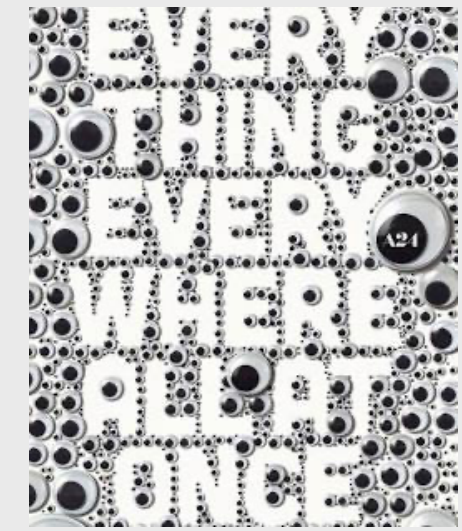




Every Correction, Everywhere, All At Once

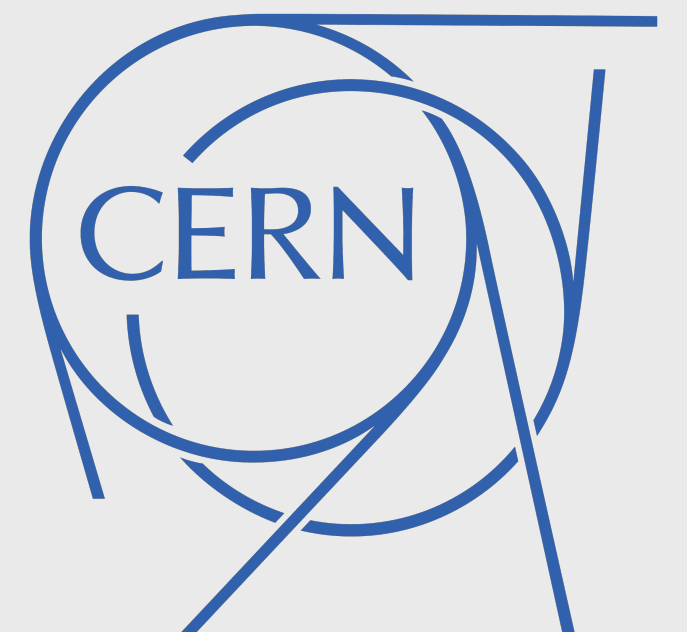
RYAN PLESTID

THEORY FELLOW - CERN

COLLABORATORS

RICHARD HILL, MARK B. WISE, PETER VANDER GRIEND, ZEHUA CAO, KAUSHIK BORAH

FUTURE DIRECTIONS IN NUCLEAR BETA DECAY AT FRIB - LANSING, MI, USA - SEPTEMBER 2025



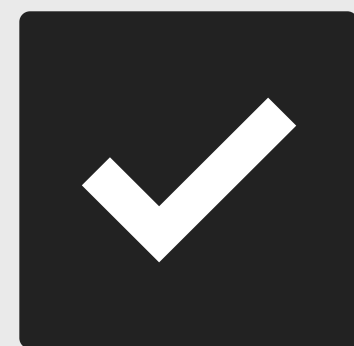


INTRODUCTION

LIT. CONVENTIONS

$$(1 + \Delta_R)(1 + \delta_{\text{NS}} - \delta_C)(1 + \delta'_R)F(Z, E)C(Z, E)$$

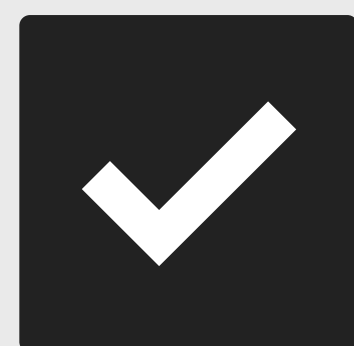
- What is the relevant physics? Should it factorize?



1-LOOP STRUCTURE

GAUGE INVARIANCE

- Formal expressions at 1-loop.
- This δ , that δ , $F(Z, E)$ and $C(Z, E)$.



HIGHER LOOP-ORDER

ISOSPIN & COULOMB

- Outer corrections to $O(Z^2\alpha^3L)$.
- Isospin breaking as a two-loop effect.

Conventional Separation

$$(1 + \Delta_R)(1 + \delta_{\text{NS}})(1 - \delta_C)(1 + \delta'_R) \times F(Z, E)$$

INNER CORRECTION

- Short-distance on a nucleon.

NUCLEAR STRUCTURE

- $\square_{W_\gamma}$ "box".

ISOSPIN BREAKING

- Wavefunction.

OUTER CORRECTION

- Diagrammatic.

FERMI FUNCTION

- Solve Dirac equation.

DIFFERENT METHODS JOINED TOGETHER.

Conventional Separation

$$(1 + \Delta_R)(1 + \delta_{NS})(1 - \delta_C)(1 + \delta'_R) \times F(Z, E)$$

INNER CORRECTION

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- Solve Dirac equation.

DOES THIS WORK TO 0.01% ACCURACY?

Conventional Separation

$$(1 + \Delta_R)(1 + \delta_{\text{NS}})(1 - \delta_C)(1 + \delta'_R) \times F(Z, E)$$

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NUCLEAR STRUCTURE

- $\square_{W_\gamma}$ "box".

ISOSPIN BREAKING

- Wavefunction.

OUTER CORRECTION

- Diagrammatic.

FERMI FUNCTION

- Solve Dirac equation.

ARE WE SURE WE AREN'T DOUBLE COUNTING?

DOES THIS WORK TO 0.01% ACCURACY?

$$(1 + \Delta_R)(1 + \delta_{\text{NS}})(1 - \delta_C)(1 + \delta'_R) \times F(Z, E)C(Z, E)$$

 Δ_R

RG EFFECTS + DISPERSIVE AND/OR LATTICE

 δ_{NS}

DIAGRAMMATIC + NUCLEAR MANY-BODY

 δ_C

COMPUTE NUCLEAR WAVEFUNCTION OVERLAPS

 δ'_R

DIAGRAMMATIC APPROACH

 $F(Z, E)C(Z, E)$

SOLVE DIRAC EQUATION FOR ELECTRON

What You Should Demand

- ***Every piece*** can be defined in terms of Feynman diagrams.
- ***Defined in a single formalism***
(no double counting).
- ***Systematic*** and can be extended to higher orders.

Outer Corrections

CONSIDER δ'_R : THE "OUTER CORRECTION"

$$\delta'_R = \frac{\alpha}{2\pi} \left(\delta_1 + \underbrace{Z\alpha\delta_2}_{\text{2ND-ORDER}} + \underbrace{Z^2\alpha^2\delta_3}_{\text{3RD-ORDER}} + \dots \right)$$

COMPARE TO FERMI FUNCTION

$$F(Z, E) = \sum_n C_n(E) (Z\alpha)^n$$



δ_2 AND δ_3 NEED TO MATCH
FERMI FUNCTION
CONVENTIONS

Isospin Breaking Correction

CONSIDER δ_C : THE "ISOSPIN CORRECTION"

$$(1 - \delta_C) \sim 1 - \#(Z\alpha)^2 + \dots$$

2ND-ORDER

TYPICALLY DEFINED WITH W.F. OVERLAP

$$(1 - \delta_C) \equiv \langle B | \tau^- | A \rangle / \sqrt{2}$$

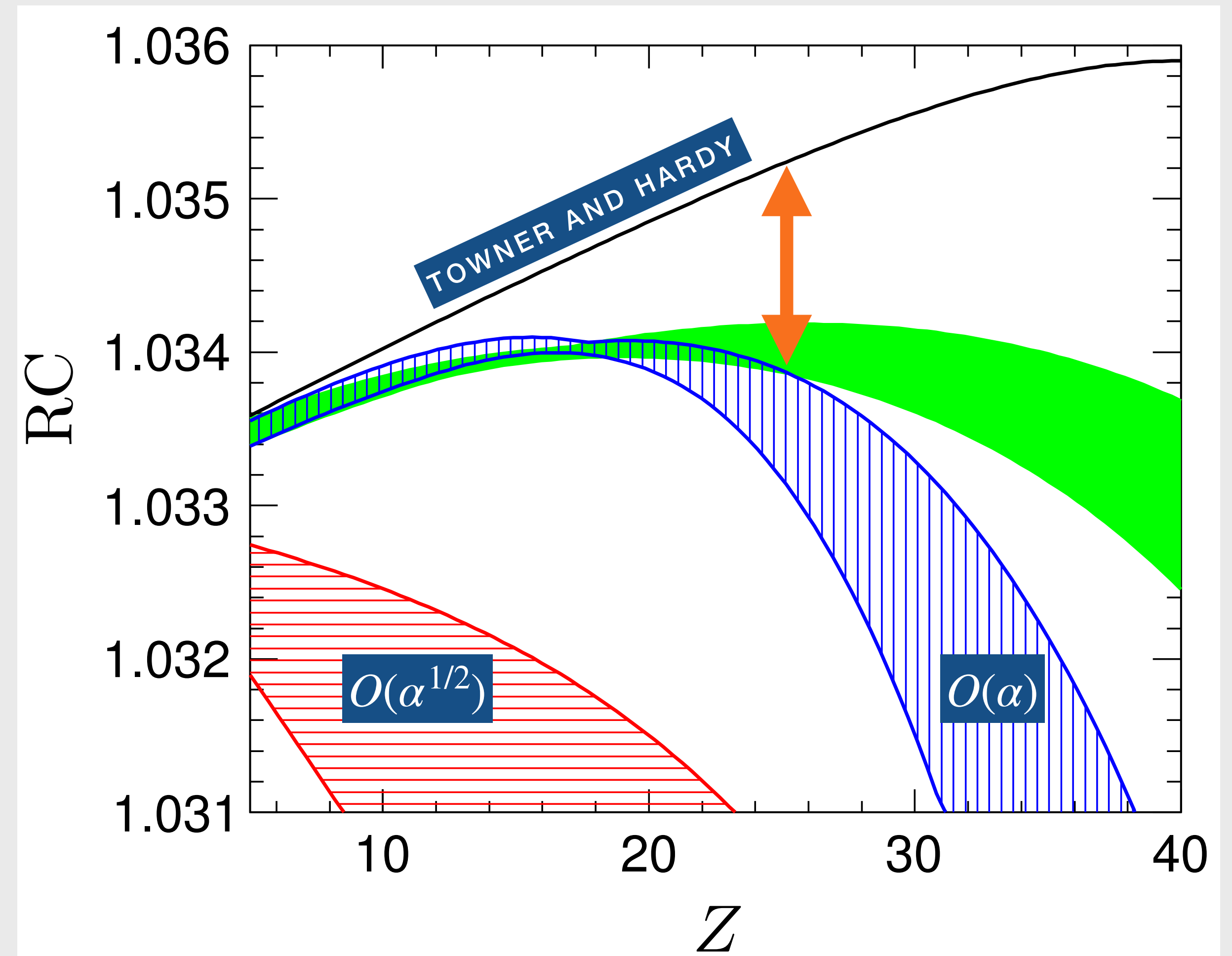
1⁺ Loops (Higher Orders)

- Everything I have described is of electromagnetic origin
- Without a single unified formalism we run the risk of double-counting or missing effects.
- Conventions matter when matching pieces together.

Numerical Impact

SHIFTING δ_3

transition	$(\Delta a) \times Z^2 \alpha^3 \log(\Lambda/m)$
$^{14}\text{O} \rightarrow ^{14}\text{N}$	-1.1×10^{-4}
$^{26m}\text{Al} \rightarrow ^{26}\text{Mg}$	-3.2×10^{-4}
$^{34}\text{Cl} \rightarrow ^{34}\text{S}$	-5.6×10^{-4}
$^{34}\text{Ar} \rightarrow ^{34}\text{Cl}$	-6.3×10^{-4}
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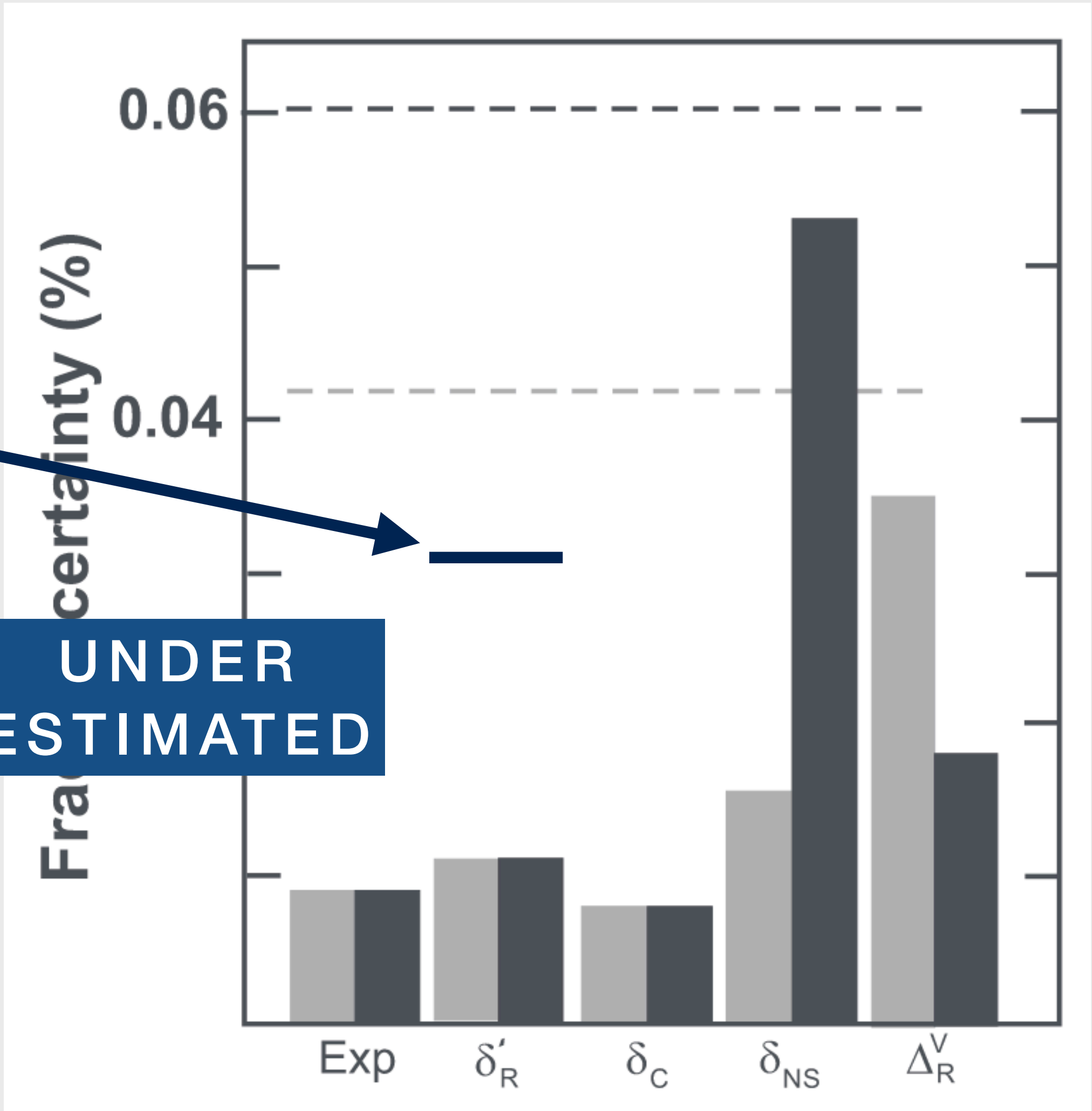


COUNTING $Z \sim \log \sim 1/\sqrt{\alpha}$

Numerical Impact

OUTER CORRECTIONS ARE
NOT "WELL UNDER CONTROL"

transition	$(\Delta a) \times Z^2 \alpha^3 \log(\Lambda/m)$
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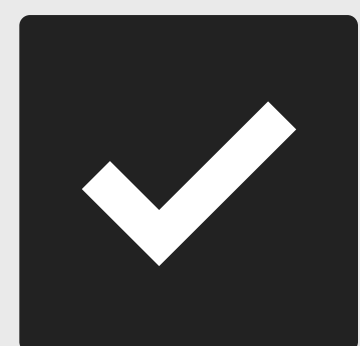
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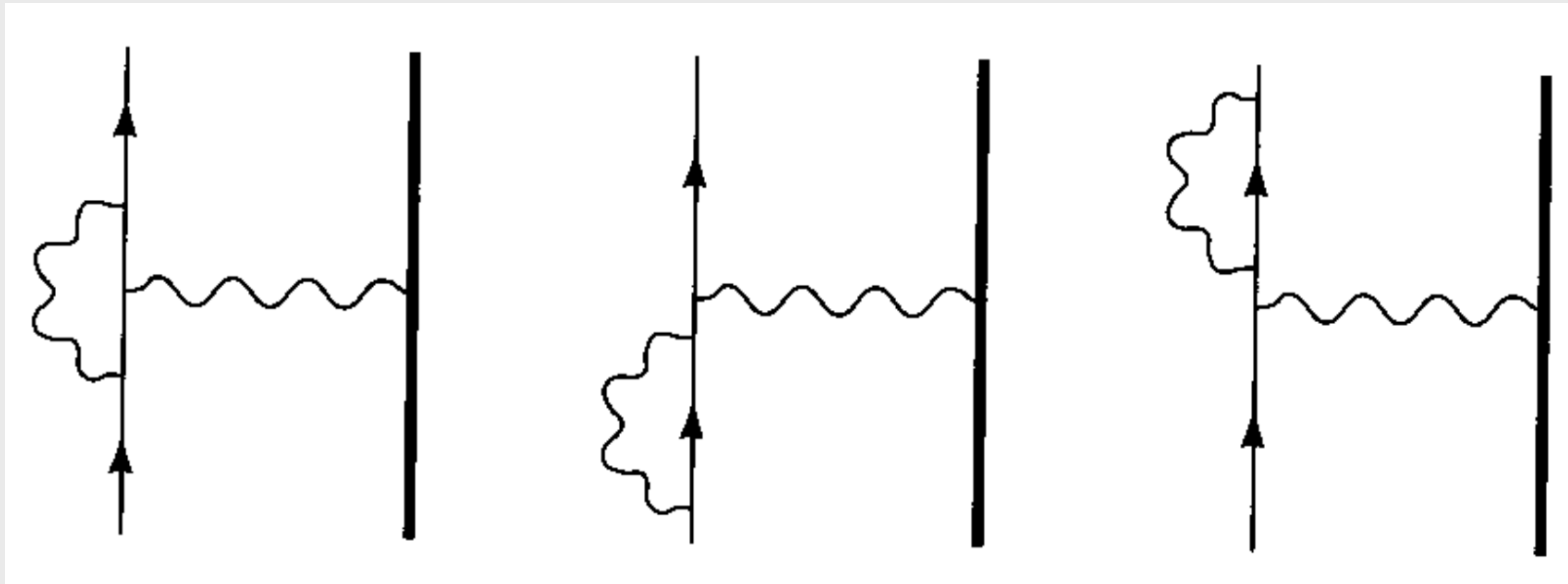
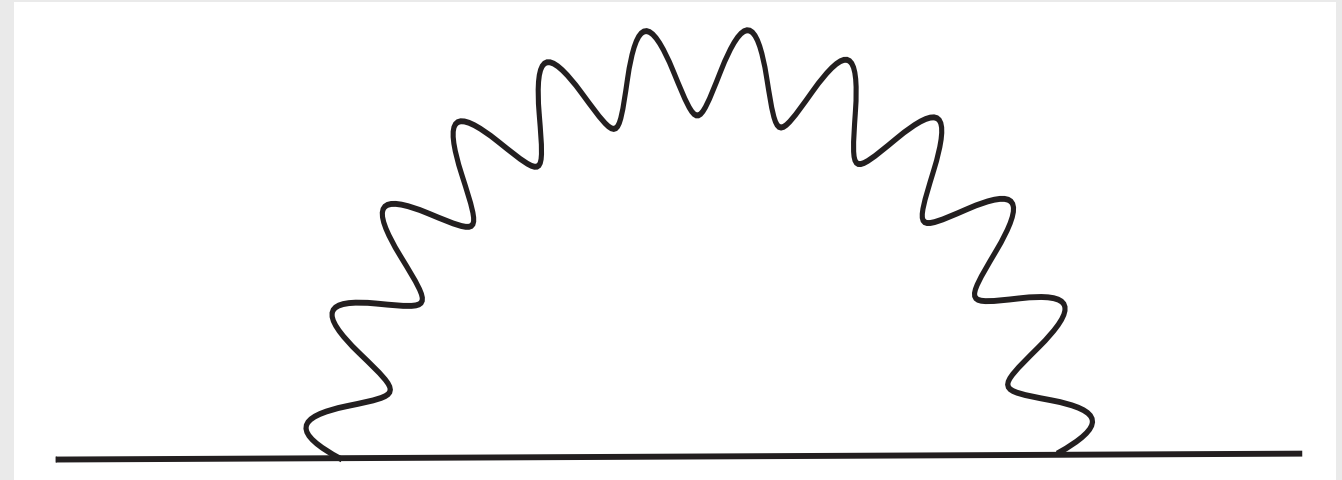
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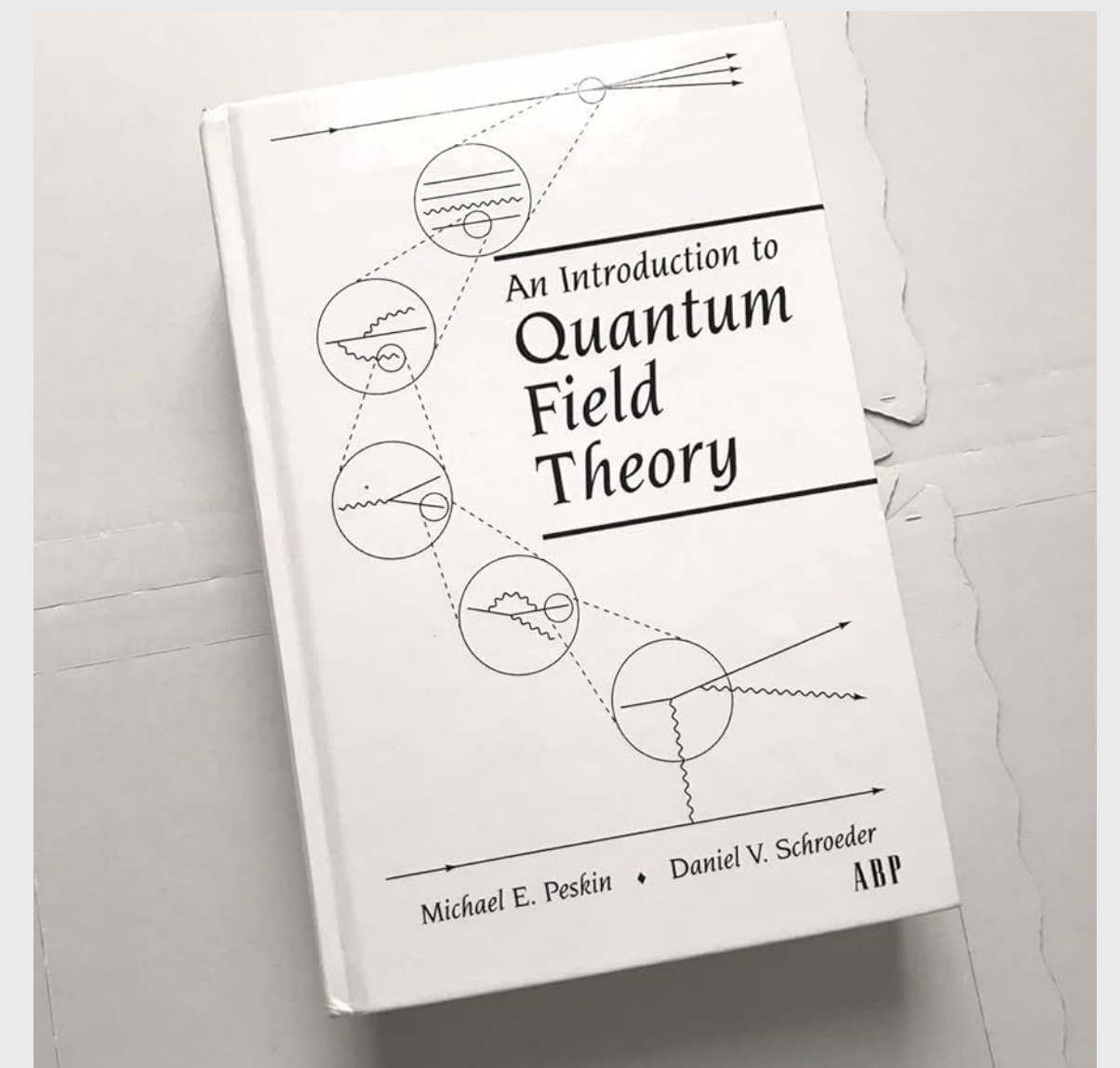
Radiative Corrections

ELECTRON SCATTERING

$$\sqrt{Z} =$$



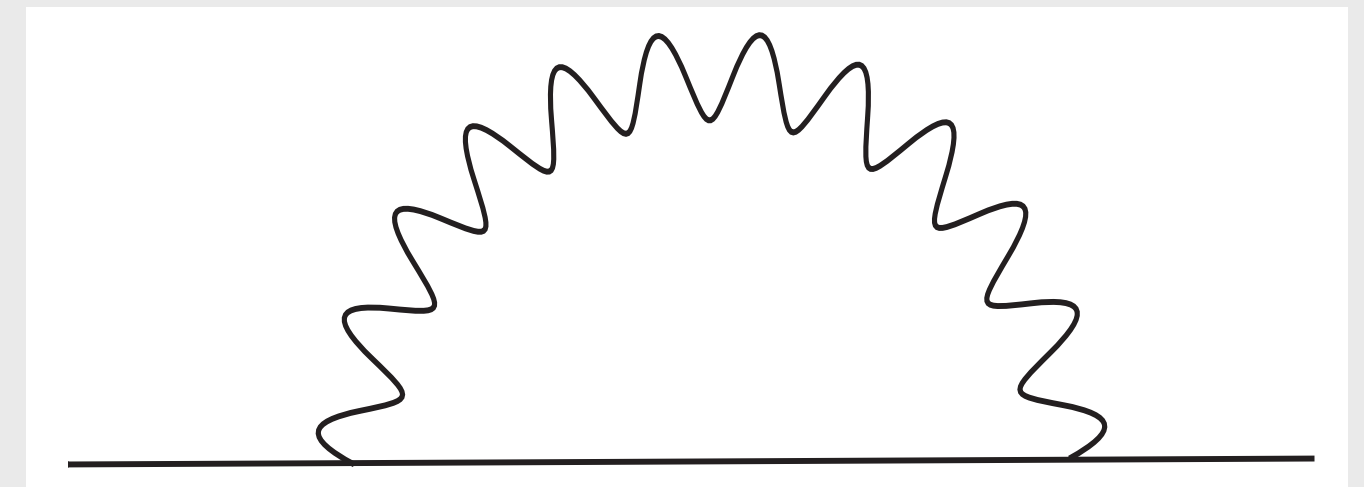
FEYNMAN RULES → INTEGRALS → ANSWERS



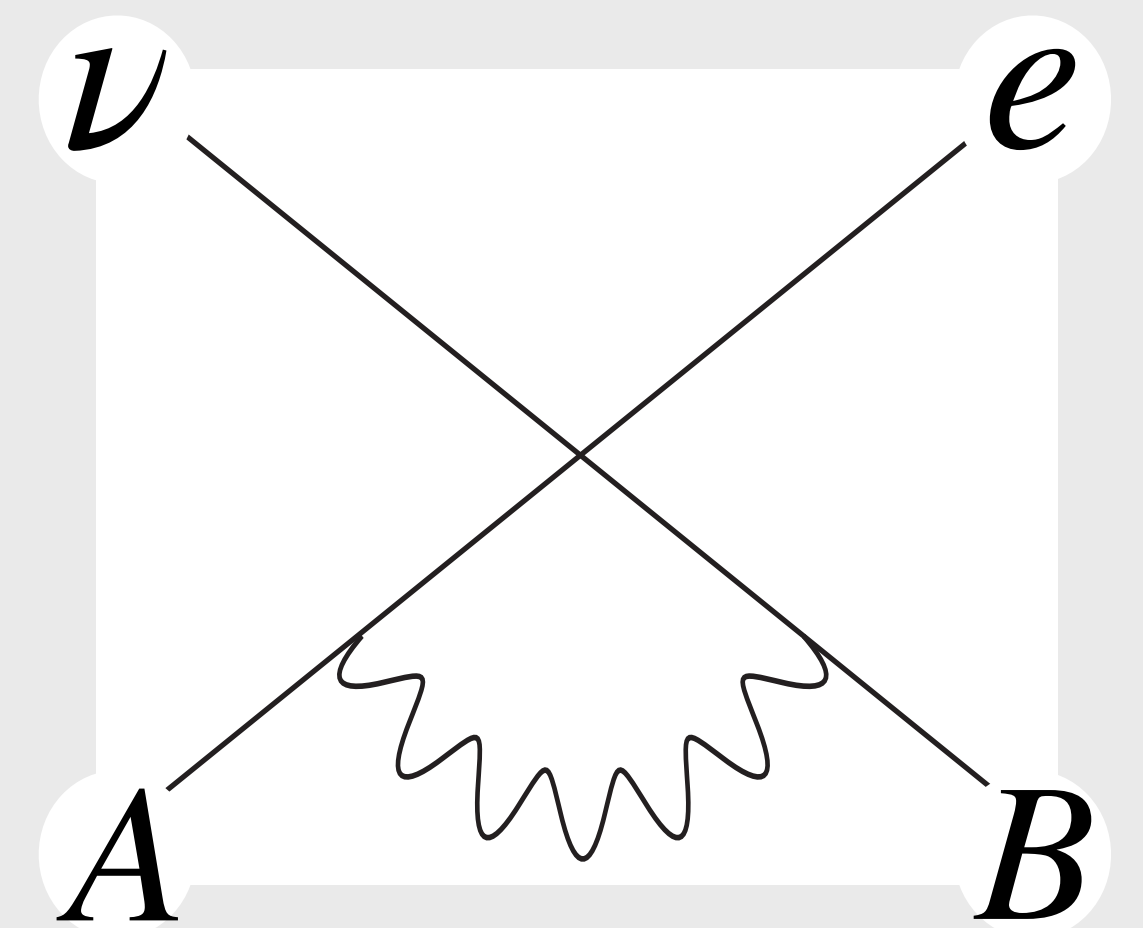
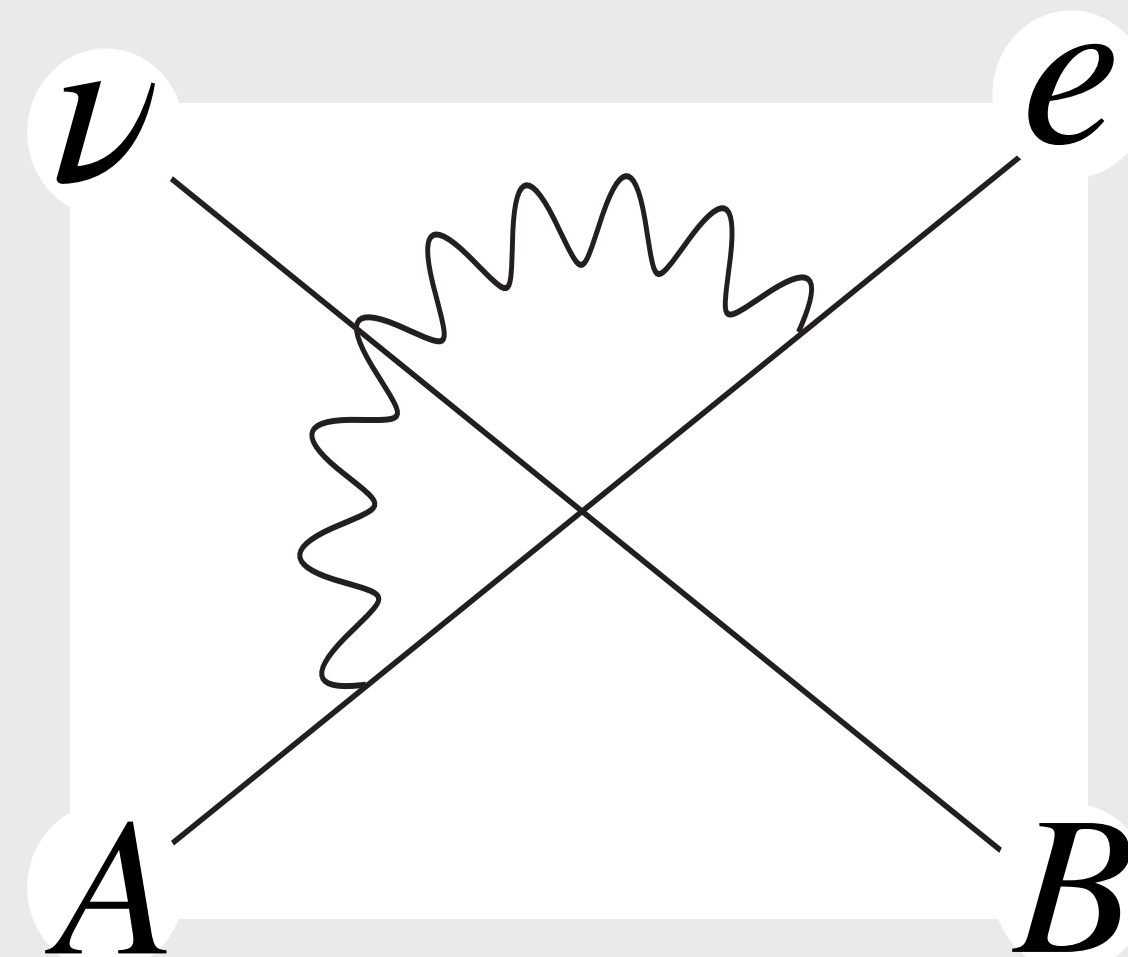
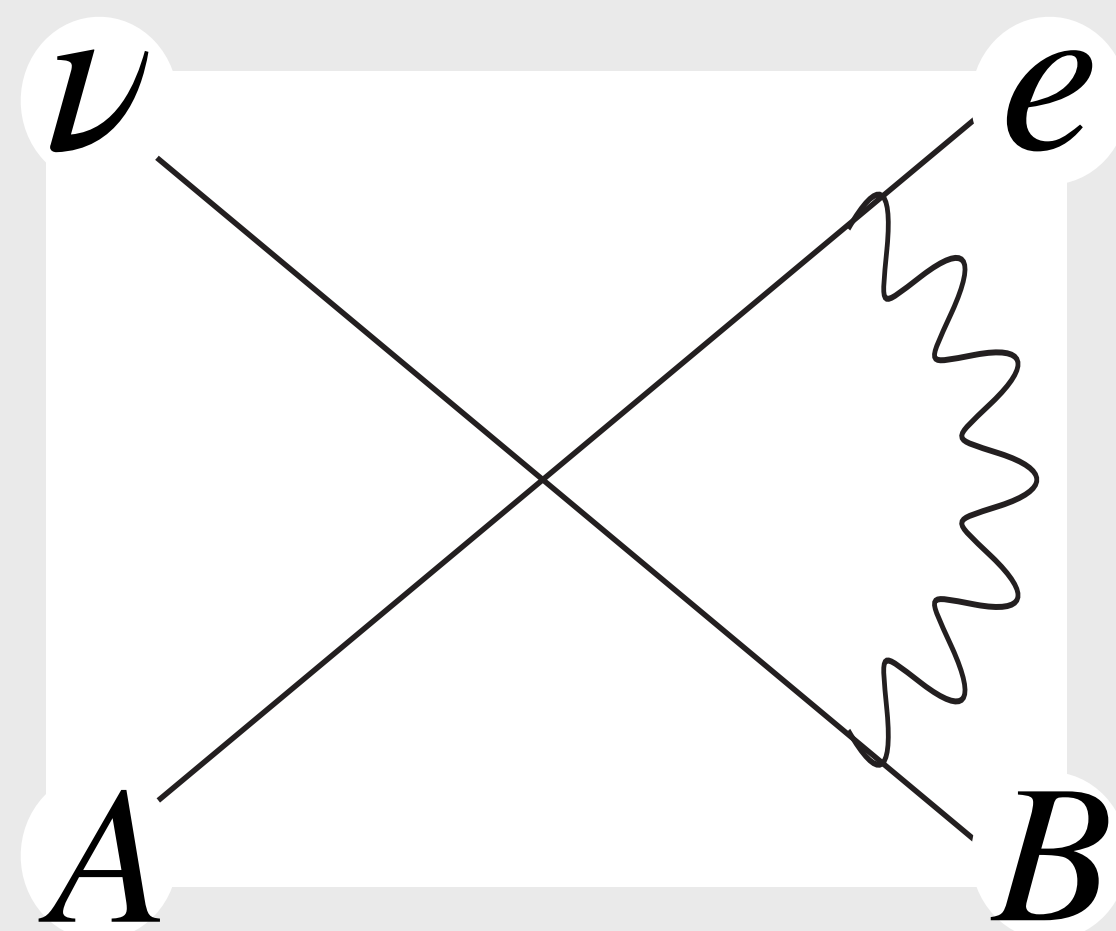
Radiative Corrections

LETS IMAGINE A & B WERE ELEMENTARY-PARTICLES

$$\mathcal{A} = \left(\sqrt{Z_e} \sqrt{Z_A} \sqrt{Z_B} \right) \times \Gamma \quad \bigg| \quad \sqrt{Z} =$$



$$\Gamma^{(1)} =$$

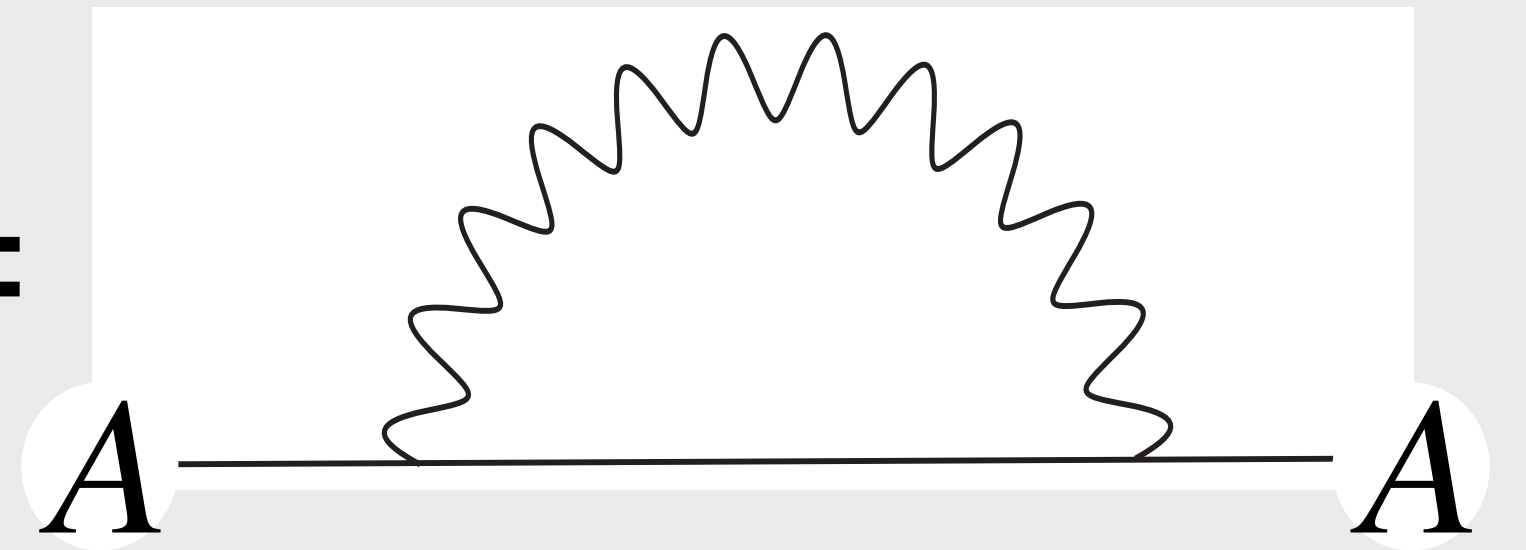


Radiative Corrections

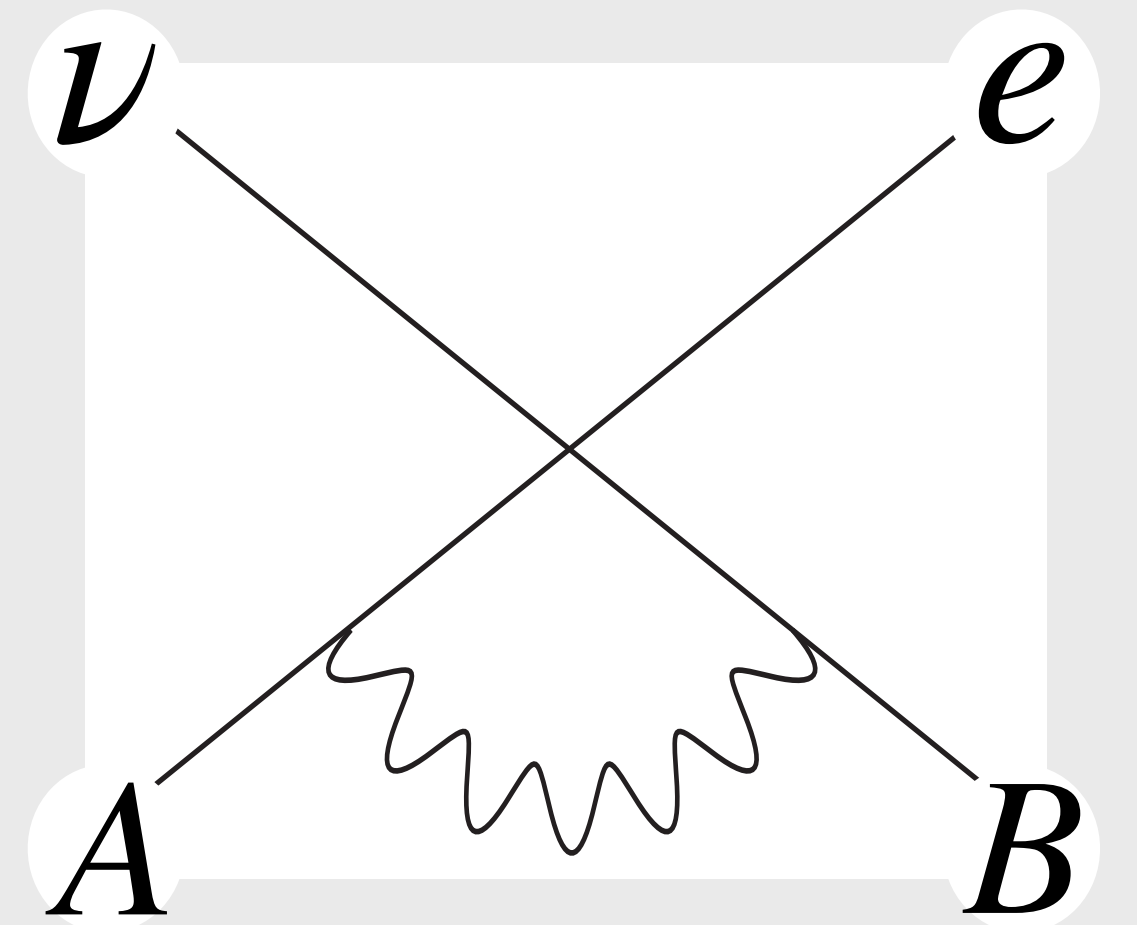
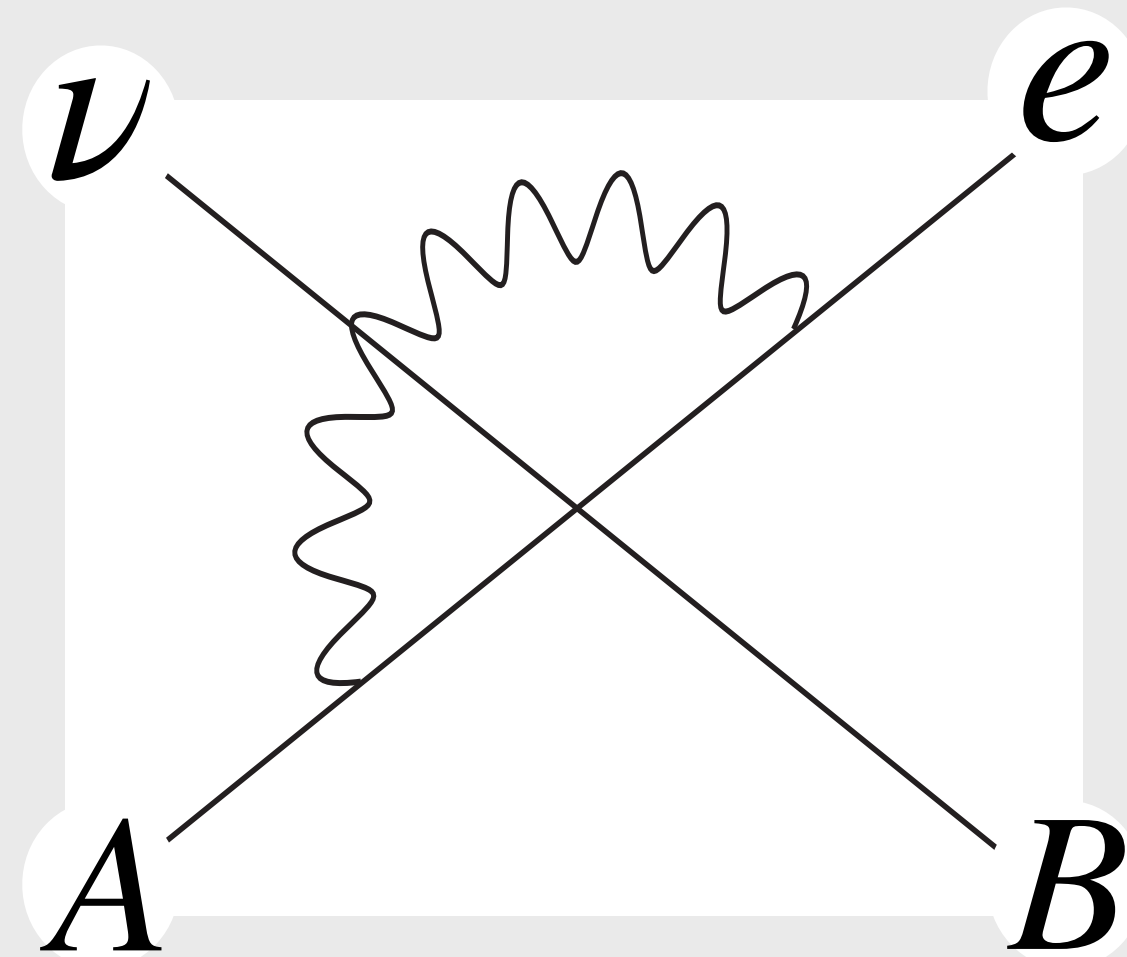
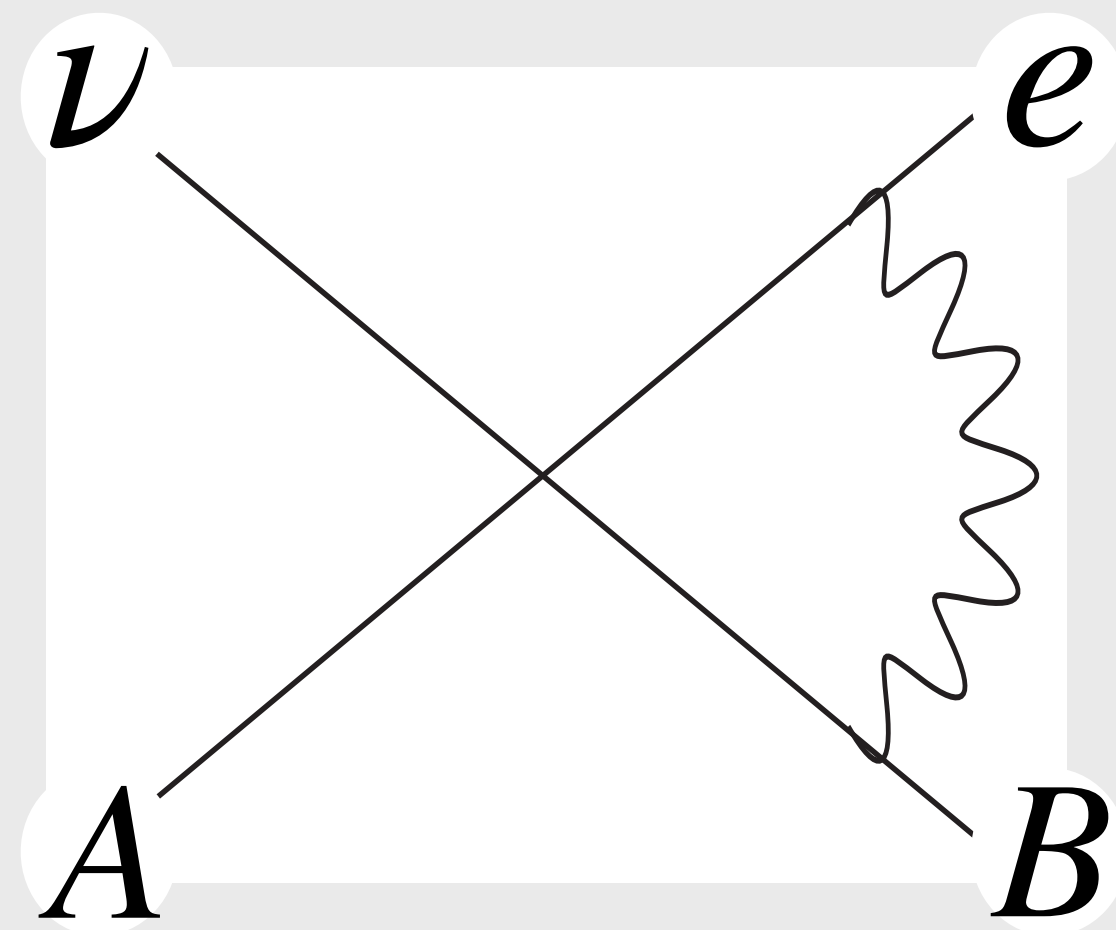
WHAT ABOUT THE REAL PROBLEM WITH NUCLEI?

- Definition of wavefunction renormalization?
- Connection to literature?

$$\sqrt{Z} =$$



$$\Gamma^{(1)} =$$



Feynman Rules

PROPAGATORS

$$\text{~~~~~} = D_{\mu\nu}(q) ,$$

$$\text{=====} = \frac{i}{p_0 - H_S + i0} ,$$

$$\text{=====} = \frac{i}{p_0 - H_S + i0} \mathbb{P}_A ,$$

VERTICES

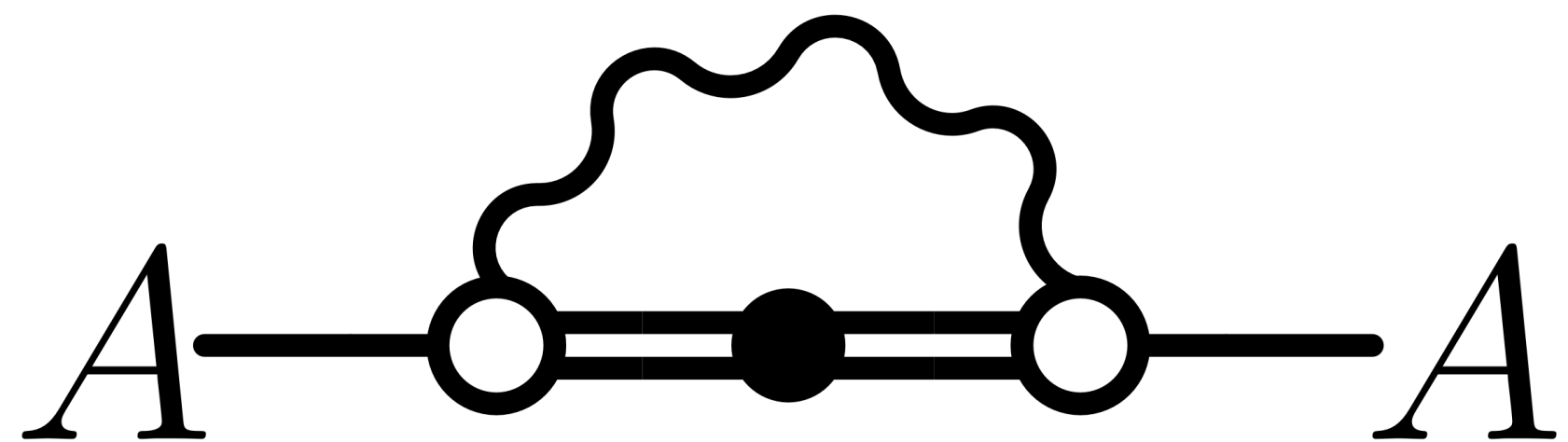
$$\text{~~~~~} \bigcirc = (ie) J_\mu(\mathbf{q}) ,$$

$$\text{~~~~~} \bigcirc \text{~~~~~} = (ie^2) S_{\mu\nu}(\mathbf{q}_1, \mathbf{q}_2) .$$

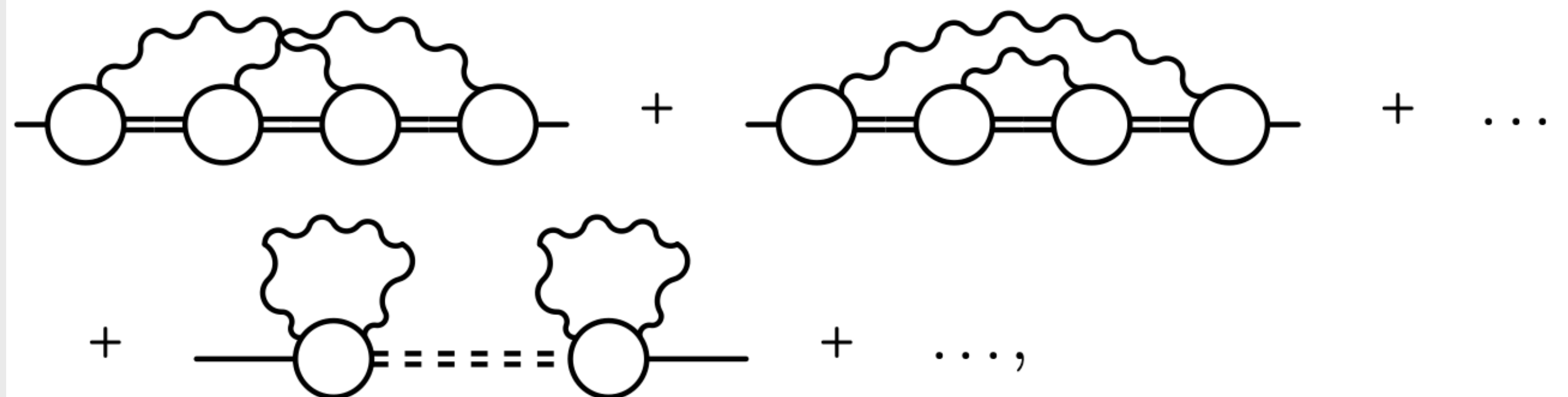
W.F. Renormalization

REDUCE TO NUCLEAR MATRIX ELEMENT

$$Z_A = 1 + e^2 \int \frac{d^4 q}{(2\pi)^4} \langle A | J_\nu(-\mathbf{q}) \left(\frac{1}{q_0 - \hat{H}_A} \right)^2 J_\mu(\mathbf{q}) | A \rangle D^{\mu\nu}(q) + \dots$$



HIGHER ORDERS

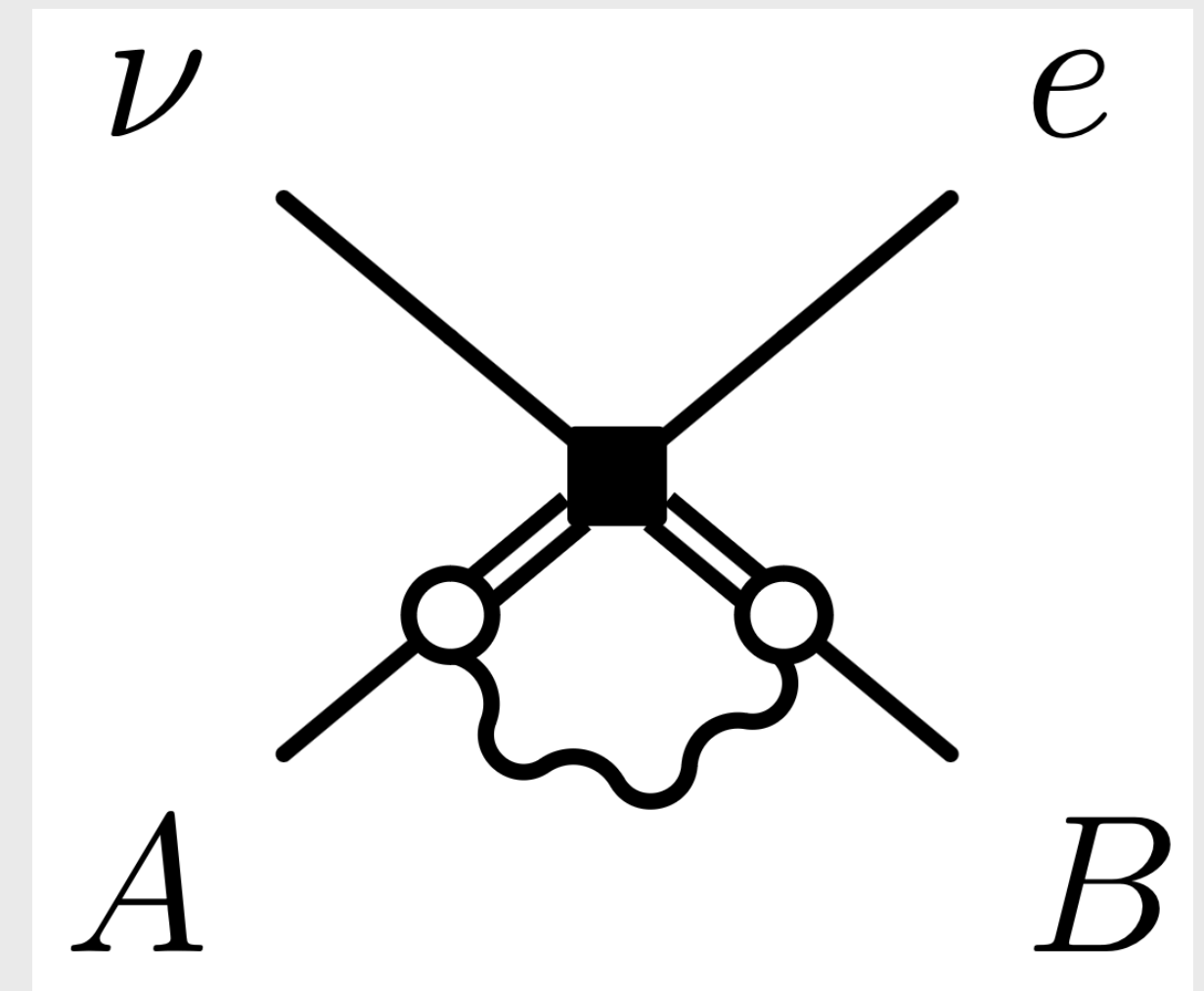


Nuclear Vertex Correction

$$e^2 \int \frac{d^4 q}{(2\pi)^4} \langle A | J_\nu(-\mathbf{q}) \frac{1}{q_0 - \hat{H}_A} \mathcal{O}(\mathbf{k}) \frac{1}{q_0 - \hat{H}_A} J_\mu(\mathbf{q}) | A \rangle D^{\mu\nu}(q) + \dots$$

- Can relate one-to-one with LSZ reduction formalism.
- Each diagram comes with a nuclear matrix element.

HOW DO WE USE THIS EFFECTIVELY?



Analysis At One-Loop

EXACT M.E.

$$(Be^+ \nu_e | \mathcal{H}_W | A) = \left[1 + V^{(1)} + B^{(1)} + \frac{1}{2} Z_e^{(1)} \right] \times \left(\begin{array}{c} \nu \\ A \end{array} \begin{array}{c} e \\ B \end{array} \right)$$

$$Z_e^{(1)} = e \text{ --- } \text{cloud} \text{ --- } e \quad ,$$

$$V^{(1)} = \left(\begin{array}{c} \nu \\ A \end{array} \begin{array}{c} e \\ B \end{array} \right) \times \left(\begin{array}{c} \nu \\ A \end{array} \begin{array}{c} e \\ B \end{array} \right)^{-1} + \frac{1}{2} \left(A \text{ --- } \text{cloud} \text{ --- } A + B \text{ --- } \text{cloud} \text{ --- } B \right)$$

$$B^{(1)} = \left(\begin{array}{c} \nu \\ A \end{array} \begin{array}{c} e \\ B \end{array} \right) + \left(\begin{array}{c} \nu \\ A \end{array} \begin{array}{c} e \\ B \end{array} \right) \times \left(\begin{array}{c} \nu \\ A \end{array} \begin{array}{c} e \\ B \end{array} \right)^{-1} .$$

Analysis At One-Loop

POINT-LIKE PROTON

$$B_p^{(1)} = \left(\begin{array}{c} \nu \\ \text{[Diagram: wavy line loop with a square vertex]} \\ p \end{array} \begin{array}{c} e \\ \\ n \end{array} \right) \times \left(\begin{array}{c} \nu \\ \text{[Diagram: straight line loop with a square vertex]} \\ p \end{array} \begin{array}{c} e \\ \\ n \end{array} \right)^{-1}$$

"SUBTRACTED"

$$\tilde{V}^{(1)} = V^{(1)} - \frac{1}{2} Z_p^{(1)}$$

$$\tilde{B}^{(1)} = B^{(1)} - B_p^{(1)},$$

CORRECTIONS ON A POINT-LIKE PROTON

$$\tilde{O}_{\text{virt.}}^{(1)} = \left(\begin{array}{c} \nu \\ \text{[Diagram: wavy line loop with a square vertex]} \\ p \end{array} \begin{array}{c} e \\ \\ n \end{array} \right) \times \left(\begin{array}{c} \nu \\ \text{[Diagram: straight line loop with a square vertex]} \\ p \end{array} \begin{array}{c} e \\ \\ n \end{array} \right)^{-1} + \frac{1}{2} \left(\begin{array}{c} (1 + \delta'_R) \\ \text{[Diagram: wavy line loop with a dot vertex]} \\ p \end{array} \begin{array}{c} e \\ \\ n \end{array} \right)$$

Identifying $F(Z, E) \times C(Z, E)$

$$\begin{aligned} \tilde{B}^{(1)} = & \frac{1}{\bar{u}(p_\nu) \Gamma v(p_e) \langle B | \mathcal{O}(\mathbf{k}) | A \rangle} \times e^2 \int \frac{d^4 q}{(2\pi)^4} D^{\mu\nu}(q) \bar{u}(p_\nu) \Gamma \frac{1}{-\not{p}_e - \not{q} - m} \gamma_\nu v(p_e) \\ & \times \left[- (2\pi i) v_\mu \delta(\omega) \langle B | \rho(\mathbf{q}) \mathcal{O}(\mathbf{k} - \mathbf{q}) | A \rangle + \right. \\ & \left. \langle B | \mathcal{J}_\mu^{(-)}(\mathbf{q}) \frac{1}{-\omega - H_B} \mathcal{O}(\mathbf{k} - \mathbf{q}) | A \rangle + \langle B | \mathcal{O}(\mathbf{k} - \mathbf{q}) \frac{1}{\omega - H_A} \mathcal{J}_\mu^{(+)}(\mathbf{q}) | A \rangle \right]. \end{aligned}$$

Identifying $F(Z, E) \times C(Z, E)$

$$\begin{aligned} \tilde{B}^{(1)} = & \frac{1}{\bar{u}(p_\nu) \Gamma v(p_e) \langle B | \mathcal{O}(\mathbf{k}) | A \rangle} \times e^2 \int \frac{d^4 q}{(2\pi)^4} D^{\mu\nu}(q) \bar{u}(p_\nu) \Gamma \frac{1}{-\not{p}_e - \not{q} - m} \gamma_\nu v(p_e) \\ & \times \left[- (2\pi i) v_\mu \delta(\omega) \langle B | \rho(\mathbf{q}) \mathcal{O}(\mathbf{k} - \mathbf{q}) | A \rangle + \dots \right] \end{aligned}$$

THIS CONTRIBUTION IS GAUGE INVARIANT

REMAINS GAUGE INVARIANT FOR EVERY INTERMEDIATE STATE

STICK IN THE GROUND-STATE

$$\langle B | \rho(\mathbf{q}) | B \rangle \langle B | \mathcal{O}(\mathbf{k} - \mathbf{q}) | A \rangle$$

Identifying $F(Z, E) \times C(Z, E)$

$$e^2 \int \frac{d^3 q}{(2\pi)^3} \bar{u}(p_\nu) \Gamma \frac{1}{-\not{p}_e - \not{q} - m} \frac{\langle B | \rho(\mathbf{q}) | B \rangle}{\mathbf{q}^2} \gamma_0 v(p_e) \langle B | \mathcal{O}(\mathbf{k} - \mathbf{q}) | A \rangle$$
$$\rightarrow \int \frac{d^3 q}{(2\pi)^3} \bar{u}(p_\nu) \Gamma \mathcal{V}_{p_e}(\mathbf{q}) \langle B | \mathcal{O}(\mathbf{k} - \mathbf{q}) | A \rangle .$$

IDENTIFY CONVOLUTION WITH COORDINATE SPACE OVERLAP

$$\int d^3 x \quad e^{-i\mathbf{p}_\nu \cdot \mathbf{x}} \rho_W(\mathbf{x}) \mathcal{V}_{p_e}(\mathbf{x}) \equiv F(Z, E) C(Z, E)$$

Analysis At One-Loop

- Can identify precisely the Fermi function and shape factor.
- Matches current algebra formalism of Sirlin and Brown (thanks Chien-Yeah for discussions).
- Makes matching onto EFT definition of outer corrections simple.



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- What is the relevant physics? Should it factorize?



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DOUBLE COUNTING

- Formal expressions at 1-loop.
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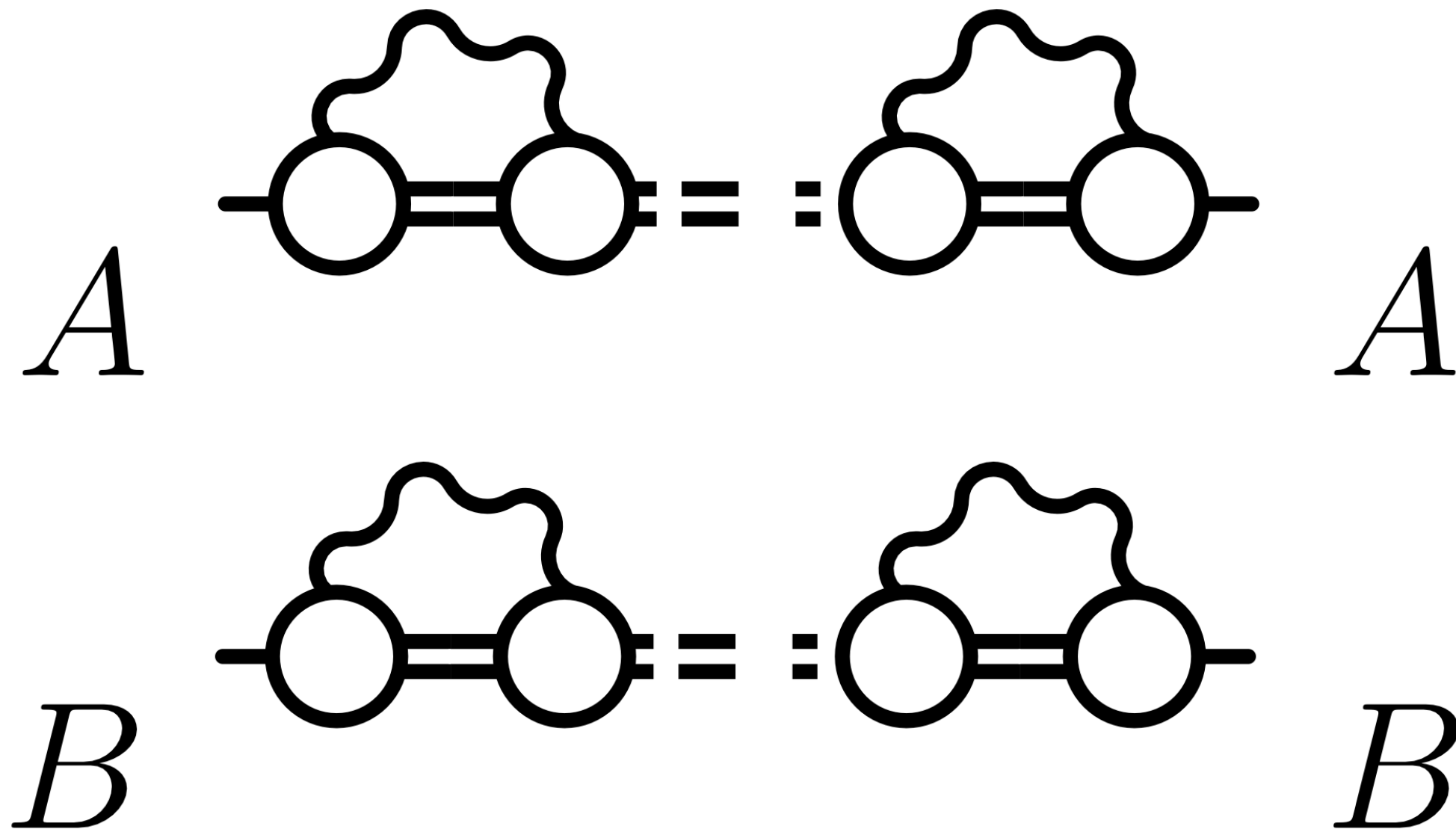
HIGHER LOOP-ORDER

ISOSPIN & COULOMB

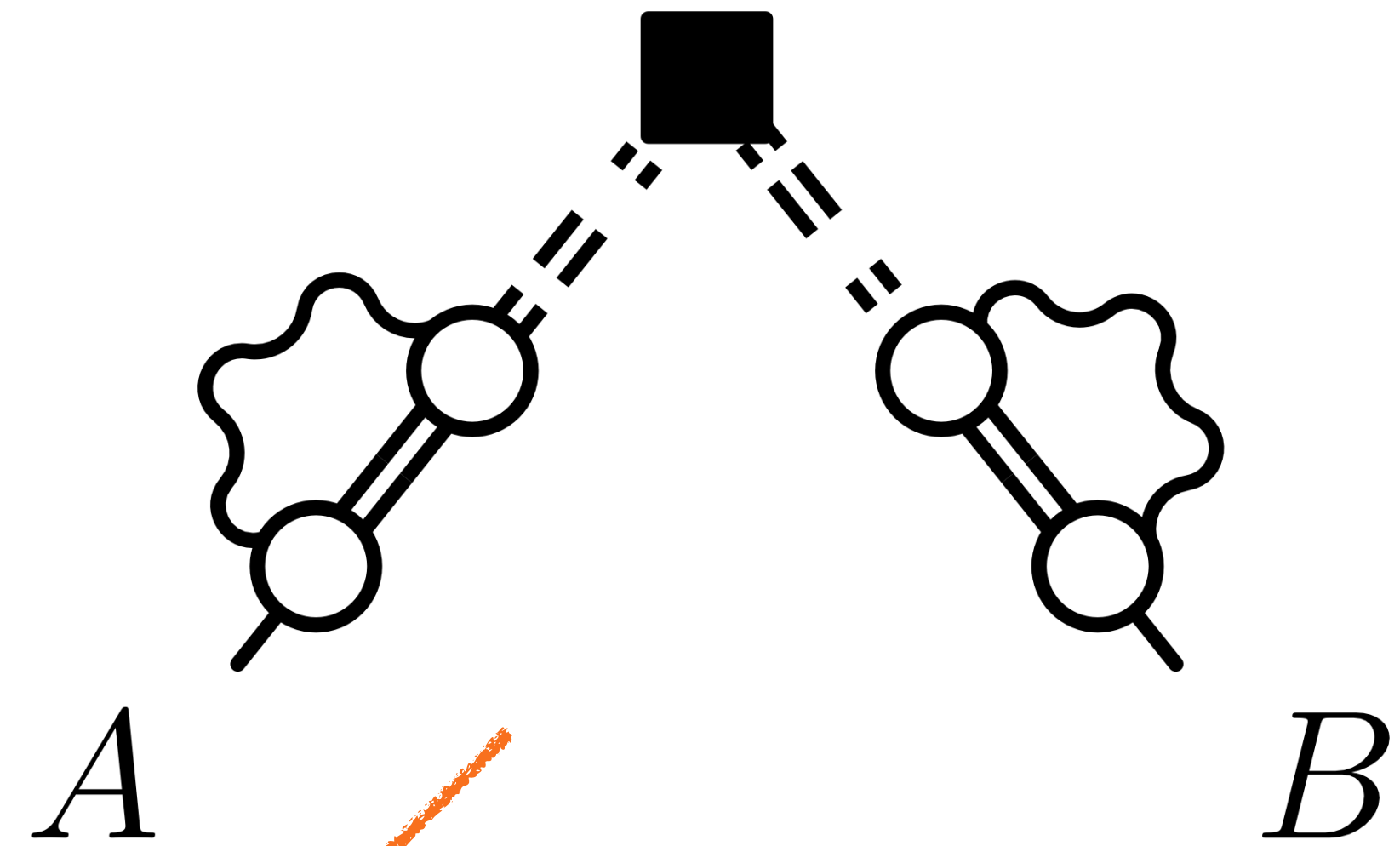
- Outer corrections to $O(Z^2\alpha^3L)$.
- Isospin breaking as a two-loop effect.

'Classic' Isospin Correction

WAVEFUNCTION RENORMALIZATION



VERTEX CORRECTION



MAPS ONE-TO-ONE AT $O(Z^2\alpha^2)$

$$M_F = \sqrt{Z_i Z_f} \left[M_0 + (f | V_c \Lambda_f \frac{1}{E_f - \Lambda_f H \Lambda_f} \tau_+ \frac{1}{E_i - \Lambda_i H \Lambda_i} \Lambda_i V_c | i) \right],$$

Mode Factorization & EFT

$$\Lambda_\chi \sim 1 \text{ GeV}$$

$$\mathcal{A} = \mathcal{C}_V(\mu) \mathcal{M}_{\text{EFT}}(\mu)$$

$$R_A^{-1} \sim 100 \text{ MeV}$$

AMPLITUDE FACTORIZES INTO WILSON
COEFFICIENT & EFT MATRIX ELEMENT

$$\mathcal{L} \supset \mathcal{C}_V(\mu) (B_\nu^\dagger \tau^- A_\nu) \bar{\psi}_e \gamma_0 P_L \psi_\nu + \dots$$

$$p_e \sim 1 \text{ MeV}$$

IN POINT-LIKE LIMIT: ONLY ONE OPERATOR

$$(1 + \Delta_R)(1 + \delta_{\text{NS}} - \delta_C)(1 + \delta'_R) \times F(Z, E)$$

Outer Corrections & Fermi Function

- Multi-loop input needed because of Z -enhancements.
- I will focus **only on leading power** here (take only leading-operator).

$$\mathcal{A} = \mathcal{C}(\mu) \mathcal{M}(\mu) + \dots$$

POINT-LIKE THEORY



$$\mathcal{C}(\mu) = \sum_{n,m} Z^n \alpha^m \mathcal{C}^{(n,m)}(\mu) \qquad \mathcal{M}(\mu) = \sum_{n,m} Z^n \alpha^m \mathcal{M}^{(n,m)}(\mu)$$

Fermi Function = Matrix Element

All orders factorization and the Coulomb problem

RICHARD J. HILL^{1,2} AND RYAN PLESTID^{1,2,3}

Field theory of the Fermi function

Richard J. Hill^{1,2,*} and Ryan Plestid^{1,2,3,†}

$$\left[\mathcal{M}_{\text{EFT}}(\mu) \right]_{\text{leading}-Z\alpha} \sim F(Z, E)$$

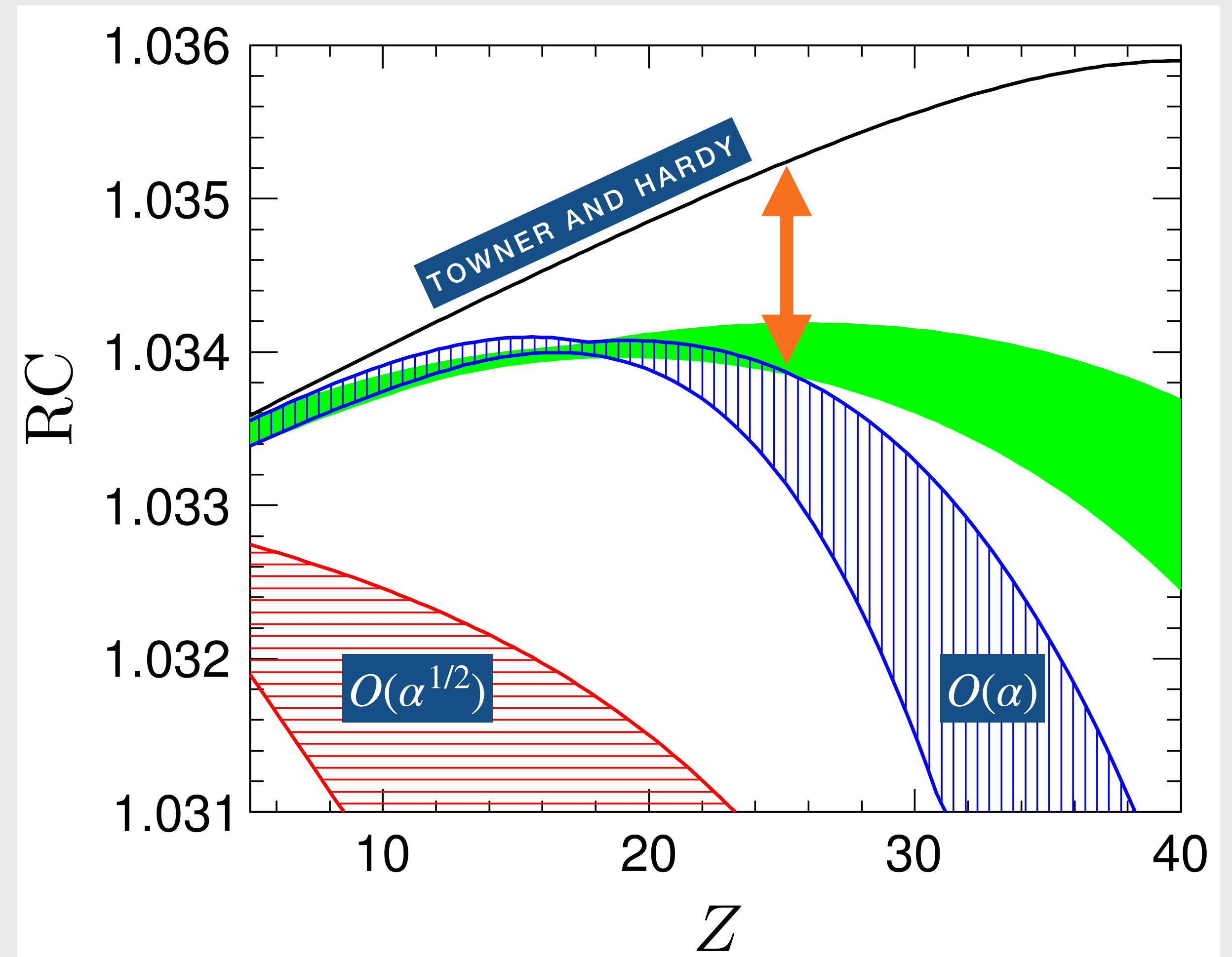
$$\frac{\left[\mathcal{M}_{\text{EFT}}(\mu) \right]}{F(Z, E)} = (1 + \delta'_R)$$

- Both objects defined in ***same formalism!***
- Easy to combine with other EFT ingredients.

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COUNTING $Z \sim \log \sim 1/\sqrt{\alpha}$

Scale-Dependence & RG

$$\frac{d}{d\mu} \mathcal{C}(\mu) = - \frac{d}{d\mu} \mathcal{M}(\mu) = \gamma_{\mathcal{C}}$$

- Controls logarithms related to "running" from $\mu \sim R^{-1}$ to $\mu \sim m_e \sim p_e$.
- Allows us to predict $Z^2 \alpha^3 L^2$ and $Z^2 \alpha^3 L$ terms.

Scale-Dependence & RG

$$\frac{d}{d\mu} \mathcal{C}(\mu) = - \frac{d}{d\mu} \mathcal{M}(\mu) = \gamma_{\mathcal{C}}$$

CAN PREDICT MANY TERMS

$$\left[\frac{C(\mu_L)/C(\mu_H)}{\exp \left[(1 - \sqrt{1 - Z^2 \alpha^2}) \log(\mu_H/\mu_L) \right]} \right]^2 = 1 + \alpha \left[\frac{3}{2\pi} L \right] + \alpha^2 \left[ZL + \frac{13}{8\pi^2} L^2 + \left(\frac{1}{3} - \frac{35}{48\pi^2} \right) L \right] \\ + \alpha^3 \left[\frac{2}{3\pi} Z^2 L^2 + \left(\frac{\pi}{6} - \frac{3}{\pi} \right) Z^2 L + \frac{13}{6\pi} ZL^2 + \frac{221}{144\pi^3} L^3 \right] + \alpha^4 \left[\frac{13}{9\pi^2} Z^2 L^3 \right] +$$

What Do We Know Now?

NEEDED

γ_0

- Known through 3-loops (complete), and leading terms at four-loops.

$\mathcal{C}^{(n,m)}(\mu)$

- Leading- $Z\alpha$ from Dirac equation. Can take $O(\alpha)$ from literature.

$\mathcal{M}^{(n,m)}(\mu)$

- Leading- $Z\alpha$ known. Results at $O(\alpha)$ well known. New input at $O(\alpha^2)$ and $O(Z\alpha^2)$ for $m_e \rightarrow 0$.

UNPUBLISHED, BUT COMING SOON



$O(Z\alpha^2)$

$O(Z^2\alpha^3)$

Neutron Decay

$$\tau_n \times |V_{ud}|^2 [1 + 3\lambda^2] [1 + \Delta_R] = \frac{5263.284(17) \text{ s}}{1 + 27.04(7) \times 10^{-3}}$$

	Without Resummation	With Resummation
1	0.3 ± 3.5 ± 2.1	34.5 ± 3.6 ± 2.2
1 + $H_V^{(1)}$	32.6 ± 0.1 ± 2.2	33.2 ± 0.004 ± 2.2
1 + $H^{(1)}$	28.8 ± 0.08 ± 0.05	29.32 ± 0.02 ± 0.01
1 + $H^{(1)}$ + $H_V^{(2)}$	29.04 ± 0.05 ± 0.05	29.31 ± 0.02 ± 0.01

Quantity	Value [10^{-3}]
Δ_R	45.37 ± 0.27
$\delta_{R,\text{static}}$	29.18 ± 0.07
δ_{recoil}	− 2.06
$\delta_{\text{rad.rec.}}$	− 0.08

- Extracted (partial) two-loop result for neutron decay from HQET literature.
- New understanding of “ π -enhanced” terms usually ascribed to Fermi-function.

$$\left| \frac{F_H(E, \mu_*)}{F_H(-E, \mu_*)} \right|^2 = \left| \frac{F_H(E, \mu_*)}{F_H(E, -\mu_* - i0)} \right|^2 = \exp \left[-X_*^2 \frac{\bar{\alpha}}{4\pi} + \frac{32}{9} n_e X_*^2 \left(\frac{\bar{\alpha}}{4\pi} \right)^2 - \frac{8}{27} n_e^2 X_*^4 \left(\frac{\bar{\alpha}}{4\pi} \right)^3 + \dots \right]$$



Conclusions

Take Home Message

- Understanding the structure of QED corrections to beta decay an important challenge.
- **All** QED related effects can be described in a single formalism (no double counting).
- Formalism is diagrammatic and easily extended to higher orders.

Take Home Message

- Understanding the structure of QED corrections to beta decay an important challenge.



NEED TO KNOW HOW TO
ORGANIZE THEORY AT
HIGHER LOOP ORDERS

What Is Needed

- Matching from chiral currents onto point-like effective theory.
- Full analysis at $O(Z\alpha^2)$ including both long-distance and short-distance parts.
- Understanding of possible higher-order corrections (draw all possible diagrams).

References

- Gauge dependence & LSZ formalism: (with M.B. Wise)
[arXiv:2505.05449](#), [arXiv:2508.05741](#) .
- Point-like theory & Fermi function: (with Richard Hill)
[arXiv:2309.15929](#), [arXiv:2309.07343](#) .
- Anomalous dimension: (with Richard Hill, & Kaushik Borah)
[arXiv:2402.13307](#).
- Neutron decay: (with Hill, Vander Griend, and Cao)
[arXiv:2501.17916](#) , [arXiv:2508.05741](#), [arXiv:25mm.xxxxx](#)