Isospin-symmetry breaking and nuclear beta decay (Theory under experimental constraints)

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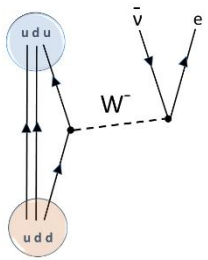


Topic program: Future directions in **Nuclear Beta Decay at FRIB**
FRIB, East Lansing, MI, USA, September 8 – 19, 2025

Isospin-symmetry breaking and nuclear beta decay

I.

- Background: Isospin Symmetry and Its Breaking in Nuclear Structure
- Shell-Model Formalism for isospin-symmetry breaking: state-of-the-art and application to nuclear structure and nuclear astrophysics

II.

- Isospin-symmetry breaking correction to Superallowed Fermi beta decay δ_C within shell model + WS or Hartree-Fock wave functions

  L. Xayavong, N. Smirnova, Phys. Rev. C97, 024324 (2018)

  L. Xayavong, N. Smirnova, Phys. Rev. C105, 044308 (2022)
- Higher-order terms in δ_C

  L. Xayavong, N. Smirnova, Phys. Rev. C109, 014317 (2024)
- Towards *ab-initio* No-core shell-model calculation of isospin-symmetry breaking correction δ_C

  L. Xayavong, Y. Lim, N. Smirnova, C. Johnson, submitted.
- Conclusions and perspectives

I. Isospin-symmetry breaking and shell model description

Isospin Symmetry

W. Heisenberg (1932)

$$\psi_n(\vec{r}) = \psi(\vec{r})\begin{pmatrix}1\\0\end{pmatrix},\ \psi_p(\vec{r}) = \psi(\vec{r})\begin{pmatrix}0\\1\end{pmatrix}$$

$t = 1/2$ $\quad$ $t = 1/2$

$t_z = +1/2$ $\quad$ $t_z = -1/2$

$SU_T(2)$

$$[\hat{t}_j, \hat{t}_k] = i\epsilon_{jkl}\hat{t}_l\,, \quad [\hat{\mathbf{t}}^2, \hat{t}_j] = 0$$

$$\hat{t}_x = \frac{1}{2}\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix},\quad \hat{t}_y = \frac{1}{2}\begin{pmatrix}0 & i\\-i & 0\end{pmatrix},\quad \hat{t}_z = \frac{1}{2}\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$$

Electric Charge operator

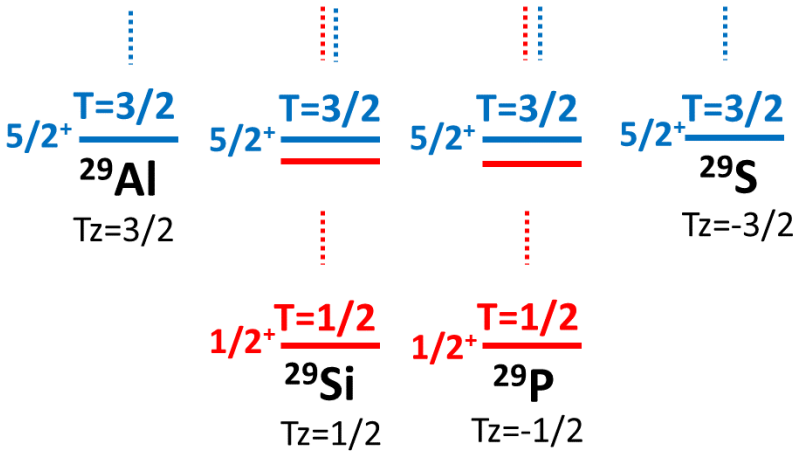
$$\hat{q} = \left(\frac{1}{2} - \hat{t}_z\right)e \quad \Longrightarrow \quad \hat{q}\psi_n(\vec{r}) = 0,\quad \hat{q}\psi_p(\vec{r}) = e\psi_p(\vec{r})$$

Operators useful for Beta decay

$$\hat{t}_\pm = \hat{t}_x \pm i\hat{t}_y\,. \quad \Longrightarrow \quad \begin{aligned}\hat{t}_+\psi_p(\vec{r}) &= \psi_n(\vec{r}), & \hat{t}_+\psi_n(\vec{r}) &= 0\\ \hat{t}_-\psi_n(\vec{r}) &= \psi_p(\vec{r}), & \hat{t}_-\psi_p(\vec{r}) &= 0\end{aligned}$$

Isospin Symmetry in a Many-Body System

E.P. Wigner (1937)



$$\hat{\mathbf{T}} = \sum_{k=1}^{A} \hat{\mathbf{t}}(k), \qquad \hat{T}_{\pm} = \sum_{k=1}^{A} \hat{t}_{\pm}(k), \quad \hat{T}_z = \sum_{k=1}^{A} \hat{t}_z(k)$$

A charge-independent Hamiltonian commutes with isospin operators

$$[\hat{H}_{nucl}, \hat{T}_{\pm}] = [\hat{H}_{nucl}, \hat{T}_z] = 0. \quad \Longrightarrow \quad |\eta T T_z\rangle$$

$$T_z = \frac{N - Z}{2},$$

$$T = T_z, T_Z + 1, \ldots, A/2$$

Consequences of isospin symmetry:

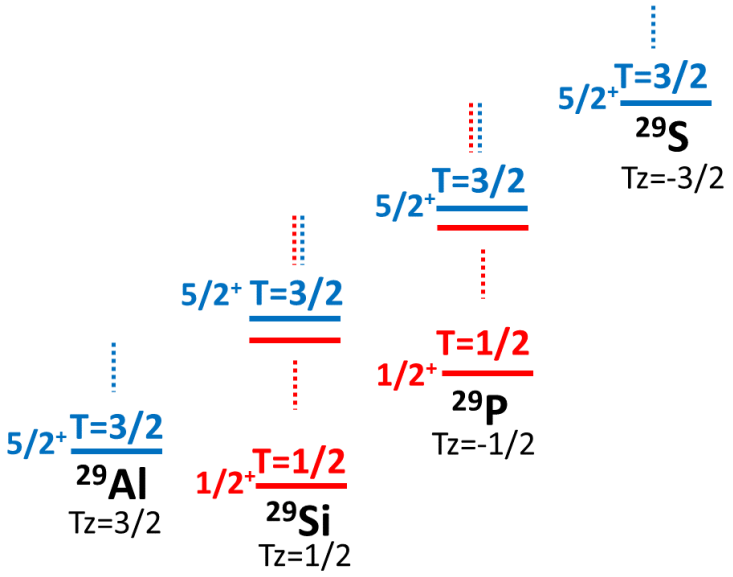
- Degenerate isobaric multiplets
- Isospin selection rules for transition operators

Dynamical breaking of Isospin Symmetry

E.P. Wigner (1957)

$$\hat{V}_{Coul} = \sum_{i<k}^{A} \left(\frac{1}{2} - \hat{t}_z(i)\right)\left(\frac{1}{2} - \hat{t}_z(k)\right) \frac{e^2}{|\vec{r}(i) - \vec{r}(k)|}$$

$$= \sum_{i<k}^{A} \left\{ \underbrace{\left[\frac{1}{4} + \frac{1}{3}\hat{\mathbf{t}}(i)\hat{\mathbf{t}}(k)\right]}_{isoscalar} - \underbrace{\frac{1}{2}(\hat{t}_z(i) + \hat{t}_z(k))}_{isovector} + \underbrace{\left[\hat{t}_z(i)\hat{t}_z(k) - \frac{1}{3}\hat{\mathbf{t}}(i)\hat{\mathbf{t}}(k)\right]}_{isotensor} \right\} \frac{e^2}{|\vec{r}(i) - \vec{r}(k)|}$$



$$\langle \eta T T_z | \hat{V}_{Coul} | \eta T T_z \rangle = E^{(0)}(\eta, T) + E^{(1)}(\eta, T) T_z + E^{(2)}(\eta, T)\left[3T_z^2 - T(T+1)\right]$$

$$\boxed{\mathcal{M}(\eta, T, T_z) = a(\eta, T) + b(\eta, T) T_z + c(\eta, T) T_z^2,}$$

$SU_T(2) \supset SO_T(2)$ Quantum numbers are conserved!

Selection rules (still hold under dynamical breaking): examples

Fermi and Gamow-Teller beta decay operators:

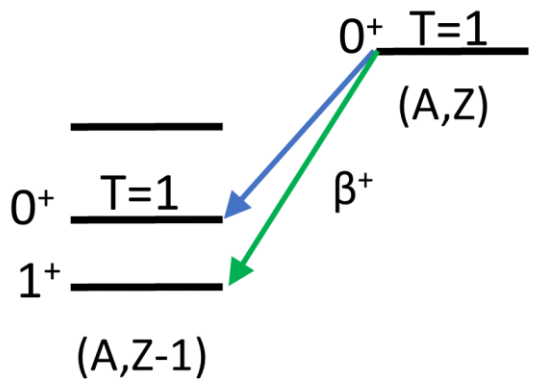
$$\hat{O}_F(\beta^\pm) = \sum_{k=1}^{A} \hat{t}_\pm(k)\,, \quad \hat{O}_{GT}(\beta^\pm) = \sum_{k=1}^{A} \hat{\boldsymbol{\sigma}}(k)\hat{t}_\pm(k)\,.$$

$$\Delta J = 0, \Delta T = 0, \Delta\pi = 0$$

$$|M_F| = \sqrt{T(T+1) - T_{iz}T_{fz}}$$

$$\Delta J = 0{,}1, \Delta T = 0{,}1, \Delta\pi = 0$$

no $J = 0 \rightarrow J = 0$

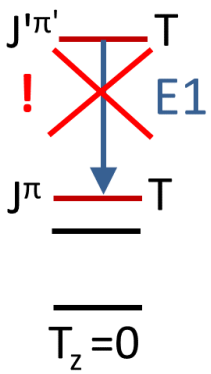


Electric dipole (E1) operator is purely isovector (one-body):

$$\hat{O}(\mathrm{E1}) = \sum_{k=1}^{A} e(k)\vec{r}(k) = \sum_{k=1}^{A}\left(\frac{1}{2} + \hat{t}_z(k)\right)e\vec{r}(k)$$

$$(T010|T0) = 0$$

Transitions between states with the same T
In N=Z nuclei are isospin-forbidden!



Selection rules for Electromagnetic operators

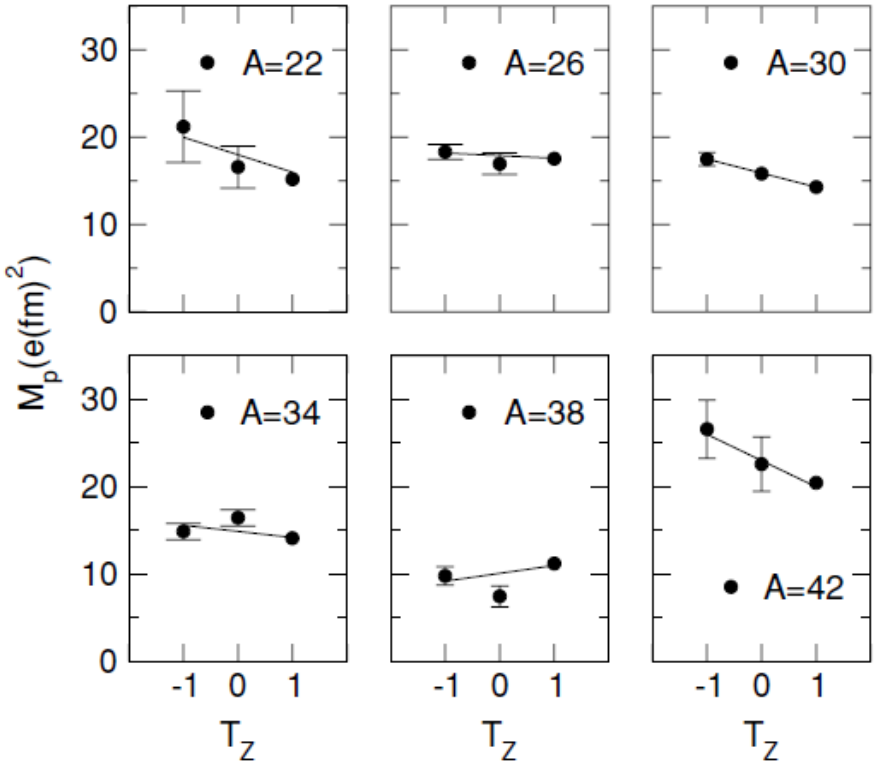
One-body transition operator (except for E1):

Warburton, Wesener (1968)

$$\hat{O}_{LM} = \hat{O}^{(0)}_{LM} + \hat{O}^{(1)}_{LM}$$

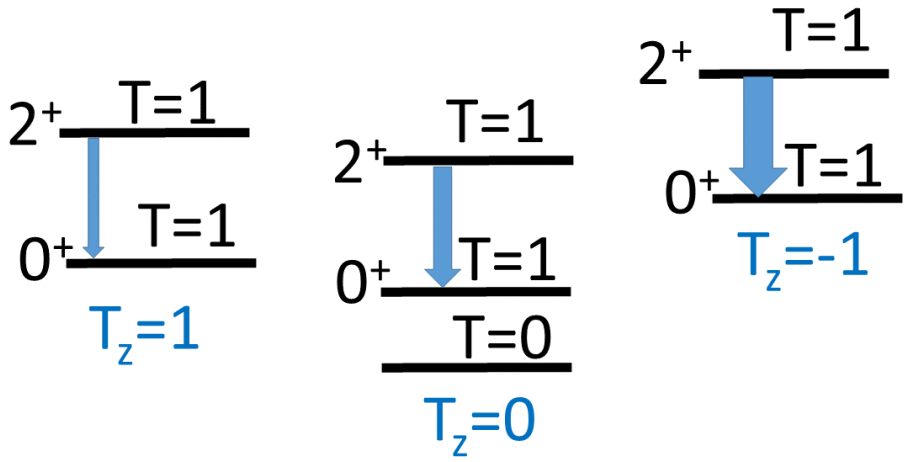
$$\langle J_f M_f; T T_z | \hat{O}_{LM} | J_i M_i; T T_z \rangle = \langle J_f M_f | \hat{O}^{(0)}_{LM} | J_i M_i \rangle + \frac{T_z}{\sqrt{T(T+1)(2T+1)}} \langle J_f M_f; T || \hat{O}^{(1)}_{LM} || J_i M_i; T \rangle .$$

$$\boxed{\langle J_f M_f T T z | \hat{O} | J_i M_i T T z \rangle = A + B T_z}$$



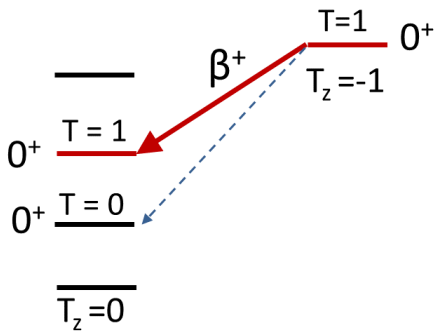
Prados-Estevez et al, PRC75 (2007)

Example of E2 transitions in an Isobaric triplet:

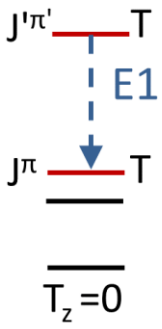


Explicit breaking of isospin symmetry

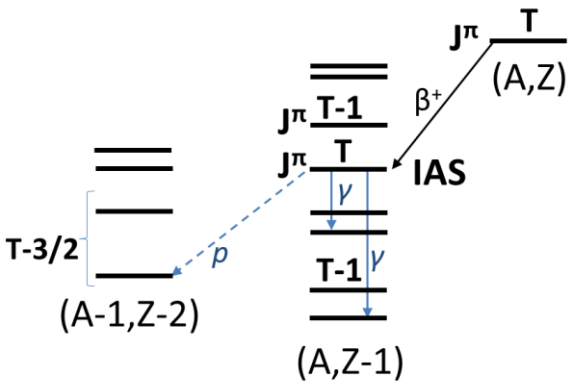
Observation of (typically weak) processes forbidden by isospin symmetry :



Forbidden Fermi beta decay



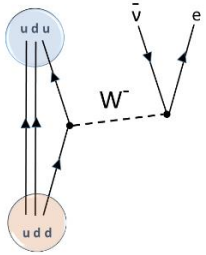
Isospin-forbidden
E1 transitions

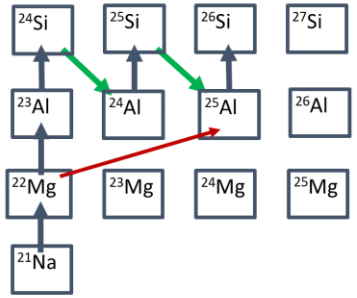


Isospin-forbidden nucleon(s) emission

These are small effects, but important for

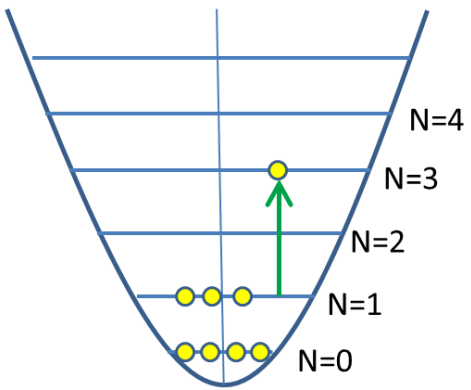
- Structure and decay modes of proton-rich nuclei
- Fundamental interaction studies (theoretical corrections to nuclear matrix elements)
- Astrophysics applications : proton/alpha capture/emission on proton-rich nuclei (p,γ), (p,α), (α,p), …





No-Core Shell model - configuration-interaction approach

$$H = T_{rel} + V_{NN}$$



$$H|\Psi_n\rangle = E_n|\Psi_n\rangle$$

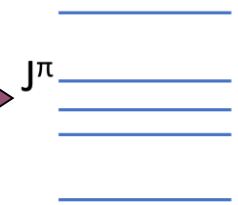
Harmonic Oscillator Basis

$$|\Psi_n\rangle = \sum_k c_{kn}\,|\Phi_k\rangle$$

$$\langle\Phi_k|\Phi_l\rangle = \delta_{kl}$$

$$\sum_{k=1}^{d} \langle\Phi_l|H|\Phi_k\rangle\, c_{kn} = E_n c_{ln}$$

$$\begin{pmatrix} H_{11} & H_{12} & \cdots & H_{1d} \\ H_{21} & H_{22} & \cdots & H_{2d} \\ \vdots & & \ddots & \\ H_{d1} & H_{d2} & \cdots & H_{dd} \end{pmatrix}$$

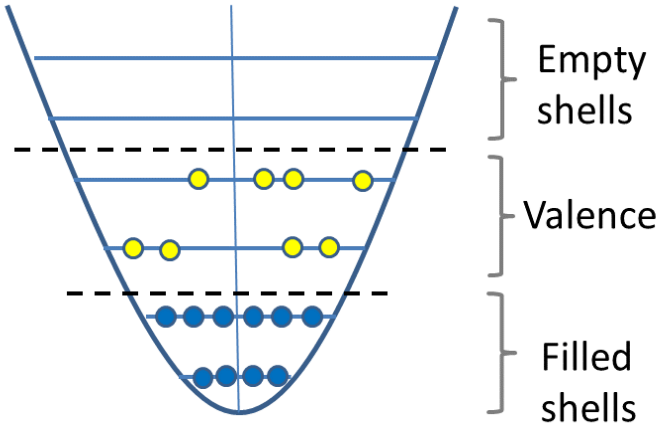


***Ab-initio* No-Core Shell Model : sufficiently large model space so that the results for A nucleons do not depend on the basis parameters (hw and Nmax)**

Conservation of symmetries of the Hamiltonian, detailed information on low-energy states and transitions

Valence-space shell model for heavier nuclei

Restricted model space



Effective operators

$$H|\Psi_n\rangle = E_n|\Psi_n\rangle \quad \longrightarrow \quad H_{eff}|\Psi_n^M\rangle = E_n|\Psi_n^M\rangle$$

$$H_{eff} = T + V_{eff} = \underbrace{(T+U)}_{\text{exactly solvable}} + \underbrace{(V_{eff} - U)}_{\text{residual interaction}} = H_0 + V_{res}$$

$$H_{eff} = \sum_\alpha \varepsilon_\alpha a_\alpha^\dagger a_\alpha + \frac{1}{4}\sum_{\alpha\beta\gamma\delta}\langle\alpha\beta|V_{res}|\delta\gamma\rangle\, a_\alpha^\dagger a_\beta^\dagger\, a_\gamma\, a_\delta$$

Empirical

Microscopic derived from NN potential

Phenomenological (constrained by the data)

- **Current status :**
- Excellent description with empirical (phenomenological) interactions
- Microscopic interactions -> recent progress and challenges

Isospin-symmetry breaking: NN interaction and light nuclei

Sources of isospin-symmetry breaking : $m_u \neq m_d$ and electromagnetic interactions between quarks

- Coulomb interaction between protons
- $M_p - M_n \approx 1.3\ MeV$ and charge-dependent forces of nuclear origin

} Nucleon level

Experimental evidence on charge-dependent components of NN forces

1) NN scattering length in the $^1S_0$ channel:

$$a_{nn} - a_{pp} = 1.65 \pm 0.60\ fm$$ ⟹ *Charge-symmetry breaking (CSB)* $$\boldsymbol{V_{pp} - V_{nn} \neq 0}$$

$$\frac{1}{2}(a_{nn} + a_{pp}) - a_{np} = 5.6 \pm 0.6\ fm$$ ⟹ *Charge-independence breaking (CIB)* $$\boldsymbol{V_{pn}^{T=1} - \frac{1}{2}(V_{pp} + V_{nn}) \neq 0}$$

2) Nolen-Schiffer anomaly (1969): mirror binding energy differences (^3H – ^3He , …)

Modern high-precision potentials are charge-dependent (CD-Bonn, Argonne V18, chiral EFT, etc

⟹ *Ab-initio approaches (NCSM, GFMC,..*

Light nuclei in *ab-initio* approaches :

A = 3,7,8: GFMC with CD interactions

Wiringa, Pastore, Pieper, Miller, PRC88 (2013)

G.A. Miller et al PR194, 1 (1990); R. Machleidt, χEFT : U. van Kolck, J.L. Friar, R. Machleidt, E.Epelbaum, …

Valence-space isospin non-conserving (INC) Shell Model

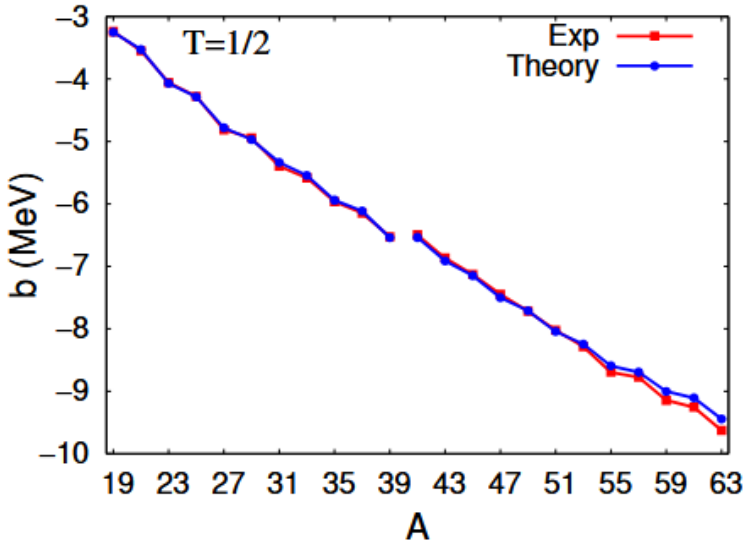
We include Coulomb + effective charge-dependent components perturbatively

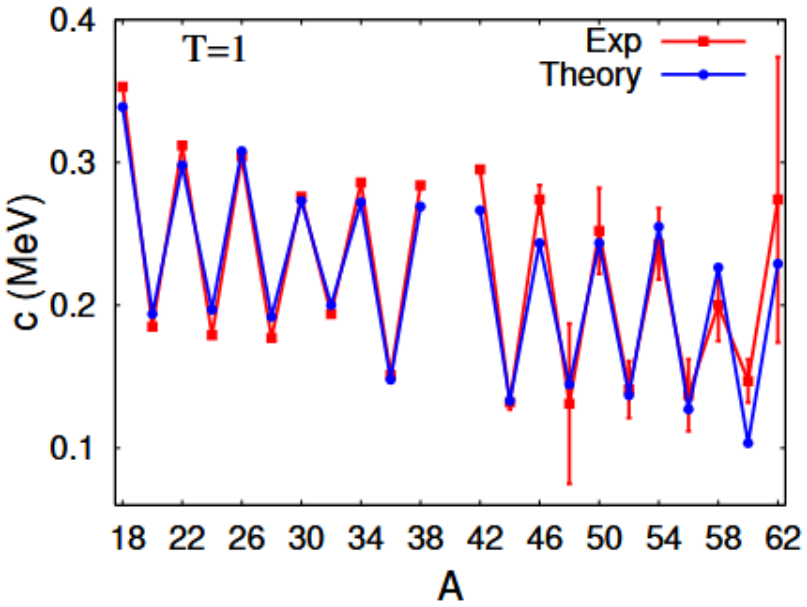
$$H_{INC} = H_0^{pn} + V_{res} + \Delta V_{CD}$$ with $$\Delta V_{CD} = V_C + V_{CSB}^{(1)} + V_{CIB}^{(2)}$$

$$\langle \eta T T_z | H_{INC} | \eta T T_z \rangle = a^{th}(\eta, T) + b^{th}(\eta, T) T_z + c^{th}(\eta, T) T_z^2$$

- **Phenomenological Hamiltonians** (still more accurate)
  - fit of isovector s.p.e's and V_{CD} (3 parameters) to experimental *b* and *c* coefficients

Ormand, Brown, NPA491 (1989); Lam, Smirnova, Caurier, PRC87 (2013); Magiligan, Brown (2020),..





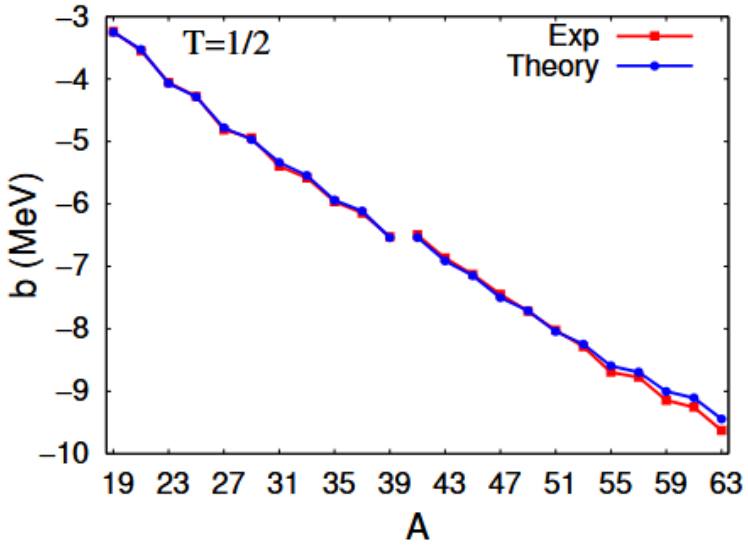
RMS errors (theory & experiment) :

	b (keV)	c (keV)	N data (b)	N data (c)
sd shell	32	11	81 (all T)	51 (T=1,3/2,2)
pf shell	50	20	g.s. (T=1/2,1)	g.s. (T=1)

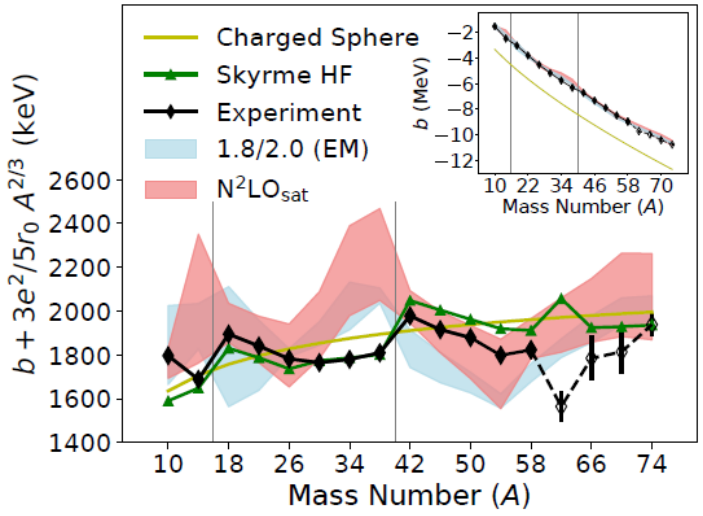
Nuclei along N=Z line: isospin-symmetry breaking

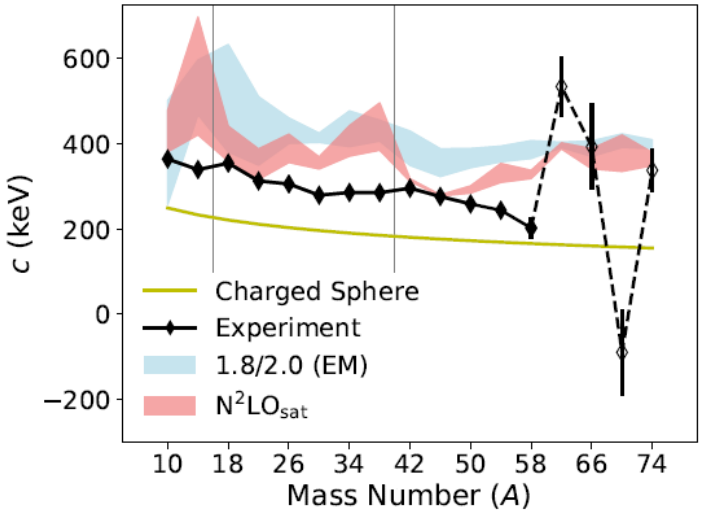
$$M(\eta, T, M_T) = a(\eta, T) + b(\eta, T)T_z + c(\eta, T)T_z^2$$

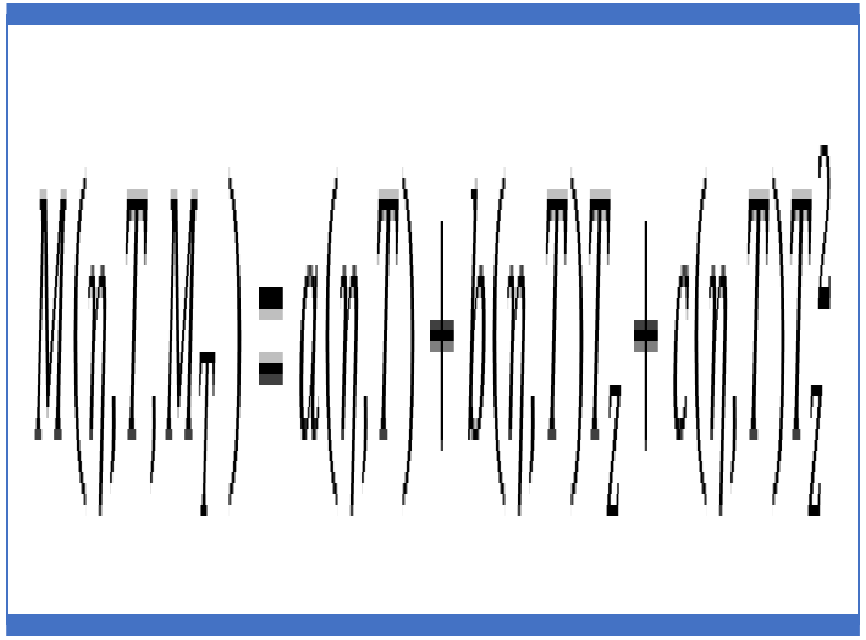
	b (keV)	c (keV)	N data (b)	N data (c)
sd shell	32	11	81 (all T)	51 (T=1,3/2,2)
pf shell	50	20	g.s. (T=1/2,1)	g.s. (T=1)



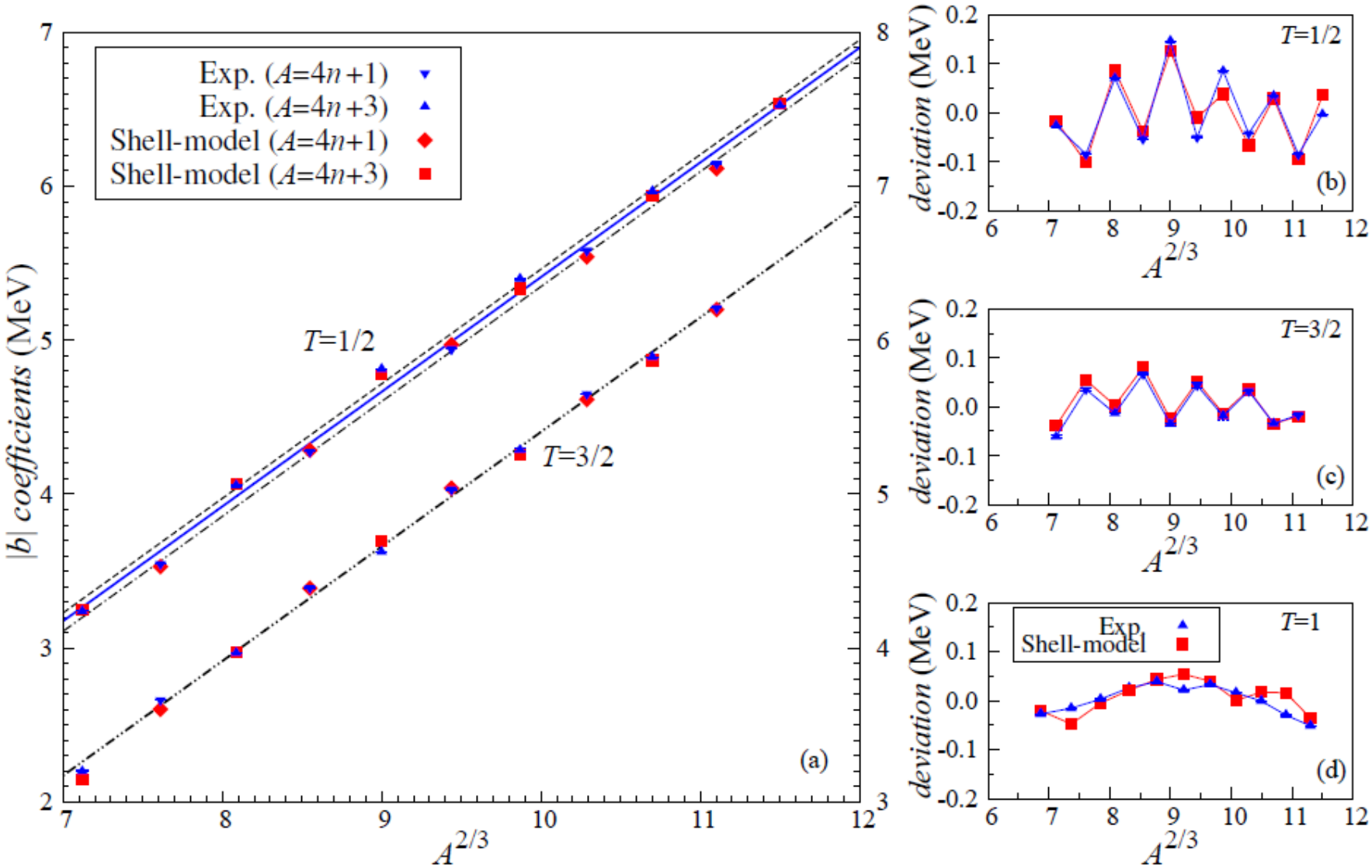
IMSRG: Martin et al, PRC104, 014324 (2021) – A=10,14,18,22,…,74







Theoretical description of isospin-symmetry breaking



Lam, Smirnova, Caurier (2013)

IMME b and c coefficients of lowest and excited multiplets

Importance for mass and excitation energies predictions

$$M_{T_z} = a + bT_z + cT_z^2$$

T=1

$$M_1 = a + b + c$$
$$M_0 = a$$
$$M_{-1} = a - b + c$$

- Prediction of masses and excited levels in proton-rich nuclei, e.g. if *Tz*>0 :

$$M_{-T_z} = M_{T_z}^{exp} + 2b^{th}T_z$$

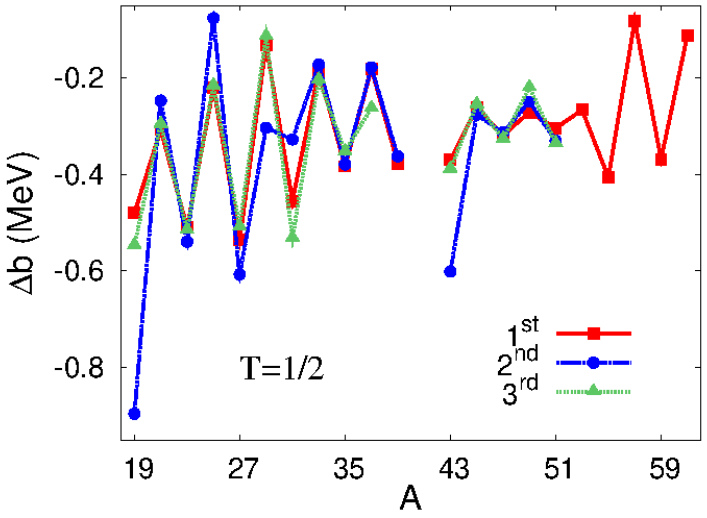
- Particular case of triplets :

$$M_{-1} = 2M_0^{exp} - M_1^{exp} + 2c^{th}$$

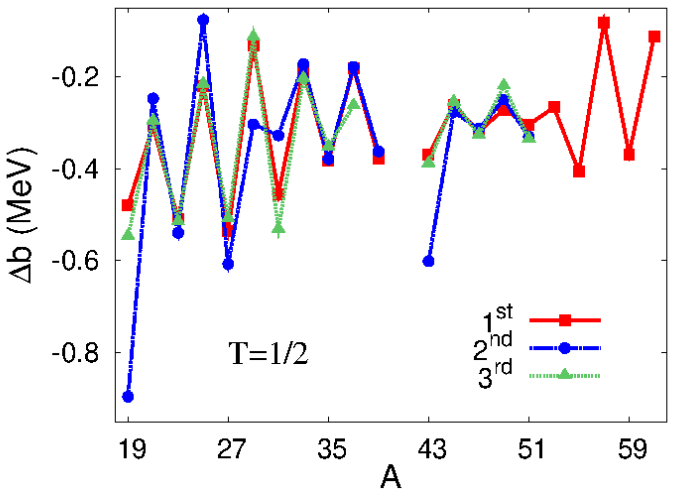
Important for nuclear astrophysics applications!

Brown, Richter (from 2011); Kaneko et al (2013)

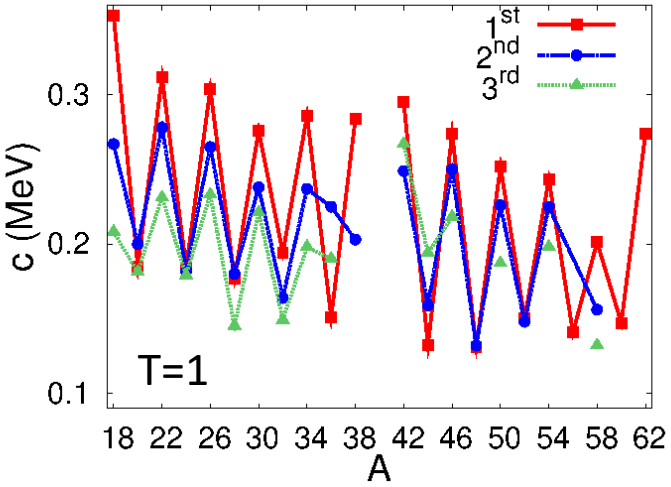
Experiment



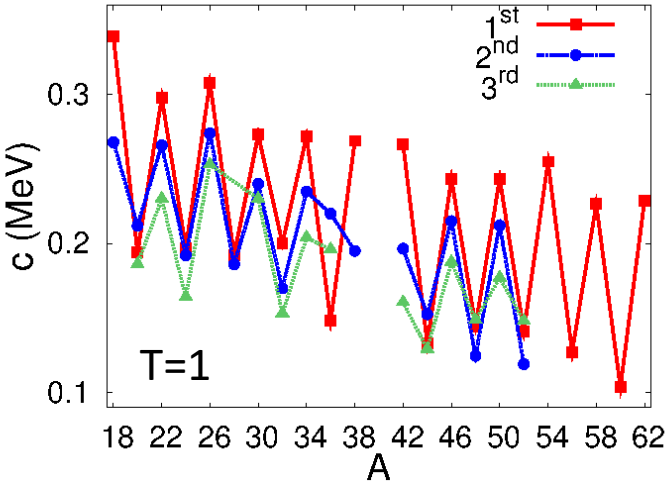
Theory



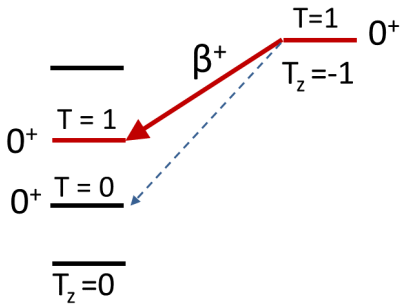
Experiment



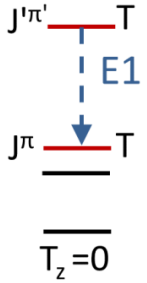
Theory



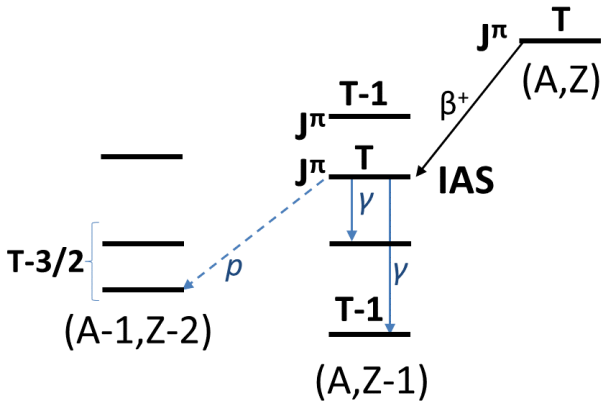
Isospin-forbidden decays and isospin mixing (?)



Isospin-forbidden Fermi beta decay



Isospin-forbidden E-M transitions

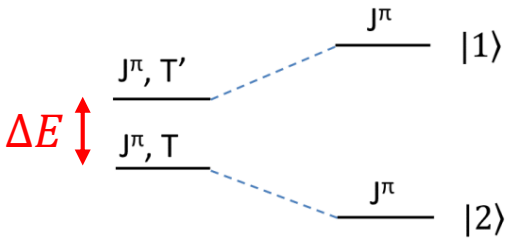


Isospin-forbidden proton emission

$$H_{INC} = H_0 + V + V_{CD}$$

⇨ ***Isospin mixing !***

How reliable can these phenomena be described theoretically?



$$|1\rangle = \sqrt{1-\alpha^2}|T\rangle + \alpha|T'\rangle$$

$$|2\rangle = \alpha|T\rangle + \sqrt{1-\alpha^2}|T'\rangle$$

$$\alpha^2 \sim \frac{\langle V_{CD}\rangle^2}{\Delta E^2}$$

Reliable (at low energies)

Challenging…

$$\langle V_{CD}\rangle = \langle T|V_{CD}|T'\rangle$$

p-shell	< 150(40) keV
sd-shell	< 100(40) keV
pf-shell	< 40(20) keV

II. Application to fundamental interaction studies on nuclei

Computation of isospin-symmetry breaking to superallowed $0^+ \rightarrow 0^+$ Fermi beta decay for the tests of CVC and Vud CKM matrix element extraction

II. Application to fundamental interactions studies

Fermions in the Standard Model

$$\begin{pmatrix} u \\ d \end{pmatrix} \quad \begin{pmatrix} c \\ s \end{pmatrix} \quad \begin{pmatrix} t \\ b \end{pmatrix}$$
$$\begin{pmatrix} e \\ \nu_e \end{pmatrix} \quad \begin{pmatrix} \mu \\ \nu_\mu \end{pmatrix} \quad \begin{pmatrix} \tau \\ \nu_\tau \end{pmatrix}$$

Testing grounds:

- at colliders: search for direct production
- at low energies in nuclear beta decay: in precision experiments

Nuclear Matrix elements are needed !

Cabibbo - Kobayashi - Maskawa (CKM) quark-mixing matrix

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

N. Cabibbo (1963); M. Kobayasi, T. Maskawa (1973).

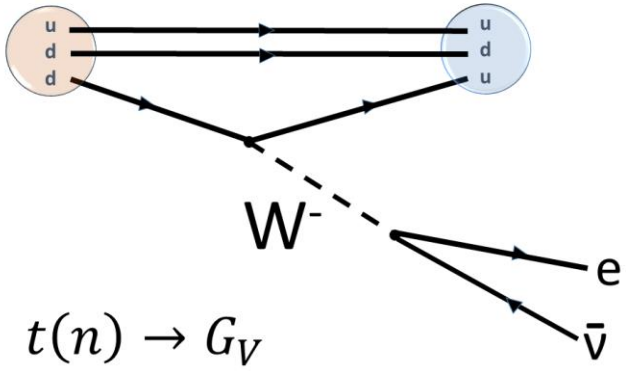
What does the Standard Model prescribe?

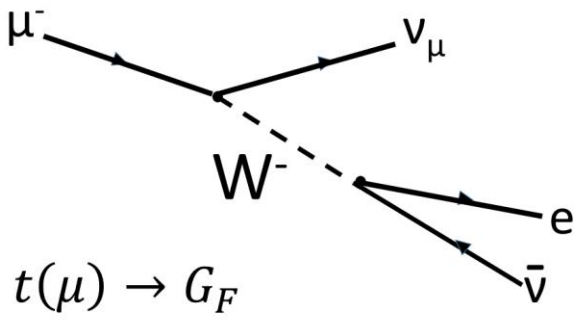
- V_{ik} to be extracted from experiment
- Standard Model: V is unitary

$$\sum_k |V_{ik}|^2 = \sum_k |V_{ki}|^2 = 1$$

$\Rightarrow$ constraints on new physics beyond the Standard Model

$|V_{ud}|$ from pion, nucleon and nuclear beta decay



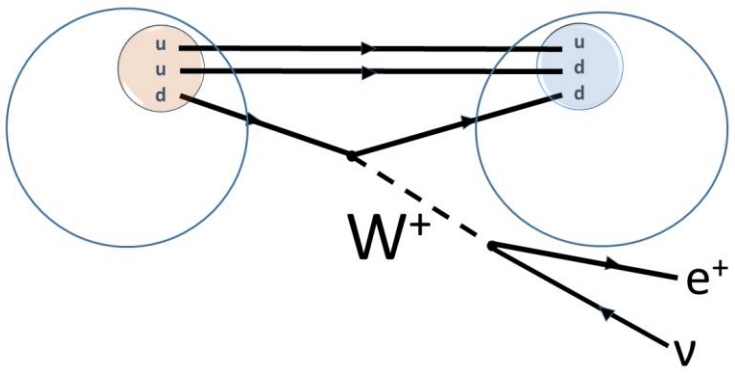


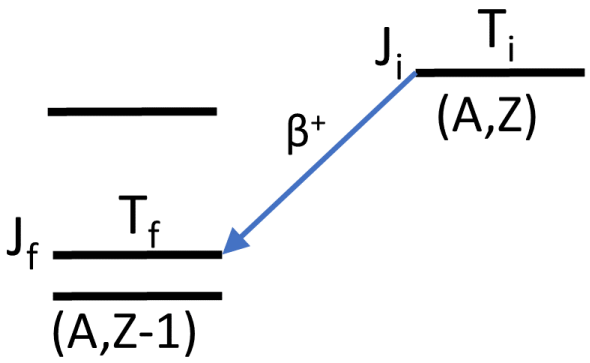
$$|V_{ud}| \quad = G_V/G_F$$

- Superallowed nuclear 0^+ -> 0^+ beta decay (nuclear structure effects)
- Mirror T=1/2 transitions (F/GT ratio, nuclear structure effects)
- Neutron decay $n \rightarrow p + e^- + \bar{\nu}_e$ (Lifetime)
- Pion decay $\pi^+ \rightarrow \pi^0 + e^+ + \nu_e$ (Branching ratio)

Current status:
J.C. Hardy, I. S. Towner, PRC102. 045501 (2020) ;
M. Gonzalez-Alonzo, O. Navillat-Cuncic, N. Severijns, PPNP104, 165 (2019)

Nuclear beta decay





Decay rate:

$$\Gamma = \frac{\ln 2}{t} = \frac{G_V^2}{K}\left[g_V^2 f_V |M_F|^2 + g_A^2 f_A |M_{GT}|^2\right]$$

$$f = \int_1^{W_0} (W - W_0)^2 p W F(Z,W) S(Z,W) dW$$

$$K = \frac{2\pi^3 \ln 2 \hbar^7}{m_e^5 c^4}$$

g_V=1, g_A≈ 1.26

$$W_0 = Q_{EC}/(m_e c^2) - 1$$

For 0+ -> 0+ beta decay:

$$f t^{0^+ \to 0^+} = \frac{K}{|M_F|^2 G_V^2}$$

Should be constant (for fixed T) – a consequence of the Conserved Vector Current (CVC) hypothesis

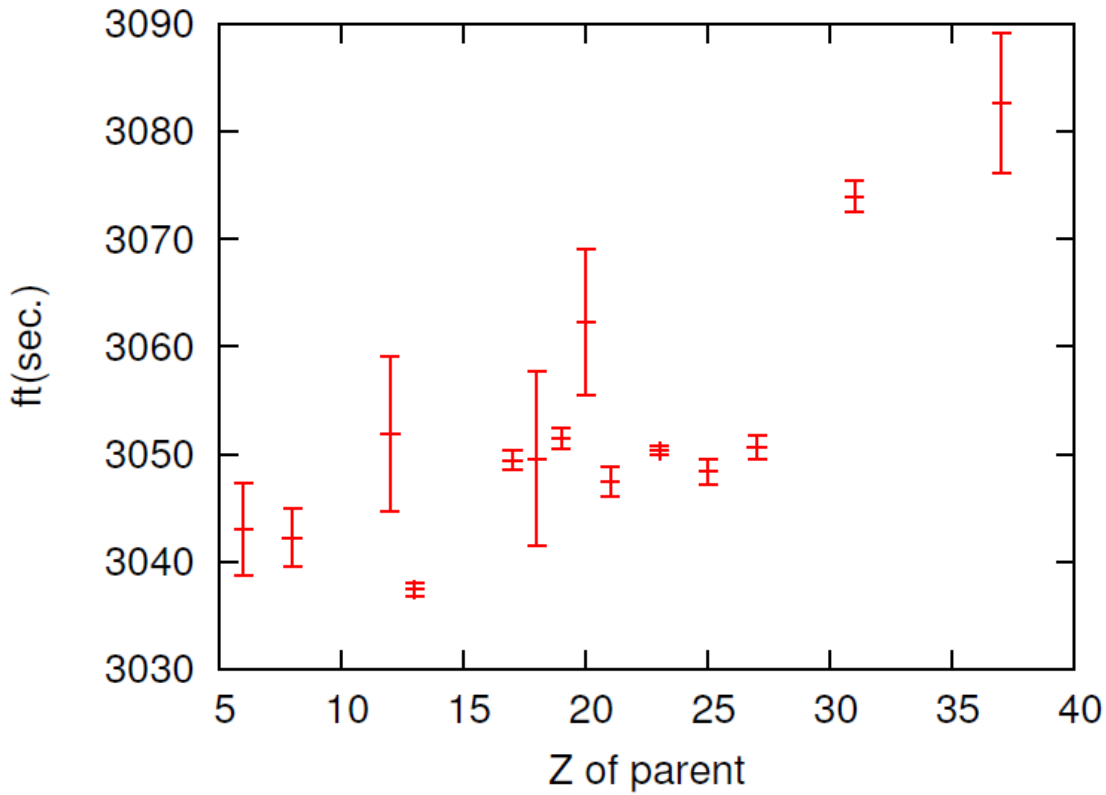
Superallowed $0^+ \rightarrow 0^+$ beta decay

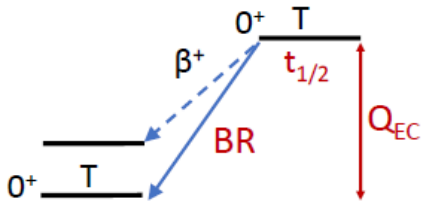
15 best known $T = 1$ emitters ($ft^{0^+\rightarrow 0^+}$-value known with a precision $\lesssim 0.4\%$):

^{10}C, ^{14}O, ^{22}Mg, ^{26m}Al, ^{26}Si, ^{34}Cl, ^{34}Ar, ^{38m}K, ^{38}Ca, ^{42}Sc, ^{46}V, ^{50}Mn, ^{54}Co, ^{62}Ga, ^{74}Rb

J.C. Hardy, I.S. Towner, PRC102, 045501 (2020)

Experimental data on $t_{½}$, BR, Q_{EC}





Statistical rate function:

$$f = f(Z, Q_{EC})$$

Partial halflife:

$$t = \frac{t_{1/2}}{BR}$$

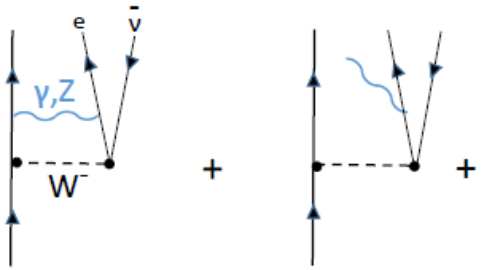
$$ft^{0^+\rightarrow 0^+} = \frac{K}{|M_F|^2 G_V^2}.$$

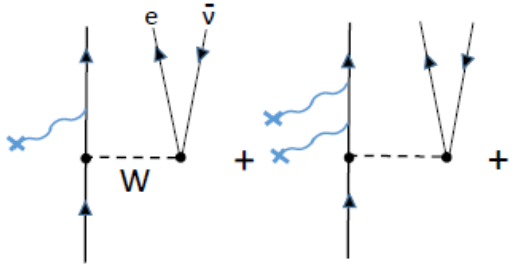
Superallowed $0^+ \rightarrow 0^+$ beta decay

Absolute *Ft* value

$$Ft^{0^+\rightarrow 0^+} \equiv ft^{0^+\rightarrow 0^+}(1+\delta'_R)(1+\delta_{NS}-\delta_C) = \frac{K}{|M_F^0|^2 G_V^2 (1+\Delta_R)}$$

J.C. Hardy, I.S. Towner, PRC102, 045501 (2020)





Radiative corrections

Much recent Interest within EFT

$\Delta_R^V = (2.454 \pm 0.019)\%$

$\delta'_R \sim (1.50 \pm \sim 0.12)\%$

$|\delta_{NS}| \lesssim 0.4\%$

Sirlin, Marciano, Jaus, Rasche, ..
Seng, Gorchtein, Cirigliano,...

Nuclear-structure correction

$|M_F|^2 = |M_F^0|^2(1-\delta_C)$

$|M_F^0|^2 = T(T+1) - T_{zi}T_{zf}$

$\delta_C \approx 0.1 - 2.0\%$

Many-body approach with charge-dependent forces

Towner, Hardy (1973 – 2020)
and many others (next slide)

Many-body approaches to isospin-symmetry breaking

- Shell model (from 60's . . .)

  W.E. Ormand, B.A. Brown et al 1985 –; S.Nakamura, K.Muto, T.Oda, 1994; A.P. Zuker, S.M. Lenzi, M.A. Bentley et al, 2001 – 2018; K. Kaneko et al, 2010 – 2018; Y.H. Lam, N. Smirnova, E. Caurier, 2013.

- Gamow shell-model, SMEC

  N. Michel, W. Nazarewicz, M. Ploszajczak, PRC82 (2010), . . .

- HF + Tamm-Dankoff, RPA, HTDA

  I. Hamamoto, H. Sagawa, N. V. Giai, J. Dobaczewski, T. Suzuki, G. Colo, J. Le Bloas, N. Auerbach et al, 1993 – 2018

- Relativistic RPA

  H. Liang, N. V. Giai, J. Meng, 2009 –

- J-projected and T-projected DFT with ISB terms

  W. Satula, J. Dobaczewski et al, 2009 – 2018, P. Baczyk et al, PLB778, 178 (2018)

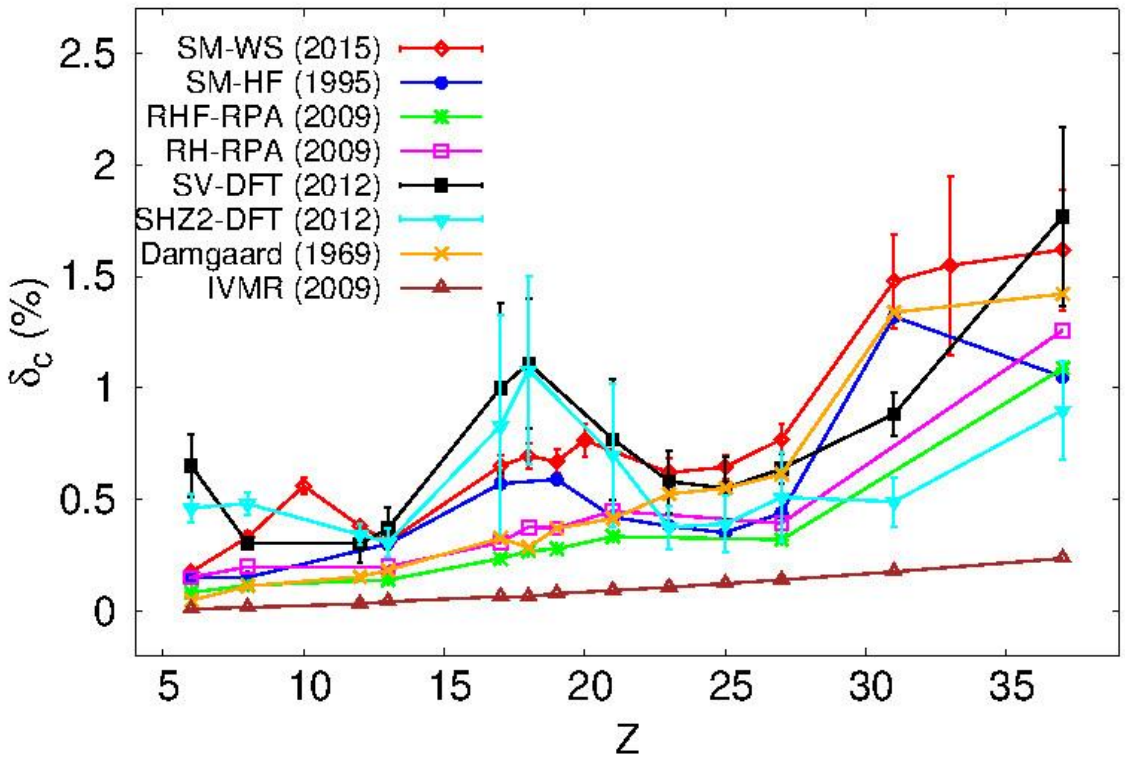
- VAP technique on the HFB basis

  A. Petrovici et al, 2008 – 2018

- Isovector giant monopole resonance

  G. Colo et al (1993); N. Auerbach, Phys. Rep. 98, 273 (1983); PRC (2009)

Isospin-symmetry breaking correction – divergence of model predictions



SM-WS : Towner, Hardy (2015)
SM-HF : Ormand, Brown (1989, 1995)
RHF-RP : Liang et al (2009)
SV/SHZ2-DFT : Satula et al (2012)
IVMR : Auerbach (2009)

CVC test
Towner, Hardy
PRC82 (2010)

Superallowed $0^+ \rightarrow 0^+$ beta decay: status by 2020

$$\mathcal{F}t = (1+\delta_R)(1+\delta_{NS}-\delta_C)ft = \frac{K}{M_0^2 G_F{}^2 |V_{ud}|^2 (1+\Delta_R)}$$

$\mathcal{F}t = 3072.24 \pm 0.57\ sec$

$\chi 2/\nu = 0.47$

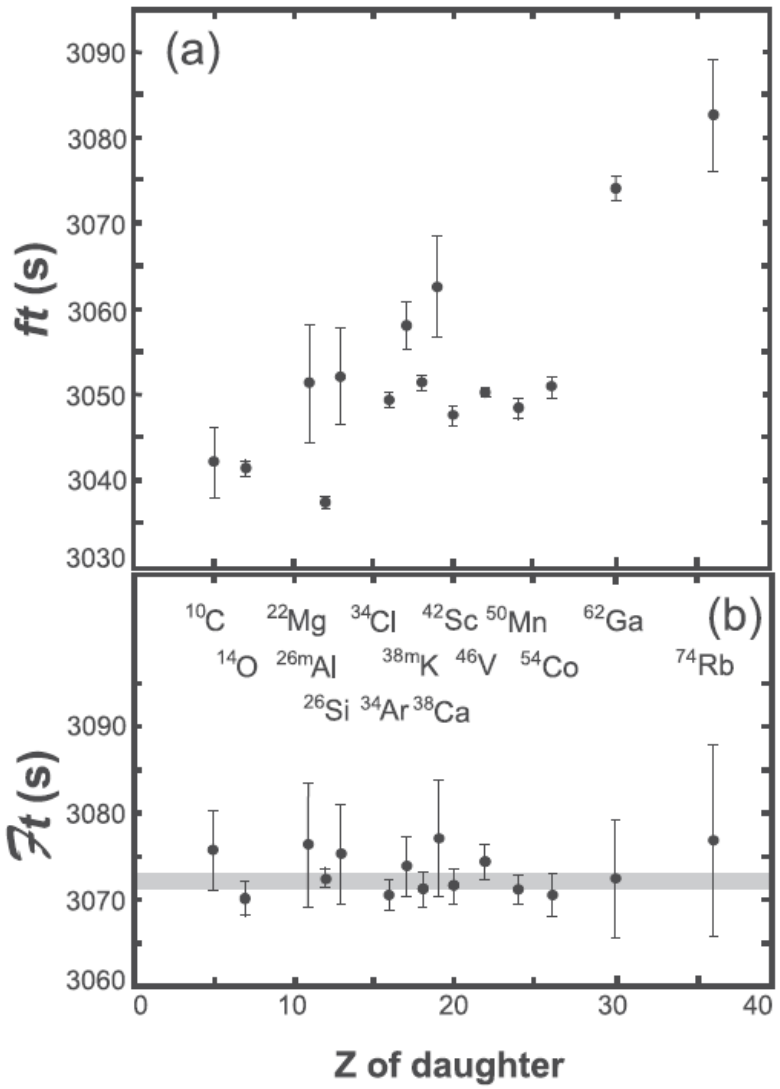
$|V_{ud}| = 0.97373(31)$

$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9985(5)$

Further revisions of radiative corrections are ongoing since 2018: for review see Seng, Gorchtein, ARNPS 74 (2023)

The purpose of our work: to revise the shell-model computation δ_C of in large-scaled calculations With realistic WS or HF wave functions

Hardy, Towner (2020)



Nuclear-structure correction to Fermi β decay

Large-scale shell model with H_{INC} + Wood-Saxon or Hartree-Fock radial wave functions

$$M_F = \langle \Psi_f | T_+ | \Psi_i \rangle = \sum_\alpha \underbrace{\left\langle \Psi_f \middle| a^\dagger_{\alpha_n} a_{\alpha_p} \middle| \Psi_i \right\rangle}_{\text{One-body transition densities}} \underbrace{\langle \alpha_n | t_+ | \alpha_p \rangle}_{\text{Radial overlap integral}}$$

$\rho_\alpha \neq \rho_\alpha^T$

$\int_0^\infty R_{\alpha_n}(r)\, R_{\alpha_p}(r) r^2 dr = \Omega_\alpha \neq 1$

Exact Fermi operator: G.A. Miller, A. Schwenk, PRC78, 035501 (2008); PRC80, 064319 (2009)

Expansion of the correction in terms of small quantities $(\rho_\alpha - \rho_\alpha^T)$ and $(1 - \Omega_\alpha)$:

$$M_F^2 = M_0^2 (1 - \delta_C) \Longrightarrow \delta_C = \underbrace{\delta_{C1} + \delta_{C2}}_{\text{LO}} + \underbrace{\delta_{C3} + \delta_{C4}}_{\text{NLO}} + \underbrace{\delta_{C5}}_{\text{N}^2\text{LO}} + \underbrace{\delta_{C6}}_{\text{N}^3\text{LO}} \Longrightarrow \delta_C \approx \delta_{C1} + \delta_{C2}$$

- From numerical evaluation, the leading-order (LO) corrections provide robust approximation !
- NLO, N2LO, N3LO corrections stay within theoretical uncertainties on δ_C

Shell-model calculations

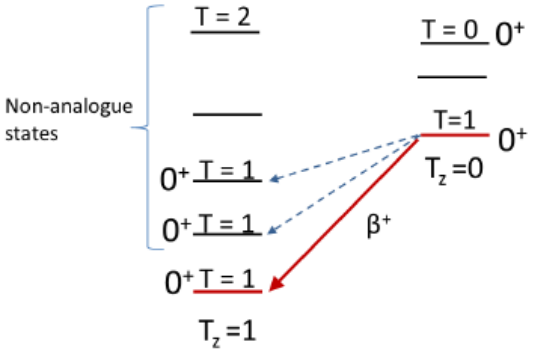
^{10}C, ^{14}O, ^{18}Ne, ^{22}Mg, ^{26m}Al, ^{26}Si, ^{34}Cl, ^{34}Ar, ^{38m}K, ^{38}Ca, ^{42}Sc, ^{42}Ti, ^{46}V, ^{50}Mn, ^{54}Co, ^{62}Ga, ^{66}As

Model spaces and effective interactions (+ charge-dependence)

- *p*-shell: CKPOT *(Cohen-Kurath, 1965)*
- $(p_{1/2}sd_{5/2})$-shell: ZBM's *(Zuker et al, 1969)*, REWIL *(Reehal, Wildenthal, 1973)*
- *sd*-shell: USD *(Wildenthal, 1984)*, USDA/USDB *(Brown, Richter, 2006)*
- $(sd_{3/2}f_{7/2}p_{3/2})$-shell: ZBM2 *(Nowacki et al, 2014)*
- *pf*-shell: KB3G *(Poves et al, 2004)*, GXPF1A *(Honma et al, 2004)*
- $pf_{5/2}g_{9/2}$: JUN45 *(Honma et al, 2009)*, MRG *(Nowacki et al, 1996)*

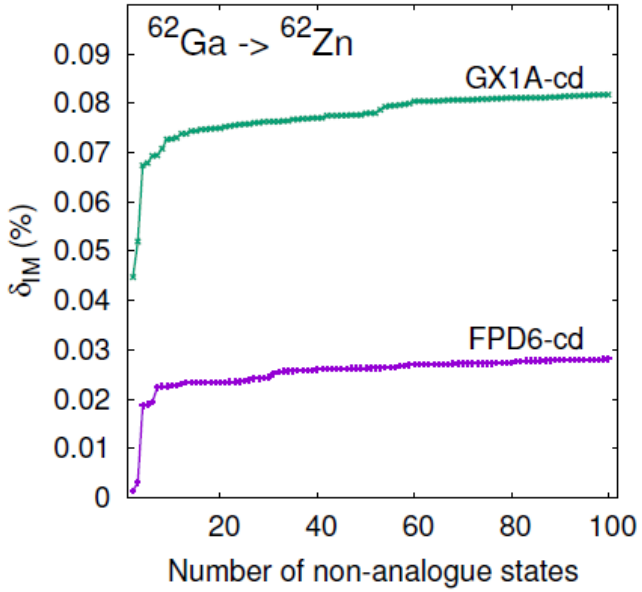
NuShellX@MSU shell-model code *(B.A. Brown, W.D.M. Rae, Nucl. Data Sheets 120, (2014)*.

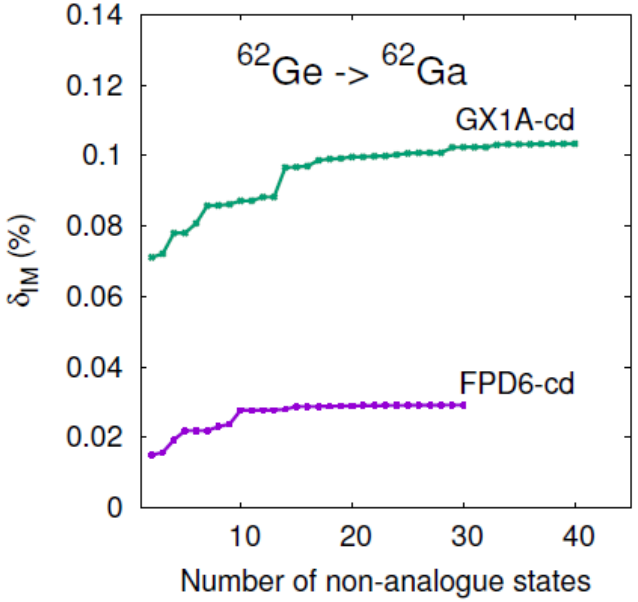
Shell-model evaluation of δ_{C1}

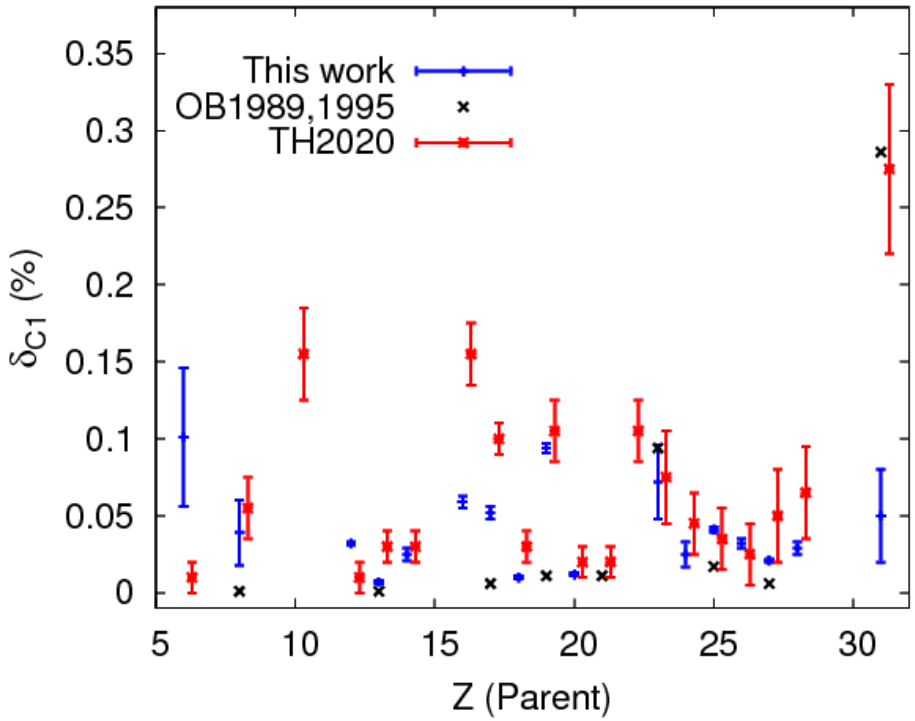


We sum over the Fermi strengths to non-analogue states!
⇒ experimental information on non-analogue strength is highly desirable!

~$(\Delta E_{th})^2 / (\Delta E_{exp})^2$







Xayavong, Smirnova, Nowacki, submitted

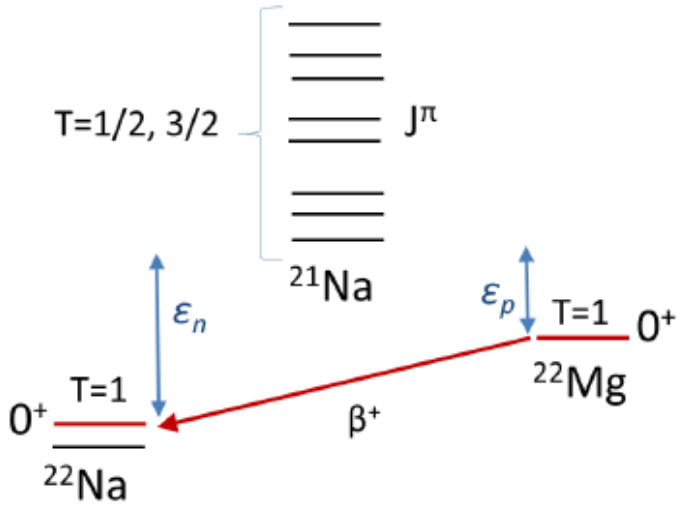
Shell-model evaluation of δ_{C2} beyond the closure approximation

$$\delta_{C2} = \frac{2}{M_F^0} \sum_{\alpha} \langle \Psi_f | a^{\dagger}_{\alpha_n} a_{\alpha_p} | \Psi_i \rangle^T (1 - \Omega_\alpha)$$

$$\Downarrow$$

$$\boxed{\delta_{C2} = \frac{2}{M_F^0} \sum_{\alpha,\pi} \langle \Psi_f | a^{\dagger}_{\alpha_n} | \pi \rangle^T \langle \pi | a_{\alpha_p} | \Psi_i \rangle^T (1 - \Omega^{\pi}_{\alpha})}$$

Experimental information on relevant spectroscopic factors and charge radii is highly desirable !



- *Spectroscopic amplitudes* (from the shell-model):

$$\langle \Psi_f | a^{\dagger}_{\alpha_n} | \pi \rangle = \frac{\langle \Psi_f || a^{\dagger}_{\alpha_n} || \pi \rangle}{\sqrt{2J_f + 1}}$$

- *Radial-overlap integrals* (from a realistic single-particle potential)

$$\Omega^{\pi}_{\alpha} = \int_0^{\infty} R^{\pi}_{\alpha_n}(r) R^{\pi}_{\alpha_p}(r) r^2 dr$$

Shell-model + Woods-Saxon evaluation of δ_{C2}

L. Xayavong, PhD thesis, U. Bordeaux (2016)

L. Xayavong, N. Smirnova, Phys. Rev. C97, 024324 (2018)

Parameterization

$$V_{WS}(r) = -\frac{V_0}{1+\exp\left(\frac{r-R}{a}\right)} - V_{ls}(r)\vec{l}\cdot\vec{\sigma} + V_C(r)$$

- A. Bohr, B.R. Mottelson modified (BM_m) from *Nuclear Structure, Vol. I.*
- N. Schwierz, I. Wiedenhöver, A. Volya (SWV) from *nucl-th:0709.3525*

$$R = r_0 A^{1/3}$$

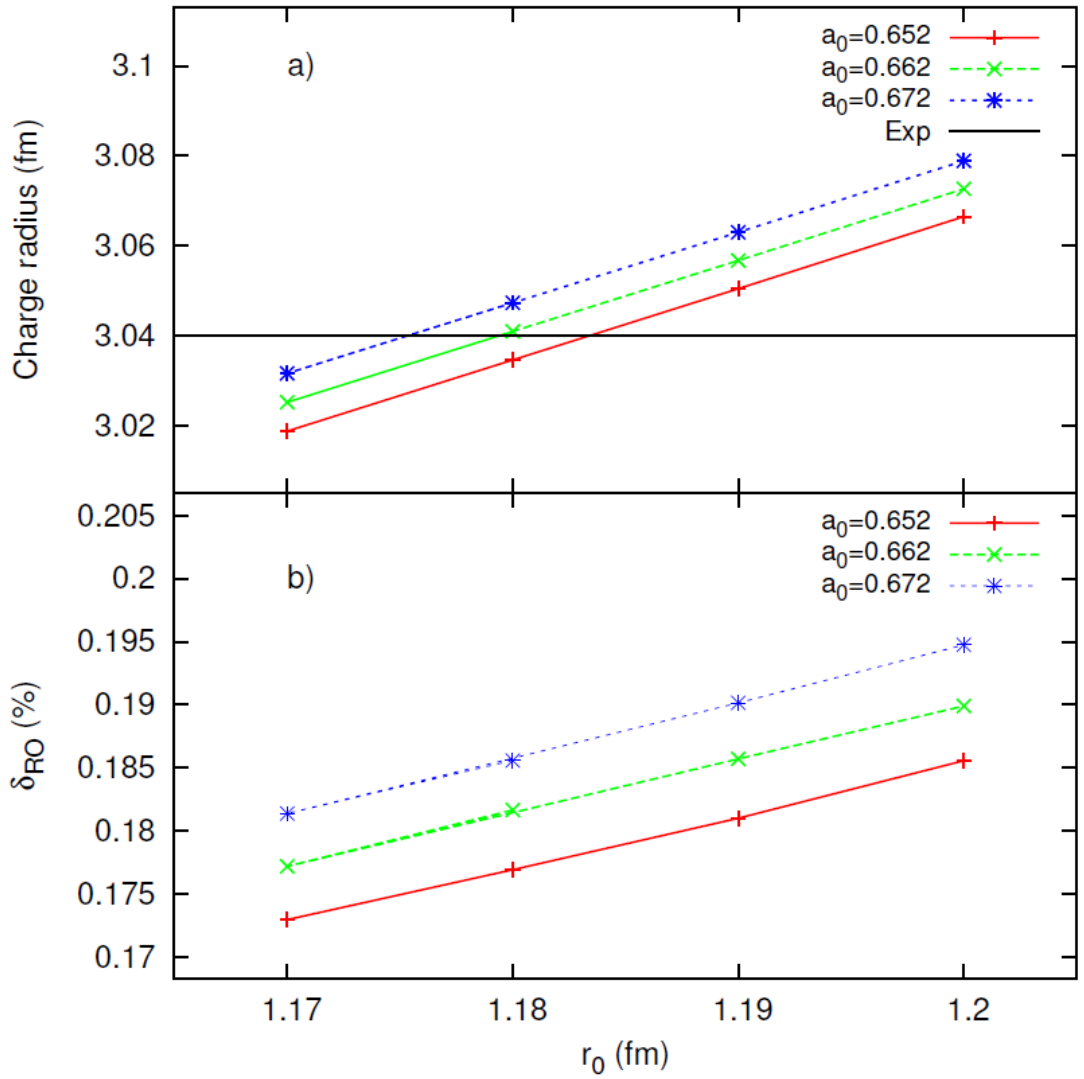
Under stringent experimental constraints:

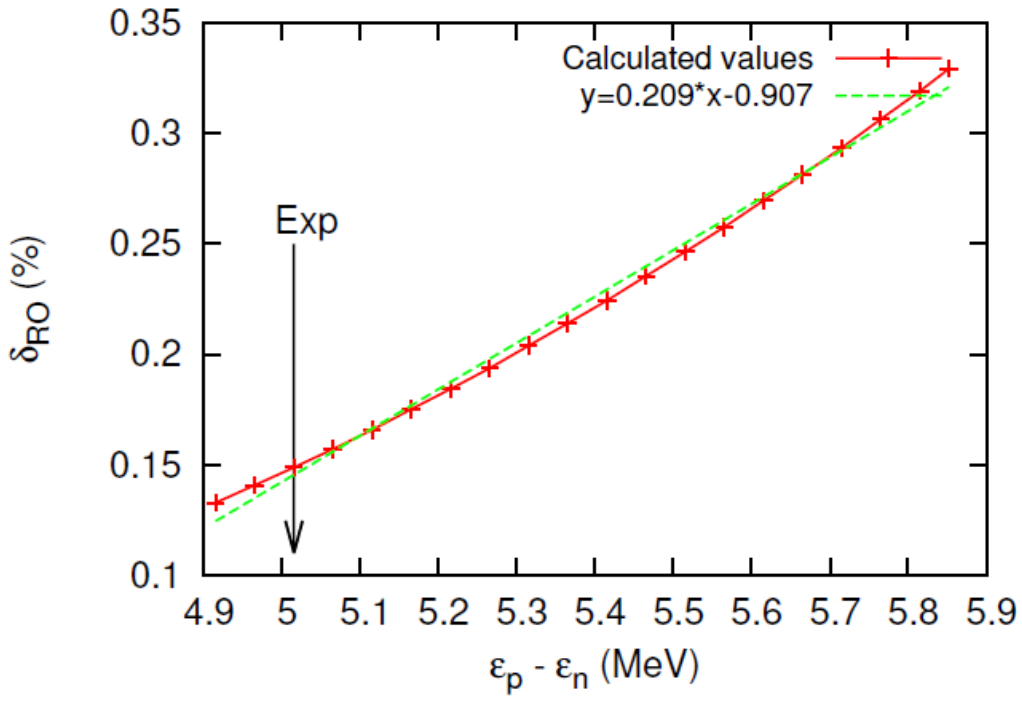
- (V_0, r_0) are adjusted simultaneously to reproduce experimental **nucleon separation energies** and **charge radii**

$$\psi(r) \rightarrow \exp\left(-\frac{\sqrt{2m|\epsilon|}r}{\hbar}\right)$$

- A new approach to nuclear charge radii beyond the closure approximation: $\langle r^2\rangle_{ch} = \frac{1}{Z}\sum_{\pi\alpha}\langle\alpha|r^2|\alpha\rangle^{\pi}|\langle\Psi_i||a^{\dagger}_{\alpha}||\pi\rangle|^2$
- Thorough **propagation of uncertainties**.

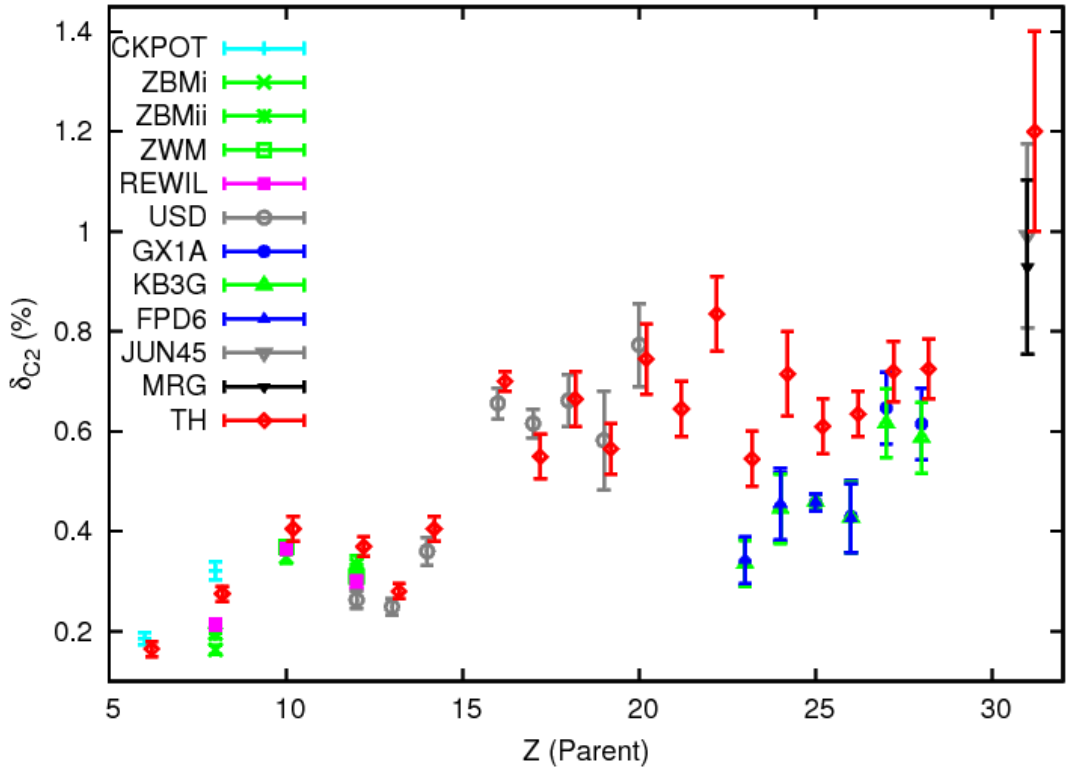
WS Adjustment





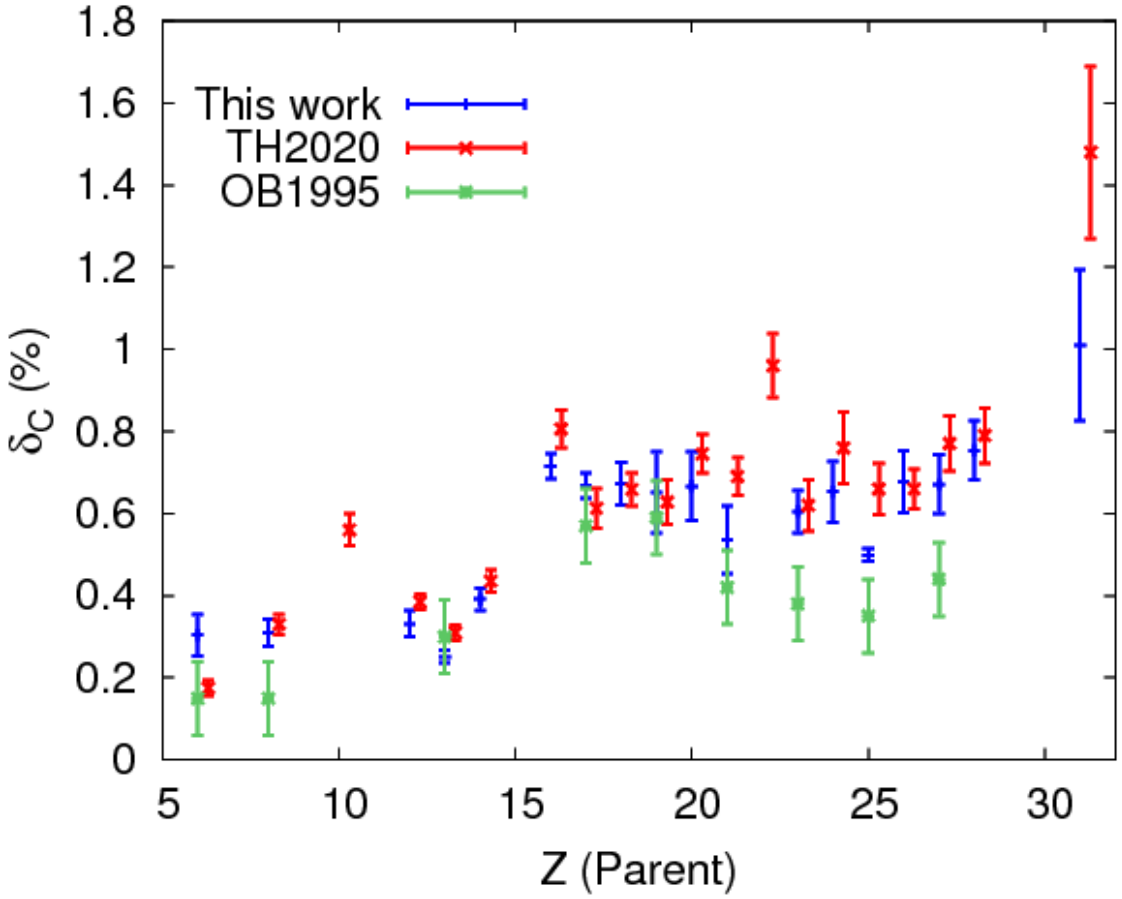
L. Xayavong, PhD thesis, U. Bordeaux (2016)

Shell-model + Woods-Saxon evaluation of δ_{C2} and δ_C



Uncertainties are mainly due to the charge radii uncertainties !

Preliminary !



Smirnova, Xayavong, Proc. NTSE2018.

Xayavong, Smirnova, Nowacki (submitted to PRC, 2025)

Consistency tests

$$\mathcal{F}t = (1+\delta_R)(1+\delta_{NS}-\delta_C)ft = \frac{K}{M_0^2 G_F{}^2 |V_{ud}|^2 (1+\Delta_R)}$$

^{10}C, ^{14}O, ^{22}Mg, ^{26m}Al, ^{26}Si, ^{34}Cl, ^{34}Ar, ^{38m}K, ^{38}Ca, ^{42}Sc, ^{46}V, ^{50}Mn, ^{54}Co, ^{62}Ga, (^{74}Rb)

- Δ_R , δ_R , δ_{NS} Radiative corrections are kept the same

Towner, Hardy (2020) $\mathcal{F}t = 3072.23(57)_{stat}\,(36)_{\delta_R'}(173)_{\delta_{NS}} sec$ χ2/ν= 0.49

This work (WS) $\mathcal{F}t = 3073.11(99)_{stat}\,(36)_{\delta_R'}(173)_{\delta_{NS}} sec$ χ2/ν= 0.63 Preliminary !

Xayavong, Smirnova, Nowacki (submitted to PRC, 2025)

Shell Model + Hartree-Fock radial wave functions for δ_{C2}

Skyrme force calculations

SkM* parameterization *J. Bartel et al, NPA386 (1982)*.

Optimization

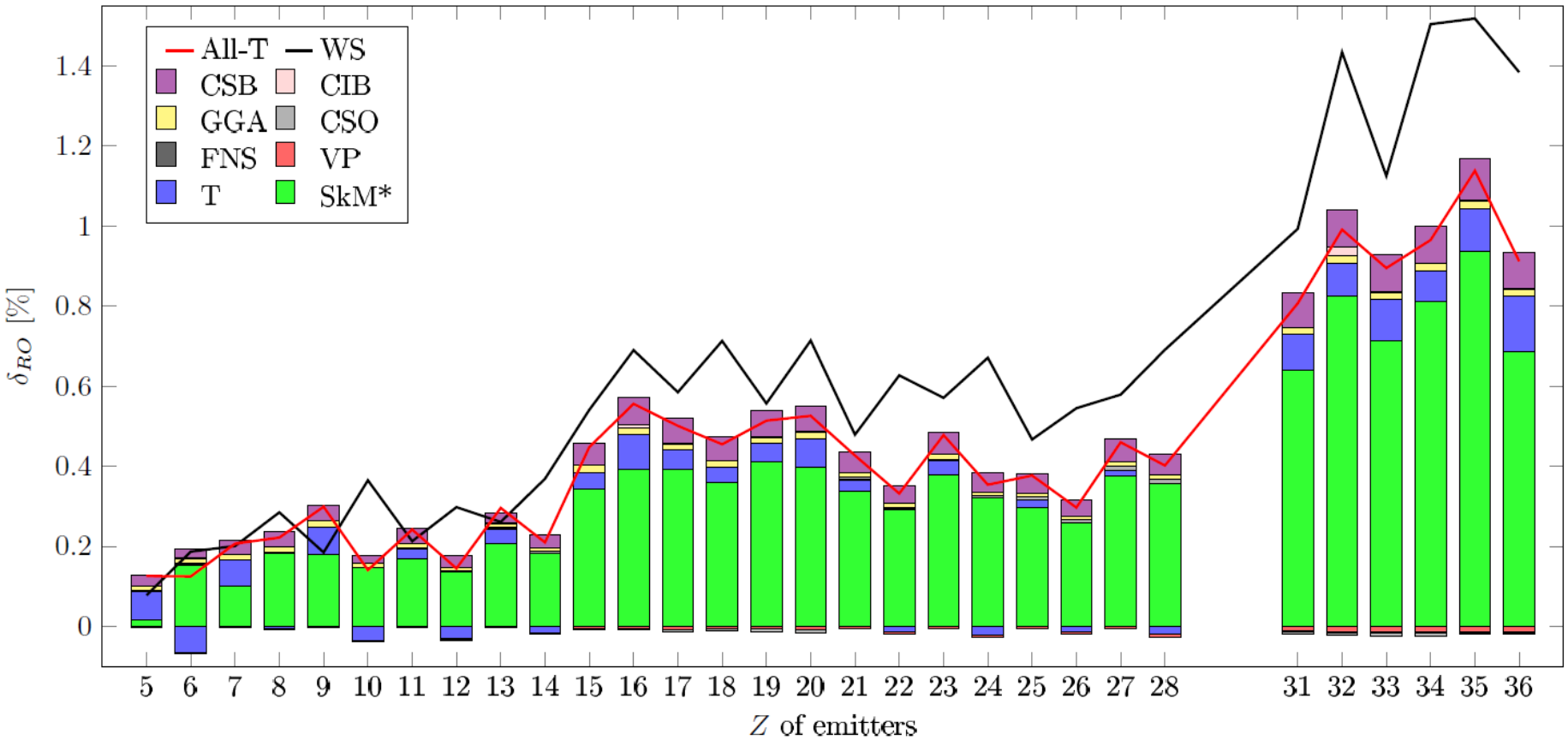
- Energy-dependent *local equivalent potential*
  C.B.Dover, N. Van Giai, NPA190 (1972):

$$V^{LE}(r,\epsilon_\alpha) = V^0(r,\epsilon_\alpha) + V^{so}(r)\langle \vec{l}\cdot\vec{s}\rangle + V_C(r) \quad \text{for} \quad R_\alpha(r) = N\sqrt{\frac{m^*(r)}{m}}\,R_\alpha^{LE}(r).$$

- Adjustment of the central term $V^0(r,\epsilon_\alpha)$ by a scaling factor to match the experimental proton and neutron separation energies
- Effects from CIB and CSB terms, spurious isospin mixing, Slater approximation for Coulomb term, two-body center-of-mass correction, finite nucleon size, etc. are investigated.

W.E. Ormand, B.A. Brown, NPA440 (1985); PRL62 (1989); PRC52 (1995)
L. Xayavong, N. Smirnova, PRC105 (2022).

Shell Model + Hartree-Fock radial wave functions for δ_{C2}



Post HF effects to be considered!

$\mathcal{F}t = 3074.81 \pm 0.93\ sec$

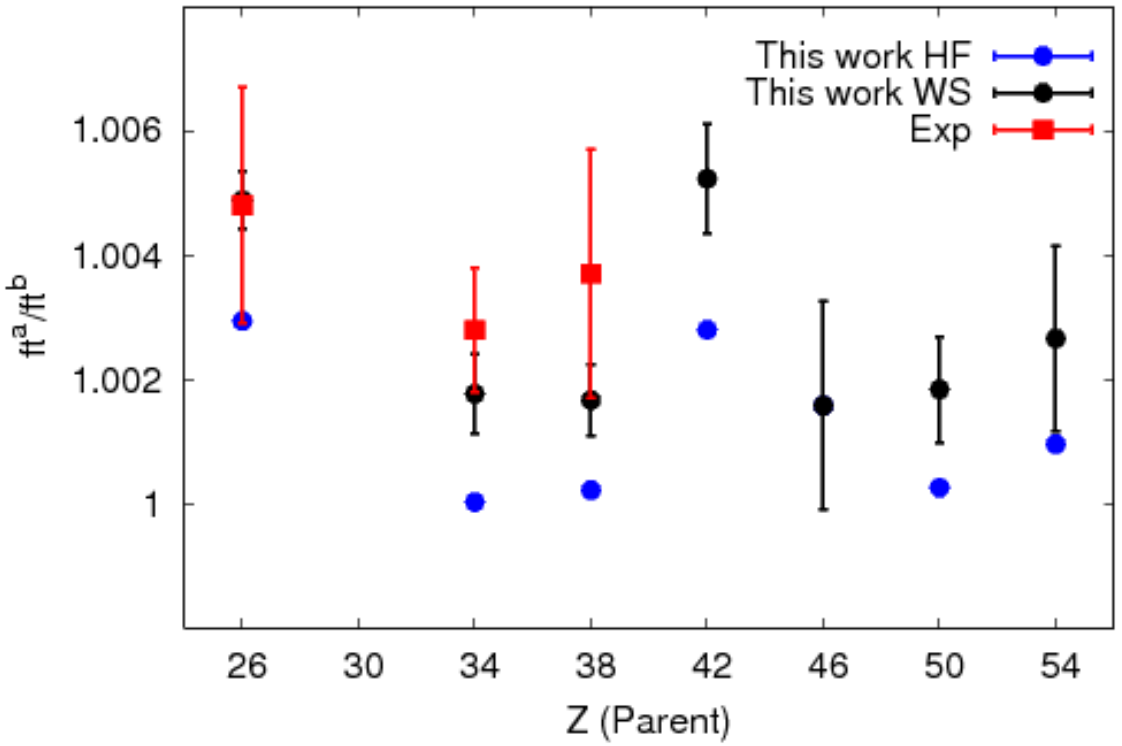
$\chi 2/\nu = 2.96$

Consistency tests

Test of the mirror ratio for triplets

$$\frac{ft^a}{ft^b} = 1 + \left(\delta_R^a - \delta_R^b\right) + \left(\delta_{NS}^a - \delta_{NS}^b\right) + \left(\delta_C^a - \delta_C^b\right)$$

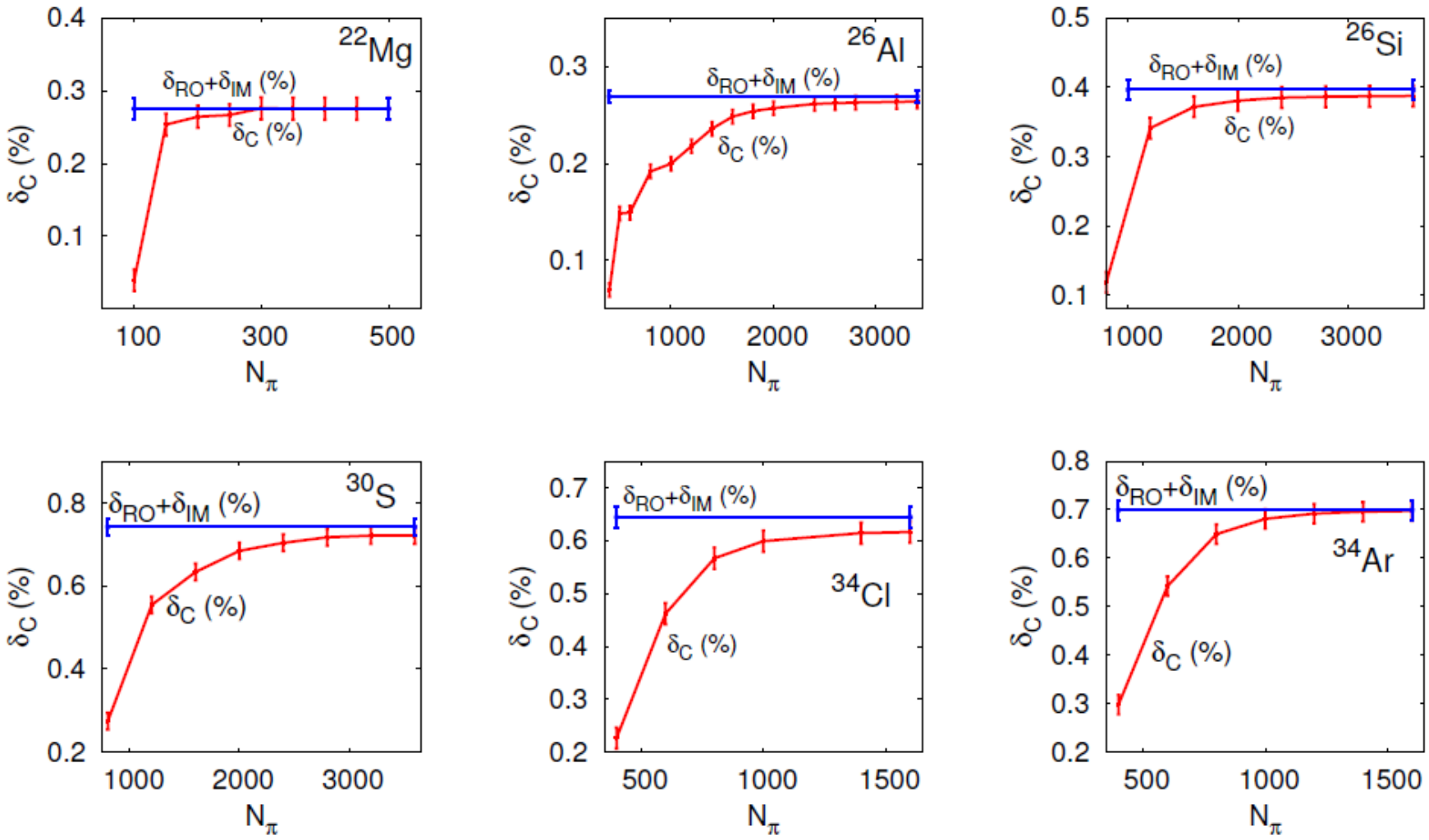
^{26m}Al, ^{26}Si, ^{34}Cl, ^{34}Ar, ^{38m}K, ^{38}Ca, ^{42}Sc, ^{42}Ti, ^{46}V, ^{46}Cr, ^{50}Mn, ^{50}Fe, ^{54}Co, ^{54}Ni



Preliminary !

Test of the separation ansatz of the isospin-symmetry breaking correction

δ_C is compared to $\delta_{C1} + \delta_{C2}$



N. Smirnova, L. Xayavong, Proc. NTSE2018.

Higher-order terms of the isospin-symmetry breaking correction

Exact evaluation of δ_C:

$$M_F = M_F^0\left[1 - \frac{1}{M_F^0}\sum_\alpha\left(\rho_\alpha^T - \rho_\alpha\right) - \frac{1}{M_F^0}\sum_\alpha \rho_\alpha^T\left(1-\Omega_\alpha\right) + \frac{1}{M_F^0}\sum_\alpha\left(\rho_\alpha^T - \rho_\alpha\right)\left(1-\Omega_\alpha\right)\right],$$

$$|M_F|^2 = |M_F^0|^2(1-\delta_C), \quad \delta_C = \sum_{i=1}^{6}\delta_{Ci}$$

Explicit expressions for the six terms:

These terms have similar structure when expressed beyond the closure approximation!

$$\delta_{C1} = \frac{2}{M_F^0}\sum_\alpha\left(\rho_\alpha^T - \rho_\alpha\right) \qquad \textbf{LO}$$

$$\delta_{C2} = \frac{2}{M_F^0}\sum_\alpha \rho_\alpha^T\left(1-\Omega_\alpha\right) \qquad \textbf{LO}$$

$$\delta_{C3} = \frac{2}{M_F^0}\sum_\alpha\left(\rho_\alpha^T - \rho_\alpha\right)\left(1-\Omega_\alpha\right) \qquad \textbf{NLO}$$

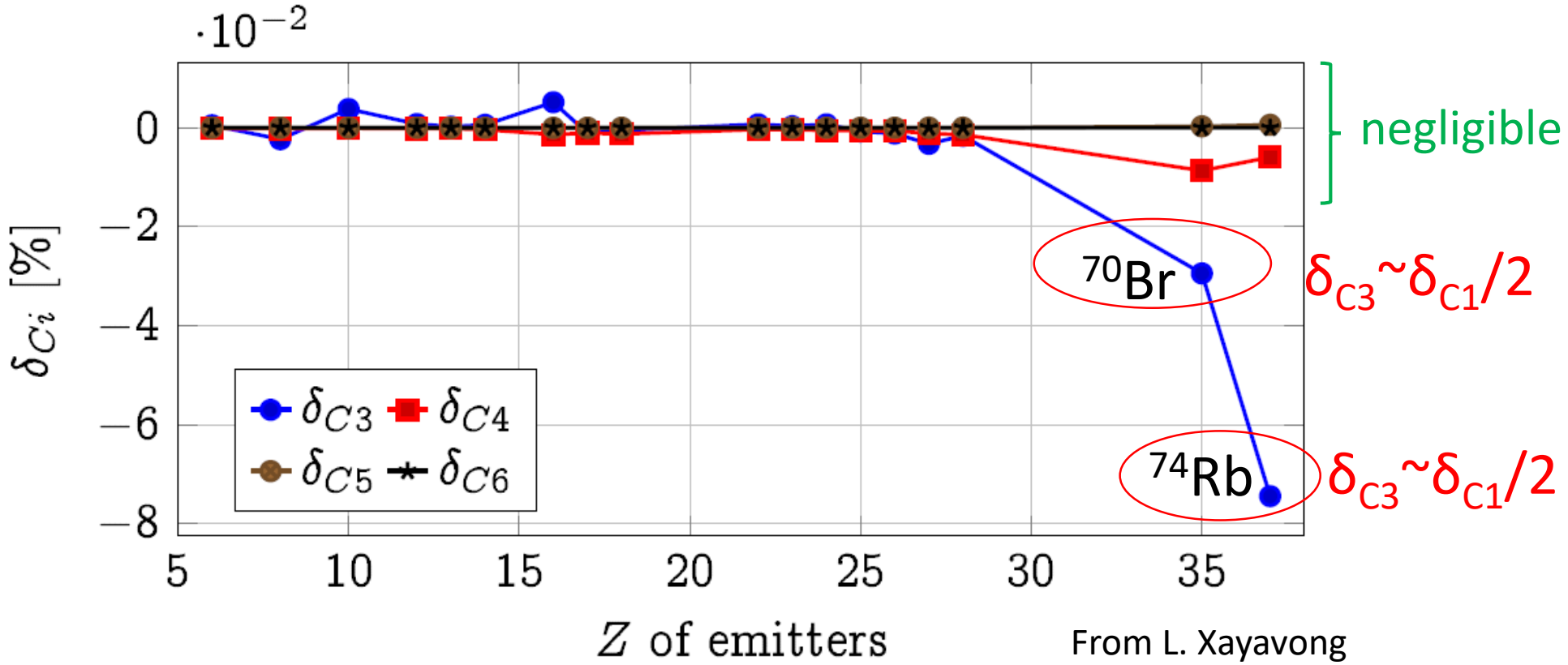
$$\delta_{C4} = -\left(\delta_{C1}+\delta_{C2}\right)^2/4 \qquad \textbf{NLO}$$

$$\delta_{C5} = -\left(\delta_{C1}+\delta_{C2}\right)\delta_{C3}/2 \qquad \textbf{N}^2\textbf{LO}$$

$$\delta_{C6} = -\delta_{C3}^2/4 \qquad \textbf{N}^3\textbf{LO}$$

Higher-order terms of the isospin-symmetry breaking correction

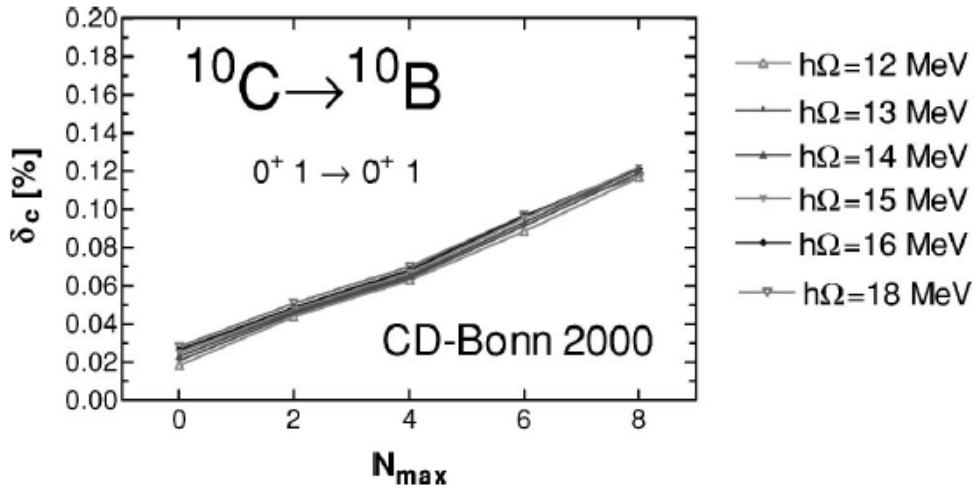
- Higher-order contribution (superallowed $0^+ \to 0^+$ Fermi transition)



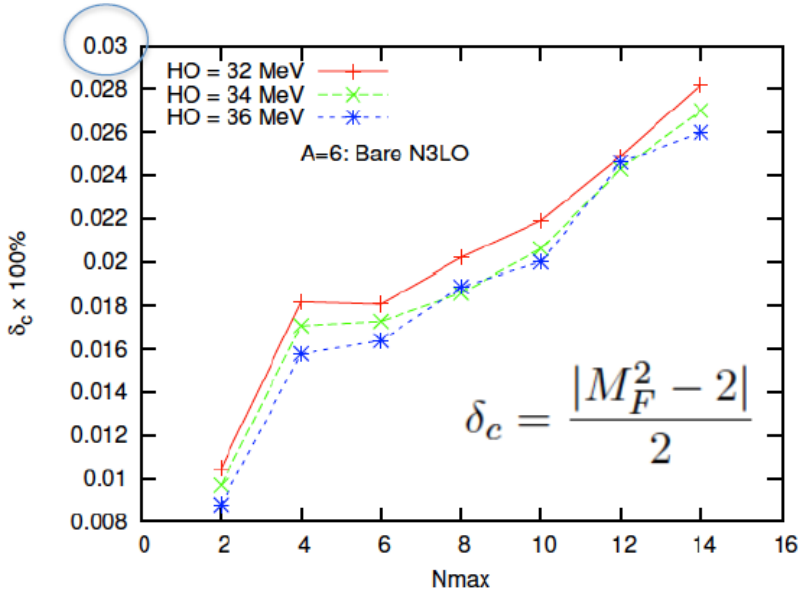
From L. Xayavong

L. Xayavong, N. Smirnova, Phys. Rev. C109, 014317 (2024)

Nuclear-structure correction to Fermi β decay: NCSM



Caurier, Navratil, Ormand, Vary (2002)



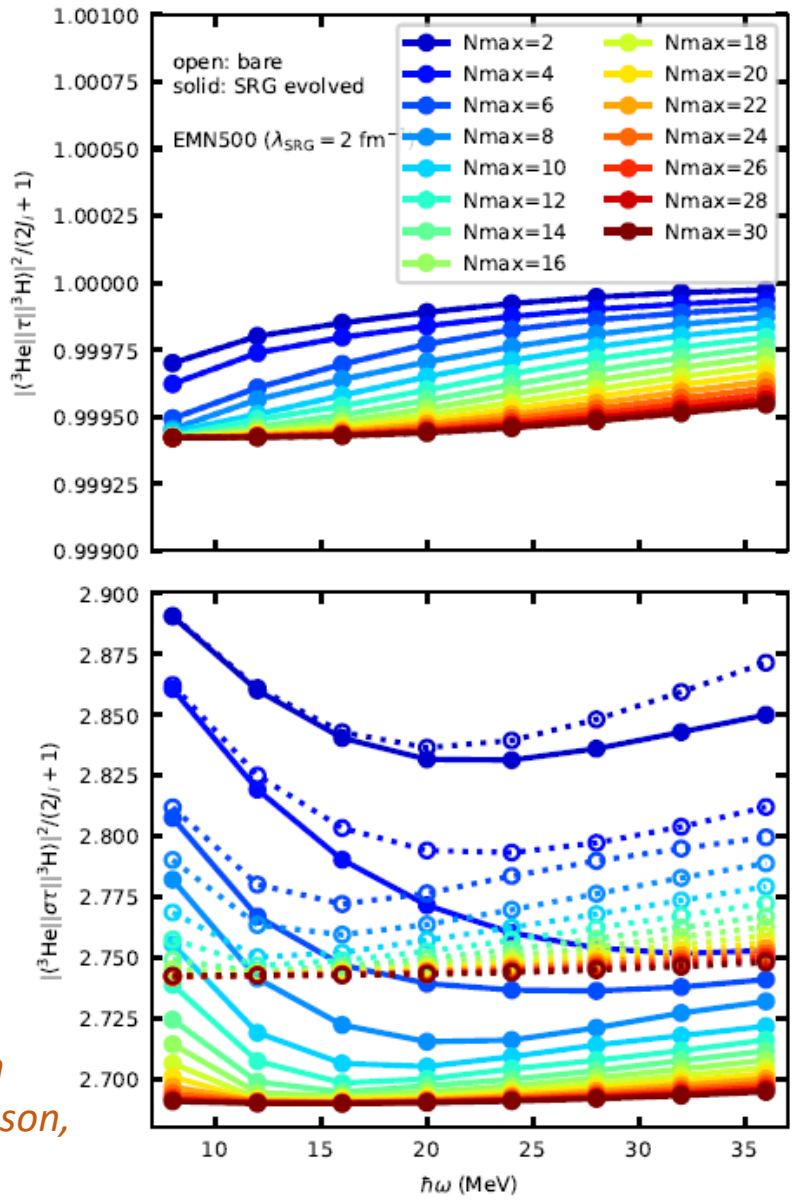
Kruse, Ormand, Johnson, Coon (2014)

Convergence?

$^3H \rightarrow {}^3He$

δ_C =0.057%

Miyagi, private communication
Xayavong, Lim, Smirnova, Johnson, submitted



Conclusions

Results :

- Phenomenological shell model provides a robust framework in the description of isospin symmetry breaking
- Large-scale calculations with global WS parametrization lead to the results similar to those of Towner and Hardy (2015, 2020)
- New calculations with the Skyrme HF wave functions + numerous effects checked. Conclusion: importance of CSB/CIB terms, of elimination of spurious isospin mixing and post-HF effects
- Higher-order terms in the correction are identified, assessed and found to be negligible

Perspectives :

- The issue of the exact Fermi operator to be investigated via ab-initio methods (L. Xayavong et al, in progress)
- High-dimensions and heavier nuclei (near A=80)
- Improved charge-dependent interactions

BACK-UP SLIDES

Theoretical description of isospin-symmetry breaking

Sources of isospin symmetry breaking (nucleon level):

- Coulomb interaction between protons
- Charge-dependent (CD) nuclear forces and Mp and Mn differences

- Ab-initio calculations: use CD NN potentials V (CD-Bonn, chiral EFT N^nLO potentials)
- Phenomenological approach: constrain effective CD forces from experiment to add to V

$$\hat{V}_{INC} = \hat{V}_{Coul} + \hat{V}_{CD} = \sum_{\lambda=0,1,2} \hat{V}^{(\lambda)}_{INC}, \quad \text{where} \quad \begin{cases} \hat{V}^{(0)}_{INC} = (v_{pp} + v_{nn} + v^{T=1}_{np})/3 \\ \hat{V}^{(1)}_{INC} = v_{pp} - v_{nn} \\ \hat{V}^{(2)}_{INC} = (v_{pp} + v_{nn})/2 - v^{T=1}_{np} \end{cases}$$

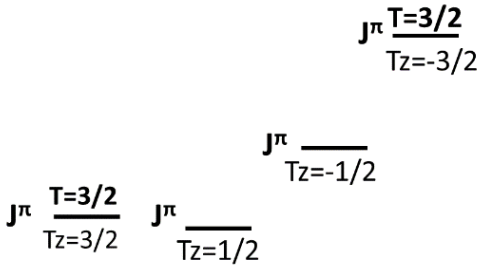
$$\langle \Psi_{TT_z} | \hat{V}_{INC} | \Psi_{TT_z} \rangle = E^{(0)}(\eta, T) + E^{(1)}(\eta, T) T_z + E^{(2)}(\eta, T) \left[3T_z^2 - T(T+1) \right]$$

Fit to experimental b and c coefficients through the valence space !

Nuclei along N=Z line: isospin-symmetry breaking

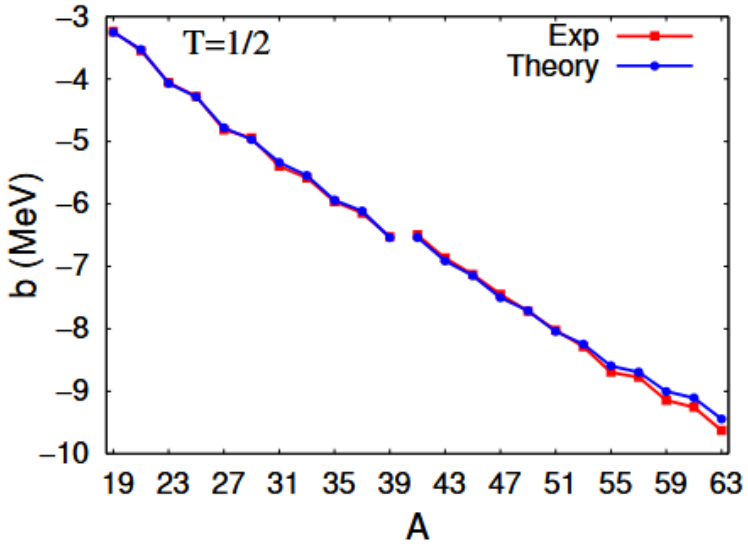
Isospin non-conserving Hamiltonian
(Coulomb + effective charge-dependent components)

$$H_{INC} = H_0 + V_{res} + V_{INC}$$



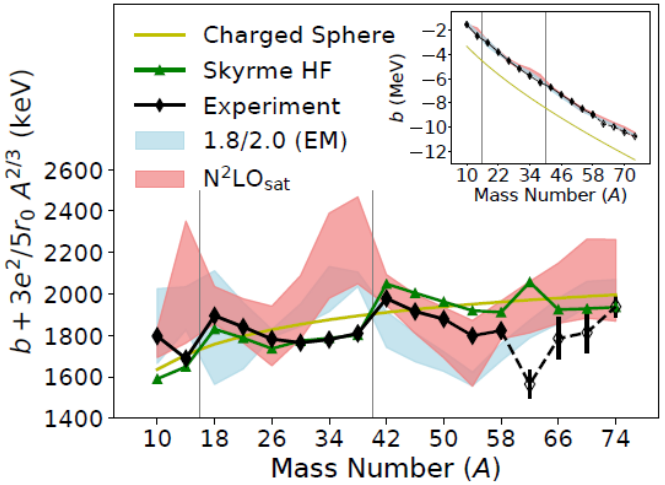
$$M(\eta, T, M_T) = a(\eta, T) + b(\eta, T)T_z + c(\eta, T)T_z^2$$

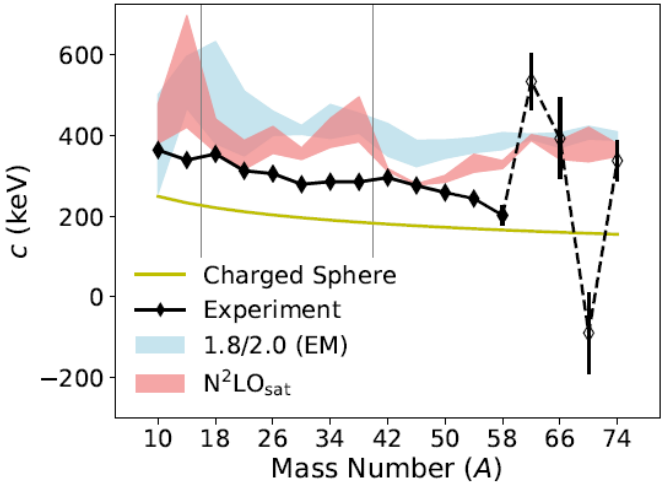
sd shell : rms(b) = 30 keV, rms(c) = 10 keV

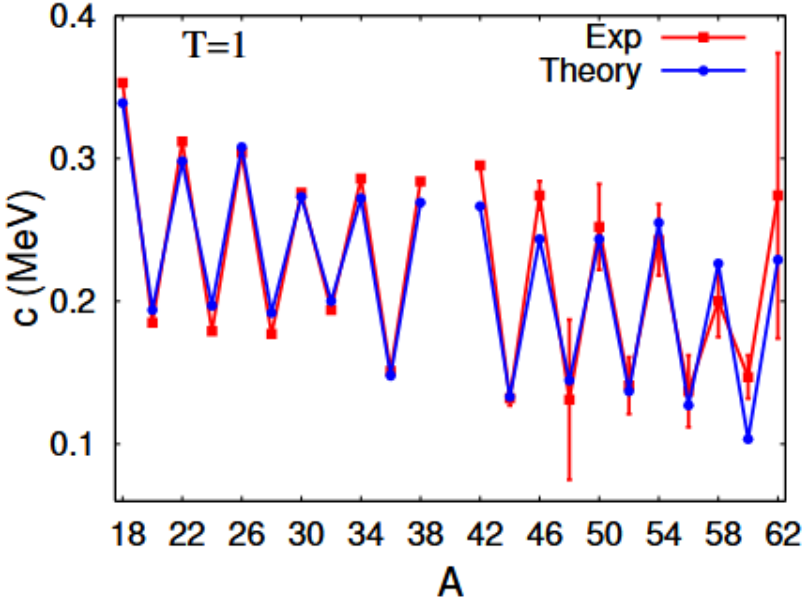


Ormand, Brown (1989), Lam, Smirnova, Caurier (2013); Magilligan, Brown (2020)

IMSRG: Martin et al, PRC104, 014324 (2021) – A=10,14,18,22,…,74

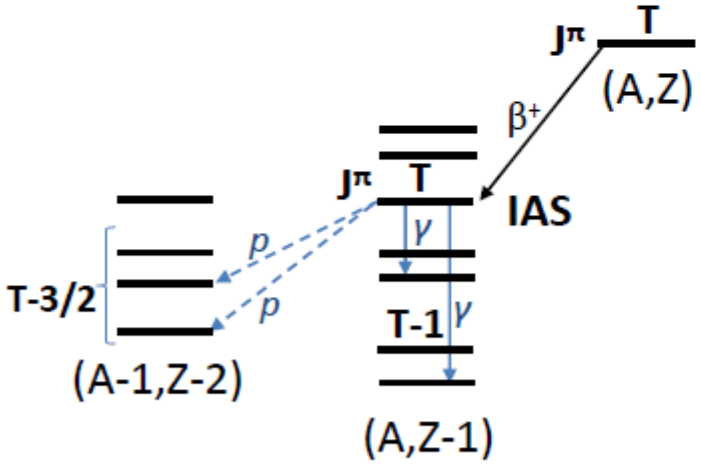


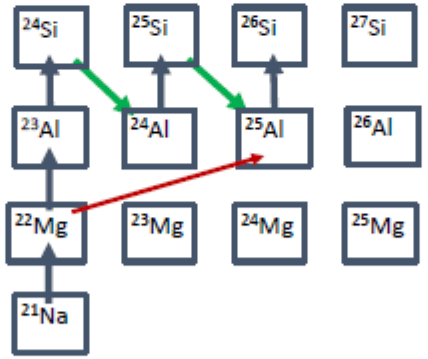


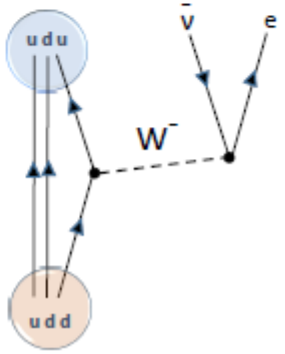


Applications of isospin-symmetry breaking

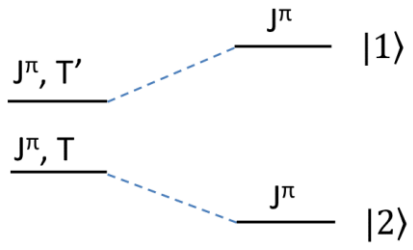
- Structure and decay modes of proton-rich nuclei and nuclei along $N = Z$
- Applications for nuclear astrophysics (masses, (p, γ), (α, p), ... reaction rates)
- Corrections to weak-interaction processes in nuclei for the tests of the Standard Model (CVC hypothesis, CKM matrix, ...)







Isospin-forbidden decays and isospin mixing



$$\alpha^2 \sim \frac{\langle V_{CD} \rangle^2}{\Delta E^2}$$

Current status :

- Reliable prediction of isospin-forbidden decay rates and of isospin mixing on theoretical grounds is challenging, because of poorly determined ΔE
- Safe predictions of Coulomb mixing matrix elements
- Efficient way: combination of theory and experiment !
- Lack of precision for the search of higher-order terms in IMME in quartets and quintets

$$M(\eta, T, T_z) = a(\eta, T) + b(\eta, T)T_z + c(\eta, T)T_z^2 + d(\eta, T)T_z^3 + e(\eta, T)T_z^4$$

Isospin-symmetry breaking: NN interaction and light nuclei

Sources of isospin-symmetry breaking : $m_u \neq m_d$ and electromagnetic interactions between quarks

- Coulomb interaction between protons
- $M_p - M_n \approx 1.3\ MeV$ and charge-dependent forces of nuclear origin

Experimental evidence on charge-dependent components of NN forces

1) NN scattering length in the $^1S_0$ channel:

$$a_{nn} - a_{pp} = 1.65 \pm 0.60\ fm$$ → *Charge-symmetry breaking (CSB)* $\boldsymbol{V_{pp} - V_{nn} \neq 0}$

$$\frac{1}{2}(a_{nn} + a_{pp}) - a_{np} = 5.6 \pm 0.6\ fm$$ → *Charge-independence breaking (CIB)* $\boldsymbol{V_{pn}^{T=1} - \frac{1}{2}(V_{pp} + V_{nn}) \neq 0}$

2) Nolen-Schiffer anomaly (1969): mirror binding energy differences ($^3H - {}^3He$, …)

Henley, Miller, 1979

Class I : $1, \boldsymbol{t}(j) \cdot \boldsymbol{t}(k)$
Class II : $t_z(j) t_z(k)$
Class III : $t_z(j) + t_z(k)$
Class IV : $t_z(j) - t_z(k), [\boldsymbol{t}(j) \times \boldsymbol{t}(k)]_z$

Ab-initio approaches →

Light nuclei in *ab-initio* approaches :
A = 3,7,8: GFMC with CD interactions
Wiringa, Pastore, Pieper, Miller, PRC88 (2013)

G.A. Miller et al PR194, 1 (1990); χEFT : U. van Kolck, J.L. Friar, E.Epelbaum, …

$$\boldsymbol{V_I > V_{II} > V_{III} > V_{IV}}$$

$$\hat{H}_{V-A} = \frac{G_V}{\sqrt{2}} \hat{j}^\dagger_\mu \hat{j}^\mu + h.c.$$

$$\begin{aligned}
\hat{j}^\dagger_\mu &= \hat{V}_\mu + \hat{A}_\mu \\
\hat{V}_\mu &= i\overline{\psi}_p \left(g_V(k^2)\gamma_\mu + \frac{g_W(k^2)}{2m_N}\sigma_{\mu\nu}k_\nu + ig_S(k^2)k_\mu \right)\psi_n \\
\hat{A}_\mu &= i\overline{\psi}_p \left(g_A(k^2)\gamma_\mu + \frac{g_T(k^2)}{2m_N}\sigma_{\mu\nu}k_\nu + ig_P(k^2)k_\mu \right)\gamma_5\psi_n
\end{aligned}$$

Fermi matrix element within the shell model

Fermi β-decay matrix element

$$H|\Psi\rangle = E|\Psi\rangle \quad \Rightarrow \quad \begin{aligned} |\Psi_i\rangle &= \sum c_{ki}|\Phi_k\rangle \\ |\Psi_f\rangle &= \sum c_{kf}|\Phi_k\rangle \end{aligned}$$

Single-particle matrix element (radial overlap)

$$\boxed{M_F = \langle\Psi_f|T_+|\Psi_i\rangle = \sum_\alpha \langle\Psi_f|a^\dagger_{\alpha_n} a_{\alpha_p}|\Psi_i\rangle\langle\alpha_n|t_+|\alpha_p\rangle}$$

One-body transition Density evaluated between initial and final nuclear states

$$\langle\Psi_f|a^\dagger_{\alpha_n} a_{\alpha_p}|\Psi_i\rangle \equiv \rho_\alpha \quad (\neq \rho^T_\alpha)$$

$$\langle\alpha_n|t_+|\alpha_p\rangle = \int_0^\infty R_{\alpha_n}(r)R_{\alpha_p}(r)r^2 dr \equiv \Omega_\alpha \quad (\neq 1)$$

$$\alpha = (n_\alpha, l_\alpha, j_\alpha, m_\alpha)$$

Miller, Schwenk
PRC (2009,2010):
Radial excitations !

- Large-scale calculations
- Global parameterization of INC forces
- Revisit WS procedure
- Implicate HF wave functions

I.S. Towner, J.C. Hardy; (1973 – 2020)
W.E. Ormand, B.A. Brown (1985 – 1995)

Fermi matrix element within the shell model

Isospin-symmetry limit

$$M_F^0 = \sum_\alpha \rho_\alpha^T \Omega_\alpha^T = \pm\sqrt{T(T+1) - T_{zi}T_{zf}}\,, \qquad \Omega_\alpha^T = 1$$

Realistic model (Coulomb and charge-symmetry breaking effective nuclear forces)

$$M_F = \sum_\alpha \rho_\alpha \Omega_\alpha$$

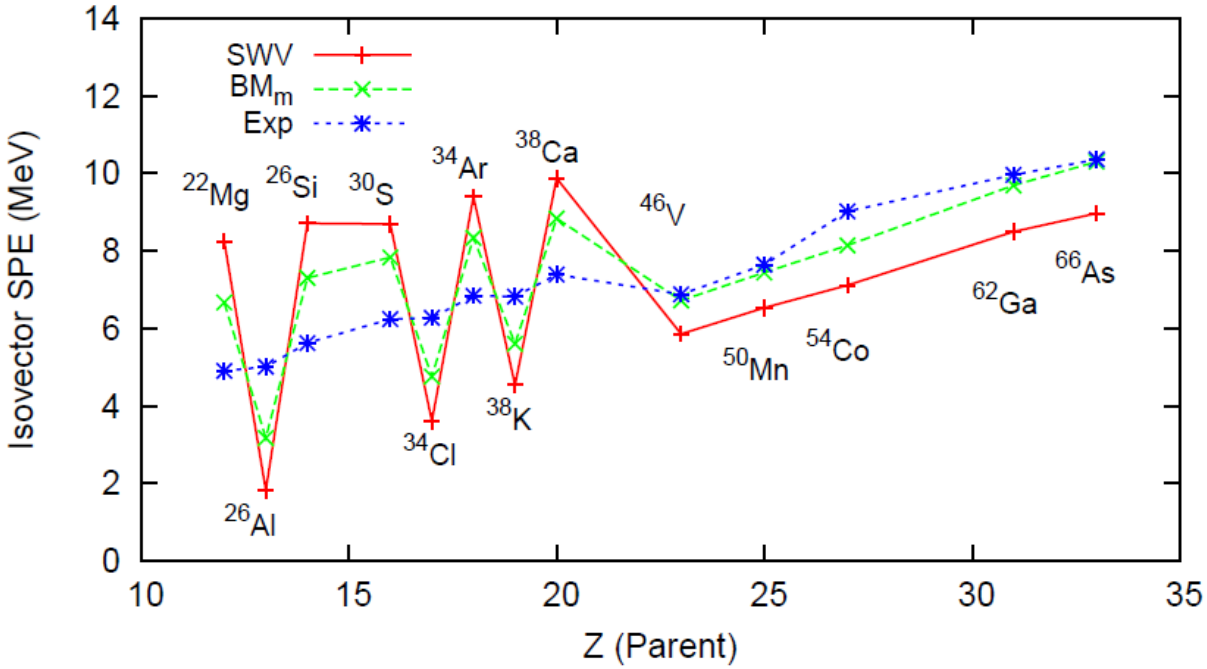
In first order perturbation theory:

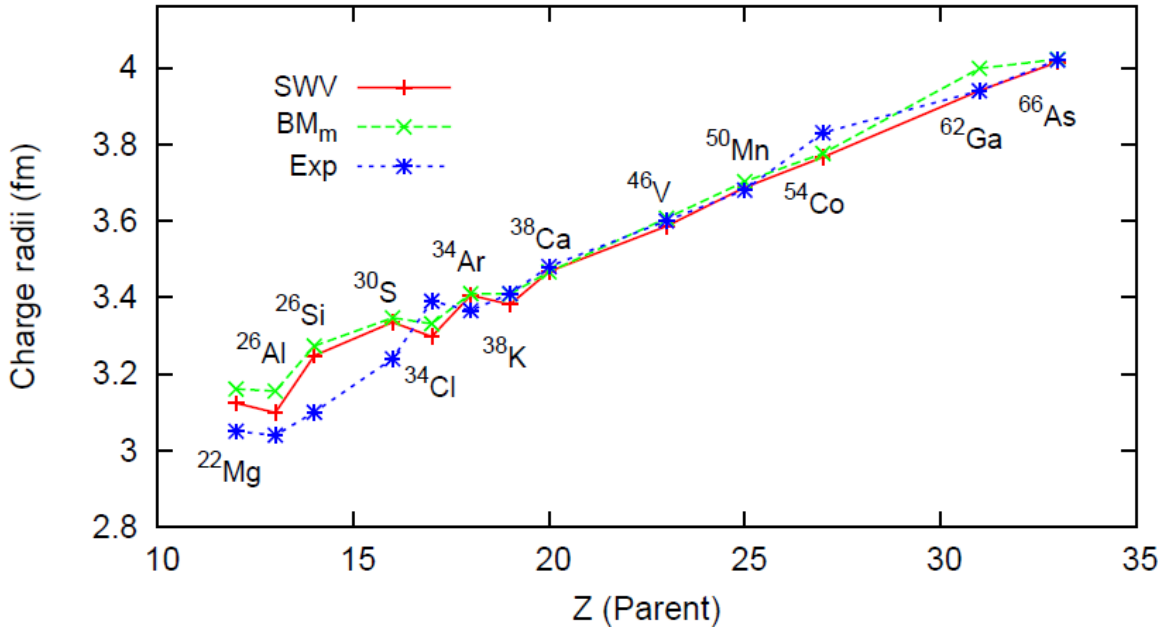
$$|M_F|^2 \approx |M_F^0|^2 \Big[1 - \underbrace{\frac{2}{M_F^0}\sum_\alpha \left(\rho_\alpha^T - \rho_\alpha\right)}_{\delta_{C1}} - \underbrace{\frac{2}{M_F^0}\sum_\alpha \rho_\alpha^T (1-\Omega_\alpha)}_{\delta_{C2}}\Big],$$

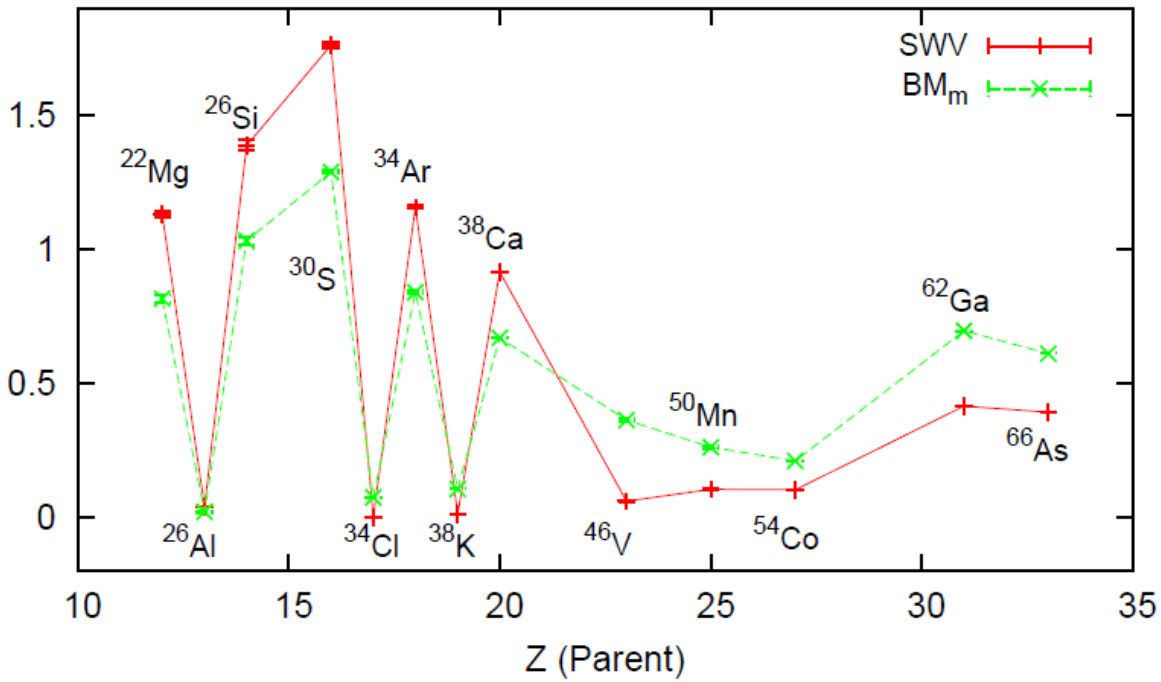
$$\delta_C \approx \delta_{C1} + \delta_{C2}$$

- δ_{C1} is the *isospin-mixing* part
- δ_{C2} is the *radial-overlap* part

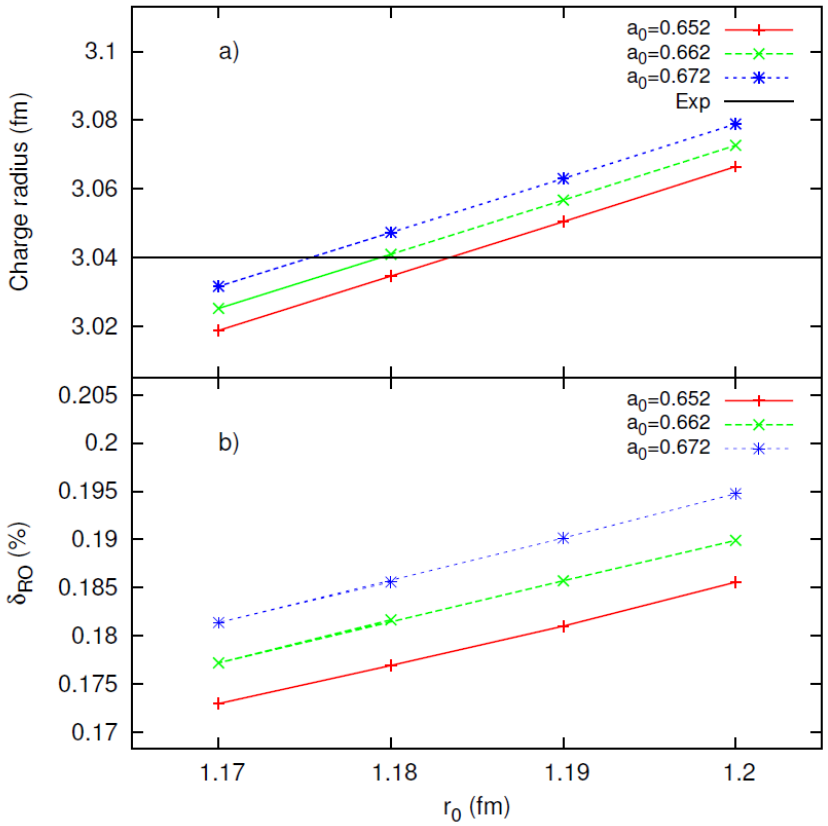
WS Non-adjusted !

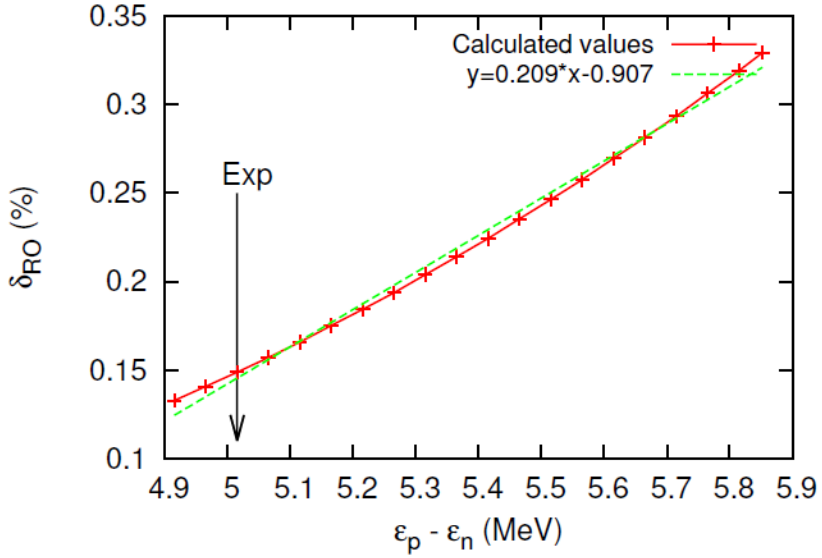


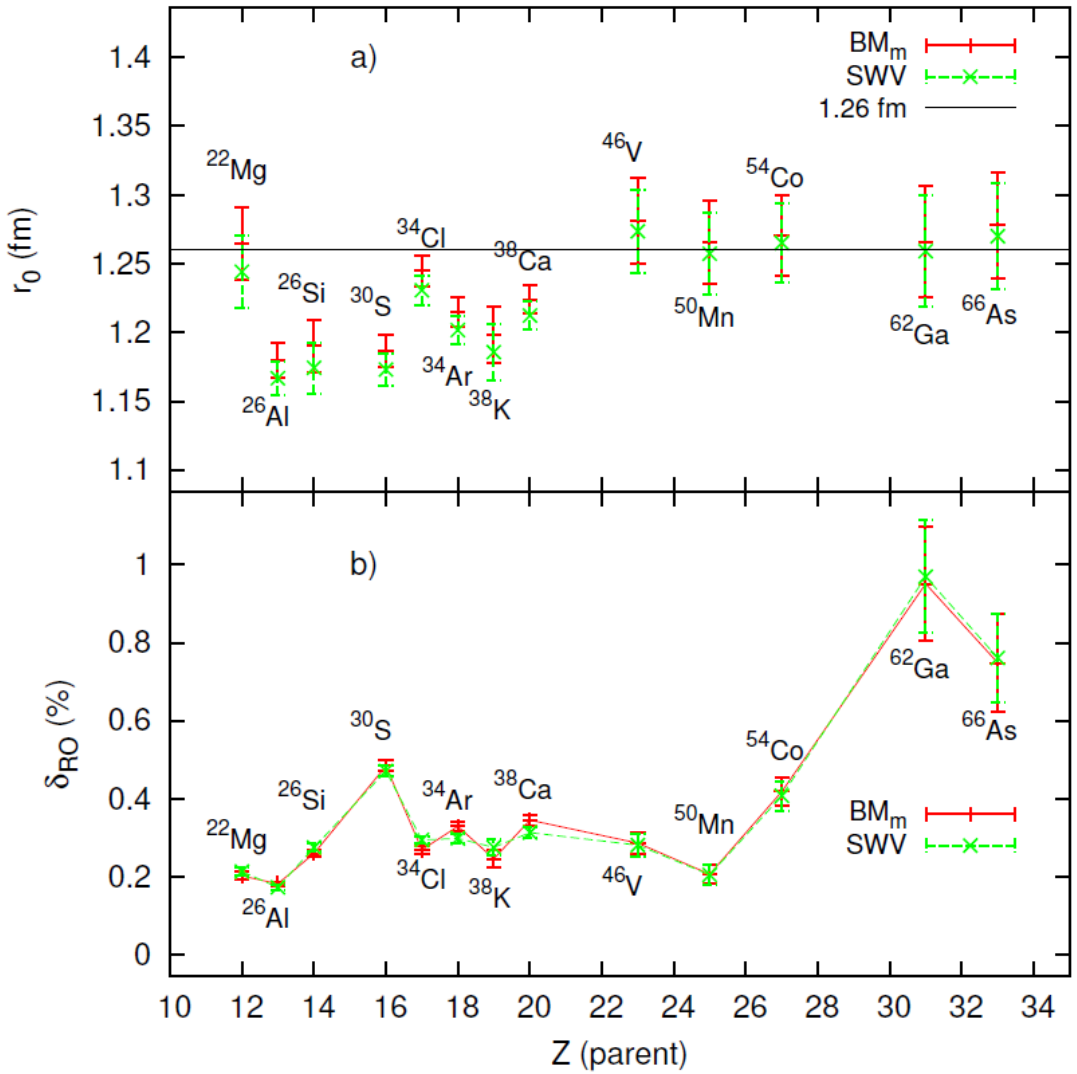




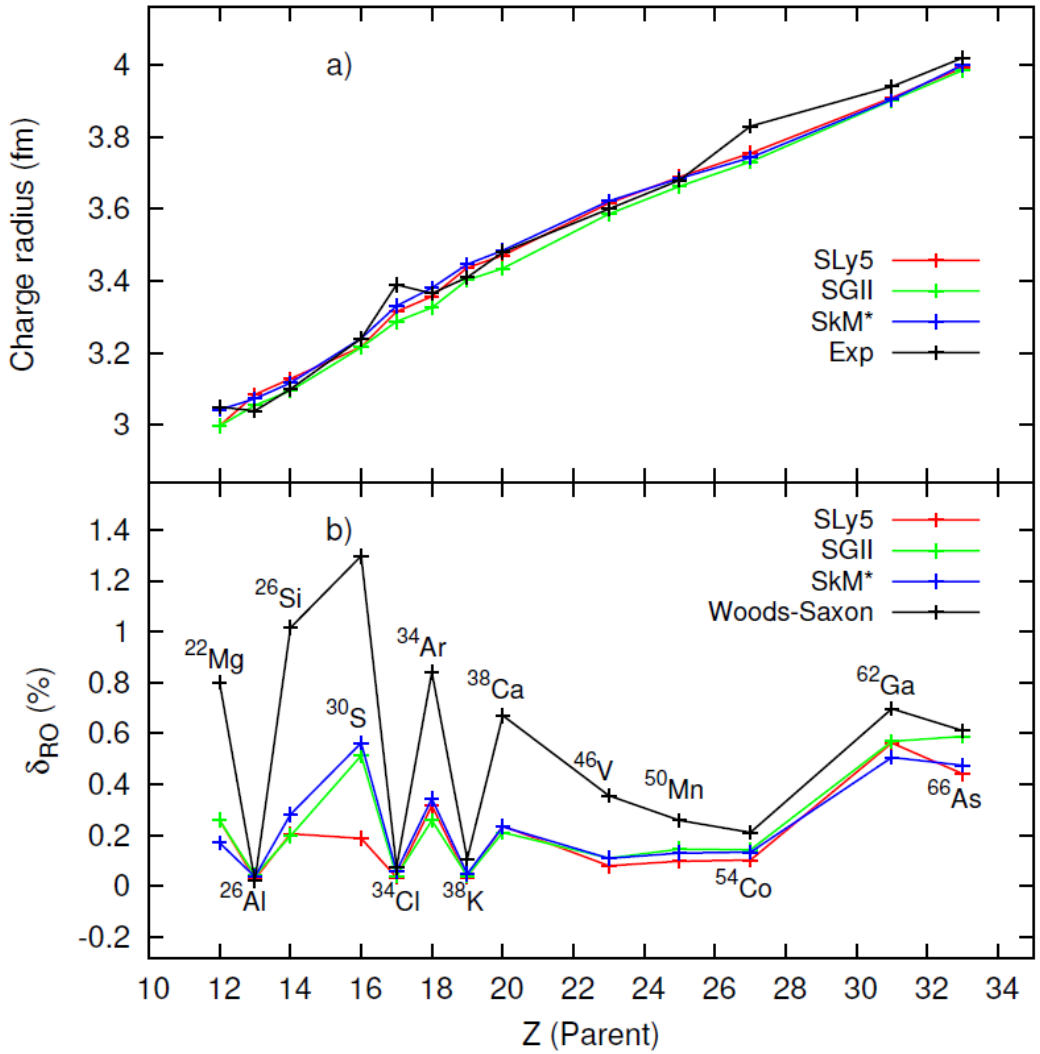
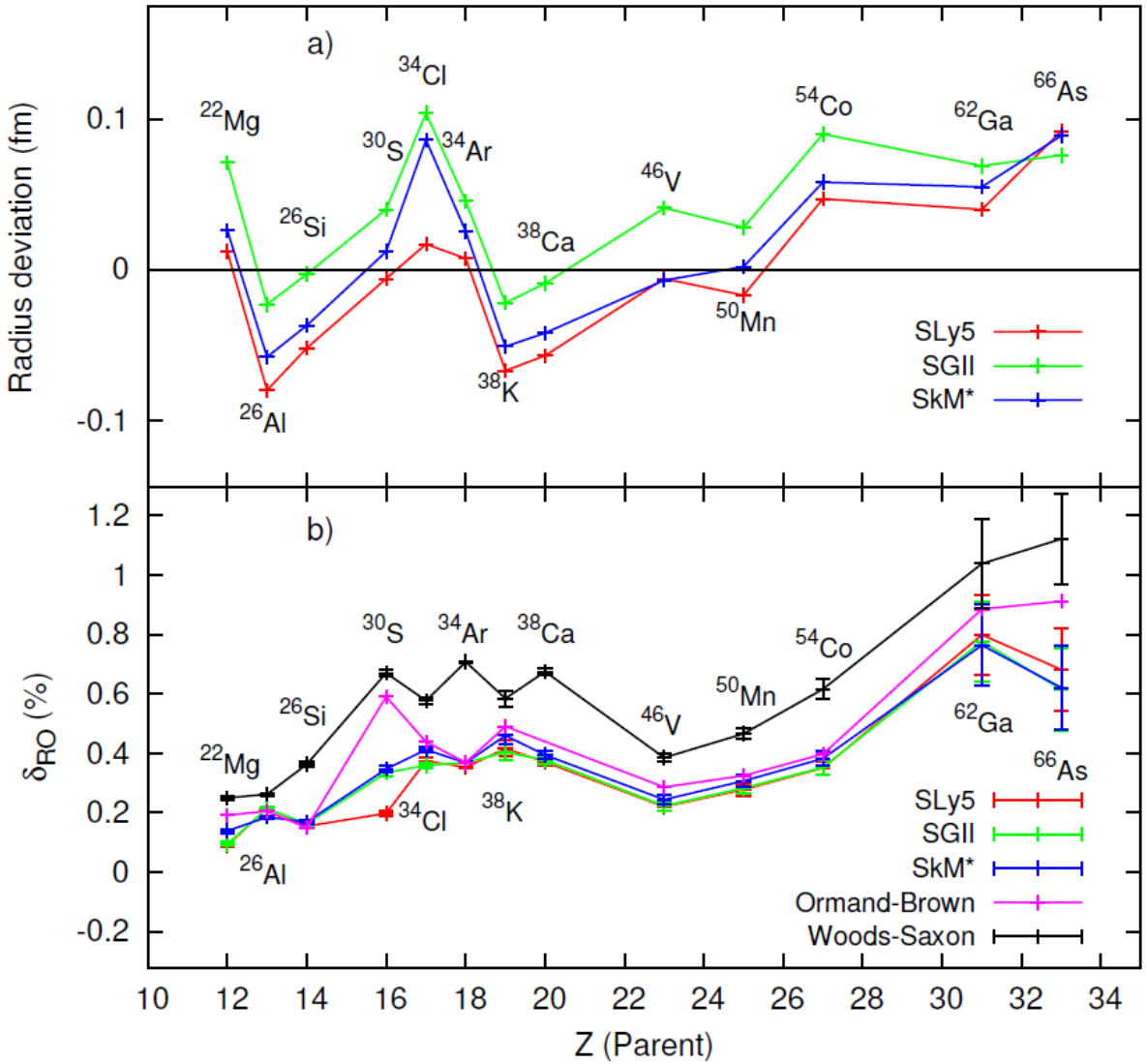
WS Adjusted !


a)
b)
a_0=0.652
a_0=0.662
a_0=0.672
Exp
Charge radius (fm)
δ_{RO} (%)
r_0 (fm)


Calculated values
y=0.209*x-0.907
Exp
δ_{RO} (%)
ε_p - ε_n (MeV)


a)
b)
BM_m
SWV
1.26 fm
^{22}Mg
^{26}Al
^{26}Si
^{30}S
^{34}Cl
^{34}Ar
^{38}K
^{38}Ca
^{46}V
^{50}Mn
^{54}Co
^{62}Ga
^{66}As
r_0 (fm)
δ_{RO} (%)
Z (parent)

WS and HF Non-adjusted !



WS and HF Adjusted !