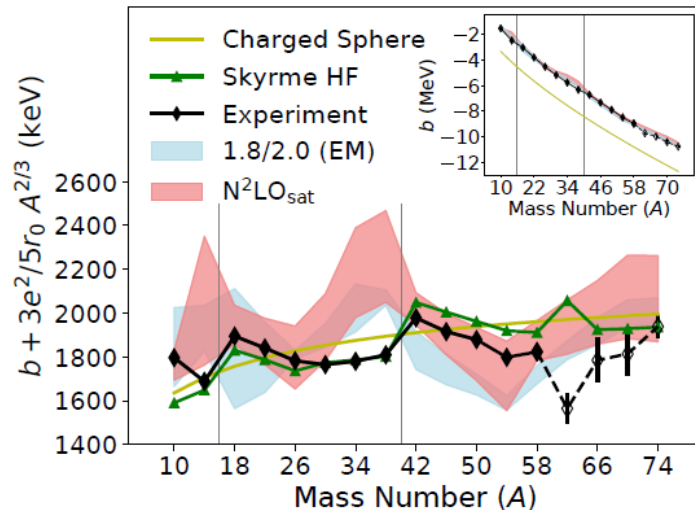
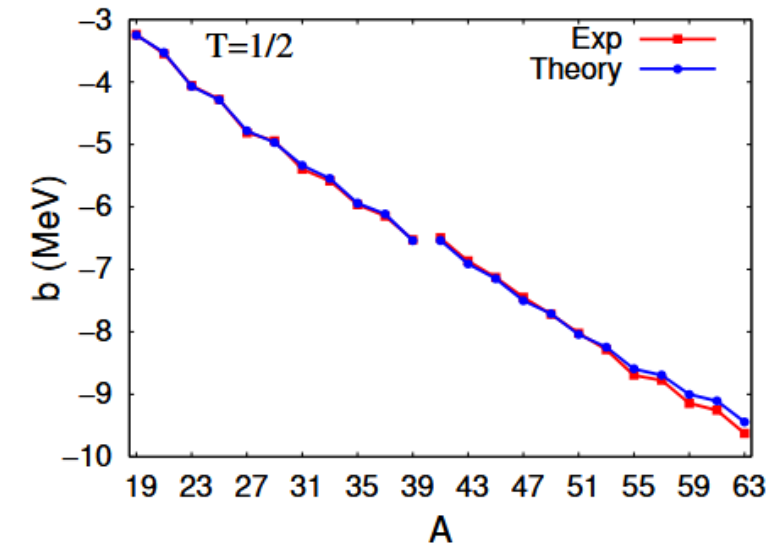


Nuclei along N=Z line: isospin-symmetry breaking

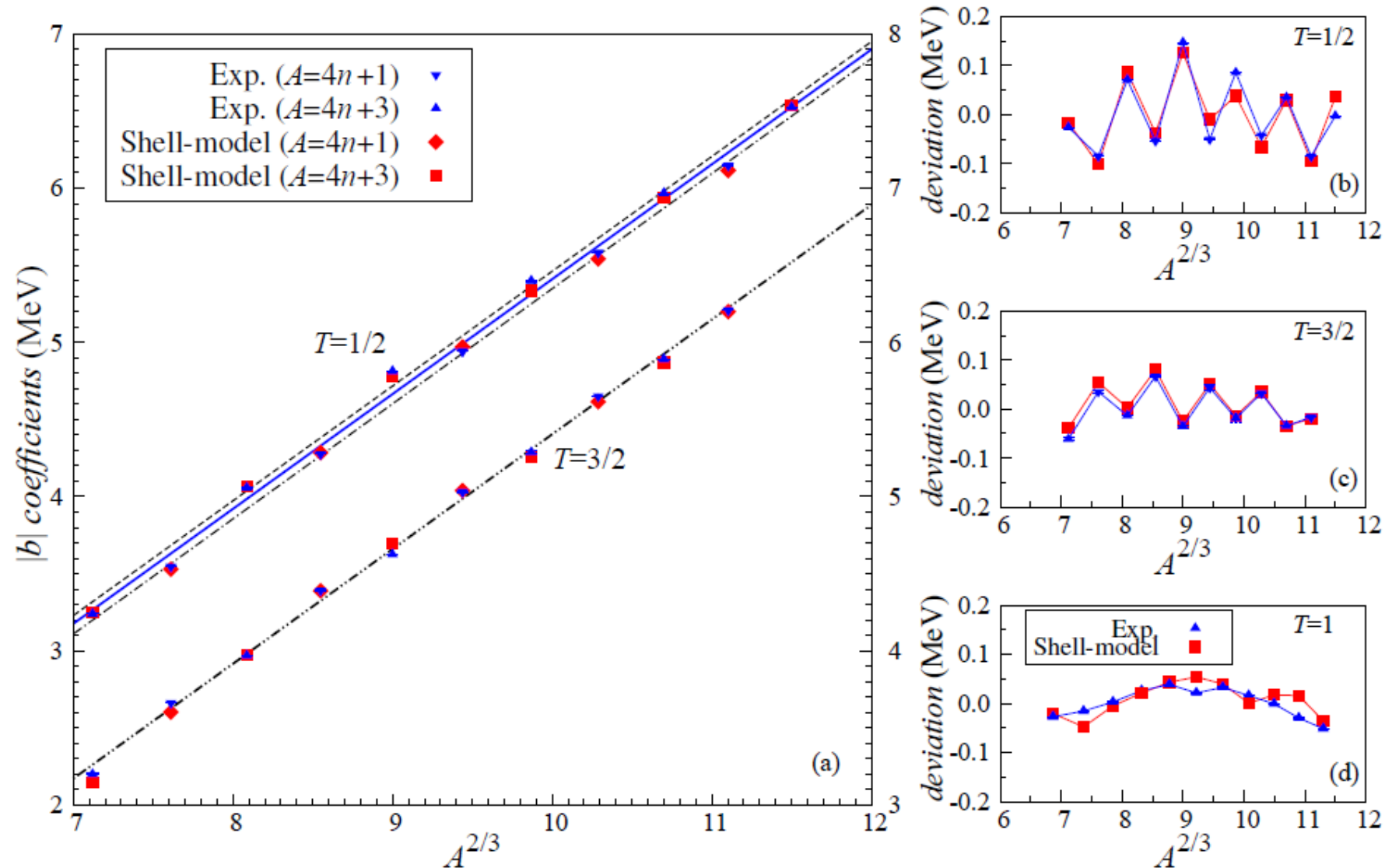
$$M(\eta, T, M_T) = a(\eta, T) + b(\eta, T)T_z + c(\eta, T)T_z^2$$

	b (keV)	c (keV)	N data (b)	N data (c)
<i>sd</i> shell	32	11	81 (all T)	51 (T=1,3/2,2)
<i>pf</i> shell	50	20	g.s. (T=1/2,1)	g.s. (T=1)

IMSRG: Martin et al, PRC104, 014324 (2021) – $A=10,14,18,22,\dots,74$



Theoretical description of isospin-symmetry breaking



Lam,
Smirnova,
Caurier (2013)

IMME b and c coefficients of lowest and excited multiplets

Importance for mass and excitation energies predictions

$$M_{T_z} = a + bT_z + cT_z^2$$

T=1

$$\begin{aligned} M_1 &= a + b + c \\ M_0 &= a \\ M_{-1} &= a - b + c \end{aligned}$$

- Prediction of masses and excited levels in proton-rich nuclei, e.g. if $T_z > 0$:

$$M_{-T_z} = M_{T_z}^{exp} + 2b^{th}T_z$$

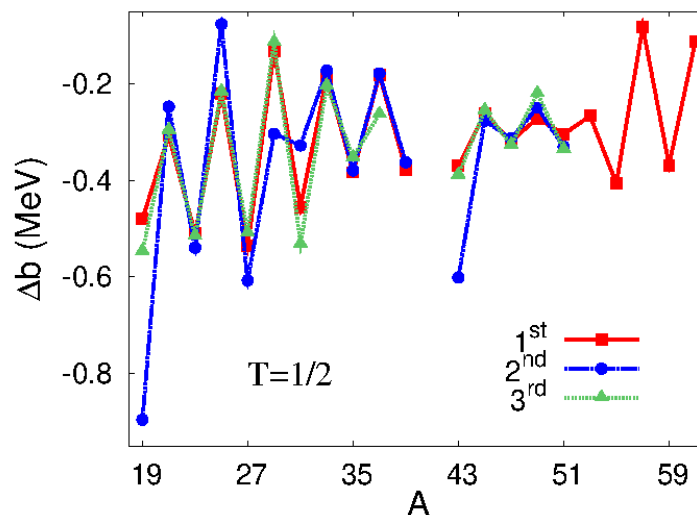
- Particular case of triplets:

$$M_{-1} = 2M_0^{exp} - M_1^{exp} + 2c^{th}$$

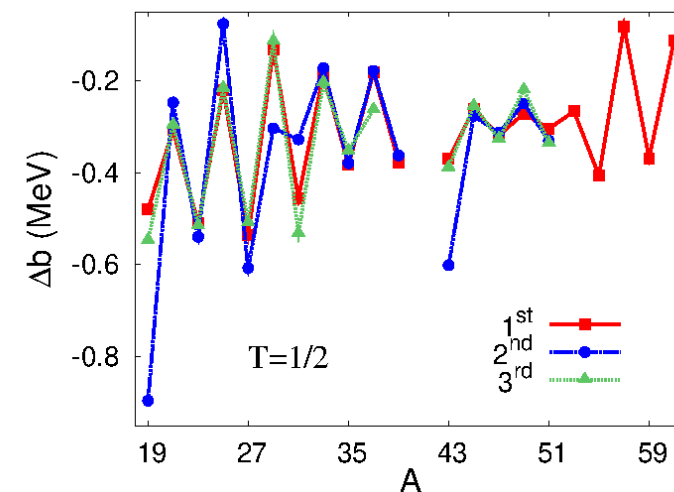
Important for nuclear astrophysics applications!

Brown, Richter (from 2011); Kaneko et al (2013)

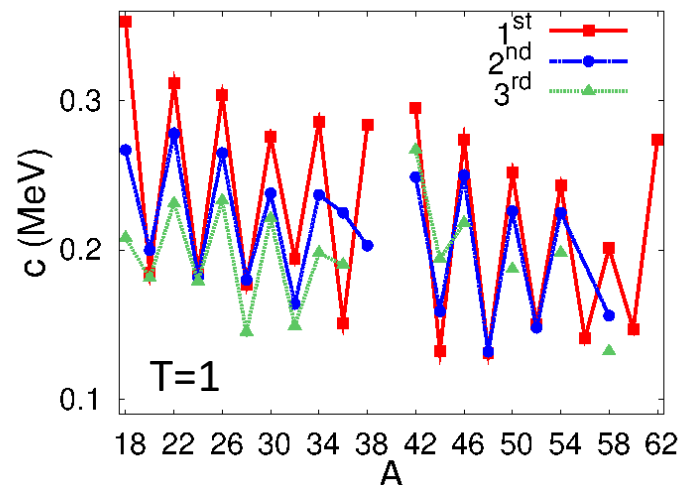
Experiment



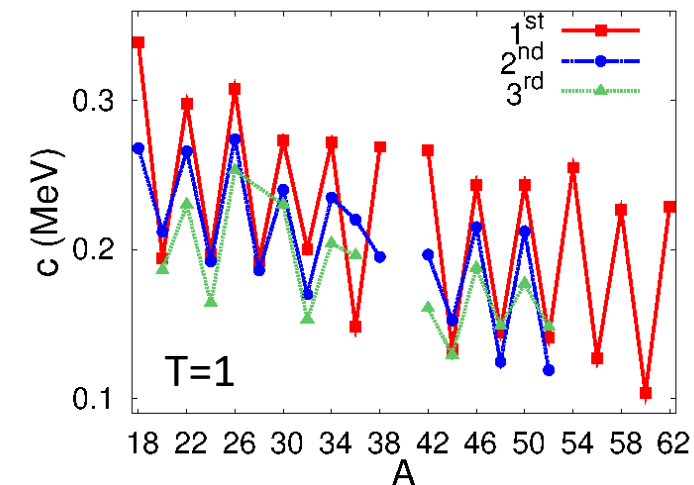
Theory



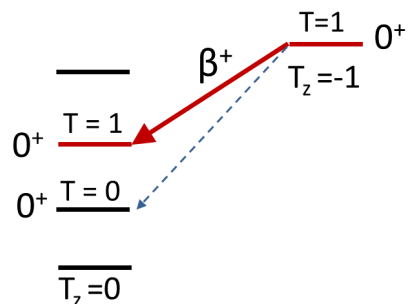
Experiment



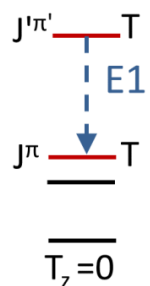
Theory



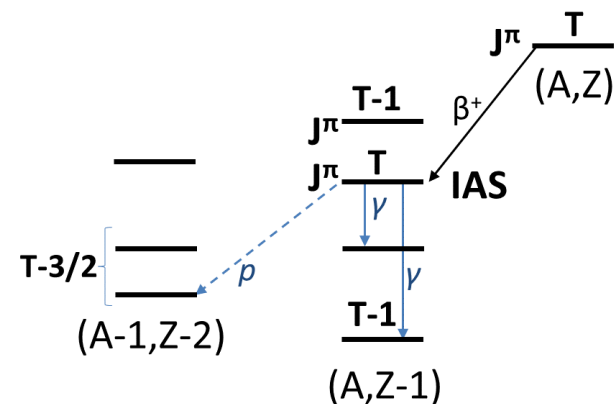
Isospin-forbidden decays and isospin mixing (?)



*Isospin-forbidden
Fermi beta decay*



*Isospin-forbidden
E-M transitions*



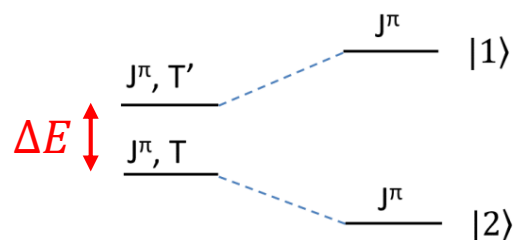
Isospin-forbidden proton emission

$$H_{INC} = H_0 + V + V_{CD}$$



Isospin mixing !

*How reliable can these phenomena be described
theoretically?*



$$|1\rangle = \sqrt{1 - \alpha^2}|T\rangle + \alpha|T'\rangle$$

$$|2\rangle = \alpha|T\rangle + \sqrt{1 - \alpha^2}|T'\rangle$$

$$\alpha^2 \sim \frac{\langle V_{CD} \rangle^2}{\Delta E^2}$$

Reliable (at low energies)

Challenging...

$$\langle V_{CD} \rangle = \langle T | V_{CD} | T' \rangle$$

p-shell	< 150(40) keV
sd-shell	< 100(40) keV
pf-shell	< 40(20) keV

II. Application to fundamental interaction studies on nuclei

**Computation of isospin-symmetry breaking to
superaligned $0^+ \rightarrow 0^+$ Fermi beta decay for the tests
of CVC and V_{ud} CKM matrix element extraction**

II. Application to fundamental interactions studies

Fermions in the Standard Model

$$\begin{pmatrix} u \\ d \end{pmatrix} \quad \begin{pmatrix} c \\ s \end{pmatrix} \quad \begin{pmatrix} t \\ b \end{pmatrix} \\ \begin{pmatrix} e \\ \nu_e \end{pmatrix} \quad \begin{pmatrix} \mu \\ \nu_\mu \end{pmatrix} \quad \begin{pmatrix} \tau \\ \nu_\tau \end{pmatrix}$$

*Cabibbo - Kobayashi - Maskawa (CKM)
quark-mixing matrix*

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

N. Cabibbo (1963); M. Kobayashi, T. Maskawa (1973).

Testing grounds:

- at colliders: search for direct production
- at low energies in nuclear beta decay: in precision experiments

**Nuclear Matrix
elements are
needed !**

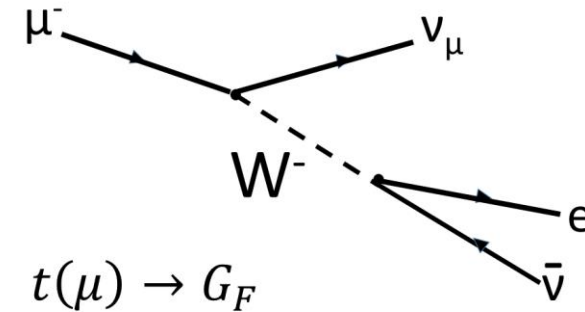
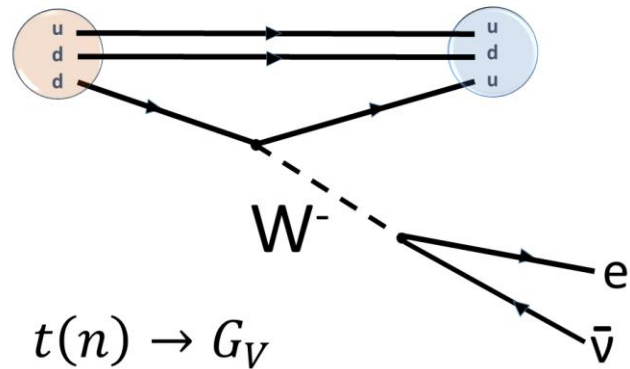
What does the Standard Model prescribe?

- V_{ik} to be extracted from experiment
- Standard Model: V is unitary

$$\sum_k |V_{ik}|^2 = \sum_k |V_{ki}|^2 = 1$$

$\Rightarrow$ constraints on new physics beyond the Standard Model

$|V_{ud}|$ from pion, nucleon and nuclear beta decay



$$|V_{ud}| = G_V/G_F$$

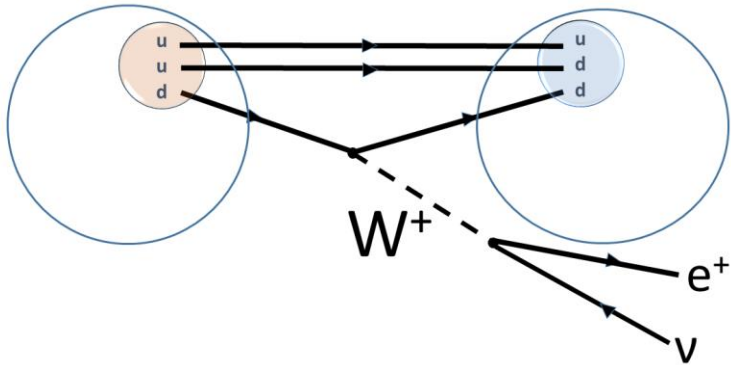
- Superaligned nuclear $0^+ \rightarrow 0^+$ beta decay (nuclear structure effects)
- Mirror $T=1/2$ transitions (F/GT ratio, nuclear structure effects)
- Neutron decay $n \rightarrow p + e^- + \bar{\nu}_e$ (Lifetime)
- Pion decay $\pi^+ \rightarrow \pi^0 + e^+ + \nu_e$ (Branching ratio)

Current status:

J.C. Hardy, I. S. Towner, PRC102. 045501 (2020) ;

M. Gonzalez-Alonso, O. Navillat-Cuncic, N. Severijns, PPNP104, 165 (2019)

Nuclear beta decay



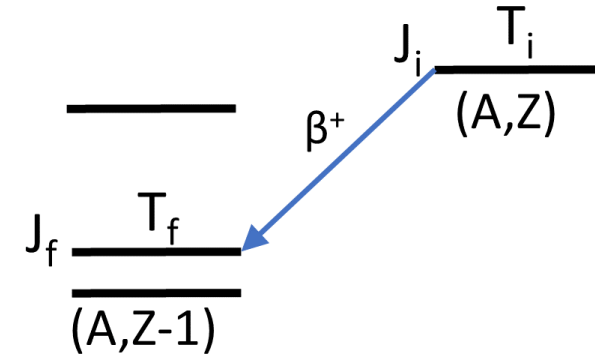
Decay rate:

$$\Gamma = \frac{\ln 2}{t} = \frac{G_V^2}{K} \left[g_V^2 f_V |M_F|^2 + g_A^2 f_A |M_{GT}|^2 \right]$$

$$f = \int_1^{W_0} (W - W_0)^2 p W F(Z, W) S(Z, W) dW$$

For $0^+ \rightarrow 0^+$ beta decay:

$$f t^{0^+ \rightarrow 0^+} = \frac{K}{|M_F|^2 G_V^2}$$



$$K = \frac{2\pi^3 \ln 2 \hbar^7}{m_e^5 c^4}$$

$$g_V=1, g_A \approx 1.26$$

$$W_0 = Q_{EC} / (m_e c^2) - 1$$

Should be constant (for fixed T) – a consequence of the Conserved Vector Current (CVC) hypothesis

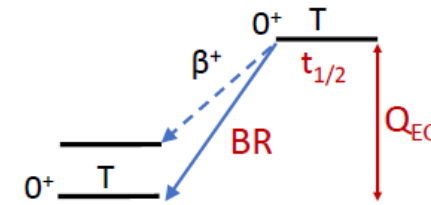
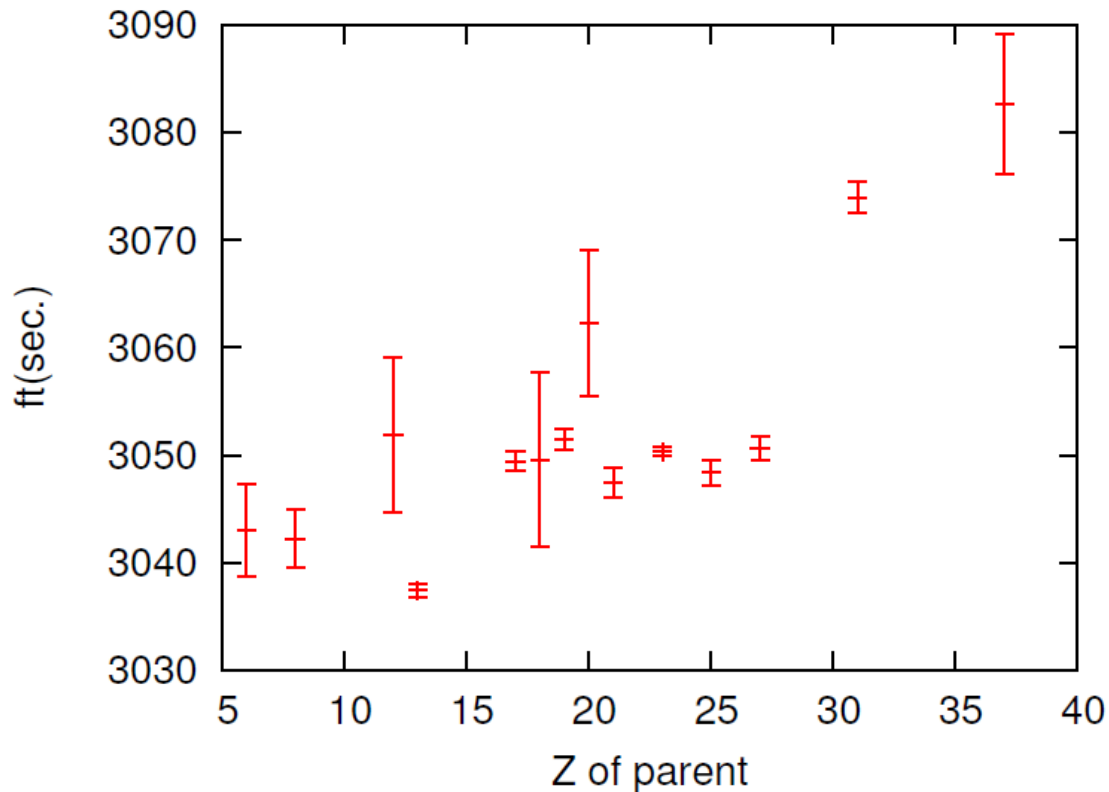
Superaligned $0^+ \rightarrow 0^+$ beta decay

15 best known $T = 1$ emitters ($ft^{0^+ \rightarrow 0^+}$ -value known with a precision $\lesssim 0.4\%$):

^{10}C , ^{14}O , ^{22}Mg , ^{26m}Al , ^{26}Si , ^{34}Cl , ^{34}Ar , ^{38m}K , ^{38}Ca , ^{42}Sc , ^{46}V , ^{50}Mn , ^{54}Co , ^{62}Ga , ^{74}Rb

J.C. Hardy, I.S. Towner, PRC102, 045501 (2020)

Experimental data on $t_{1/2}$, BR, Q_{EC}



Statistical rate function:

$$f = f(Z, Q_{EC})$$

Partial half-life:

$$t = \frac{t_{1/2}}{BR}$$

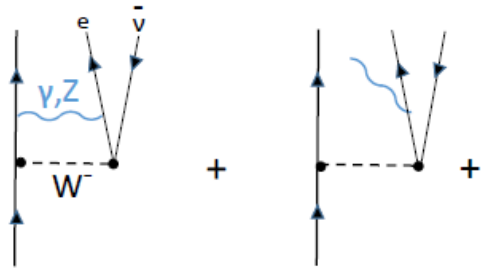
$$ft^{0^+ \rightarrow 0^+} = \frac{K}{|M_F|^2 G_V^2}$$

Superaligned $0^+ \rightarrow 0^+$ beta decay

Absolute Ft value

$$Ft^{0^+ \rightarrow 0^+} \equiv ft^{0^+ \rightarrow 0^+} (1 + \delta'_R)(1 + \delta_{NS} - \delta_C) = \frac{K}{|M_F^0|^2 G_V^2 (1 + \Delta_R)}$$

J.C. Hardy, I.S. Towner, PRC102, 045501 (2020)

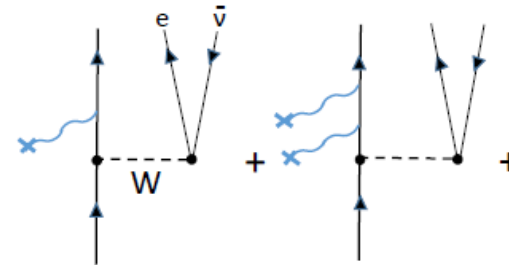


Radiative corrections

**Much recent
Interest within
EFT**

$$\begin{aligned} \Delta_R^V &= (2.454 \pm 0.019)\% \\ \delta'_R &\sim (1.50 \pm \sim 0.12)\% \\ |\delta_{NS}| &\lesssim 0.4\% \end{aligned}$$

*Sirlin, Marciano, Jaus, Rasche, ..
Seng, Gorchtein, Cirigliano,...*



Nuclear-structure correction

$$\begin{aligned} |M_F|^2 &= |M_F^0|^2 (1 - \delta_C) \\ |M_F^0|^2 &= T(T+1) - T_{zi} T_{zf} \\ \delta_C &\approx 0.1 - 2.0\% \end{aligned}$$

*Towner, Hardy (1973 – 2020)
and many others (next slide)*

**Many-body
approach with
charge-dependent
forces**

Many-body approaches to isospin-symmetry breaking

- Shell model (from 60's . . .)

W.E. Ormand, B.A. Brown et al 1985 –; S.Nakamura, K.Muto, T.Oda, 1994; A.P. Zuker, S.M. Lenzi, M.A. Bentley et al, 2001 – 2018; K. Kaneko et al, 2010 – 2018; Y.H. Lam, N. Smirnova, E. Caurier, 2013.

- Gamow shell-model, SMEC

N. Michel, W. Nazarewicz, M. Ploszajczak, PRC82 (2010), . . .

- HF + Tamm-Dankoff, RPA, HTDA

I. Hamamoto, H. Sagawa, N. V. Giai, J. Dobaczewski, T. Suzuki, G. Colo, J. Le Bloas, N. Auerbach et al, 1993 – 2018

- Relativistic RPA

H. Liang, N. V. Giai, J. Meng, 2009 –

- J -projected and T -projected DFT with ISB terms

W. Satula, J. Dobaczewski et al, 2009 – 2018, P. Baczyk et al, PLB778, 178 (2018)

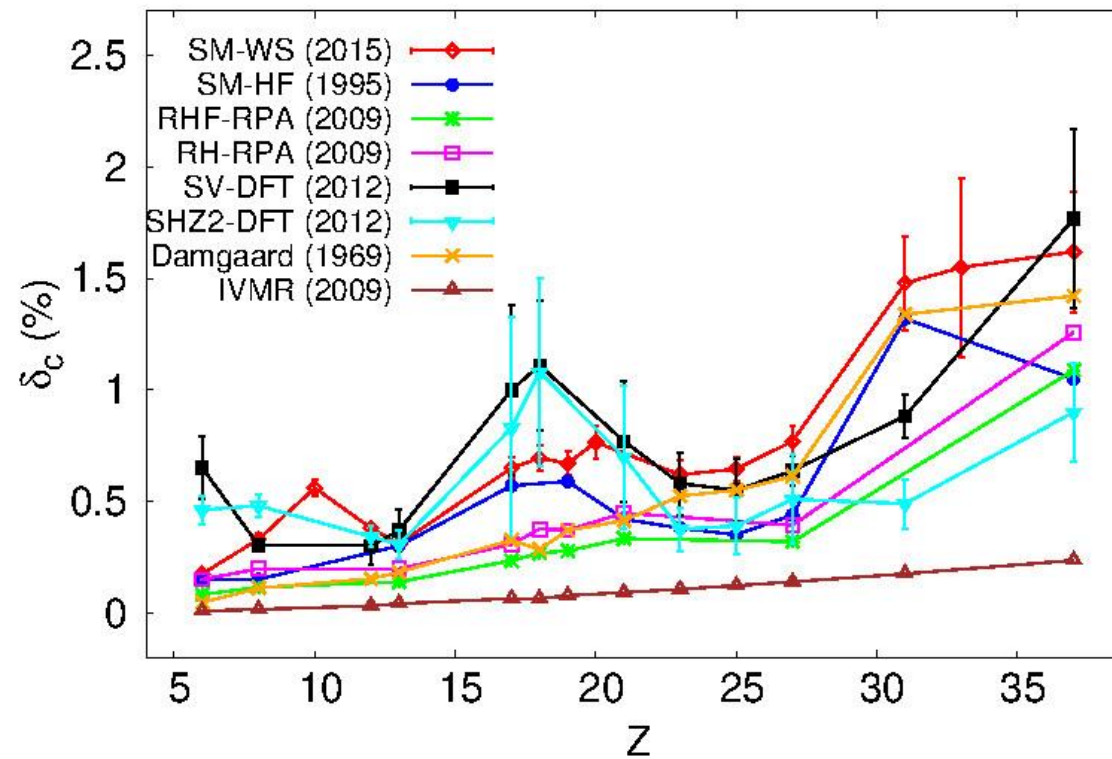
- VAP technique on the HFB basis

A. Petrovici et al, 2008 – 2018

- Isovector giant monopole resonance

G. Colo et al (1993); N. Auerbach, Phys. Rep. 98, 273 (1983); PRC (2009)

Isospin-symmetry breaking correction – divergence of model predictions



SM-WS : Towner, Hardy (2015)
SM-HF : Ormand, Brown (1989, 1995)
RHF-RP : Liang et al (2009)
SV/SHZ2-DFT : Satula et al (2012)
IVMR : Auerbach (2009)

CVC test
Towner, Hardy
PRC82 (2010)

Superaligned $0^+ \rightarrow 0^+$ beta decay: status by 2020

$$\mathcal{F}t = (1 + \delta_R)(1 + \delta_{NS} - \delta_C)ft = \frac{K}{M_0^2 G_F^2 |V_{ud}|^2 (1 + \Delta_R)}$$

$$\mathcal{F}t = 3072.24 \pm 0.57 \text{ sec}$$

$$\chi^2/\nu = 0.47$$

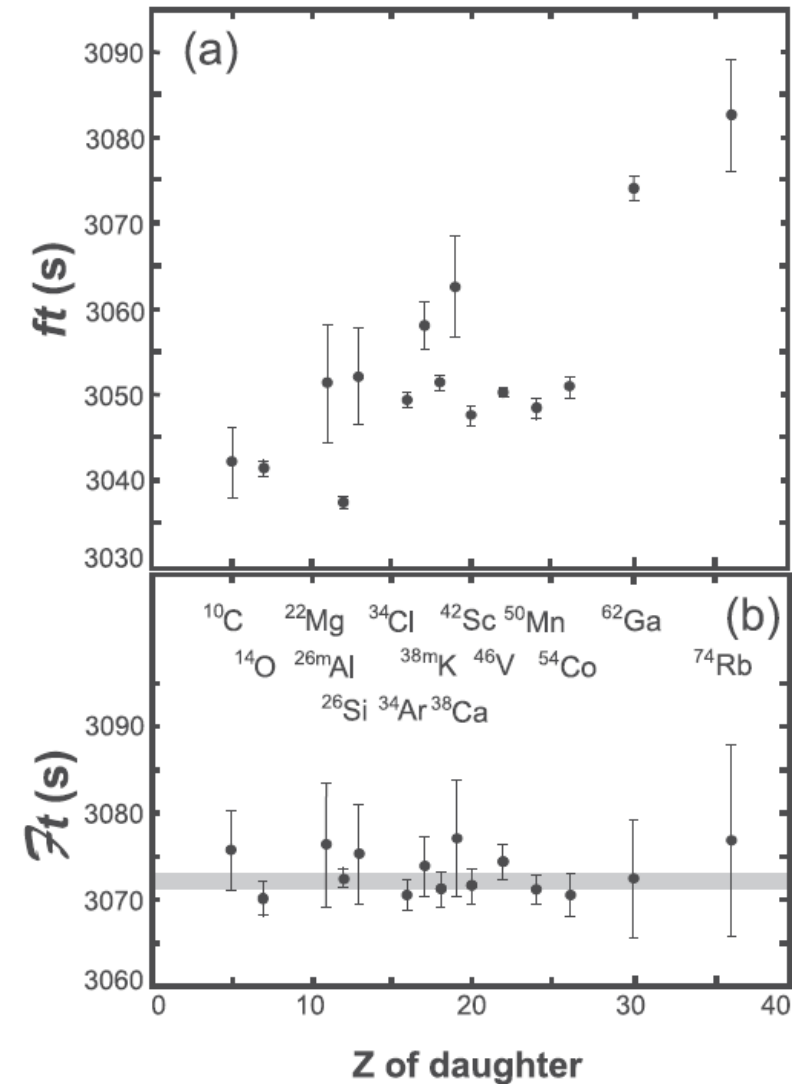
$$|V_{ud}| = 0.97373(31)$$

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9985(5)$$

*Further revisions of radiative corrections
are ongoing since 2018: for review see
Seng, Gorchtein, ARNPS 74 (2023)*

**The purpose of our work: to revise the shell-model
computation δ_C of in large-scaled calculations
With realistic WS or HF wave functions**

Hardy, Towner (2020)



Nuclear-structure correction to Fermi β decay

Large-scale shell model with H_{INC} + Wood-Saxon or Hartree-Fock radial wave functions

$$M_F = \langle \Psi_f | T_+ | \Psi_i \rangle = \sum_{\alpha} \underbrace{\langle \Psi_f | a_{\alpha_n}^{\dagger} a_{\alpha_p} | \Psi_i \rangle}_{\text{One-body transition densities}} \underbrace{\langle \alpha_n | t_+ | \alpha_p \rangle}_{\text{Radial overlap integral}}$$

$$\rho_{\alpha} \neq \rho_{\alpha}^T$$

One-body
transition
densities

Radial
overlap
integral

$$\int_0^{\infty} R_{\alpha_n}(r) R_{\alpha_p}(r) r^2 dr = \Omega_{\alpha} \neq 1$$

*Exact Fermi operator: G.A. Miller, A. Schwenk,
PRC78, 035501 (2008); PRC80, 064319 (2009)*

Expansion of the correction in terms of small quantities

$$(\rho_{\alpha} - \rho_{\alpha}^T)$$

and $(1 - \Omega_{\alpha})$:

$$M_F^2 = M_0^2(1 - \delta_C)$$



$$\delta_C = \delta_{C1} + \delta_{C2} + \delta_{C3} + \delta_{C4} + \delta_{C5} + \delta_{C6}$$

LO
NLO
N²LO
N³LO



$$\delta_C \approx \delta_{C1} + \delta_{C2}$$

- From numerical evaluation, the leading-order (LO) corrections provide robust approximation !
- NLO, N2LO, N3LO corrections stay within theoretical uncertainties on δ_C

Shell-model calculations

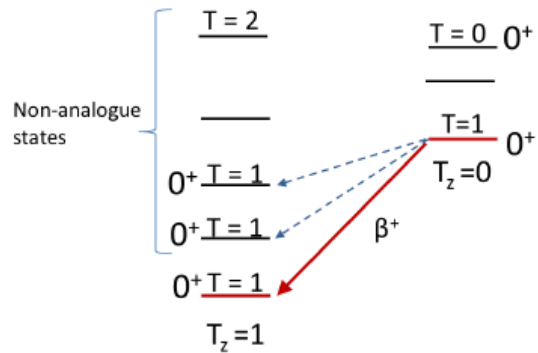
^{10}C , ^{14}O , ^{18}Ne , ^{22}Mg , ^{26m}Al , ^{26}Si , ^{34}Cl , ^{34}Ar , ^{38m}K , ^{38}Ca , ^{42}Sc , ^{42}Ti , ^{46}V , ^{50}Mn , ^{54}Co , ^{62}Ga , ^{66}As

Model spaces and effective interactions (+ charge-dependence)

- p -shell: CKPOT (*Cohen-Kurath, 1965*)
- $(p_{1/2}sd_{5/2})$ -shell: ZBM's (*Zuker et al, 1969*), REWIL (*Reehal, Wildenthal, 1973*)
- sd -shell: USD (*Wildenthal, 1984*), USDA/USDB (*Brown, Richter, 2006*)
- $(sd_{3/2}f_{7/2}p_{3/2})$ -shell: ZBM2 (*Nowacki et al, 2014*)
- pf -shell: KB3G (*Poves et al, 2004*), GXPF1A (*Honma et al, 2004*)
- $pf_{5/2}g_{9/2}$: JUN45 (*Honma et al, 2009*), MRG (*Nowacki et al, 1996*)

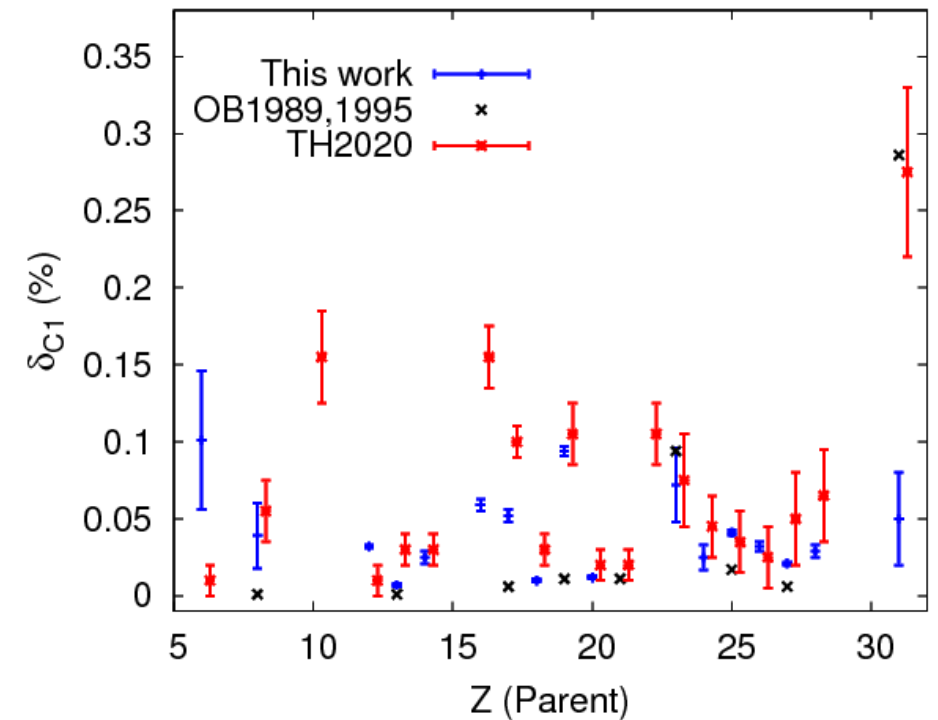
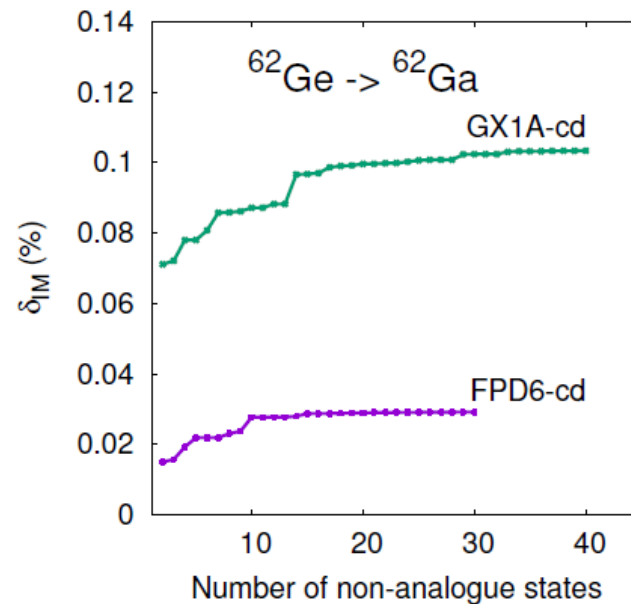
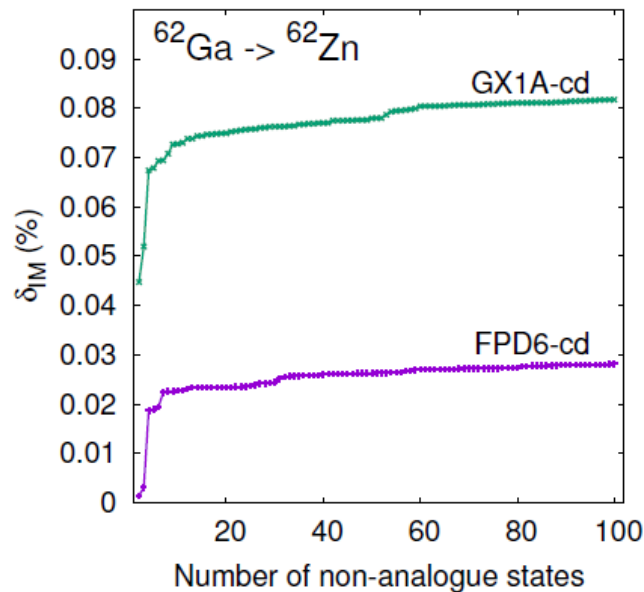
NuShellX@MSU shell-model code (*B.A. Brown, W.D.M. Rae, Nucl. Data Sheets 120, (2014).*

Shell-model evaluation of δ_{C1}



We sum over the Fermi strengths to non-analogue states!
 $\Rightarrow$ experimental information on non-analogue strength is highly desirable!

$$\sim (\Delta E_{th})^2 / (\Delta E_{exp})^2$$



Xayavong, Smirnova, Nowacki, submitted

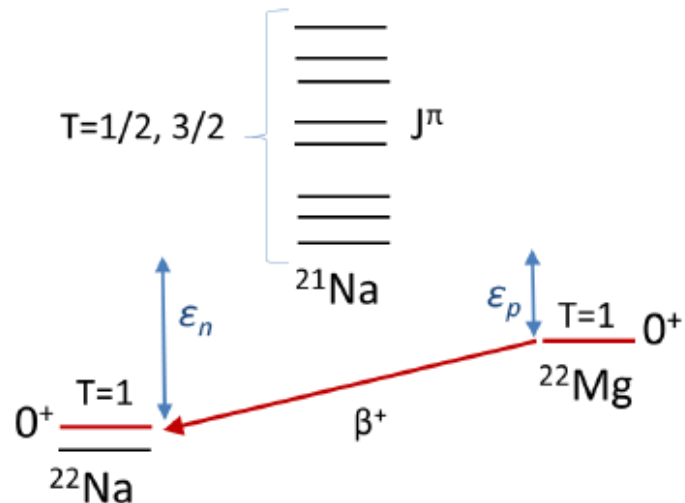
Shell-model evaluation of δ_{C2} beyond the closure approximation

$$\delta_{C2} = \frac{2}{M_F^0} \sum_{\alpha} \langle \Psi_f | a_{\alpha n}^{\dagger} a_{\alpha p} | \Psi_i \rangle^T (1 - \Omega_{\alpha})$$

$\Downarrow$

Experimental information on relevant spectroscopic factors and charge radii is highly desirable !

$$\delta_{C2} = \frac{2}{M_F^0} \sum_{\alpha, \pi} \langle \Psi_f | a_{\alpha n}^{\dagger} | \pi \rangle^T \langle \pi | a_{\alpha p} | \Psi_i \rangle^T (1 - \Omega_{\alpha}^{\pi})$$



- Spectroscopic amplitudes (from the shell-model):

$$\langle \Psi_f | a_{\alpha n}^{\dagger} | \pi \rangle = \frac{\langle \Psi_f || a_{\alpha n}^{\dagger} || \pi \rangle}{\sqrt{2J_f + 1}}$$

- Radial-overlap integrals (from a realistic single-particle potential)

$$\Omega_{\alpha}^{\pi} = \int_0^{\infty} R_{\alpha n}^{\pi}(r) R_{\alpha p}^{\pi}(r) r^2 dr$$

Shell-model + Woods-Saxon evaluation of δ_{C2}

L. Xayavong, PhD thesis, U. Bordeaux (2016)

L. Xayavong, N. Smirnova, Phys. Rev. C97, 024324 (2018)

Parameterization

$$V_{WS}(r) = -\frac{V_0}{1 + \exp\left(\frac{r-R}{a}\right)} - V_{ls}(r)\vec{l} \cdot \vec{\sigma} + V_C(r)$$

- A. Bohr, B.R. Mottelson modified (BM_m) from *Nuclear Structure, Vol. I*.
- N. Schwierz, I. Wiedenhöver, A. Volya (SWV) from *nucl-th:0709.3525* $R = r_0 A^{1/3}$

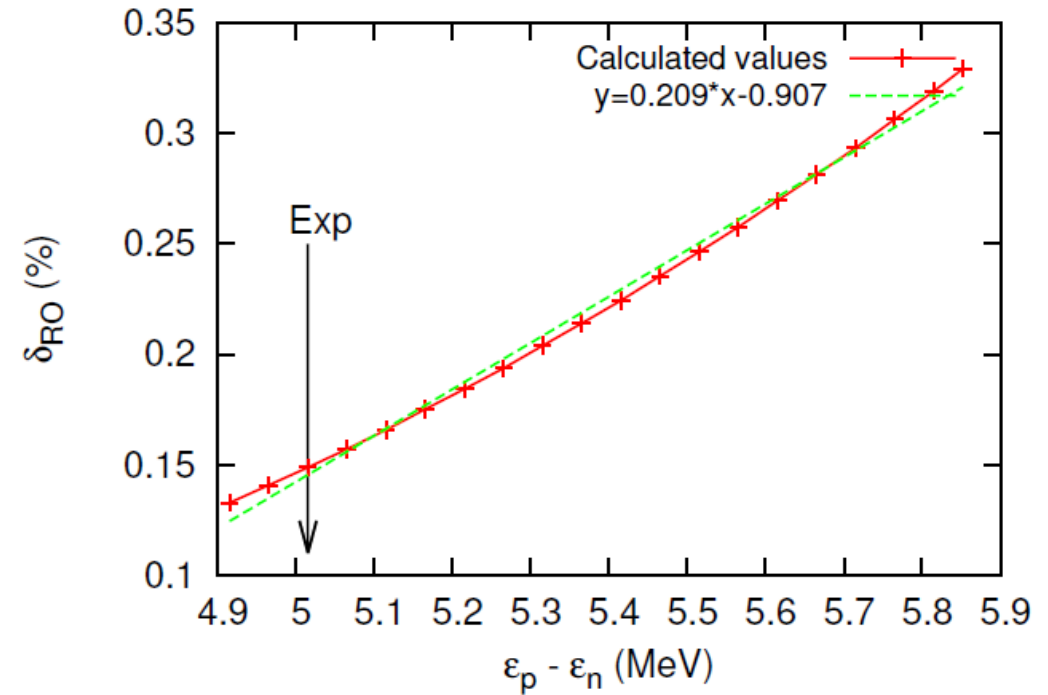
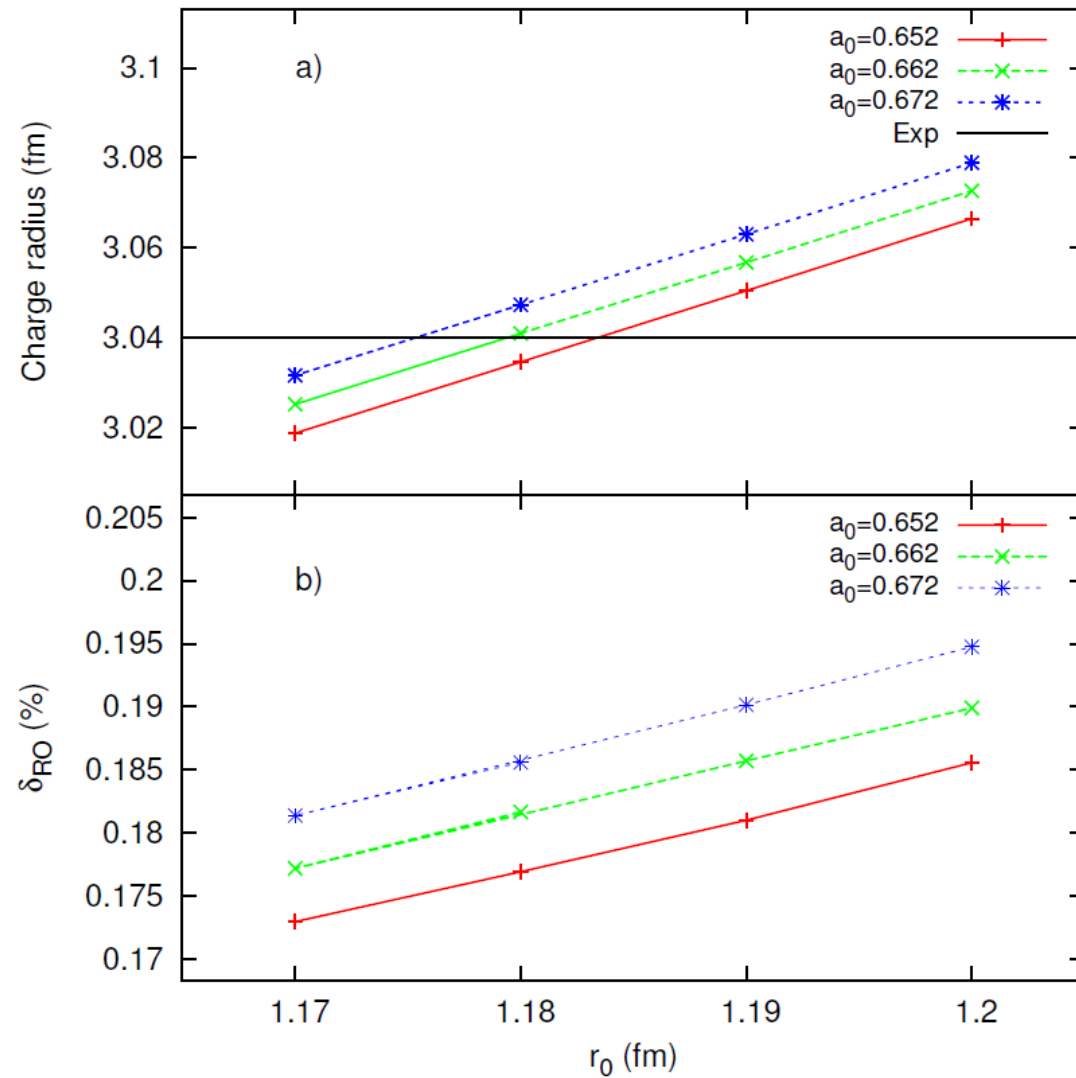
Under stringent experimental constraints:

- (V_0, r_0) are adjusted simultaneously to reproduce experimental **nucleon separation energies** and **charge radii**

$$\psi(r) \rightarrow \exp\left(-\frac{\sqrt{2m|\epsilon|r}}{\hbar}\right)$$

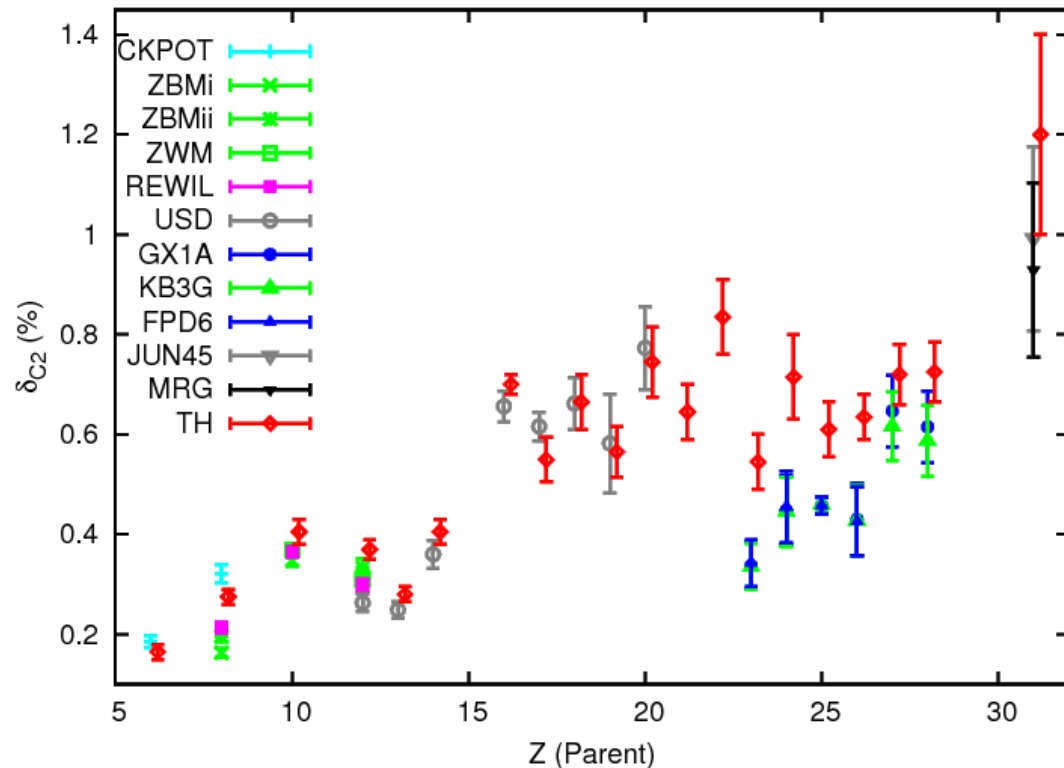
- A new approach to nuclear charge radii beyond the closure approximation:
$$\langle r^2 \rangle_{ch} = \frac{1}{Z} \sum_{\pi\alpha} \langle \alpha | r^2 | \alpha \rangle^\pi |\langle \Psi_i | | a_\alpha^\dagger | | \pi \rangle|^2$$
- Thorough **propagation of uncertainties**.

WS Adjustment



L. Xayavong, PhD thesis, U. Bordeaux (2016)

Shell-model + Woods-Saxon evaluation of δ_{C2} and δ_C

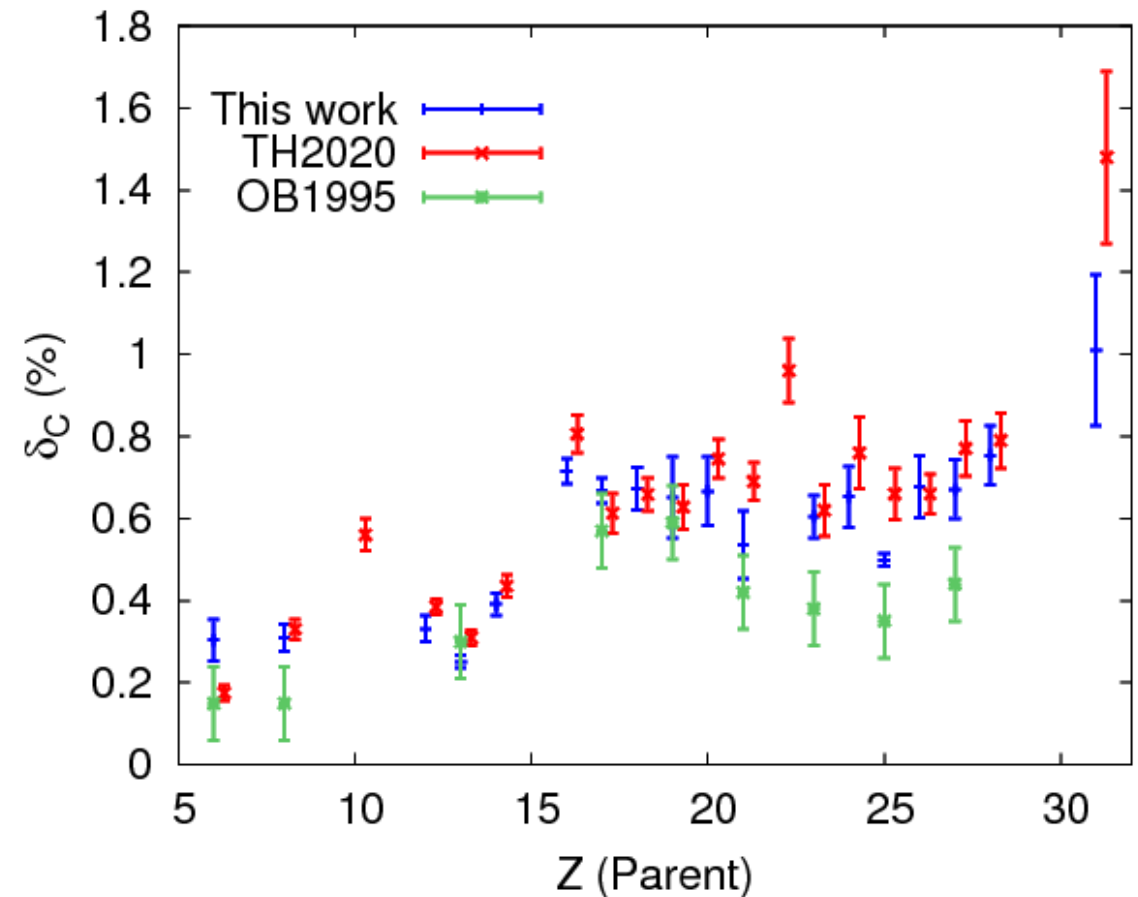


Uncertainties are mainly due to the charge radii uncertainties if we keep diffuseness constant

Smirnova, Xayavong, Proc. NTSE2018.

Xayavong, Smirnova, Nowacki (submitted to PRC, 2025)

Preliminary !



$a=0.66(10)\text{fm}$

Consistency tests

$$\mathcal{F}t = (1 + \delta_R)(1 + \delta_{NS} - \delta_C)ft = \frac{K}{M_0^2 G_F^2 |V_{ud}|^2 (1 + \Delta_R)}$$

^{10}C , ^{14}O , ^{22}Mg , $^{26\text{m}}\text{Al}$, ^{26}Si , ^{34}Cl , ^{34}Ar , $^{38\text{m}}\text{K}$, ^{38}Ca , ^{42}Sc , ^{46}V , ^{50}Mn , ^{54}Co , ^{62}Ga , (^{74}Rb)

□ Δ_R , δ_R , δ_{NS} Radiative corrections are kept the same

Towner, Hardy (2020)

$$\mathcal{F}t = 3072.23(57)_{\text{stat}} (36)_{\delta'_R} (173)_{\delta_{NS}} \text{sec}$$

$$\chi^2/\nu = 0.49$$

This work (WS)

$$\mathcal{F}t = 3073.11(99)_{\text{stat}} (36)_{\delta'_R} (173)_{\delta_{NS}} \text{sec}$$

$$\chi^2/\nu = 0.63$$

Preliminary !

Xayavong, Smirnova, Nowacki (submitted to PRC, 2025)

Shell Model + Hartree-Fock radial wave functions for δ_{C2}

Skyrme force calculations

SkM* parameterization *J. Bartel et al, NPA386 (1982).*

Optimization

- Energy-dependent *local equivalent potential*

C.B.Dover, N. Van Giai, NPA190 (1972):

$$V^{LE}(r, \epsilon_\alpha) = V^0(r, \epsilon_\alpha) + V^{so}(r) \langle \vec{l} \cdot \vec{s} \rangle + V_C(r) \quad \text{for} \quad R_\alpha(r) = N \sqrt{\frac{m^*(r)}{m}} R_\alpha^{LE}(r).$$

- Adjustment of the central term $V^0(r, \epsilon_\alpha)$ by a scaling factor to match the experimental proton and neutron separation energies
- Effects from CIB and CSB terms, spurious isospin mixing, Slater approximation for Coulomb term, two-body center-of-mass correction, finite nucleon size, etc. are investigated.

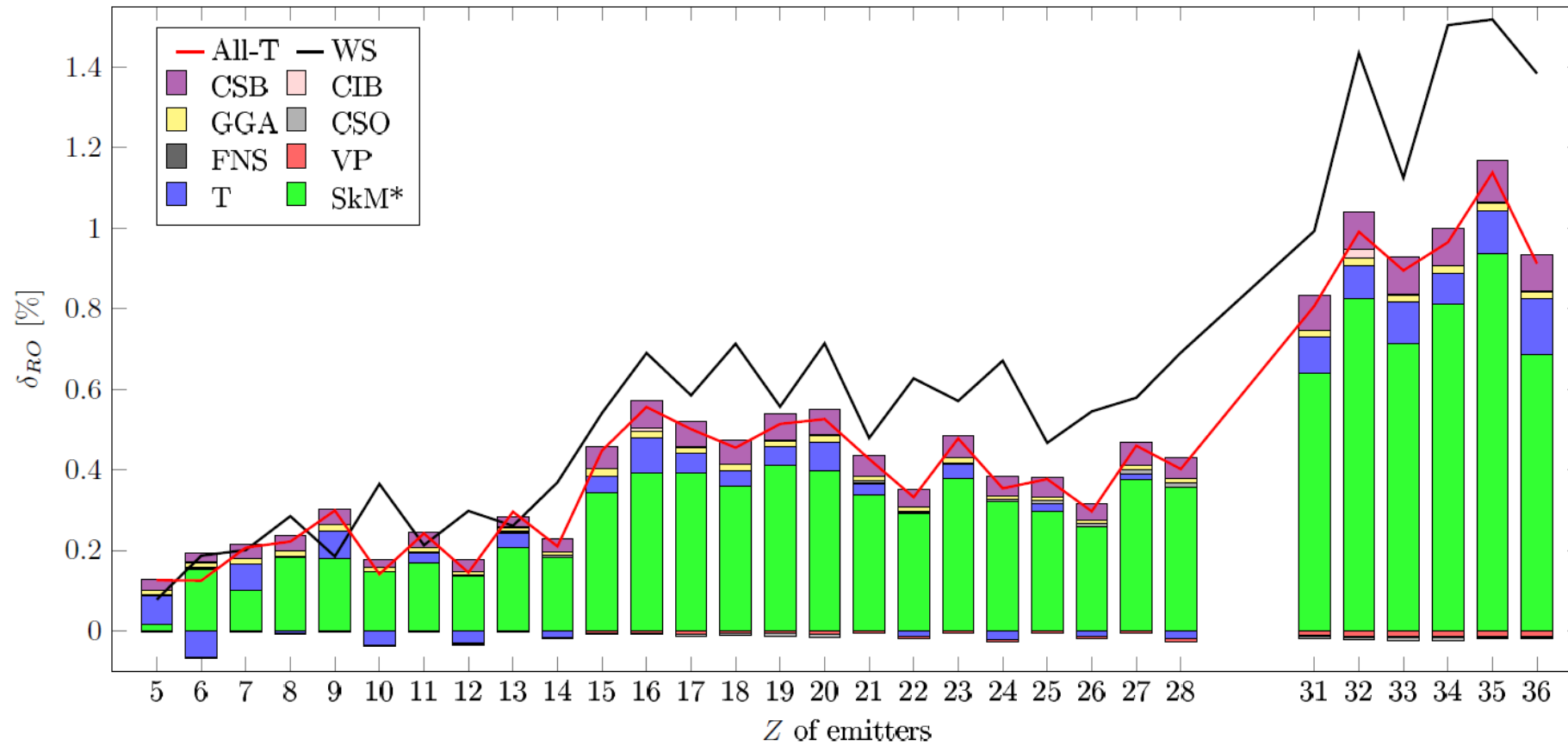
W.E. Ormand, B.A. Brown, NPA440 (1985); PRL62 (1989); PRC52 (1995)

L. Xayavong, N. Smirnova, PRC105 (2022).

Shell Model + Hartree-Fock radial wave functions for δ_{C2}

L. XAYAVONG AND N. A. SMIRNOVA

PHYSICAL REVIEW C **105**, 044308 (2022)



Post HF effects to be considered!

$$\mathcal{F}t = 3074.81 \pm 0.93 \text{ sec}$$

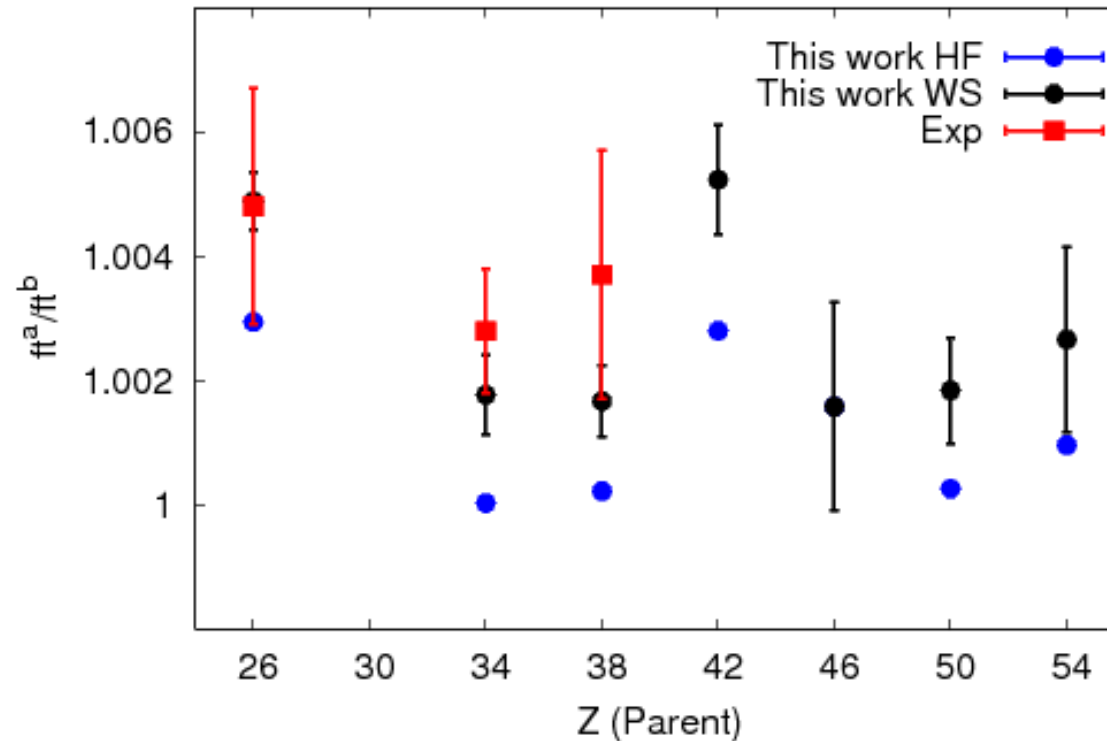
$$\chi^2/\nu = 2.96$$

Consistency tests

Test of the mirror ratio for triplets

$$\frac{ft^a}{ft^b} = 1 + (\delta_R^a - \delta_R^b) + (\delta_{NS}^a - \delta_{NS}^b) + (\delta_C^a - \delta_C^b)$$

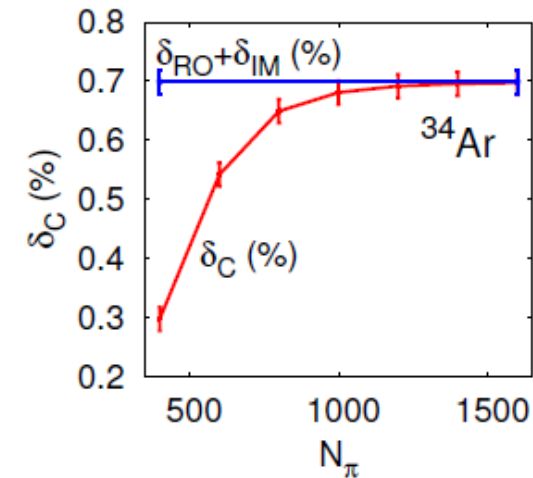
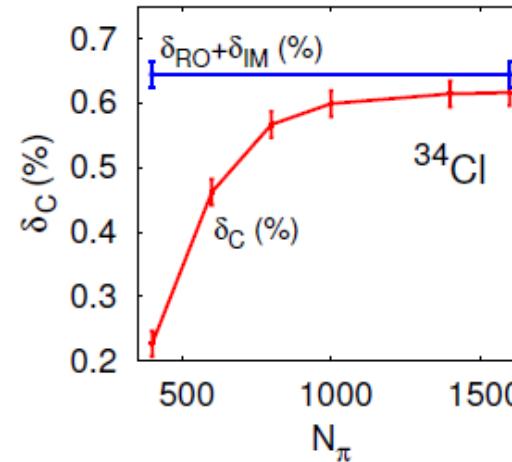
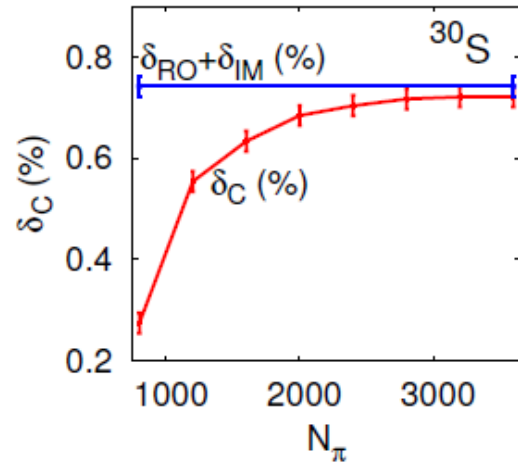
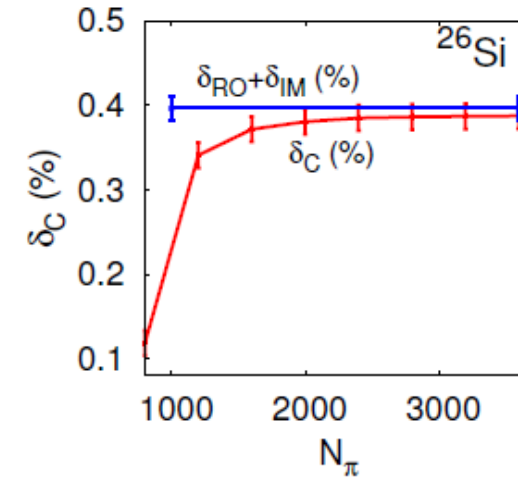
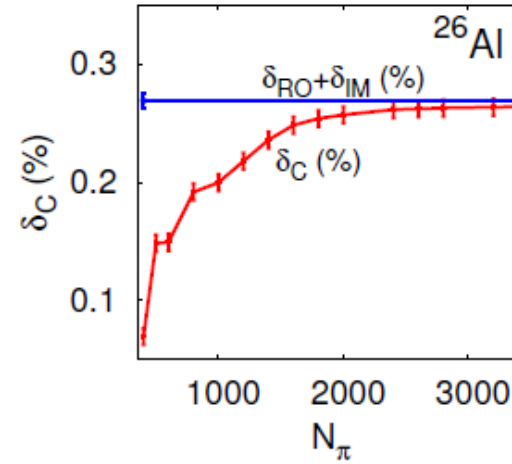
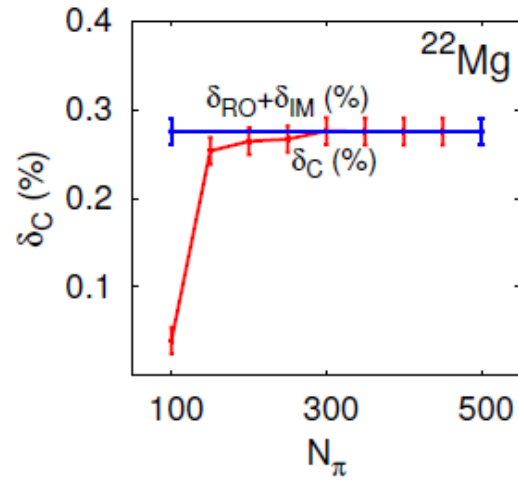
^{26m}Al , ^{26}Si , ^{34}Cl , ^{34}Ar , ^{38m}K , ^{38}Ca , ^{42}Sc , ^{42}Ti , ^{46}V , ^{46}Cr , ^{50}Mn , ^{50}Fe , ^{54}Co , ^{54}Ni



Preliminary !

Test of the separation ansatz of the isospin-symmetry breaking correction

δ_C is compared to $\delta_{C1} + \delta_{C2}$



Higher-order terms of the isospin-symmetry breaking correction

Exact evaluation of δ_C :

$$M_F = M_F^0 \left[1 - \frac{1}{M_F^0} \sum_{\alpha} \left(\rho_{\alpha}^T - \rho_{\alpha} \right) - \frac{1}{M_F^0} \sum_{\alpha} \rho_{\alpha}^T (1 - \Omega_{\alpha}) + \frac{1}{M_F^0} \sum_{\alpha} \left(\rho_{\alpha}^T - \rho_{\alpha} \right) (1 - \Omega_{\alpha}) \right],$$

$$|M_F|^2 = |M_F^0|^2 (1 - \delta_C), \quad \delta_C = \sum_{i=1}^6 \delta_{Ci}$$

Explicit expressions for the six terms:

These terms have similar structure when expressed beyond the closure approximation!

$$\delta_{C1} = \frac{2}{M_F^0} \sum_{\alpha} \left(\rho_{\alpha}^T - \rho_{\alpha} \right) \quad \text{LO}$$

$$\delta_{C2} = \frac{2}{M_F^0} \sum_{\alpha} \rho_{\alpha}^T (1 - \Omega_{\alpha}) \quad \text{LO}$$

$$\delta_{C3} = \frac{2}{M_F^0} \sum_{\alpha} \left(\rho_{\alpha}^T - \rho_{\alpha} \right) (1 - \Omega_{\alpha}) \quad \text{NLO}$$

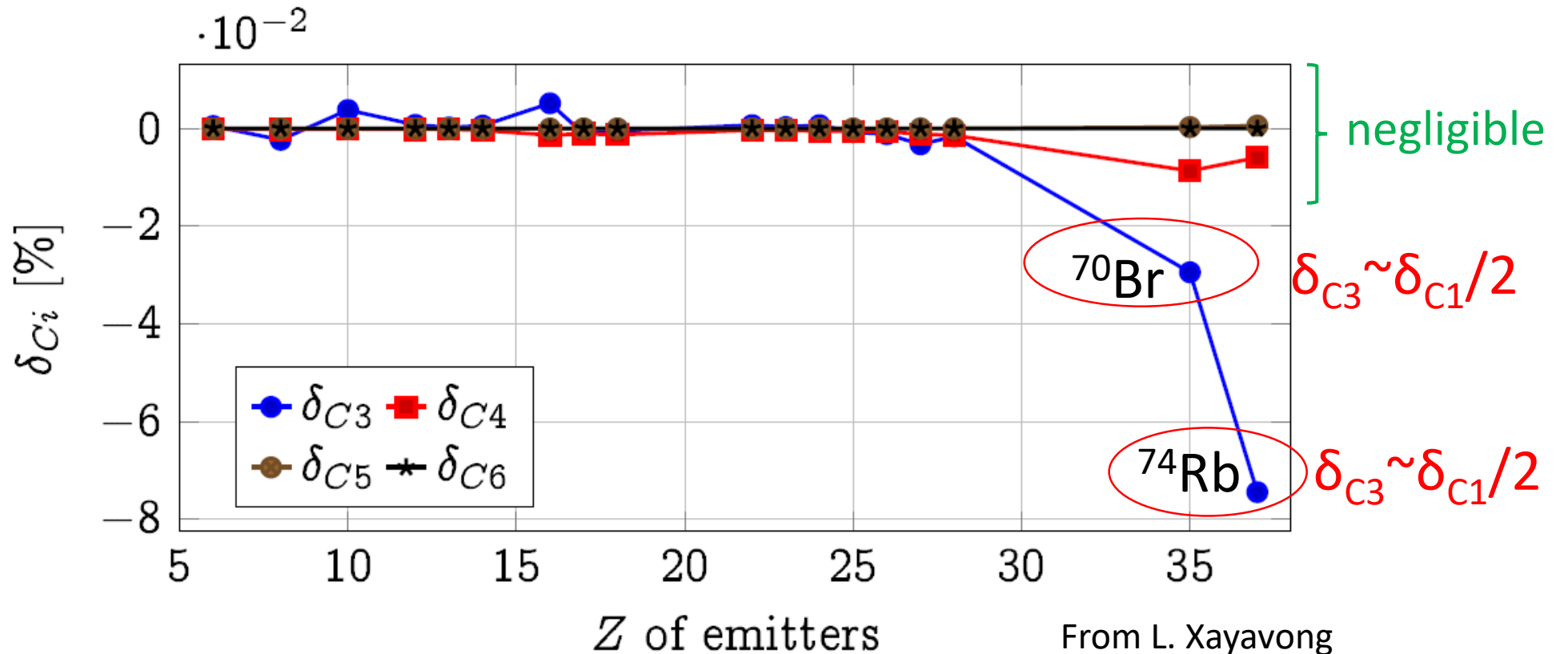
$$\delta_{C4} = -(\delta_{C1} + \delta_{C2})^2 / 4 \quad \text{NLO}$$

$$\delta_{C5} = -(\delta_{C1} + \delta_{C2}) \delta_{C3} / 2 \quad \text{N}^2\text{LO}$$

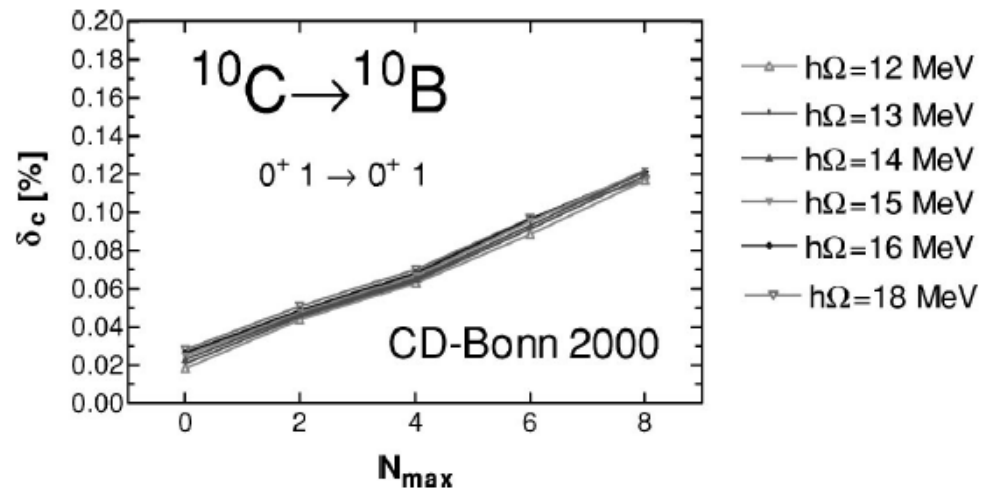
$$\delta_{C6} = -\delta_{C3}^2 / 4 \quad \text{N}^3\text{LO}$$

Higher-order terms of the isospin-symmetry breaking correction

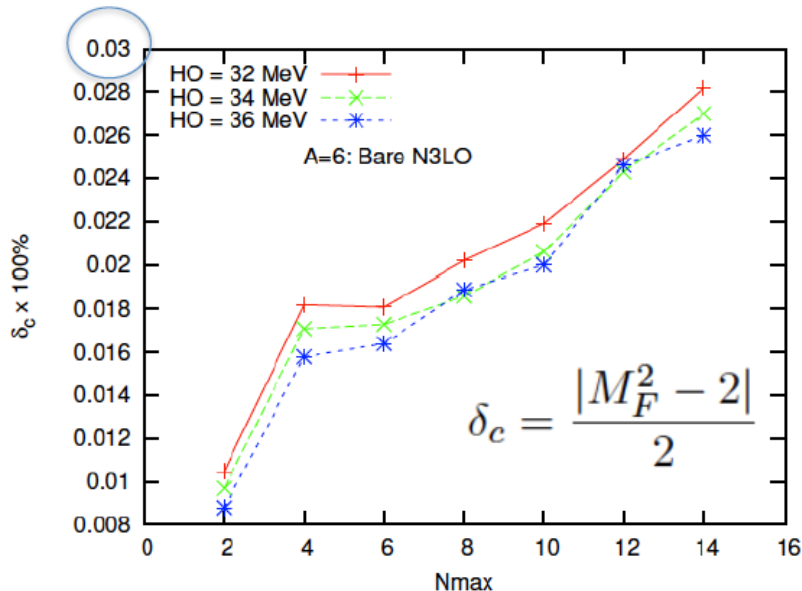
- Higher-order contribution (superallowed $0^+ \rightarrow 0^+$ Fermi transition)



Nuclear-structure correction to Fermi β decay: NCSM



Caurier, Navratil, Ormand, Vary (2002)



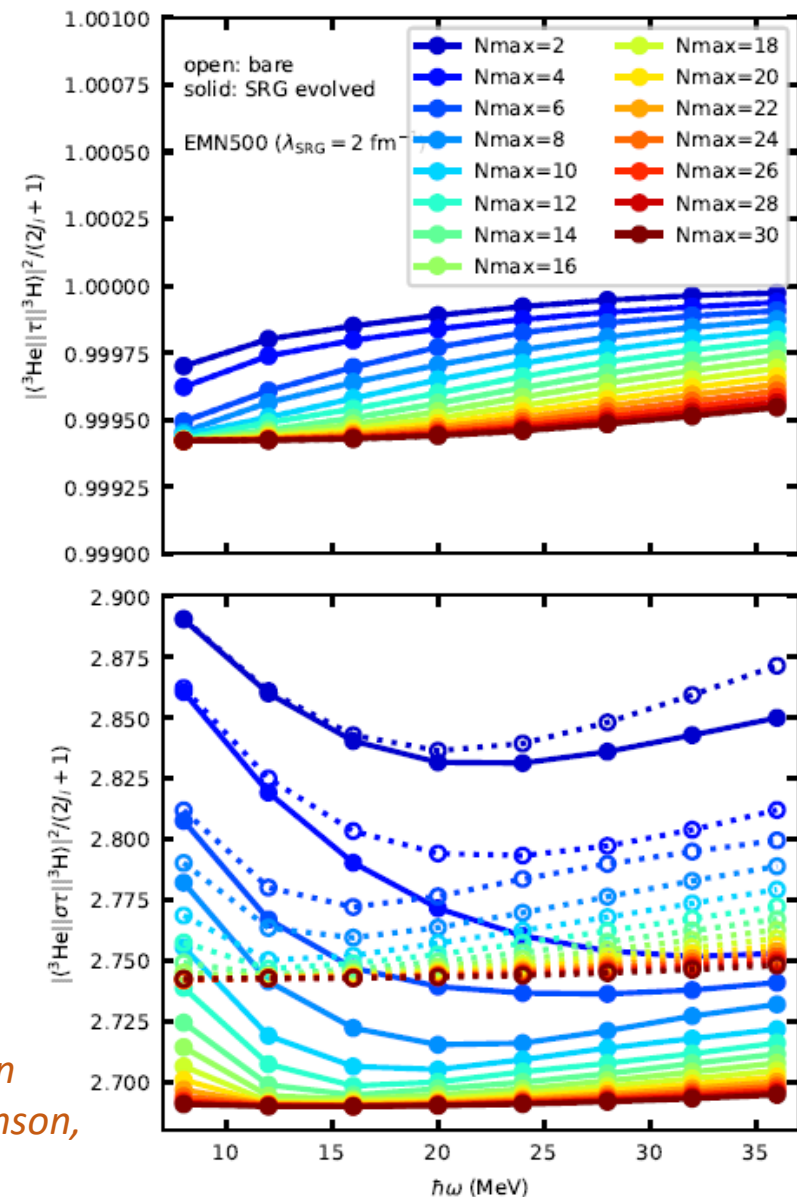
Kruse, Ormand, Johnson, Coon (2014)

Convergence?



$$\delta_c = 0.057\%$$

Miyagi, private communication
Xayavong, Lim, Smirnova, Johnson,
submitted



Conclusions

Results :

- ❑ Phenomenological shell model provides a robust framework in the description of isospin symmetry breaking
- ❑ Large-scale calculations with global WS parametrization lead to the results similar to those of Towner and Hardy (2015, 2020)
- ❑ New calculations with the Skyrme HF wave functions + numerous effects checked. Conclusion: importance of CSB/CIB terms, of elimination of spurious isospin mixing and post-HF effects
- ❑ Higher-order terms in the correction are identified, assessed and found to be negligible

Perspectives :

- ❑ The issue of the exact Fermi operator to be investigated via ab-initio methods (L. Xayavong et al, in progress)
- ❑ High-dimensions and heavier nuclei (near $A=80$)
- ❑ Improved charge-dependent interactions

BACK-UP SLIDES

Theoretical description of isospin-symmetry breaking

Sources of isospin symmetry breaking (nucleon level):

- Coulomb interaction between protons
 - Charge-dependent (CD) nuclear forces and Mp and Mn differences
-
- Ab-initio calculations: use CD NN potentials V (CD-Bonn, chiral EFT NⁿLO potentials)
 - Phenomenological approach: constrain effective CD forces from experiment to add to V

$$\hat{V}_{INC} = \hat{V}_{Coul} + \hat{V}_{CD} = \sum_{\lambda=0,1,2} \hat{V}_{INC}^{(\lambda)}, \quad \text{where} \quad \begin{cases} \hat{V}_{INC}^{(0)} = (v_{pp} + v_{nn} + v_{np}^{T=1})/3 \\ \hat{V}_{INC}^{(1)} = v_{pp} - v_{nn} \\ \hat{V}_{INC}^{(2)} = (v_{pp} + v_{nn})/2 - v_{np}^{T=1} \end{cases}$$

$$\langle \Psi_{TT_z} | \hat{V}_{INC} | \Psi_{TT_z} \rangle = E^{(0)}(\eta, T) + E^{(1)}(\eta, T) T_z + E^{(2)}(\eta, T) [3T_z^2 - T(T+1)]$$



Fit to experimental b and c coefficients through the valence space !

Nuclei along N=Z line: isospin-symmetry breaking

Isospin non-conserving Hamiltonian

(Coulomb + effective charge-dependent components)

$$H_{INC} = H_0 + V_{res} + V_{INC}$$

$$J^\pi \frac{T=3/2}{T_z=-3/2}$$

$$J^\pi \frac{T=3/2}{T_z=1/2}$$

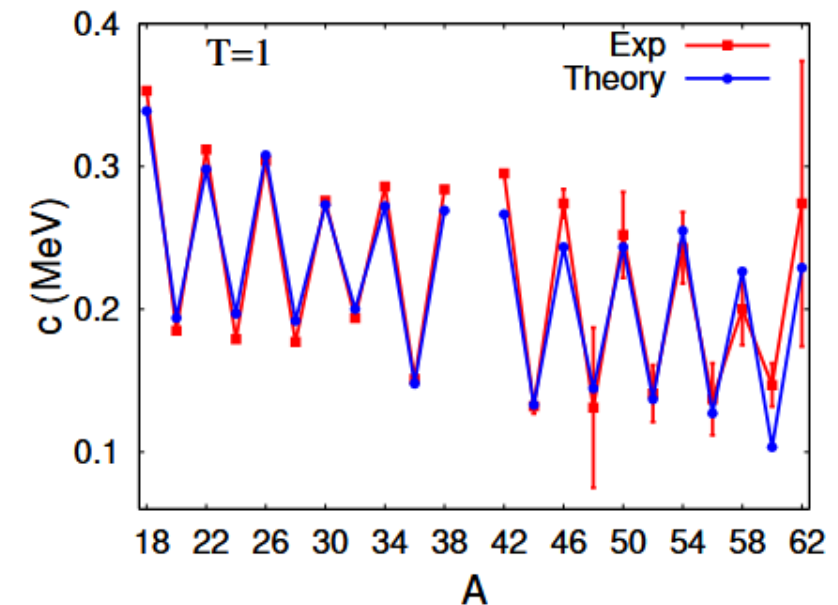
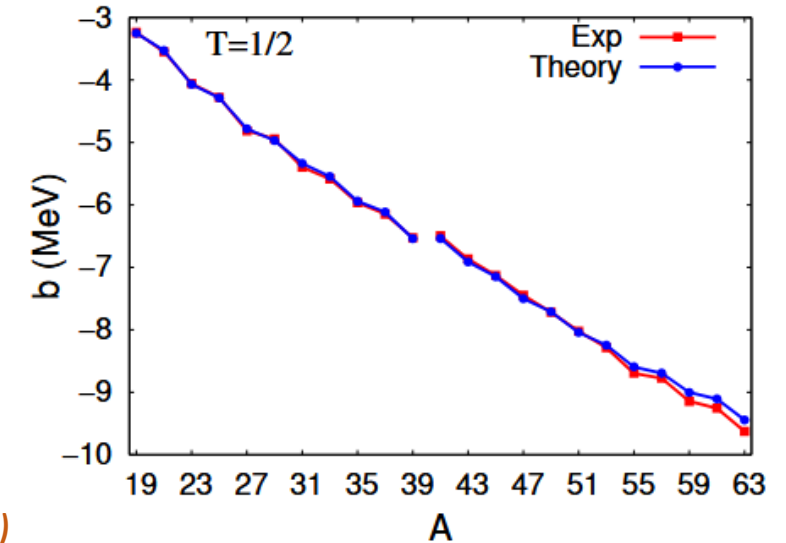
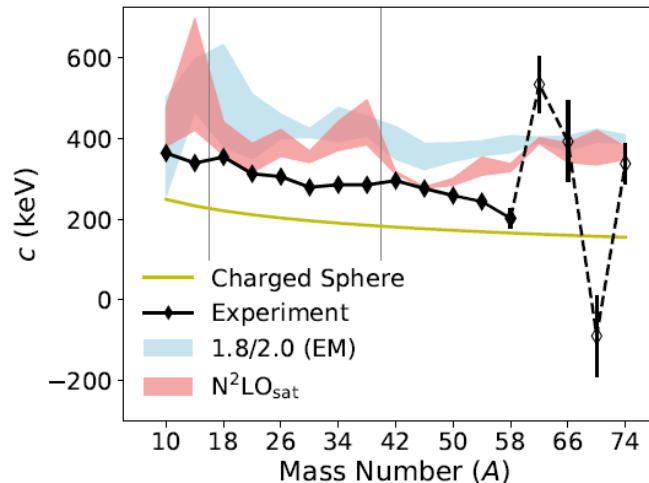
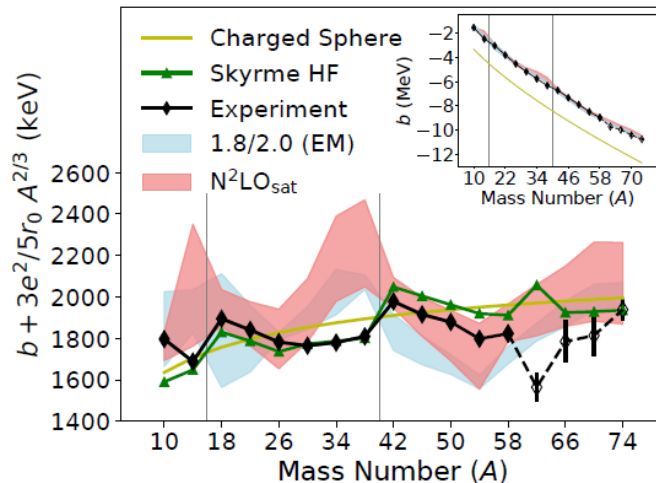
$$J^\pi \frac{T=3/2}{T_z=3/2}$$

$$M(\eta, T, M_T) = a(\eta, T) + b(\eta, T)T_z + c(\eta, T)T_z^2$$

sd shell : rms(b) = 30 keV, rms(c) = 10 keV

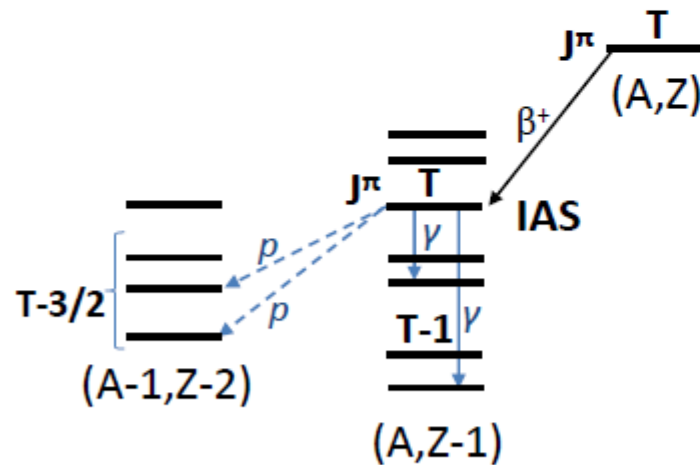
Ormand, Brown (1989), Lam, Smirnova, Caurier (2013); Magilligan, Brown (2020)

IMSRG: Martin et al, PRC104, 014324 (2021) – A=10,14,18,22,...,74

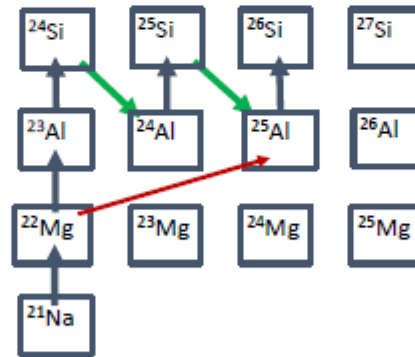


Applications of isospin-symmetry breaking

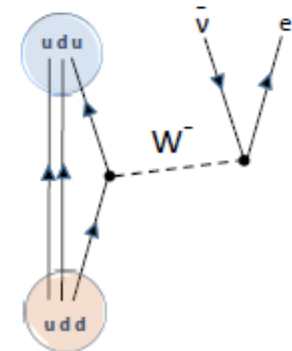
- Structure and decay modes of proton-rich nuclei and nuclei along $N = Z$



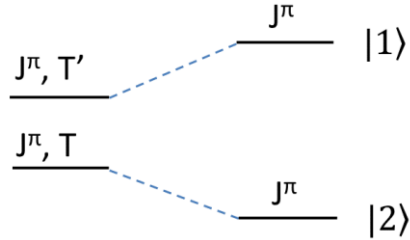
- Applications for nuclear astrophysics (masses, (p, γ) , (α, p) , ... reaction rates)



- Corrections to weak-interaction processes in nuclei for the tests of the Standard Model (CVC hypothesis, CKM matrix, ...)



Isospin-forbidden decays and isospin mixing



$$\alpha^2 \sim \frac{\langle V_{CD} \rangle^2}{\Delta E^2}$$

Current status :

- ❑ Reliable prediction of isospin-forbidden decay rates and of isospin mixing on **theoretical grounds** is challenging, because of poorly determined ΔE
- ❑ Safe predictions of **Coulomb mixing matrix elements**
- ❑ Efficient way: combination of **theory** and **experiment** !
- ❑ Lack of precision for the search of higher-order terms in IMME in quartets and quintets

$$M(\eta, T, T_z) = a(\eta, T) + b(\eta, T)T_z + c(\eta, T)T_z^2 + d(\eta, T)T_z^3 + e(\eta, T)T_z^4$$

Isospin-symmetry breaking: NN interaction and light nuclei

Sources of isospin-symmetry breaking : $m_u \neq m_d$ and electromagnetic interactions between quarks

- Coulomb interaction between protons
- $M_p - M_n \approx 1.3 \text{ MeV}$ and charge-dependent forces of nuclear origin

}

Experimental evidence on charge-dependent components of NN forces

1) NN scattering length in the $^1S_0$ channel:

$$a_{nn} - a_{pp} = 1.65 \pm 0.60 \text{ fm}$$

⇒ Charge-symmetry breaking (CSB)

$$V_{pp} - V_{nn} \neq 0$$

$$\frac{1}{2}(a_{nn} + a_{pp}) - a_{np} = 5.6 \pm 0.6 \text{ fm}$$

⇒ Charge-independence breaking (CIB)

$$V_{pn}^{T=1} - \frac{1}{2}(V_{pp} + V_{nn}) \neq 0$$

2) Nolen-Schiffer anomaly (1969): mirror binding energy differences ($^3\text{H} - ^3\text{He}$, ...)

Henley, Miller, 1979

Class I : $1, \mathbf{t}(j) \cdot \mathbf{t}(k)$

Class II : $t_z(j)t_z(k)$

Class III : $t_z(j) + t_z(k)$

Class IV : $t_z(j) - t_z(k), [\mathbf{t}(j) \times \mathbf{t}(k)]_z$

Ab-initio approaches



Light nuclei in *ab-initio* approaches :

$A = 3, 7, 8$: GFMC with CD interactions

Wiringa, Pastore, Pieper, Miller, PRC88 (2013)

G.A. Miller et al PR194, 1 (1990); χ EFT : U. van Kolck, J.L. Friar, E. Epelbaum, ...

$$V_I > V_{II} > V_{III} > V_{IV}$$

$$\hat{H}_{V-A} = \frac{G_V}{\sqrt{2}} \hat{j}_\mu^\dagger \hat{j}^\mu + h.c.$$

$$\hat{j}_\mu^\dagger = \hat{V}_\mu + \hat{A}_\mu$$

$$\hat{V}_\mu = i\bar{\psi}_p \left(g_V(k^2) \gamma_\mu + \frac{g_W(k^2)}{2m_N} \sigma_{\mu\nu} k_\nu + i g_S(k^2) k_\mu \right) \psi_n$$

$$\hat{A}_\mu = i\bar{\psi}_p \left(g_A(k^2) \gamma_\mu + \frac{g_T(k^2)}{2m_N} \sigma_{\mu\nu} k_\nu + i g_P(k^2) k_\mu \right) \gamma_5 \psi_n$$

Fermi matrix element within the shell model

Fermi β -decay matrix element

$$H|\Psi\rangle = E|\Psi\rangle \Rightarrow \begin{aligned} |\Psi_i\rangle &= \sum c_{ki} |\Phi_k\rangle \\ |\Psi_f\rangle &= \sum c_{kf} |\Phi_k\rangle \end{aligned}$$

Single-particle matrix
element (radial overlap)

$$M_F = \langle \Psi_f | T_+ | \Psi_i \rangle = \sum_{\alpha} \langle \Psi_f | a_{\alpha_n}^{\dagger} a_{\alpha_p} | \Psi_i \rangle \langle \alpha_n | t_+ | \alpha_p \rangle$$

$$\langle \Psi_f | a_{\alpha_n}^{\dagger} a_{\alpha_p} | \Psi_i \rangle \equiv \rho_{\alpha} \quad (\neq \rho_{\alpha}^T)$$

One-body transition
Density evaluated
between initial and
final nuclear states

$$\langle \alpha_n | t_+ | \alpha_p \rangle = \int_0^{\infty} R_{\alpha_n}(r) R_{\alpha_p}(r) r^2 dr \equiv \Omega_{\alpha} \quad (\neq 1)$$

$$\alpha = (n_{\alpha}, l_{\alpha}, j_{\alpha}, m_{\alpha})$$

Miller, Schwenk
PRC (2009,2010):
Radial excitations !

- Large-scale calculations
- Global parameterization of INC forces
- Revisit WS procedure
- Implicate HF wave functions

I.S. Towner, J.C. Hardy; (1973 – 2020)

W.E. Ormand, B.A. Brown (1985 – 1995)

Fermi matrix element within the shell model

Isospin-symmetry limit

$$M_F^0 = \sum_{\alpha} \rho_{\alpha}^T \Omega_{\alpha}^T = \pm \sqrt{T(T+1) - T_{zi} T_{zf}}, \quad \Omega_{\alpha}^T = 1$$

Realistic model (Coulomb and charge-symmetry breaking effective nuclear forces)

$$M_F = \sum_{\alpha} \rho_{\alpha} \Omega_{\alpha}$$

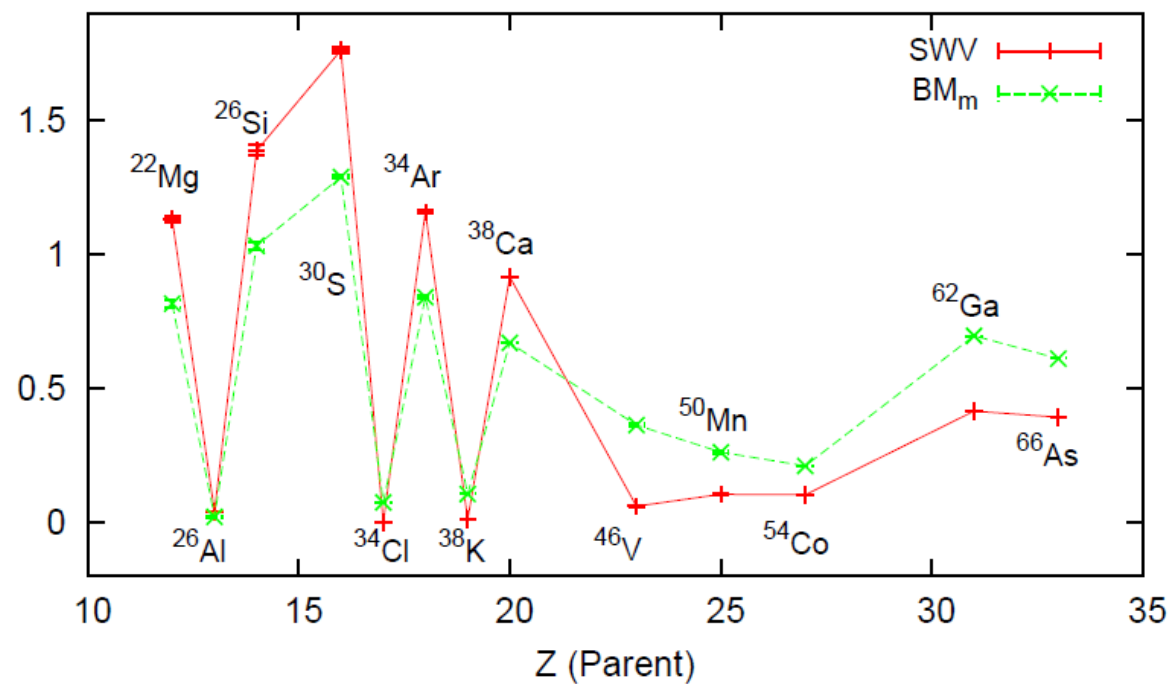
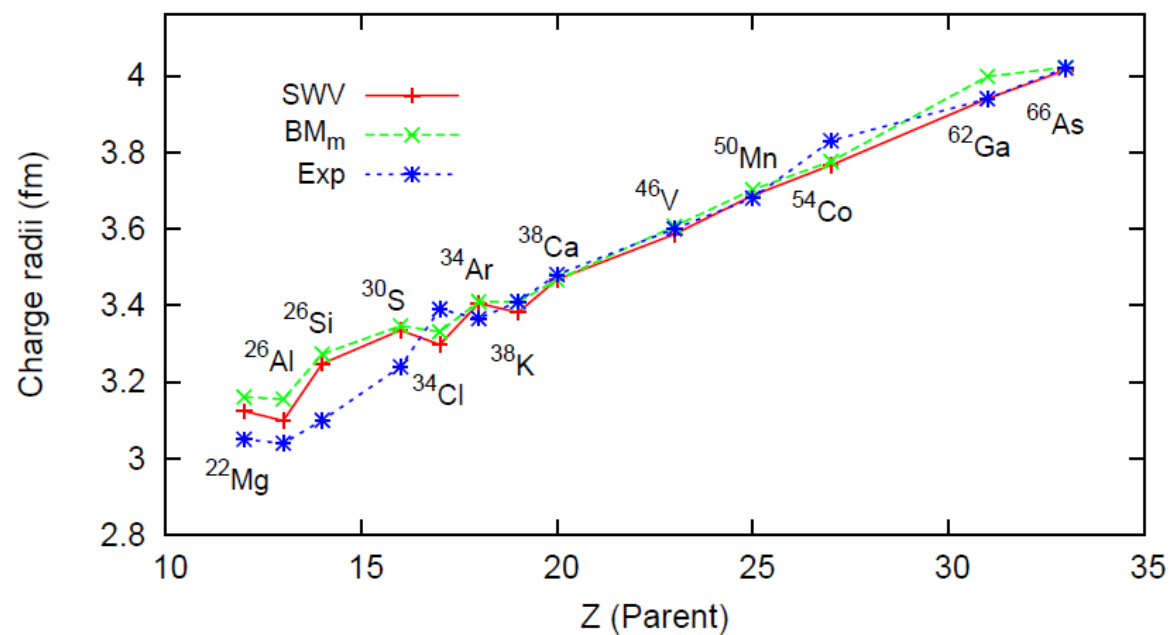
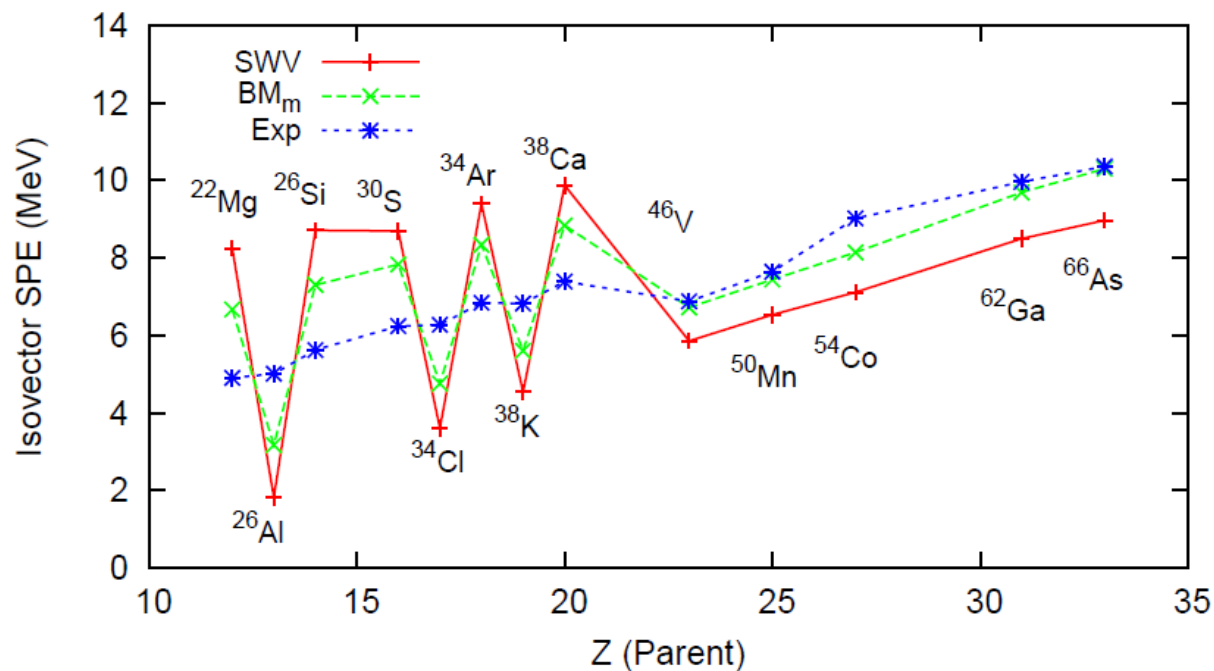
In first order perturbation theory:

$$|M_F|^2 \approx |M_F^0|^2 \left[1 - \underbrace{\frac{2}{M_F^0} \sum_{\alpha} (\rho_{\alpha}^T - \rho_{\alpha})}_{\delta_{C1}} - \underbrace{\frac{2}{M_F^0} \sum_{\alpha} \rho_{\alpha}^T (1 - \Omega_{\alpha})}_{\delta_{C2}} \right],$$

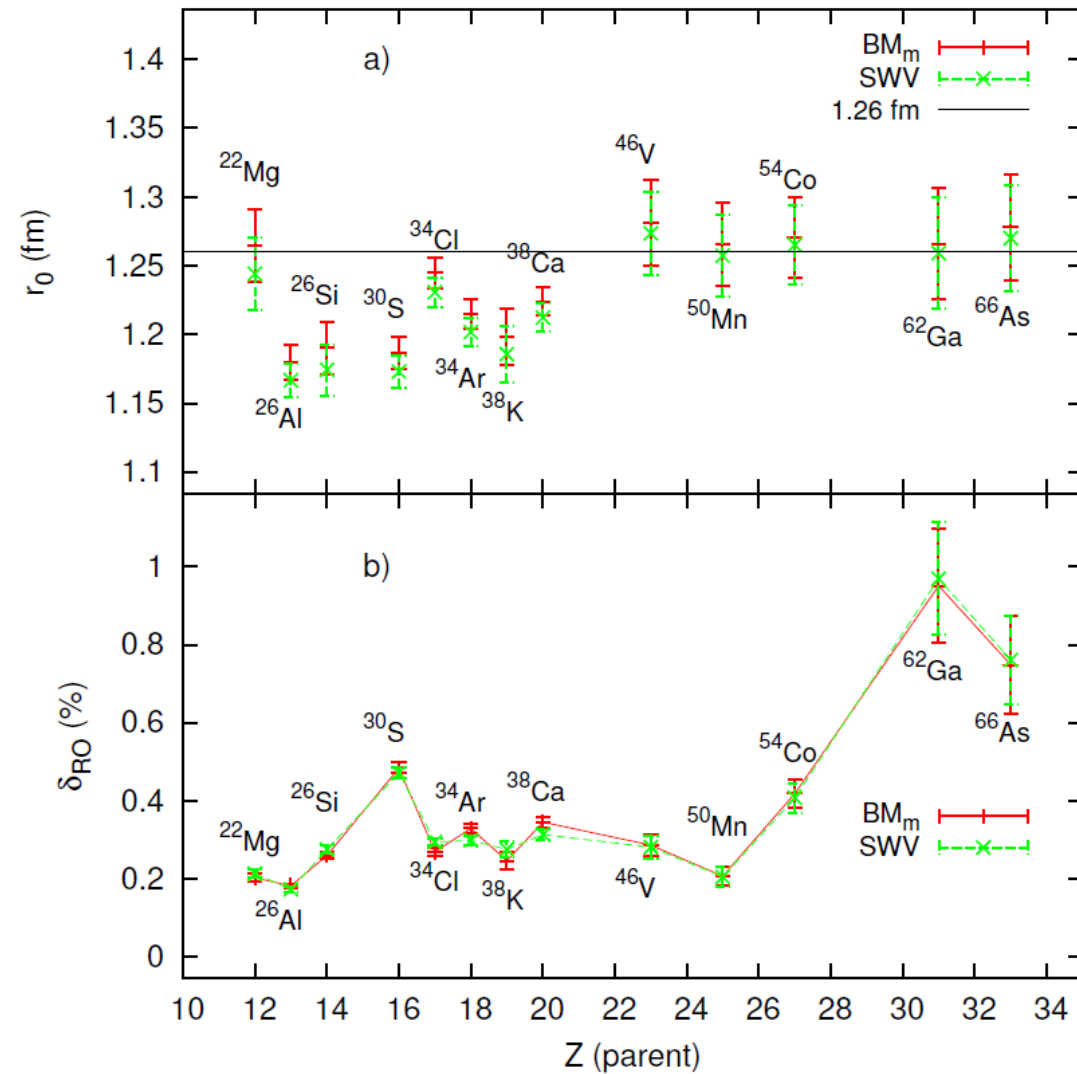
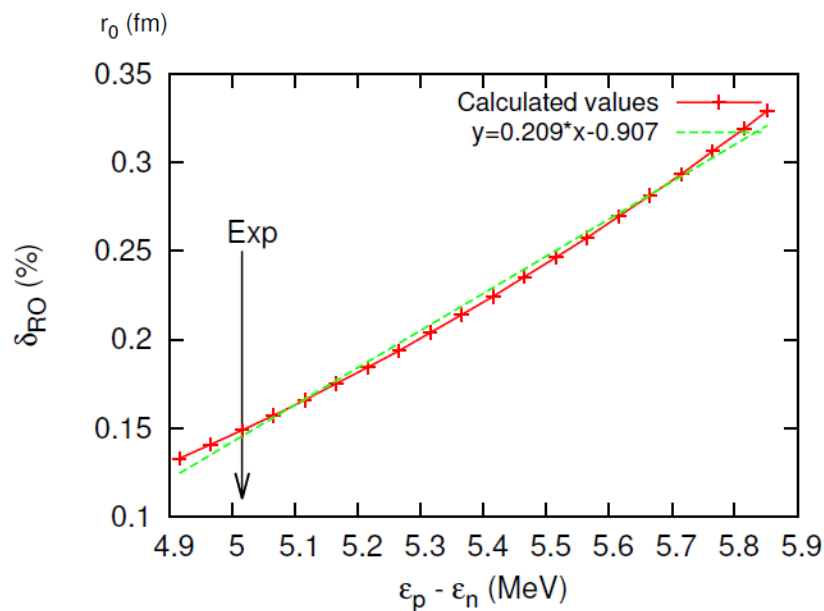
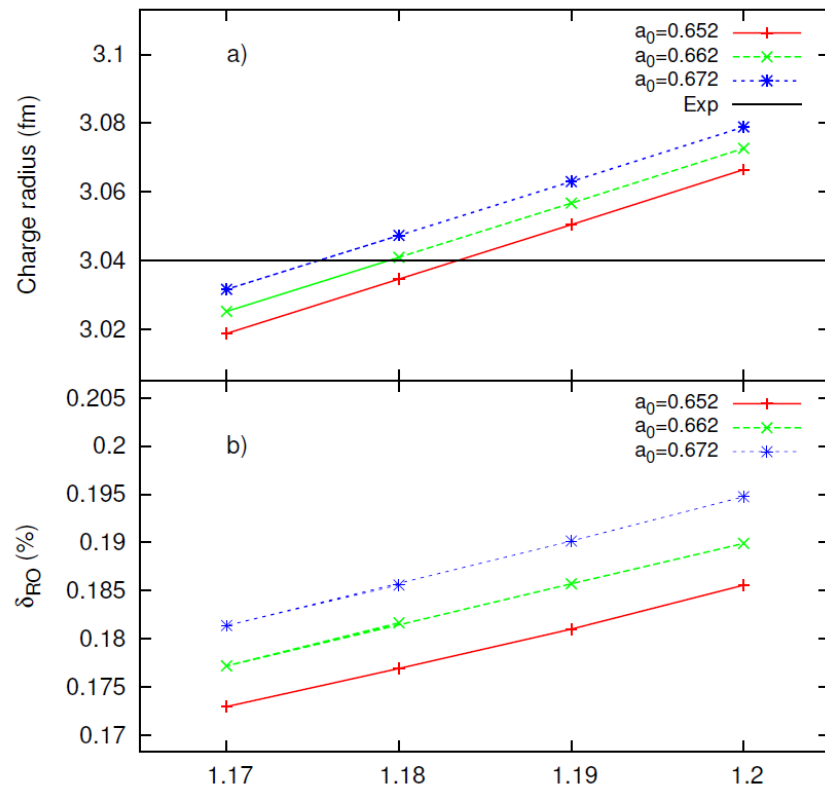
$$\delta_C \approx \delta_{C1} + \delta_{C2}$$

- δ_{C1} is the *isospin-mixing* part
- δ_{C2} is the *radial-overlap* part

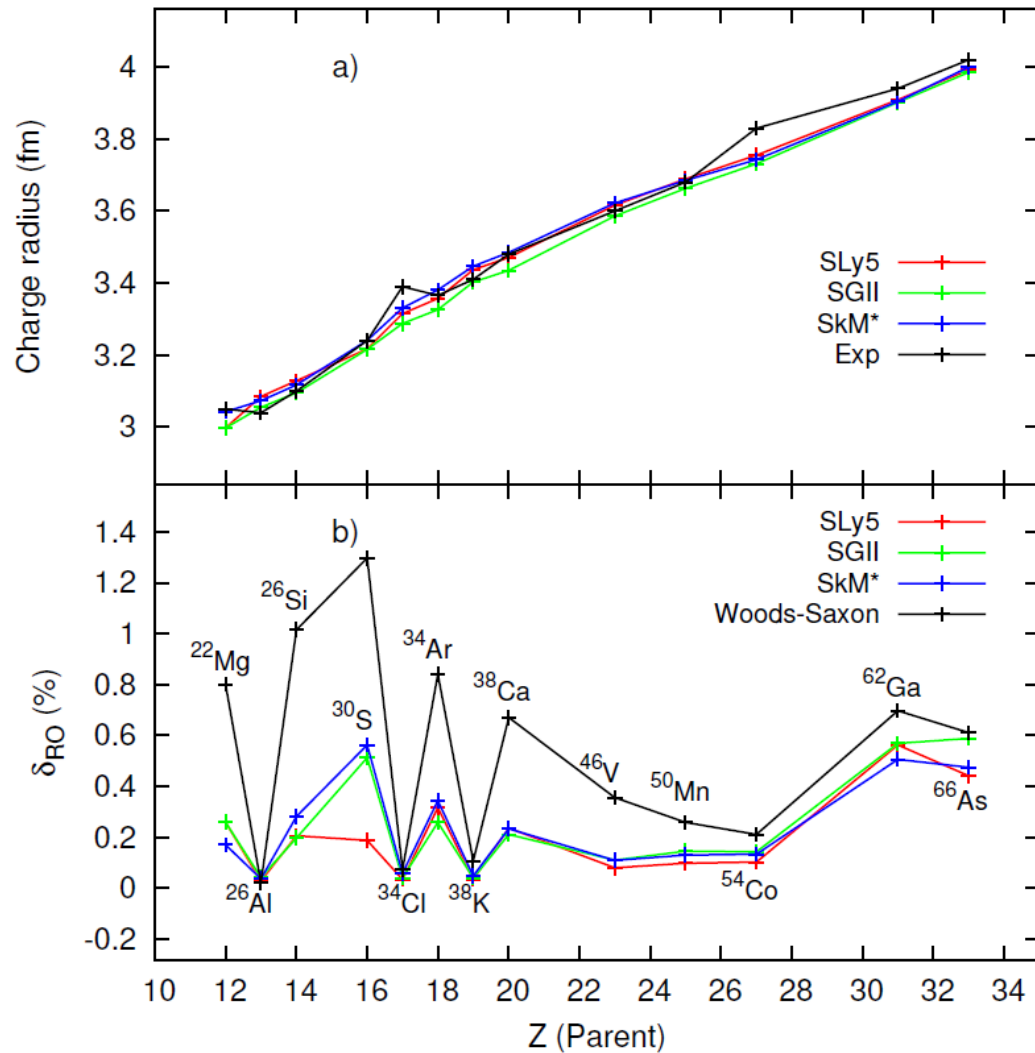
WS Non-adjusted !



WS Adjusted !



WS and HF Non-adjusted !



WS and HF Adjusted !

