



University
of Victoria

An *ab initio* framework for the modelling of electroweak processes in light nuclei

Michael Gennari

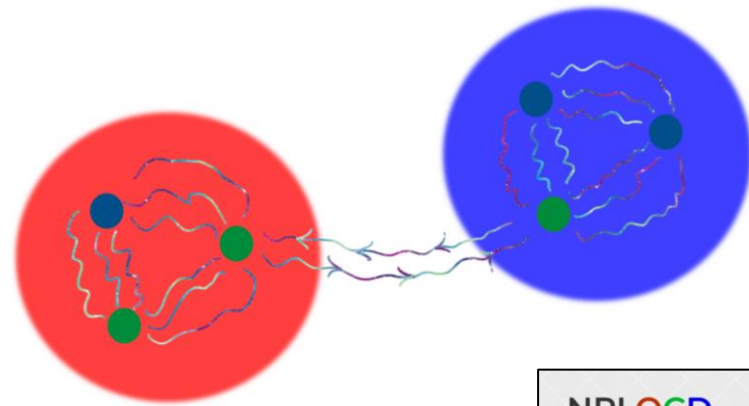
Collaborators

D. A. Najera, L. Jokiniemi, M. Drissi, C.-Y. Seng,
M. Gorchtein, P. Navratil



TECHNISCHE
UNIVERSITÄT
DARMSTADT

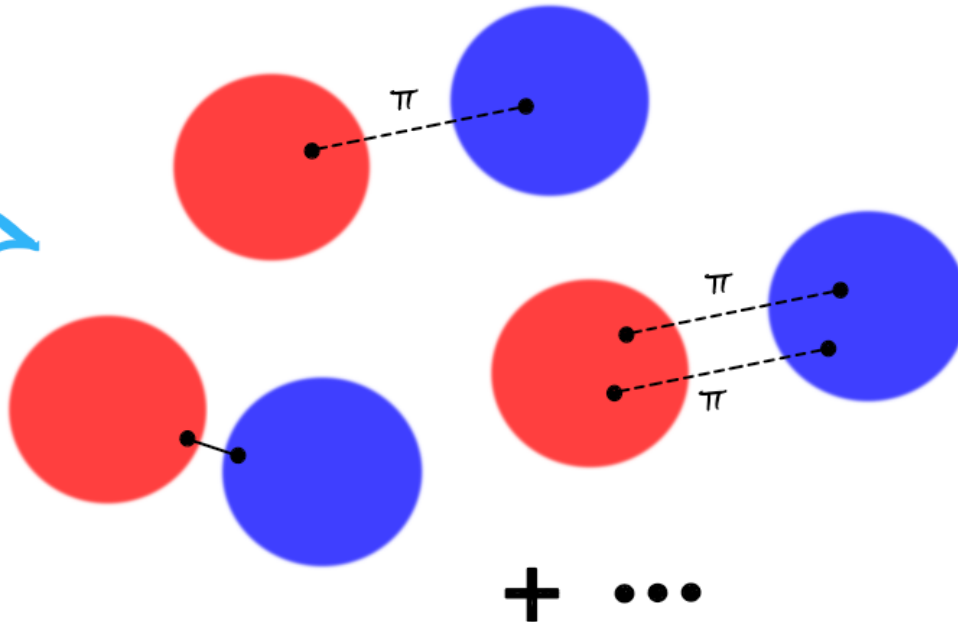
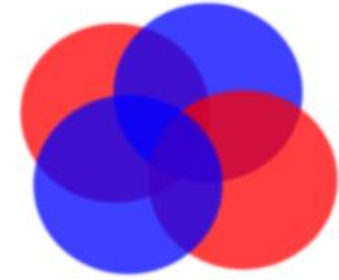




NPLQCD
Nuclear Physics with Lattice QCD

$SU(3)_C$

$$V = \sum_{i < j}^A V_{NN}^{(ij)} + \sum_{i < j < k}^A V_{NNN}^{(ijk)} + \dots$$



$$SU(2)_L \times SU(2)_R \sim SO(4)$$

Physica **96A** (1979) 327–340 © North-Holland Publishing Co.

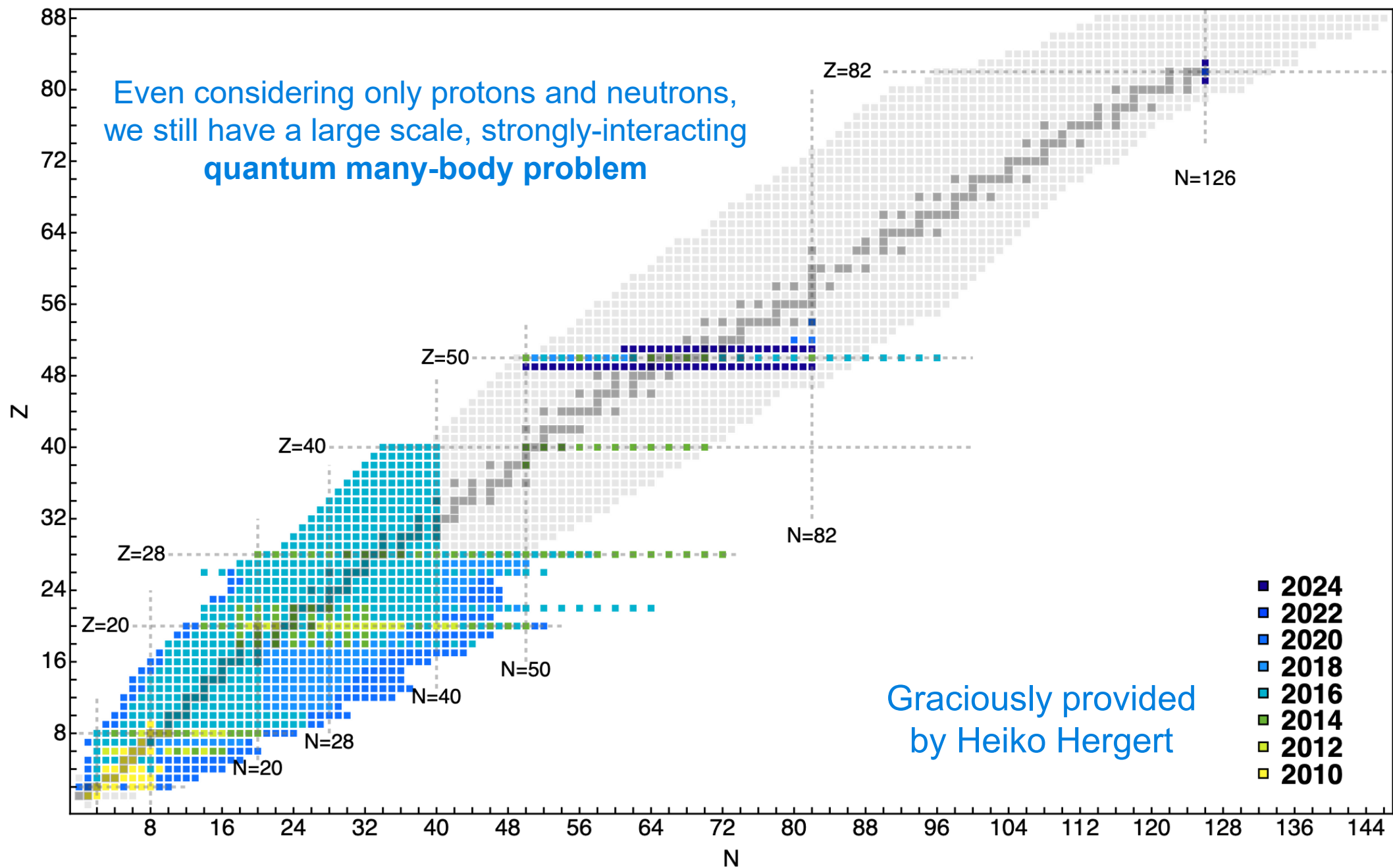
PHENOMENOLOGICAL LAGRANGIANS*

STEVEN WEINBERG

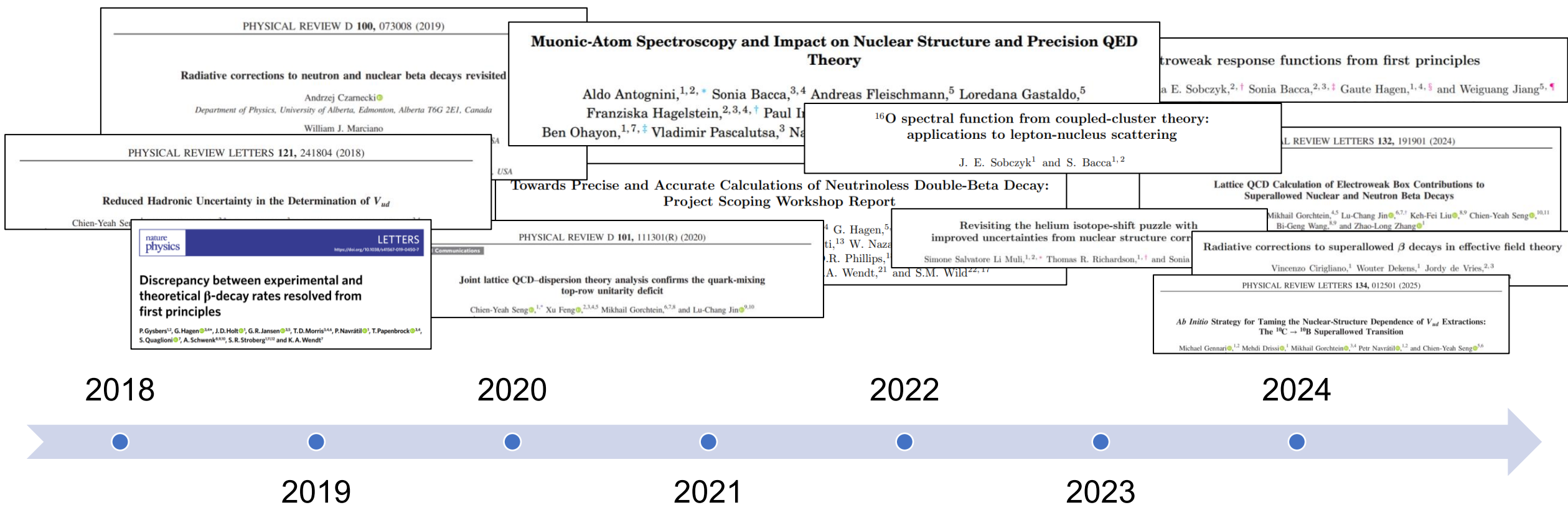
Lyman Laboratory of Physics, Harvard University

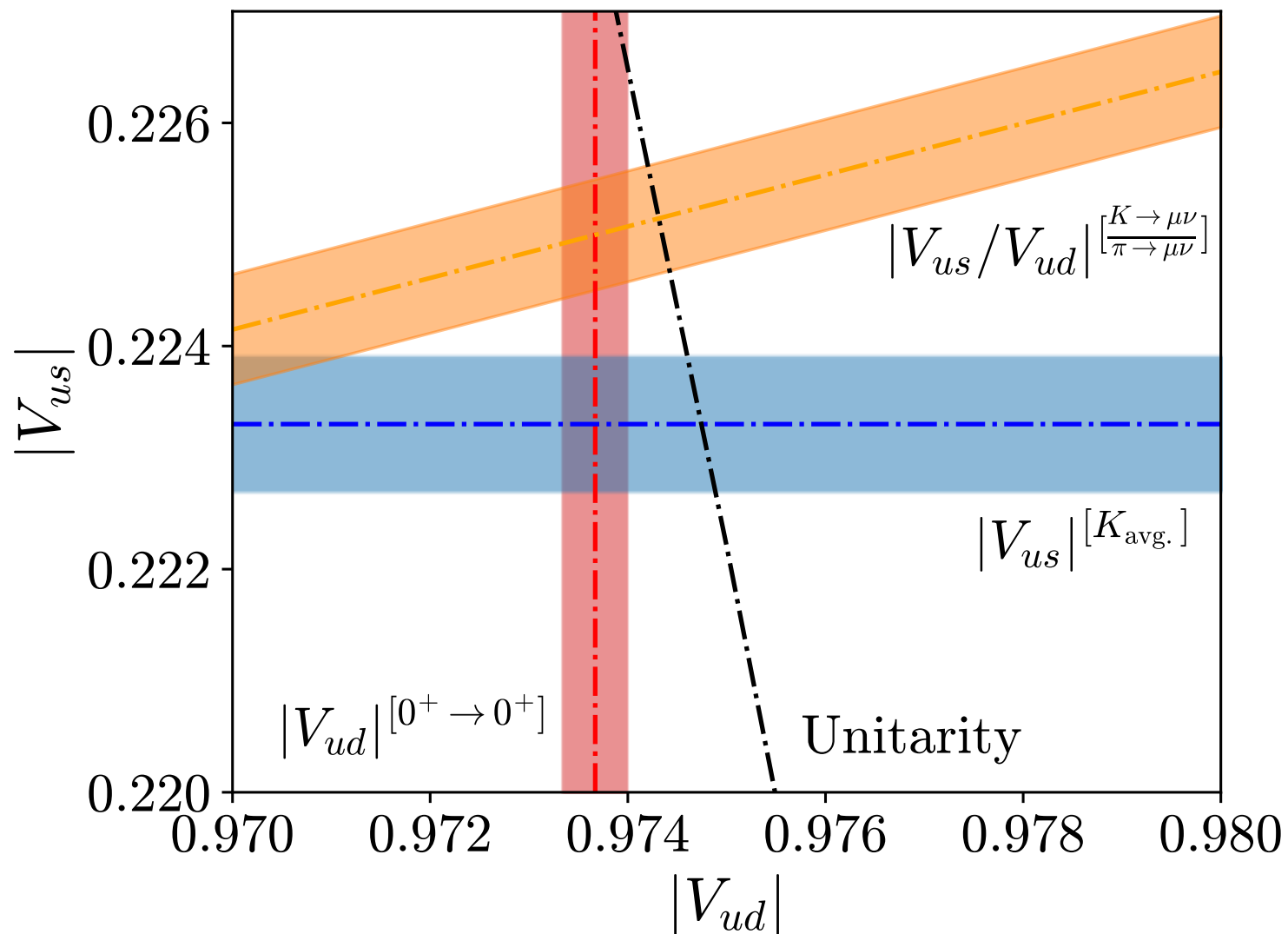
and

Harvard-Smithsonian Center for Astrophysics, Cambridge, Massachusetts 02138, USA

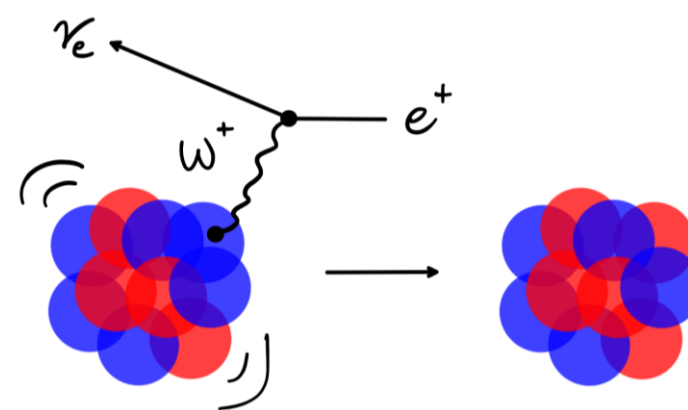
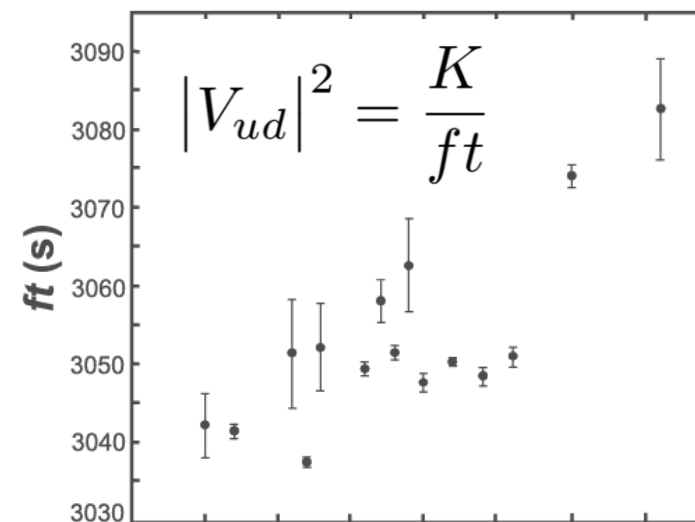


We are entering an era in which the **precision** modelling of strongly-interacting many-body systems is becoming possible

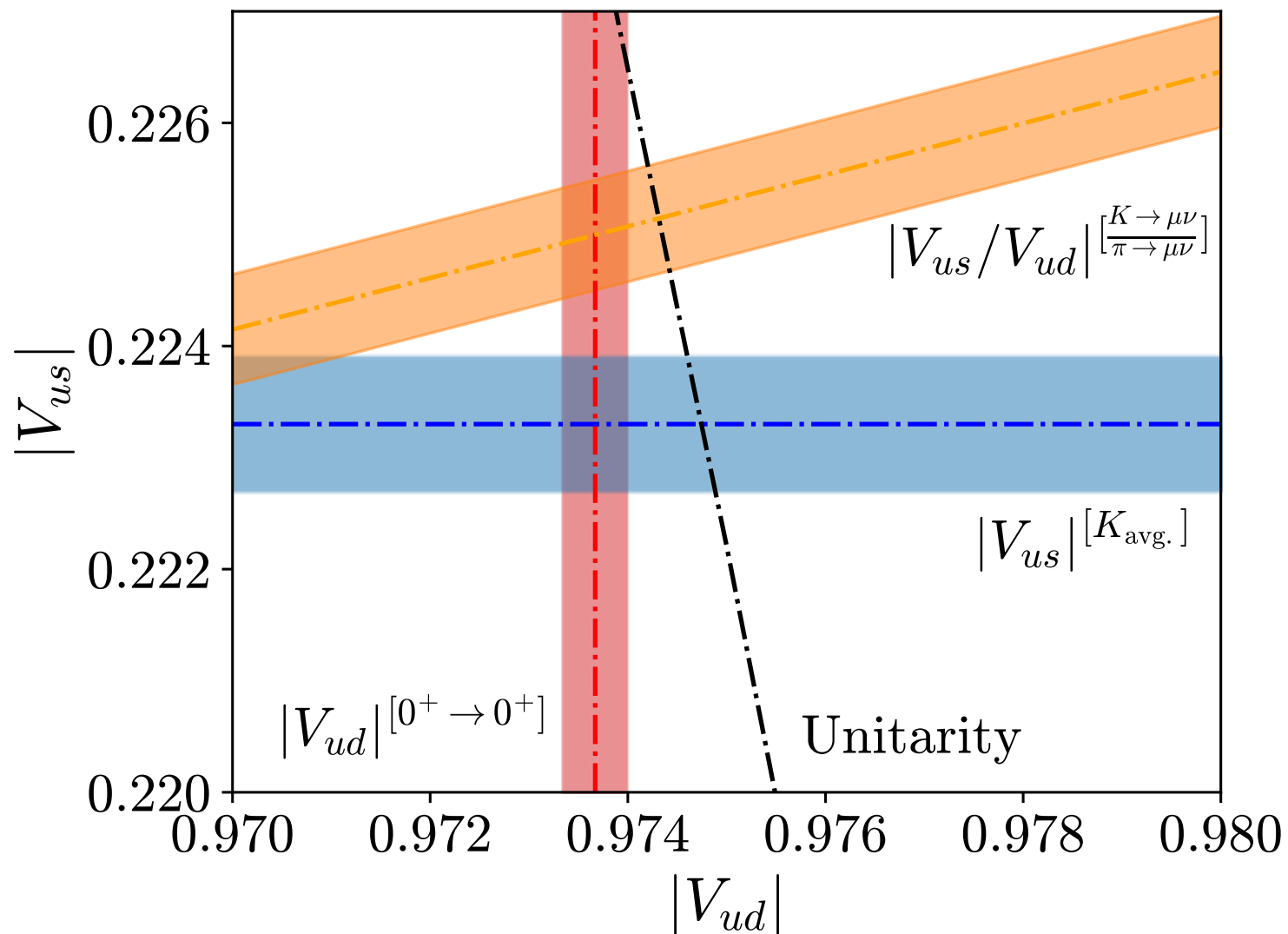




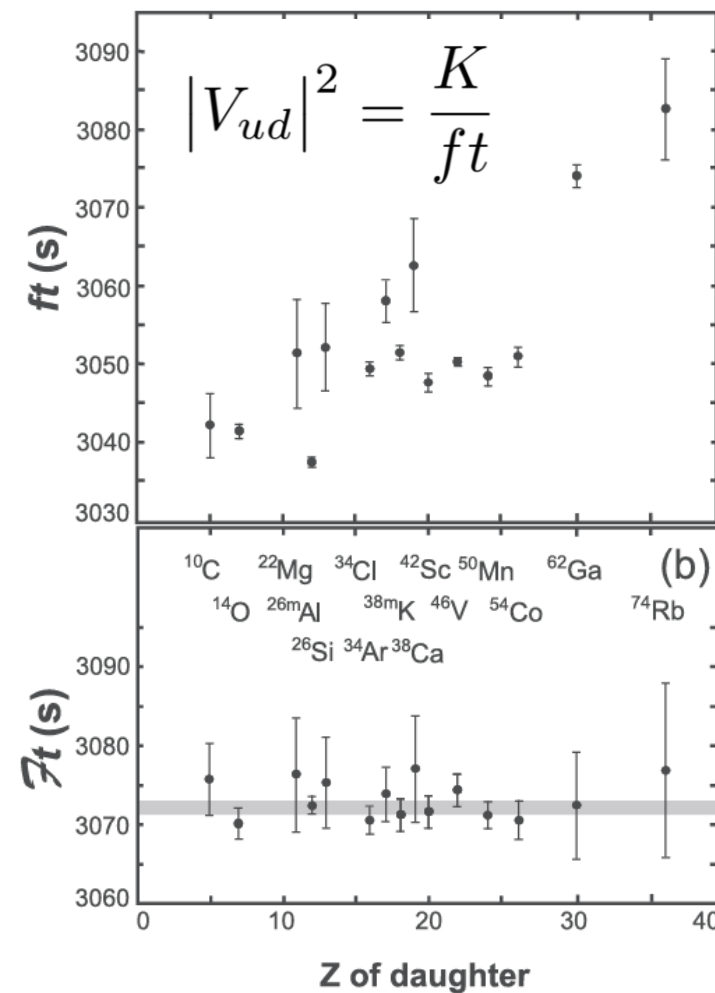
Hardy et al. PRC **102**, 045501 (2020)



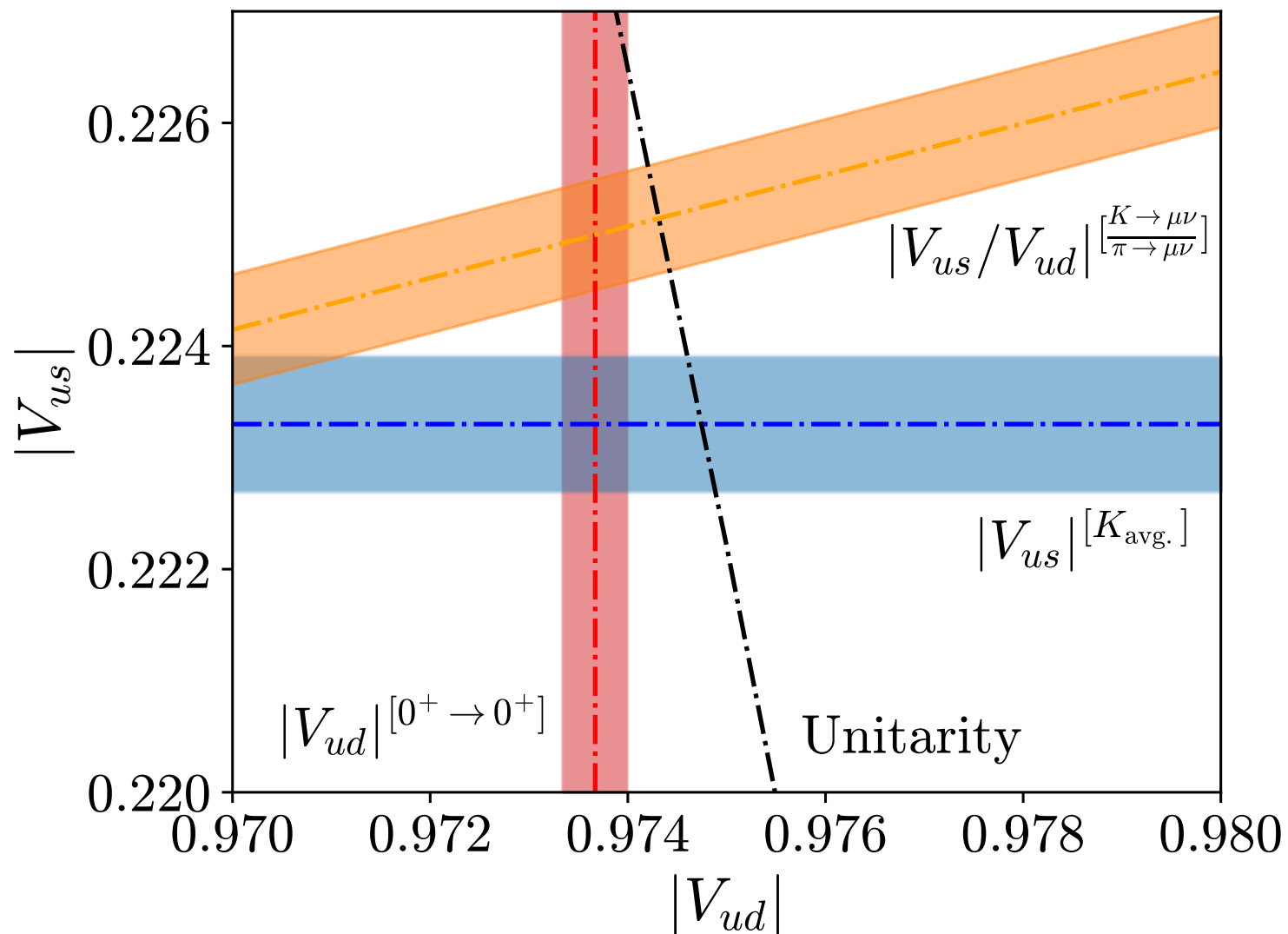
Requires nuclear theory



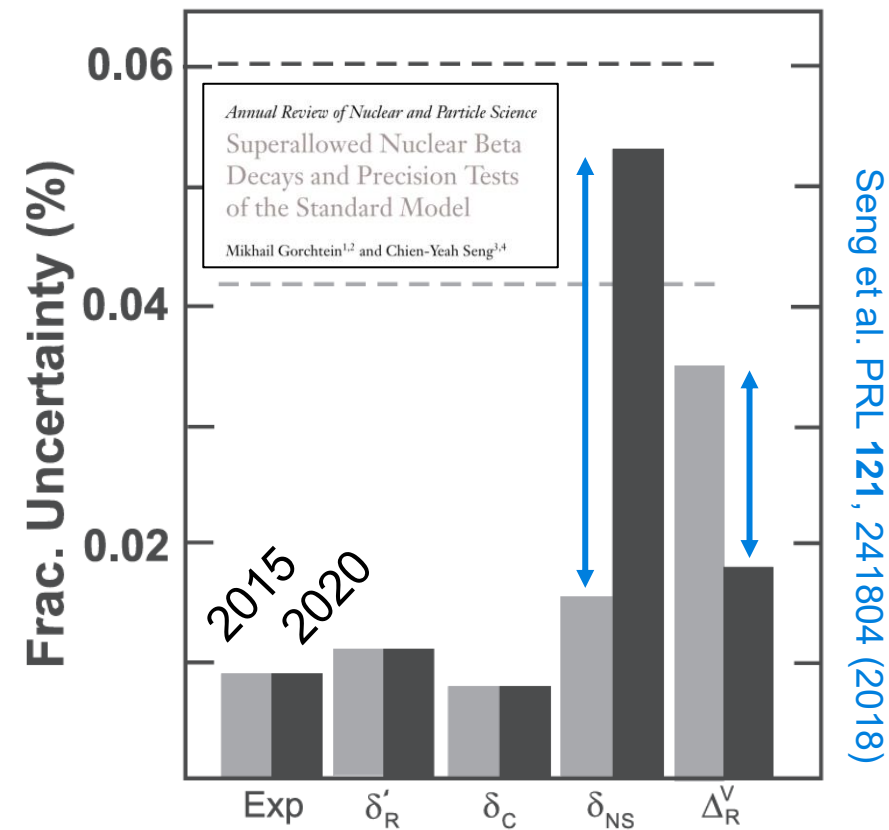
Hardy et al. PRC **102**, 045501 (2020)



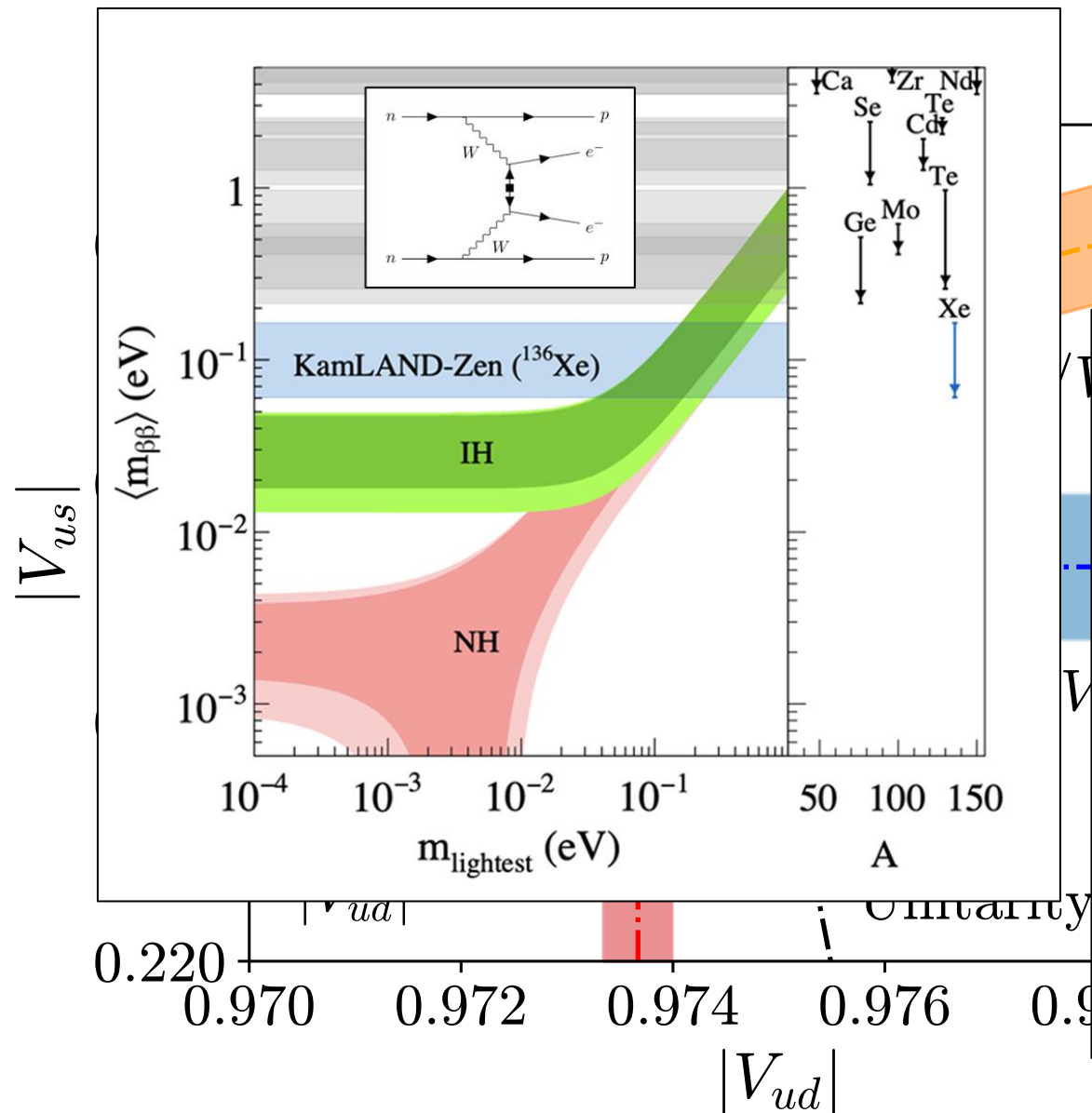
Requires nuclear theory



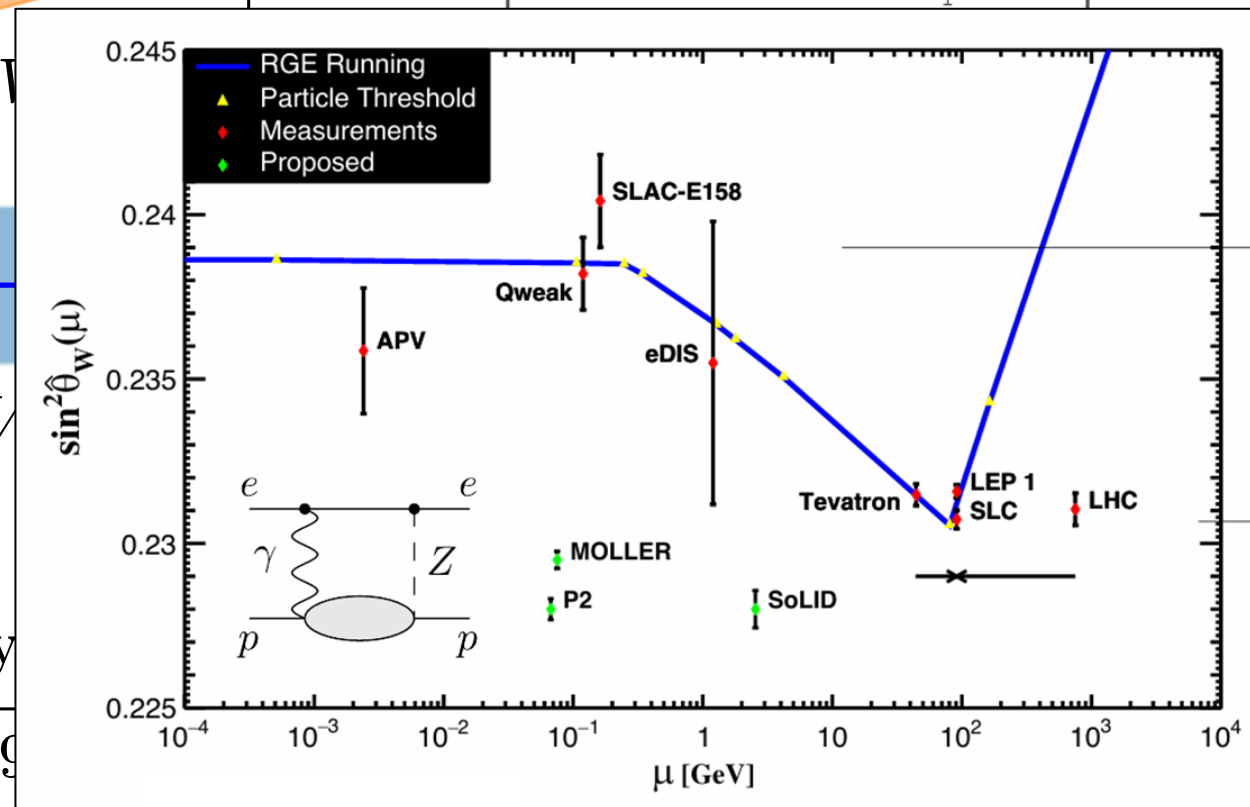
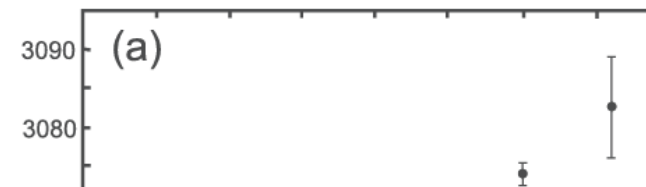
Hardy et al. PRC **102**, 045501 (2020)



Requires nuclear theory



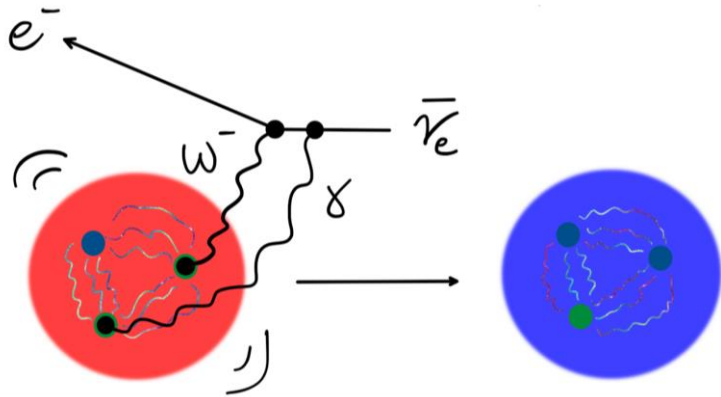
Hardy et al. PRC **102**, 045501 (2020)



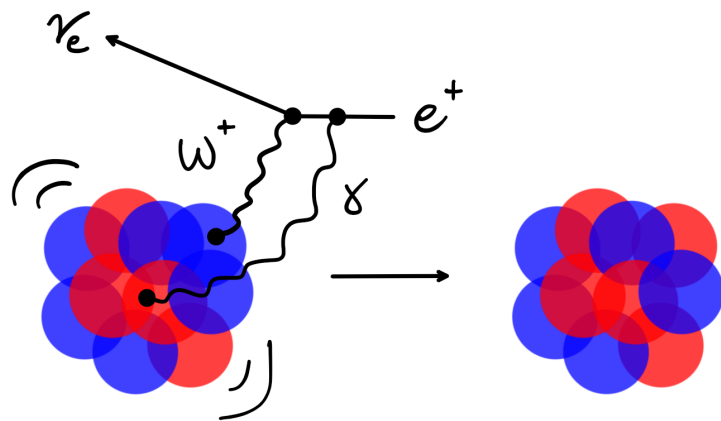
Requires nuclear theory

Nuclear Beta Decay in the Standard Model

9



$$|V_{ud}|^2 = \frac{K}{\mathcal{F}t (1 + \Delta_R^V)}$$

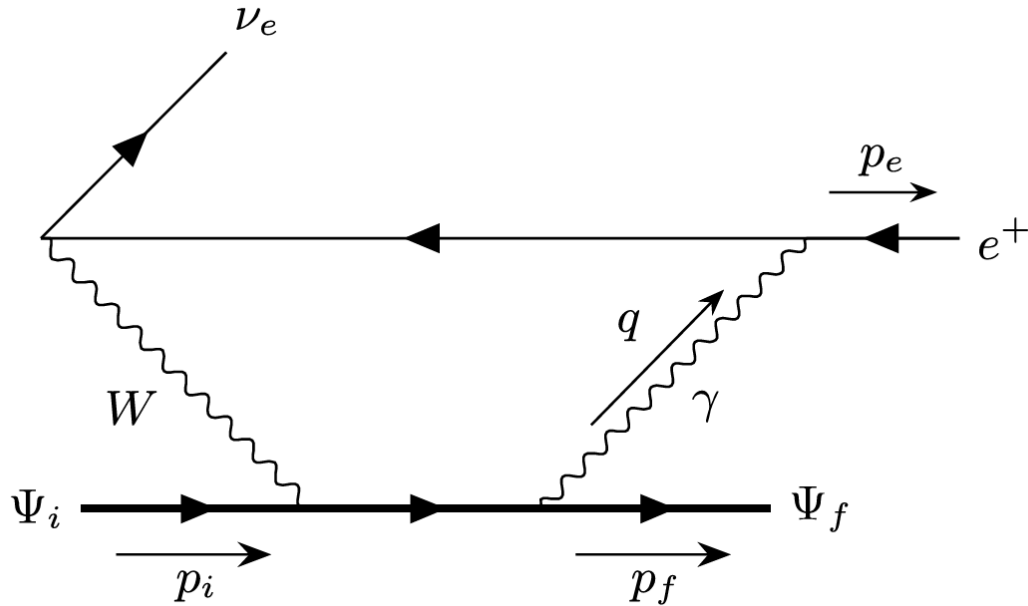


$$\mathcal{F}t = ft(1 + \delta'_R)(1 - \delta_C + \delta_{NS})$$

Accounts for isospin symmetry breaking and
electroweak radiative corrections

One-Loop Radiative Correction

10



Convenient approximations

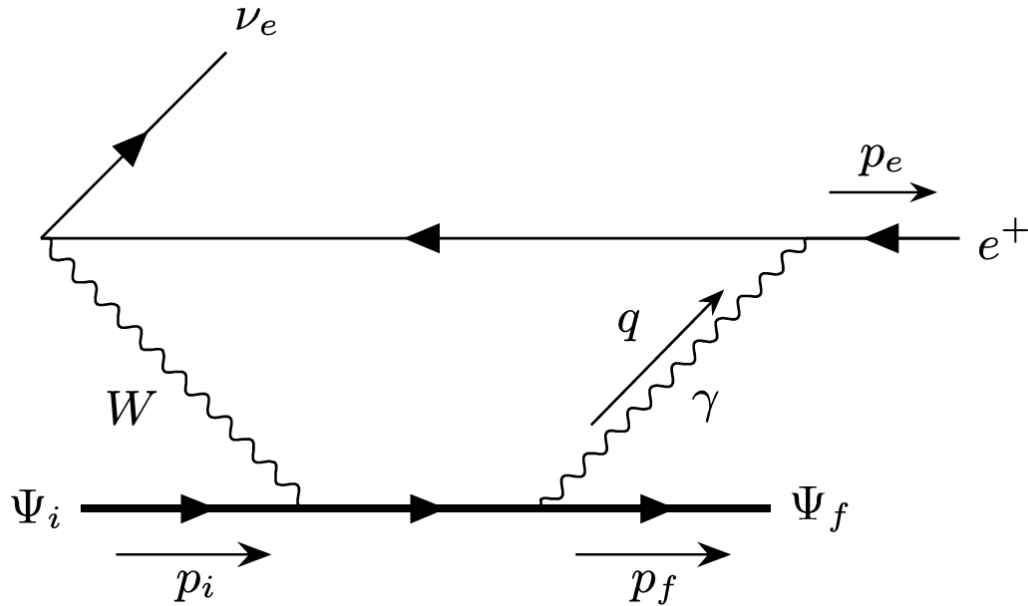
- Forward scattering limit
- Neglect recoil of final nucleus

$$\delta \mathcal{M}_{\text{one-loop}} = \square_{\gamma W}(E_e) \mathcal{M}_{\text{tree}}$$

$$T^{\mu\nu}(p_f, p_i; q) = \langle \Phi_f ; p_f | \left\{ \frac{1}{2} \int d^4x e^{iq \cdot x} \text{T} \left[J_{\text{em}}^\mu(x) J_W^\nu(0)^\dagger \right] \right\} | \Phi_i ; p_i \rangle$$

One-Loop Radiative Correction

11



Convenient approximations

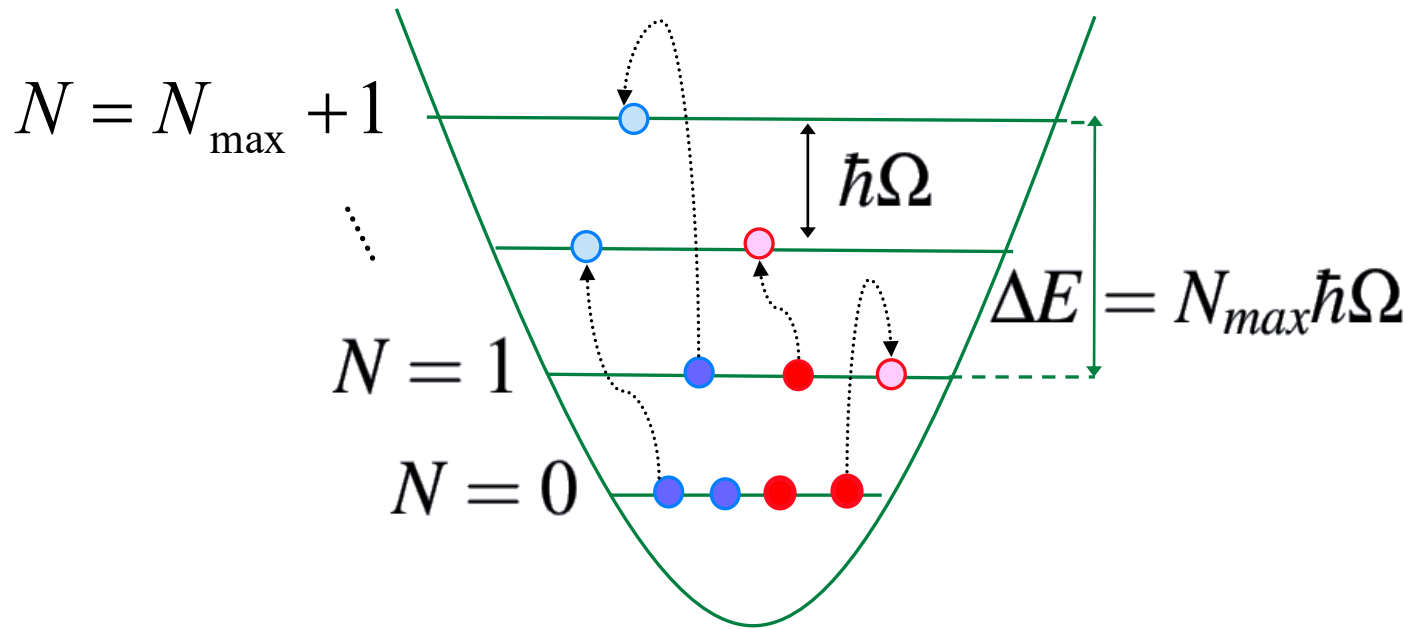
- Forward scattering limit
- Neglect recoil of final nucleus

$$\delta \mathcal{M}_{\text{one-loop}} = \square_{\gamma W}(E_e) \mathcal{M}_{\text{tree}}$$

$$\square_{\gamma W}^b(E_e) = \frac{e^2}{M} \int \frac{d^4 q}{(2\pi)^4} R_W(q) R_e(q) R_\gamma(q) \left(M \frac{p_e \cdot q}{p \cdot p_e} - \frac{q^2}{\nu} \right) \frac{T_3(\nu, |\vec{q}|)}{f_+}$$

We use the ab initio no-core shell model, a quasi-exact approach for modelling nuclei as composite structures of nucleons interacting via internucleonic forces

12



$$N = \sum_i 2n_i + l_i \leq N_{\text{LPC}} + N_{\max}$$

We use the ab initio no-core shell model, a quasi-exact approach for modelling nuclei as composite structures of nucleons interacting via
internucleonic forces

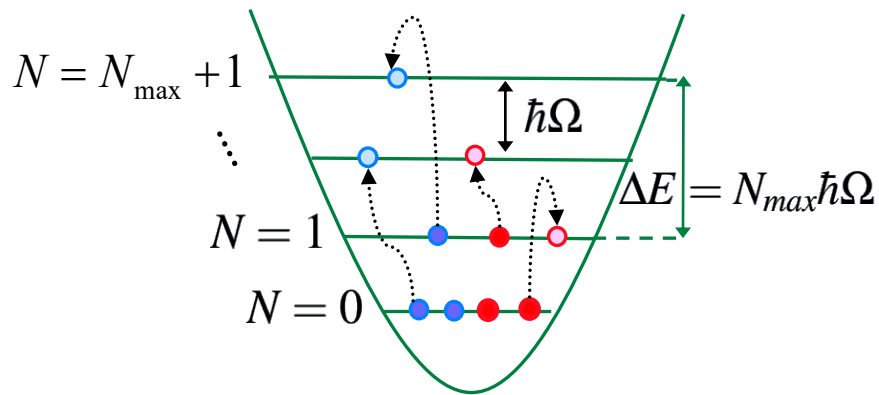
$$\mathcal{F}(\mathcal{H}) = \bigoplus_A [\mathcal{H}^{\otimes A}] = \mathbb{1} \oplus \mathcal{H} \oplus [\mathcal{H} \otimes \mathcal{H}] \oplus \dots$$

Anti-symmetrized products of many-body harmonic oscillator states

$$|\Psi_A^{J^\pi T}\rangle = \sum_{N=0}^{N_{max}} \sum_{\alpha} c_{N\alpha}^{J^\pi T} |\Phi_{N\alpha}^{J^\pi T}\rangle$$

We use the ab initio no-core shell model, a quasi-exact approach for modelling nuclei as composite structures of nucleons interacting via
internucleonic forces

$$H = \frac{1}{A} \sum_{i < j}^A \frac{(\vec{p}_i - \vec{p}_j)^2}{2m_N} + \sum_{i < j}^A V_{NN}^{(ij)} + \sum_{i < j < k}^A V_{NNN}^{(ijk)} + \dots$$



Anti-symmetrized products of many-body harmonic oscillator states

$$|\Psi_A^{J^\pi T}\rangle = \sum_{N=0}^{N_{\max}} \sum_{\alpha} c_{N\alpha}^{J^\pi T} |\Phi_{N\alpha}^{J^\pi T}\rangle$$

Barrett et al. PPNP **69** (2013), pp.181-131

Journal of Research of the National Bureau of Standards Vol. 45, No. 4, October 1950 Research Paper 2133
An Iteration Method for the Solution of the Eigenvalue Problem of Linear Differential and Integral Operators¹
 By Cornelius Lanczos

The inverse of a linear operator

Roger Haydock
 Cavendish Laboratory, Madingley Road, Cambridge CB3 0HE, UK

Received 10 May 1974, in final form 21 June 1974

We use the ab initio no-core shell model, a quasi-exact approach for modelling nuclei as composite structures of nucleons interacting via internucleonic forces

Lanczos Algorithm

$$H |\eta_0\rangle = \alpha_0 |\eta_0\rangle + \beta_0 |\eta_1\rangle$$

$$H |\eta_1\rangle = \beta_0 |\eta_0\rangle + \alpha_1 |\eta_1\rangle + \beta_1 |\eta_2\rangle$$

$$H |\eta_2\rangle = \beta_1 |\eta_1\rangle + \alpha_2 |\eta_2\rangle + \beta_2 |\eta_3\rangle$$

$$H |\eta_3\rangle = \beta_2 |\eta_2\rangle + \alpha_3 |\eta_3\rangle + \beta_3 |\eta_4\rangle$$

$$\vdots$$

$$E = P^{-1} H_{\text{Lanczos}} P$$

We use the ab initio no-core shell model, a quasi-exact approach for modelling nuclei as composite structures of nucleons interacting via
internucleonic forces

Lanczos Strengths Method

$$\mathcal{A}_{fi} = \langle \Phi_f | O_2 (z - H)^{-1} O_1 | \Phi_i \rangle = \langle \Phi_f | O_2 | \Phi_R \rangle$$

$$(z - H) | \Phi_R \rangle = O | \Phi_i \rangle$$

Method for extracting many-body
resolvent amplitudes

We use the ab initio no-core shell model, a quasi-exact approach for modelling nuclei as composite structures of nucleons interacting via internucleonic forces

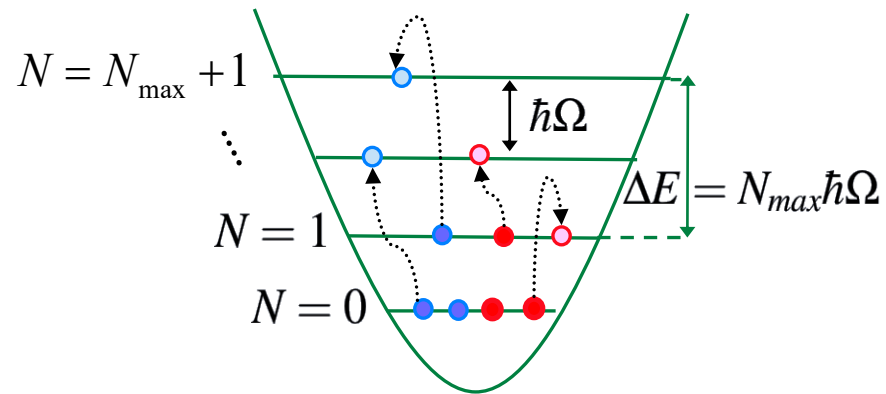
Lanczos Strengths Method

$$\langle \Psi_n | O_1 | \Psi_i \rangle = \left| \langle \Psi_i | O_1^\dagger O_1 | \Psi_i \rangle \right| \langle \eta_n | P^\dagger | \eta_0 \rangle$$

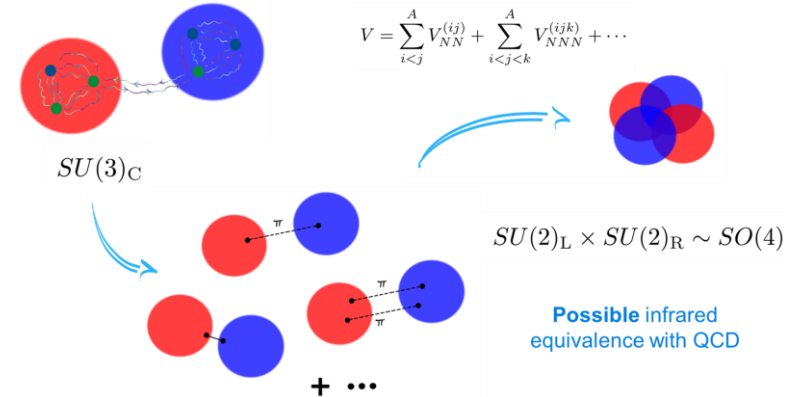
$$\langle \Psi_f | O_2 | \Psi_n \rangle = \left| \langle \Psi_f | O_2^\dagger O_2 | \Psi_f \rangle \right| \sum_m \langle \eta_m | P | \eta_n \rangle \langle \zeta_0 | \eta_m \rangle$$

We use the ab initio no-core shell model, a quasi-exact approach for modelling nuclei as composite structures of nucleons interacting via
internucleonic forces

No-core Shell Model



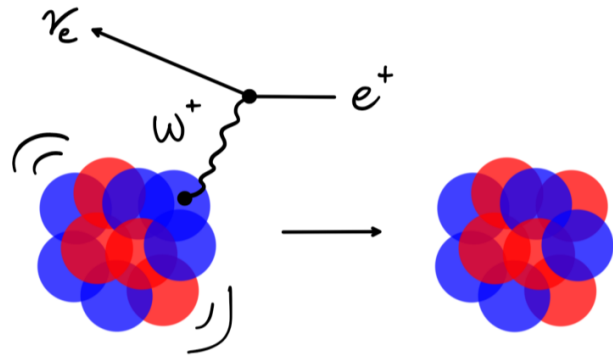
Chiral Effective Field Theory



Lanczos Strengths Method

$$(z - H)|\Phi_R\rangle = O|\Phi_i\rangle$$

We use the ab initio no-core shell model, a quasi-exact approach for modelling nuclei as composite structures of nucleons interacting via
internucleonic forces



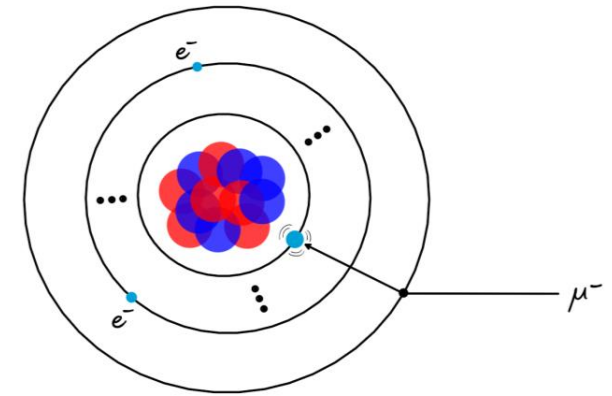
Super-allowed beta decay

Gennari et al.
PRL **134**, 012501 (2025)

PHYSICAL REVIEW LETTERS **134**, 012501 (2025)

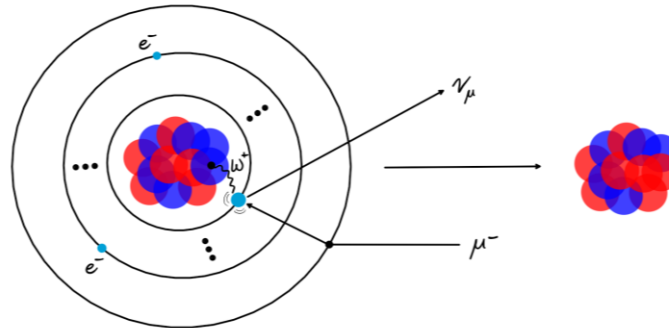
Ab Initio Strategy for Taming the Nuclear-Structure Dependence of V_{ud} Extractions:
The $^{10}\text{C} \rightarrow ^{10}\text{B}$ Superallowed Transition

Michael Gennari^{1,2}, Mehdi Drissi¹, Mikhail Gorchtein^{3,4}, Petr Navrátil^{1,2} and Chien-Yeah Seng^{5,6}



Muonic atoms

Drissi et al. In prep.



Muon capture

Najera et al. In progress.

We use the ab initio no-core shell model, a quasi-exact approach for modelling nuclei as composite structures of nucleons interacting via
internucleonic forces

No resolution for Compton amplitude above pion threshold,
thus δ_{NS} matched with the free nucleon Born contribution **only**

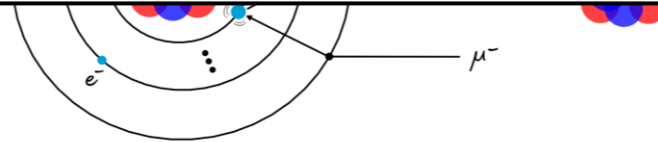
$$\delta_{\text{NS}} = 2 \left\{ \left(\square_{\gamma W}^{b,\text{nuc}} \right)_{\text{a.i.}} - \left(\square_{\gamma W}^{b,n} \right)_{\text{el}} + \delta \left(\square_{\gamma W}^{b,n} \right)_{\text{sh}} \right\}$$

Gennari et al. PRL **134**, 012501

PHYSICAL REVIEW LETTERS **134**, 012501 (2025)

Ab Initio Strategy for Taming the Nuclear-Structure Dependence of V_{ud} Extractions:
The $^{10}\text{C} \rightarrow ^{10}\text{B}$ Superaligned Transition

Michael Gennari^{1,2}, Mehdi Drissi¹, Mikhail Gorchtein^{3,4}, Petr Navrátil^{1,2} and Chien-Yeah Seng^{5,6}



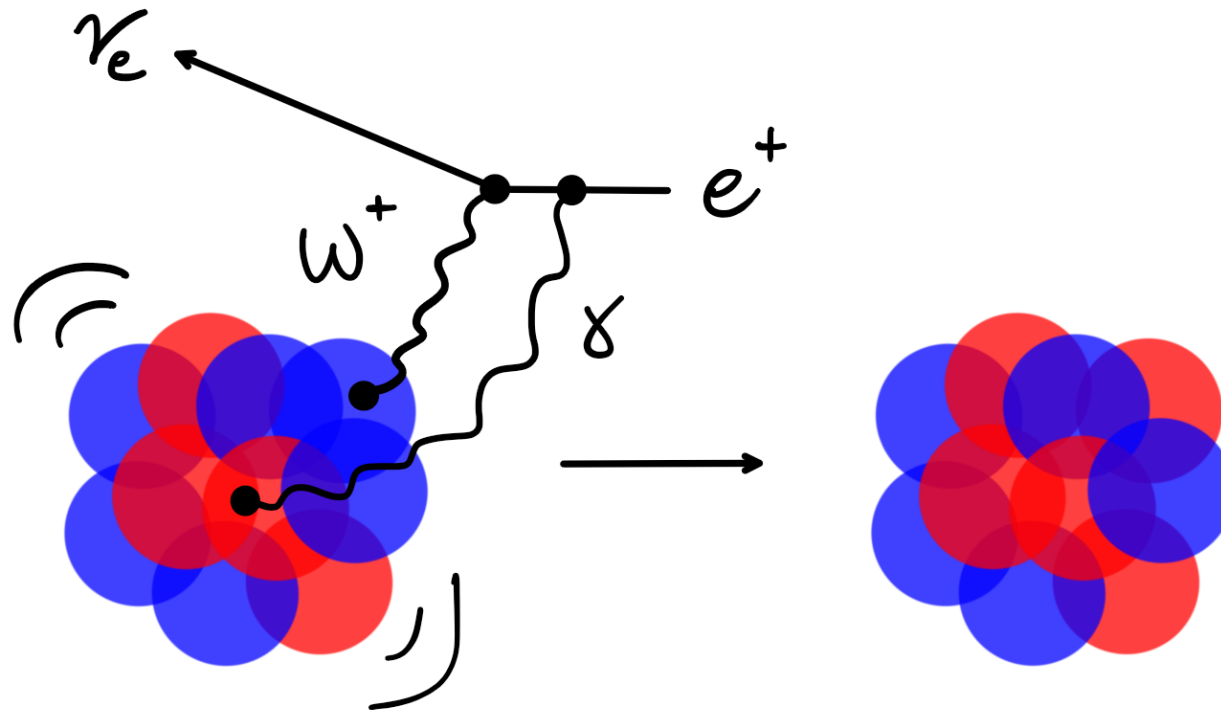
Muon capture

Najera et al. In progress.

Amplitude and Pole Structure

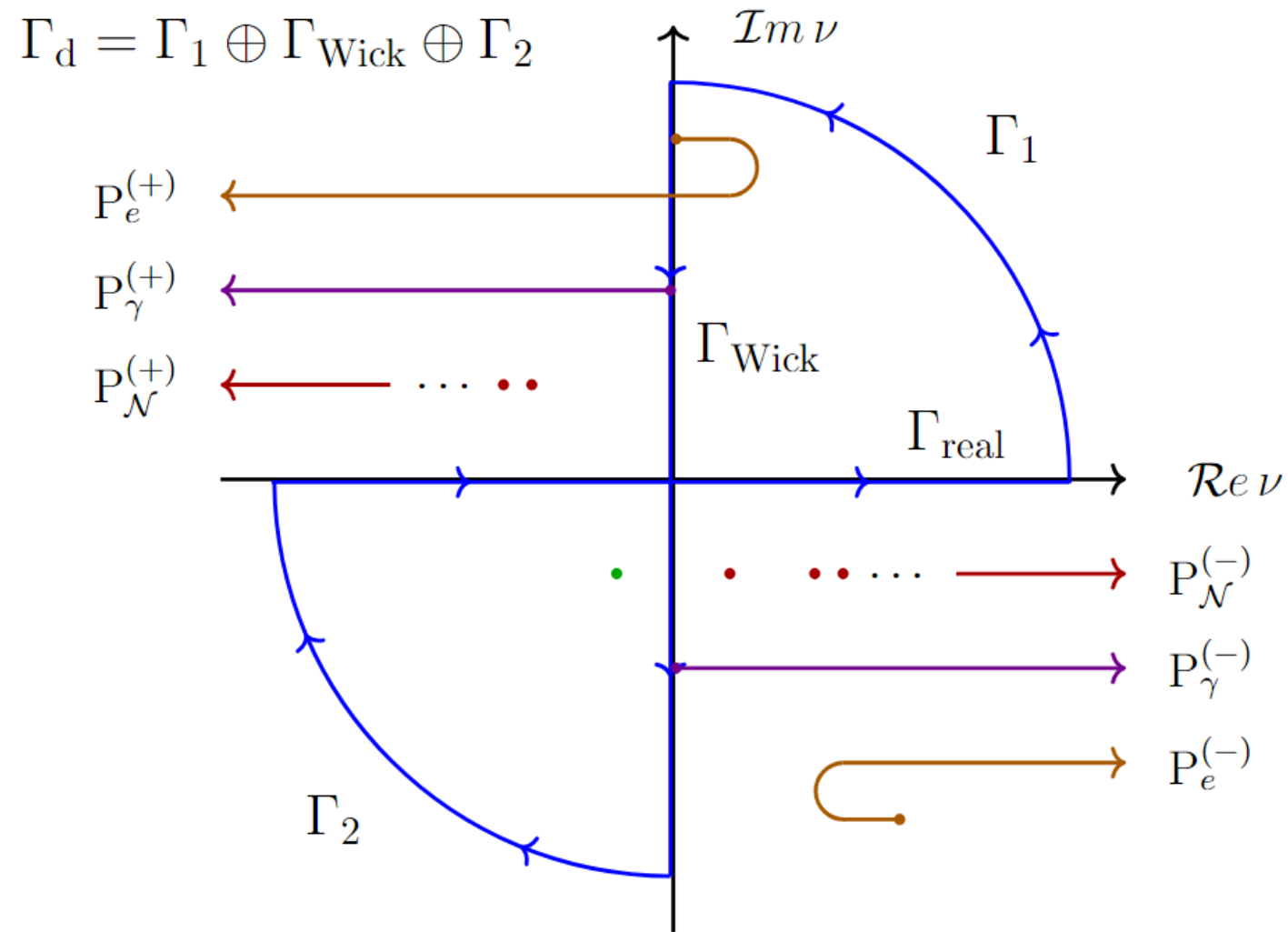
21

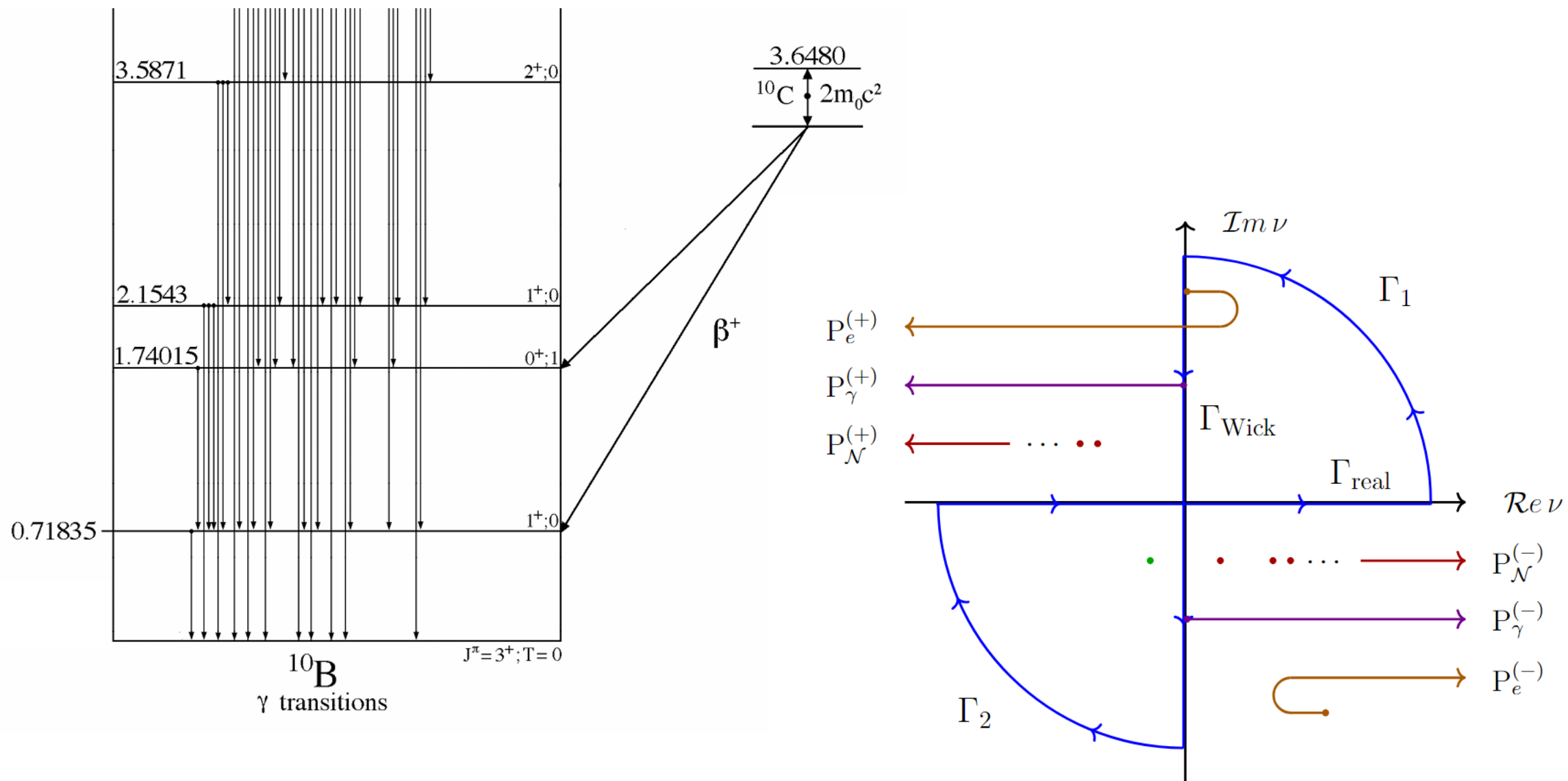
$$\mathcal{A}_{fi} = \langle \Phi_f | O_2 (z - H)^{-1} O_1 | \Phi_i \rangle$$

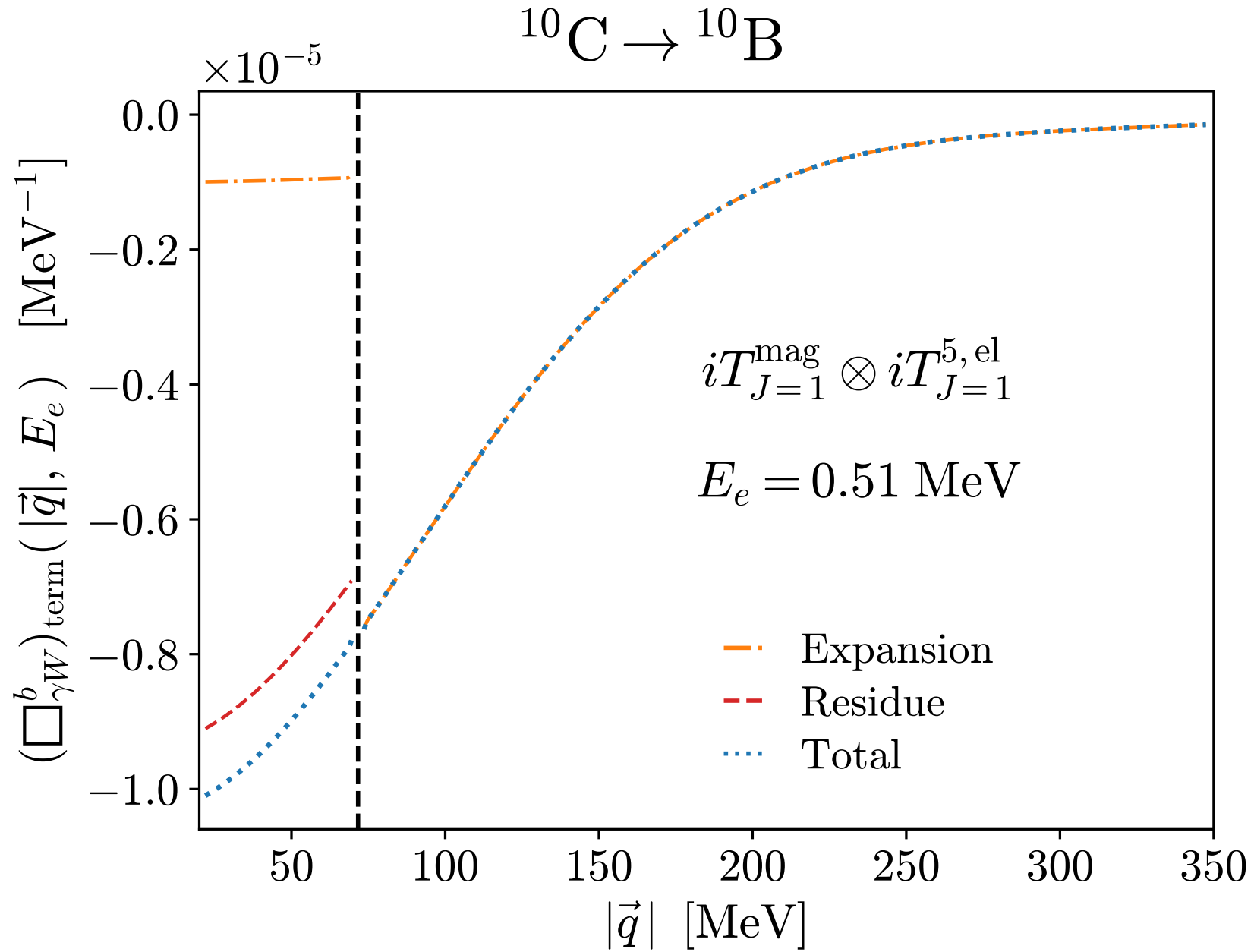


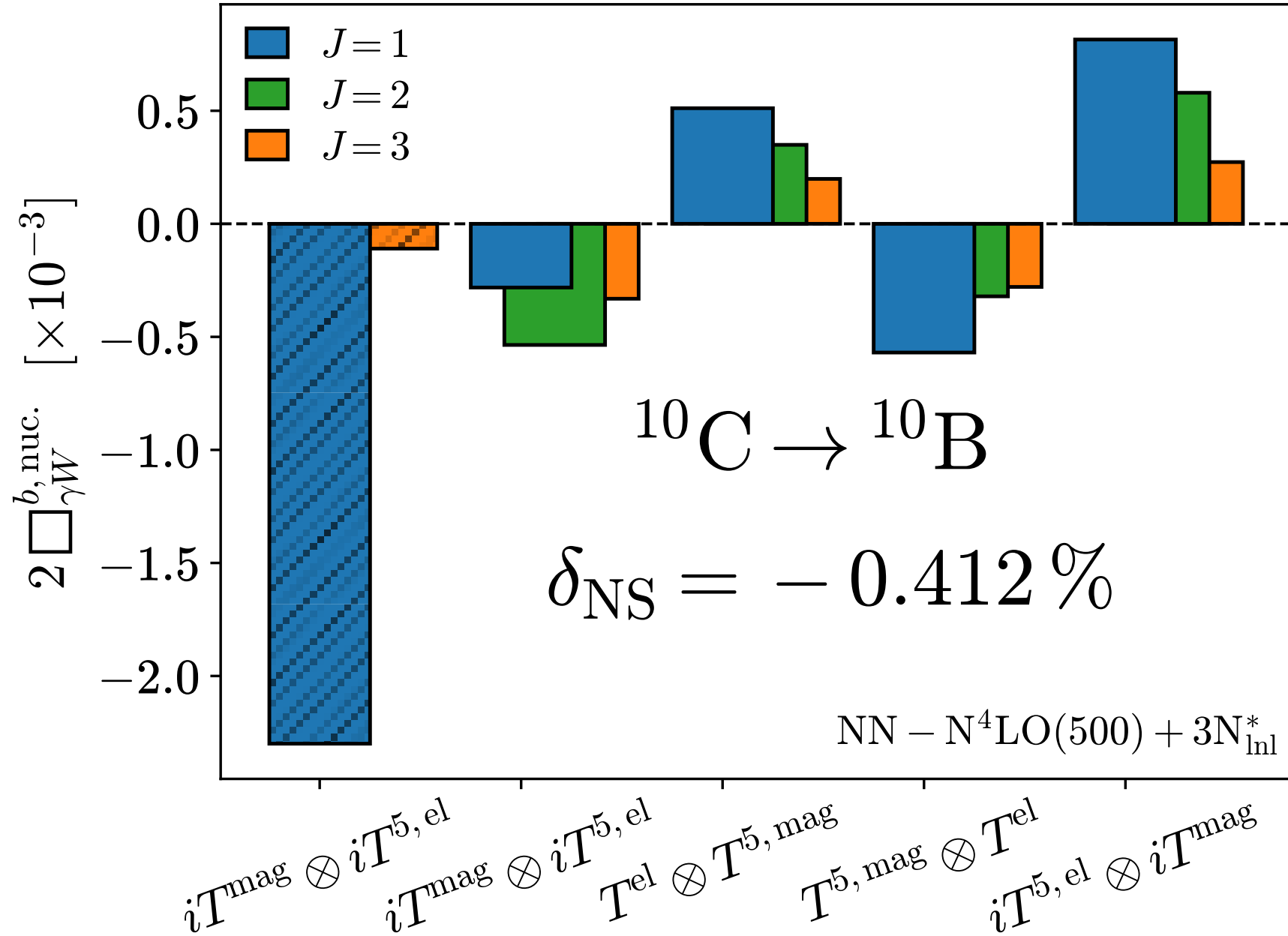
Amplitude and Pole Structure

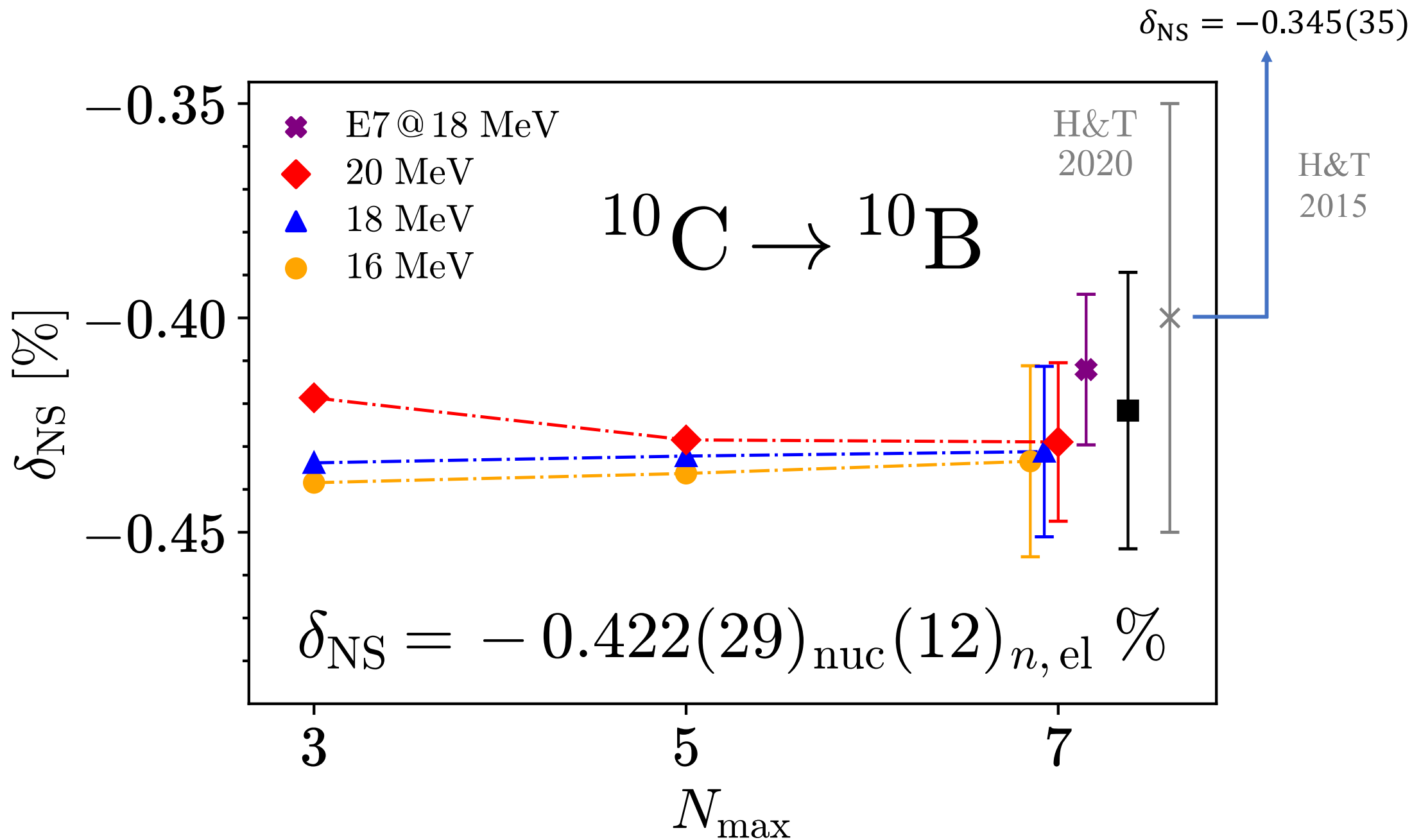
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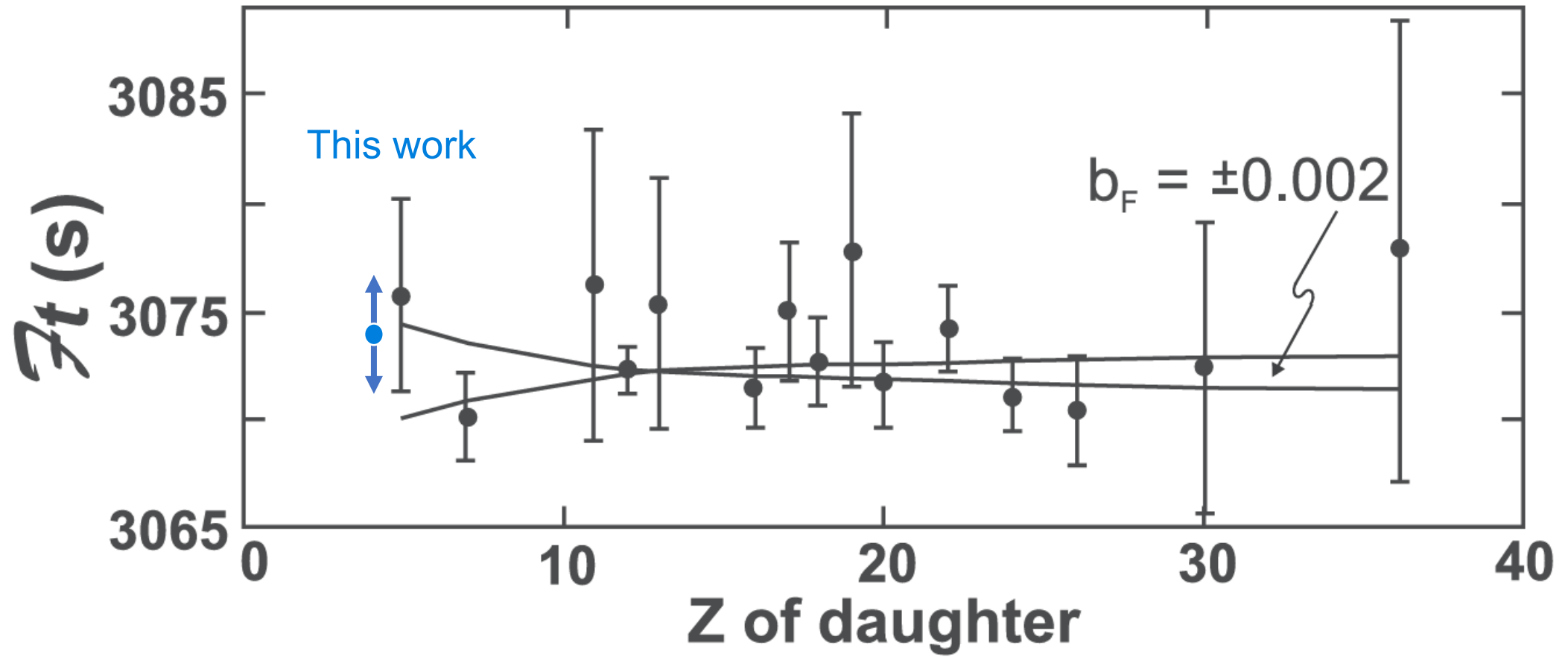


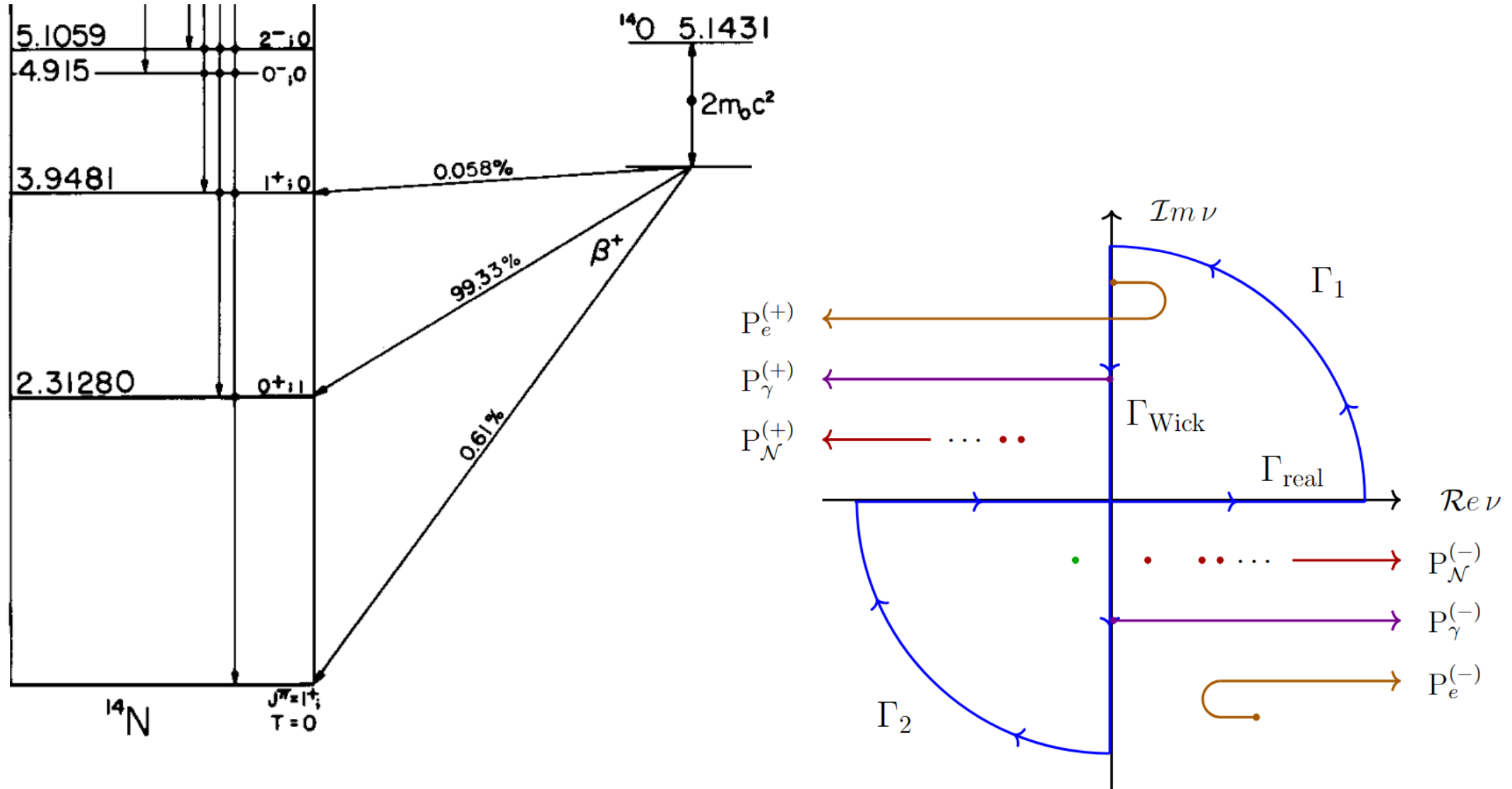


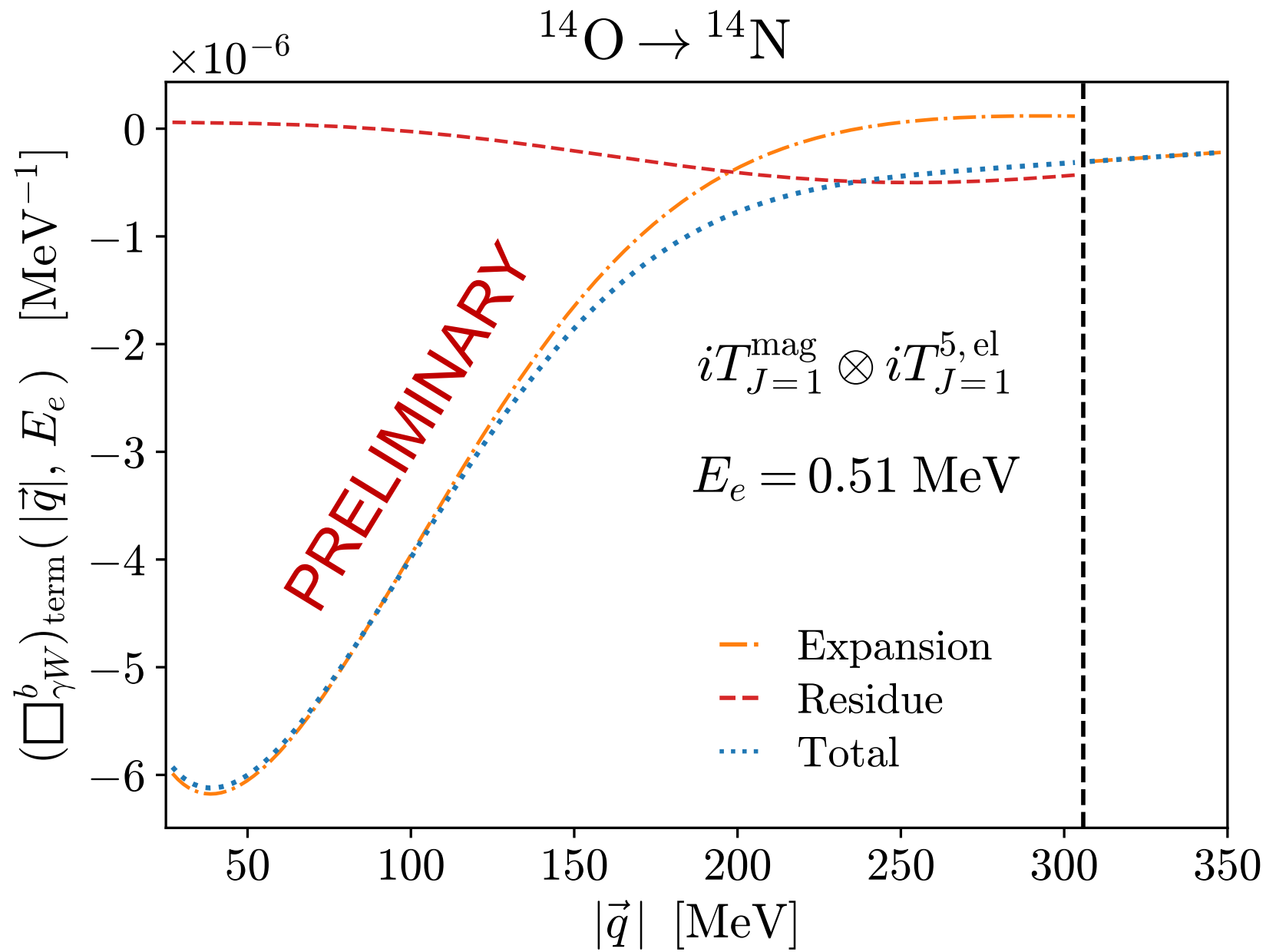


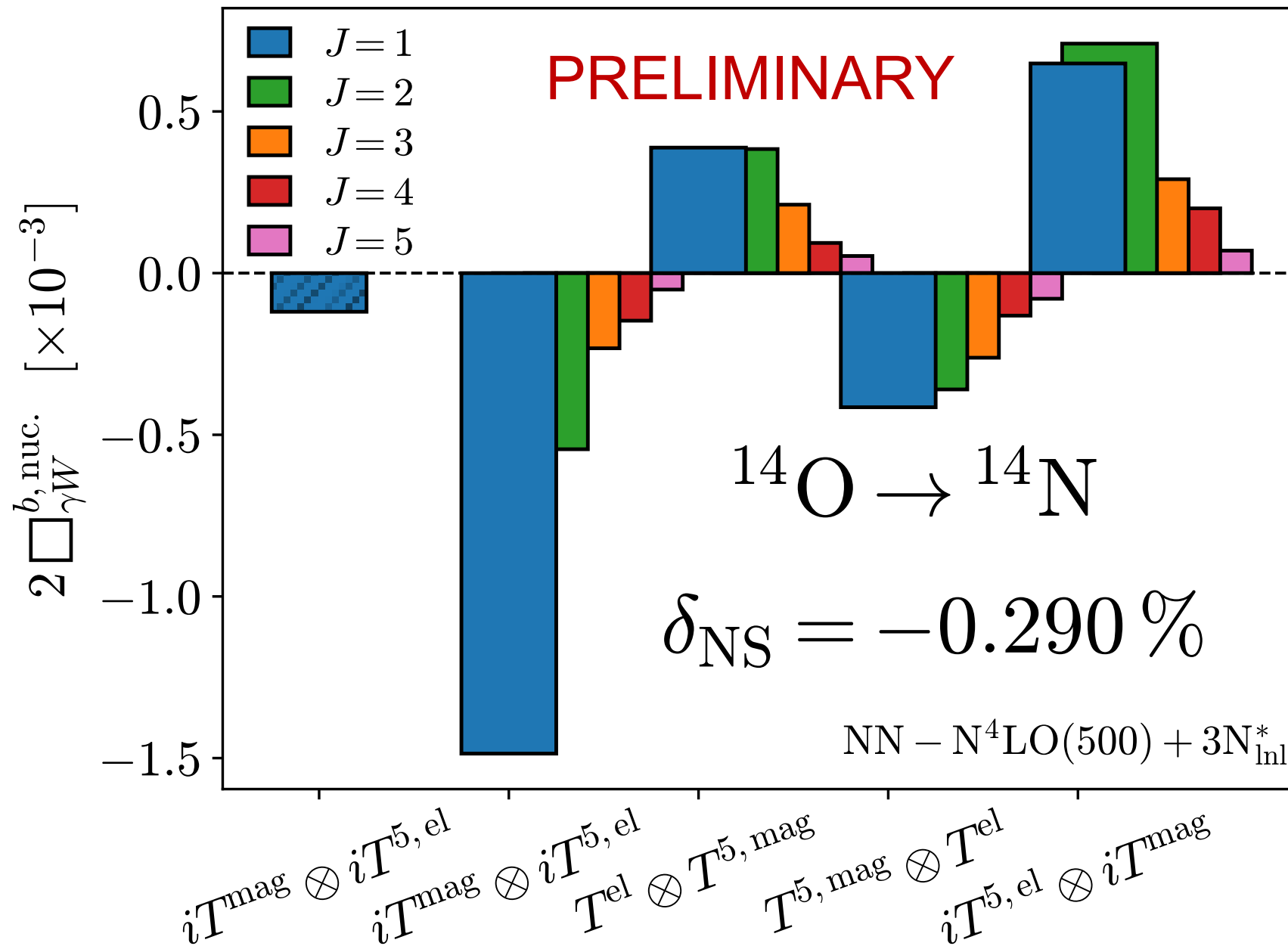


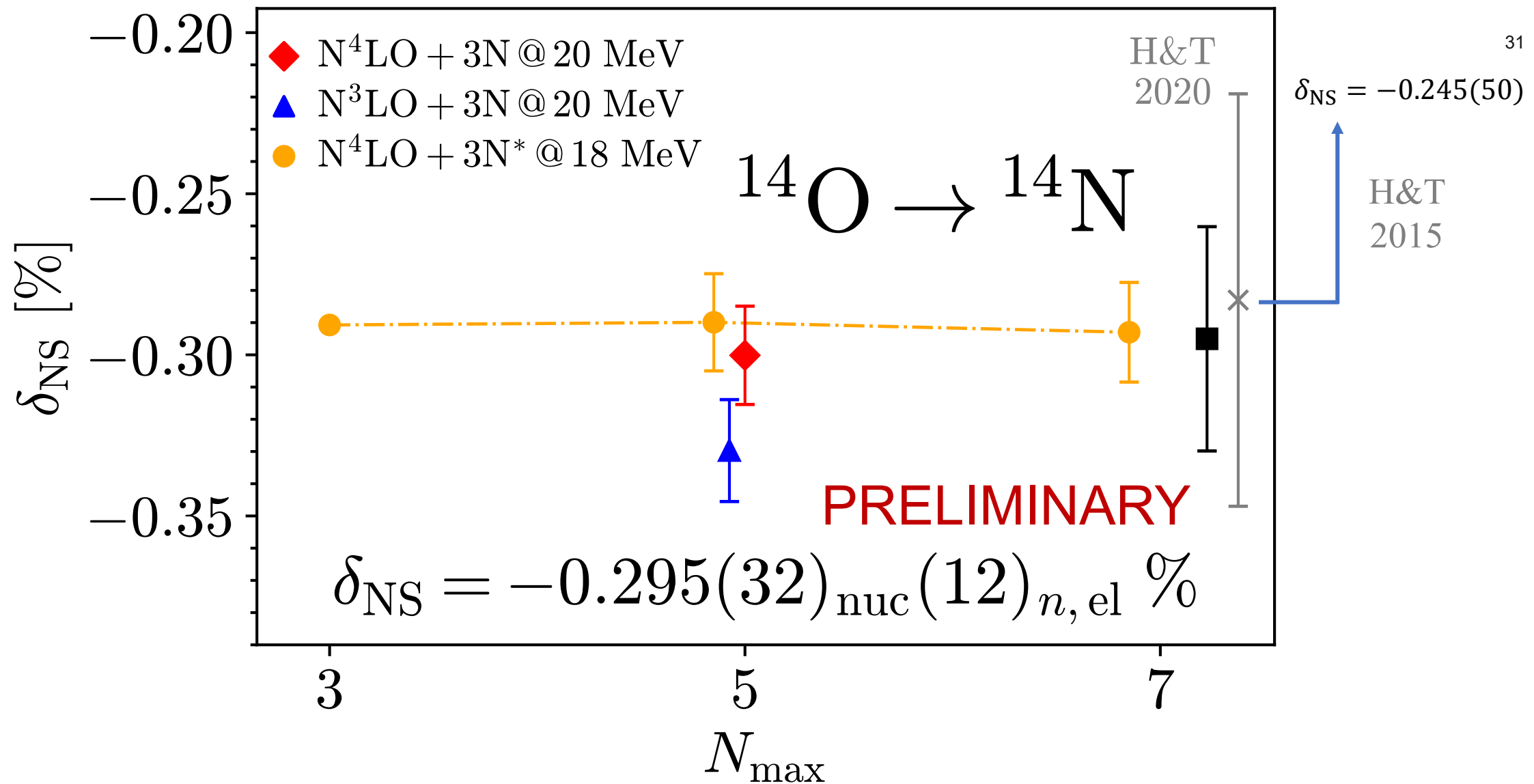




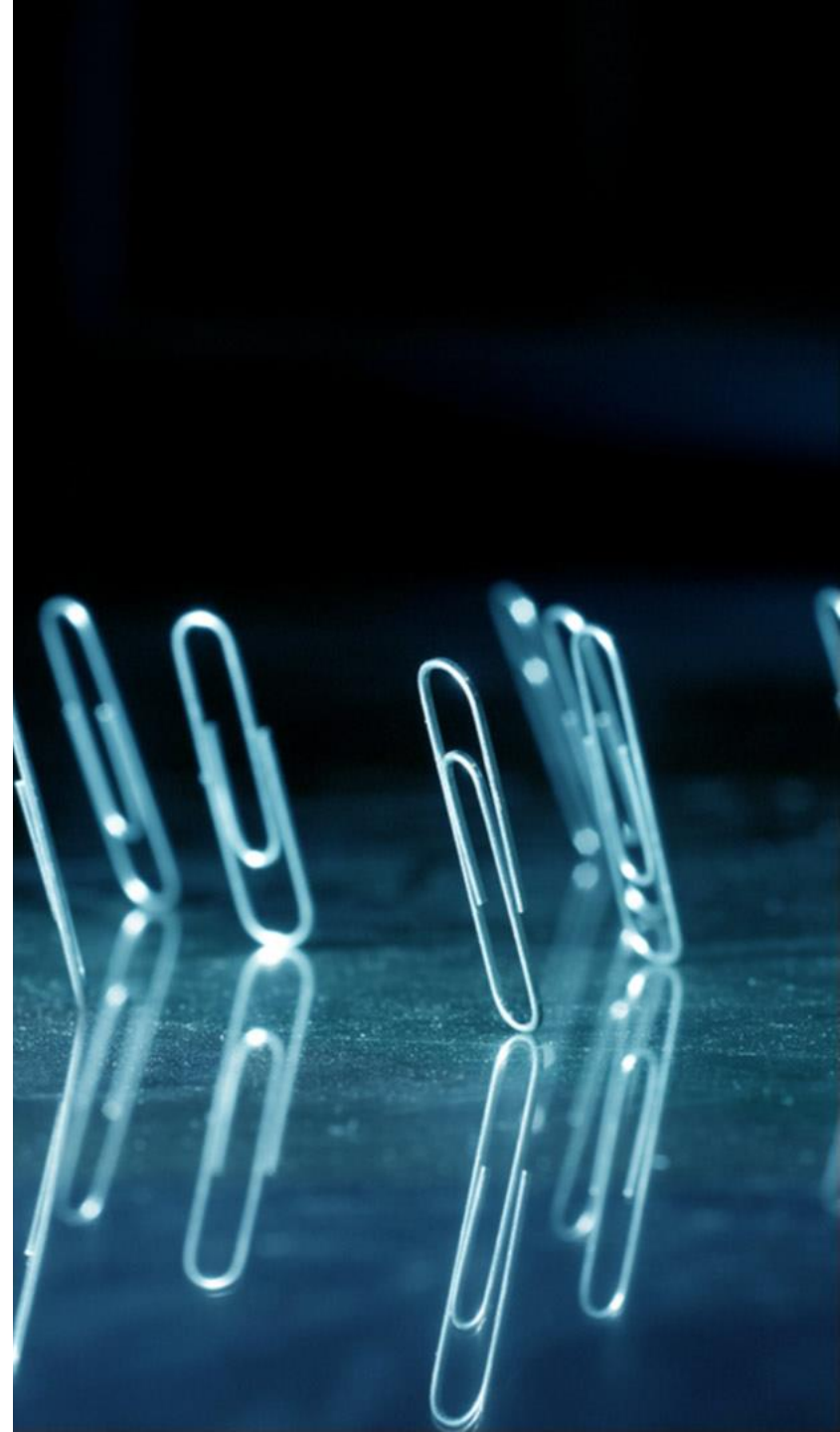








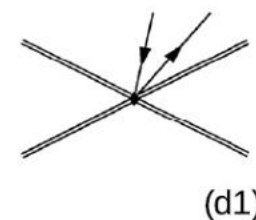
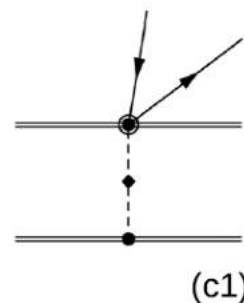
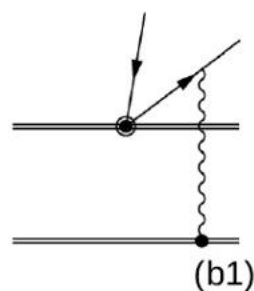
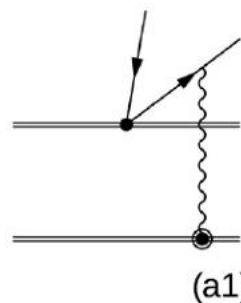
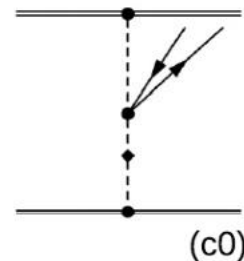
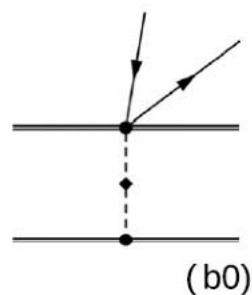
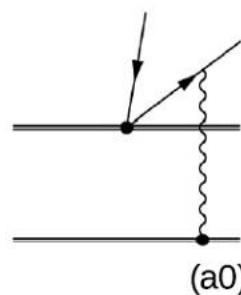
In the Near Future



Constraints on LECs of EFTs for Beta Decay



Courtesy of C.-Y. Seng



$$\delta_{\text{NS}} = \frac{2}{M_F} \langle \mathcal{V}_0^{\text{mag}}(\vec{r}) + \mathcal{V}_0^{\text{rec},1}(\vec{r}) + \mathcal{V}_0^{\text{CT}}(\vec{r}) \rangle_{fi}$$

$$\mathcal{A}_{fi} = \langle \Phi_f | O_2 (z - H)^{-1} O_1 | \Phi_i \rangle$$

Large Cost
Reduction in
our Formalism

PHYSICAL REVIEW C **72**, 065501 (2005)

Piecewise moments method: Generalized Lanczos technique for nuclear response surfaces

Wick C. Haxton,¹ Kenneth M. Nollett,² and Kathryn M. Zurek¹

¹*Institute for Nuclear Theory and Department of Physics, University of Washington, Seattle, Washington 98195, USA*

²*Physics Division, Argonne National Laboratory, Argonne, Illinois 60439, USA*

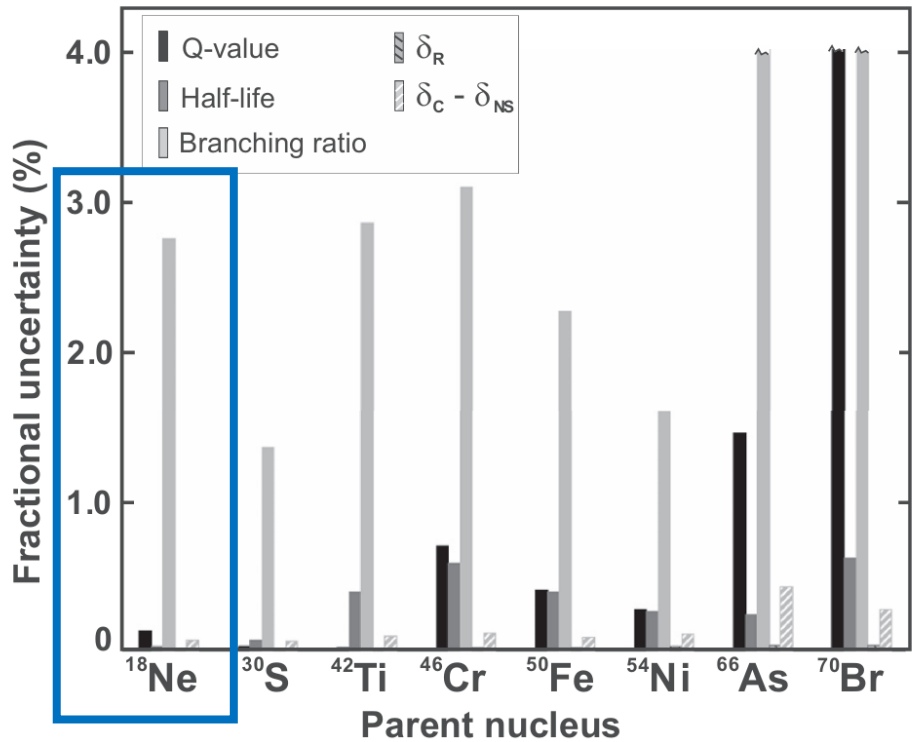
(Received 22 August 2005; published 29 December 2005)

Uncertainty
Quantification

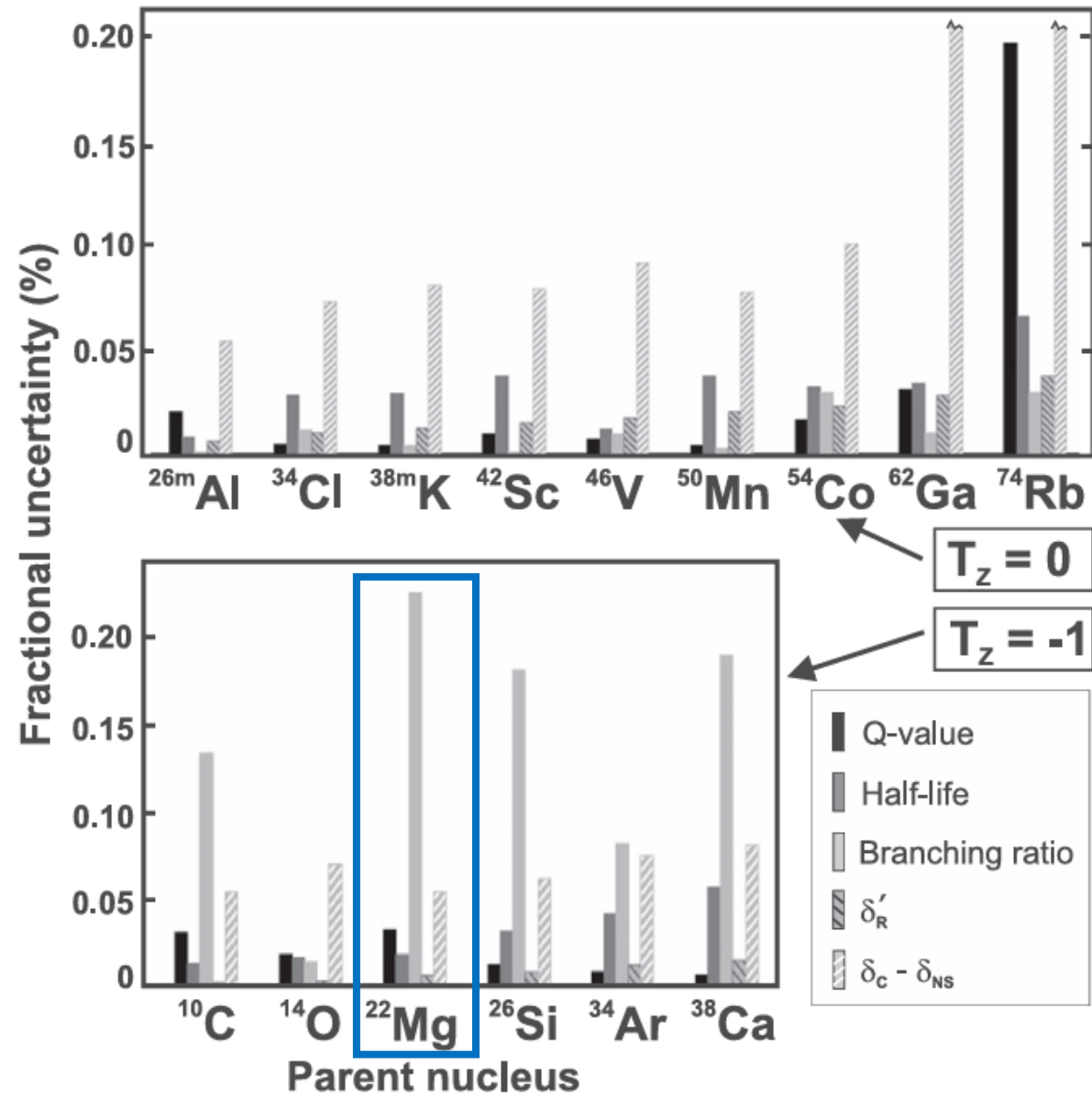
Do We Fully Understand the Symmetric
Lanczos Algorithm Yet? *

Beresford N. Parlett †

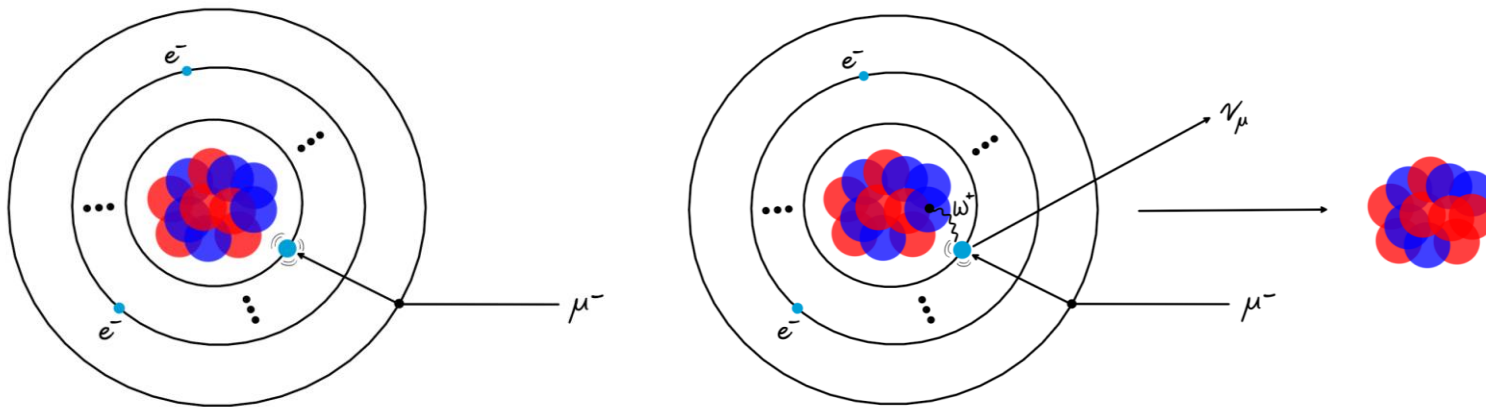
Future Measurements?



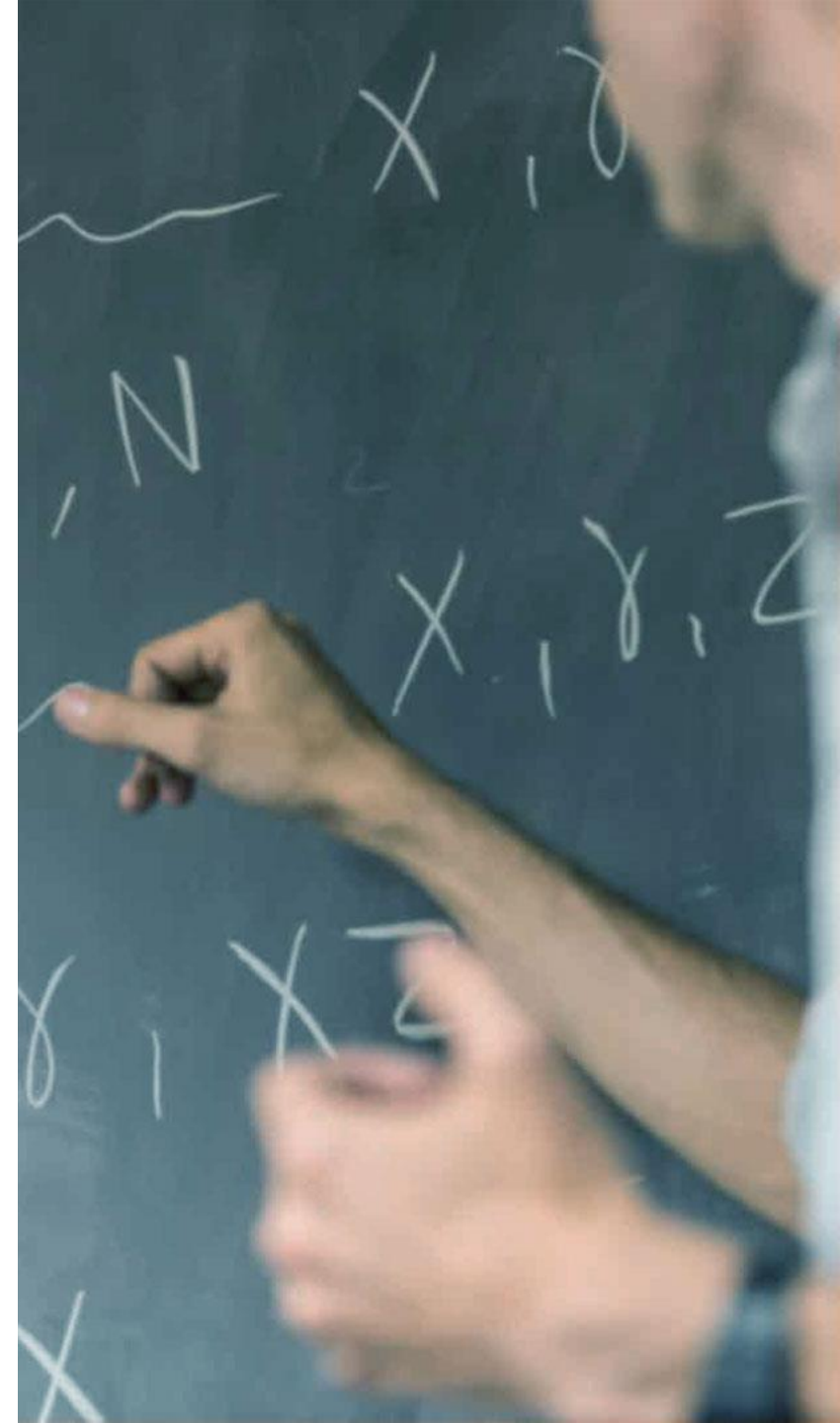
Branching ratio
measurements?



Muonic Atoms and Ordinary Muon Capture

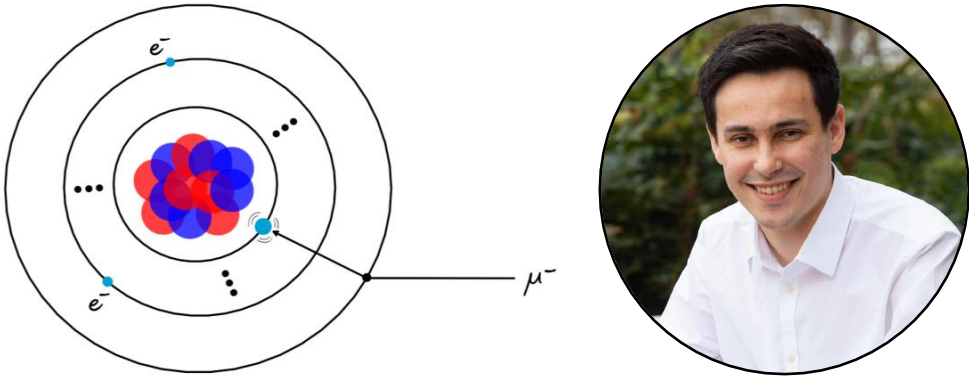


Drissi et al. In prep.
Najera et al. In progress.

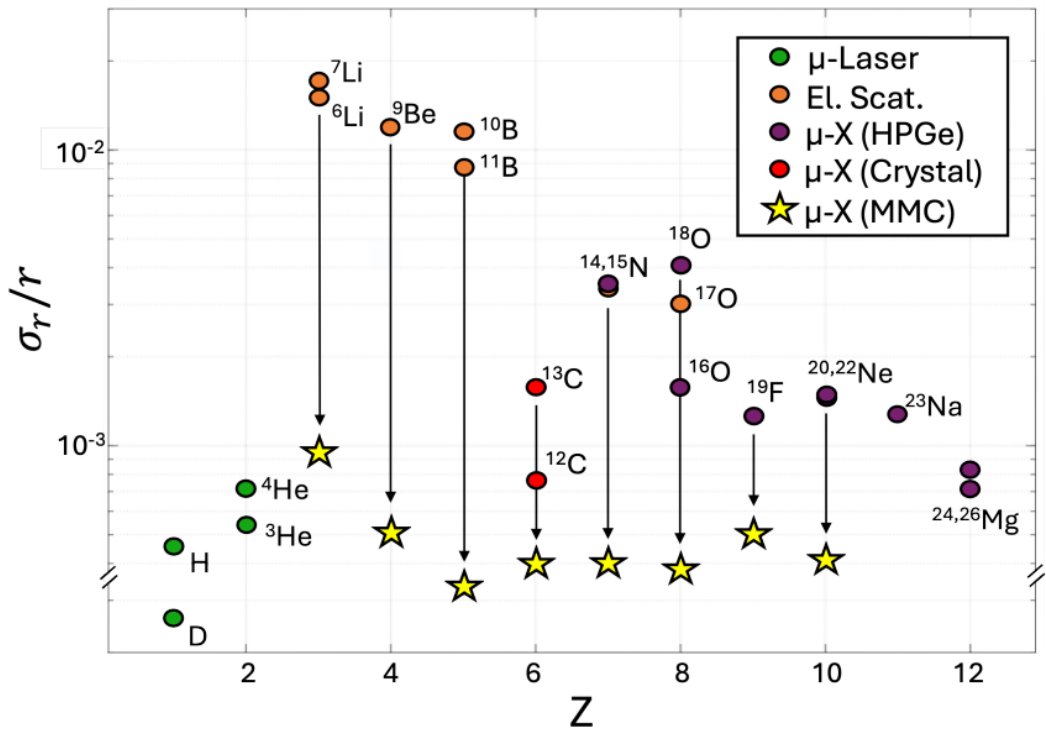


Muonic Atoms for Charge Radii

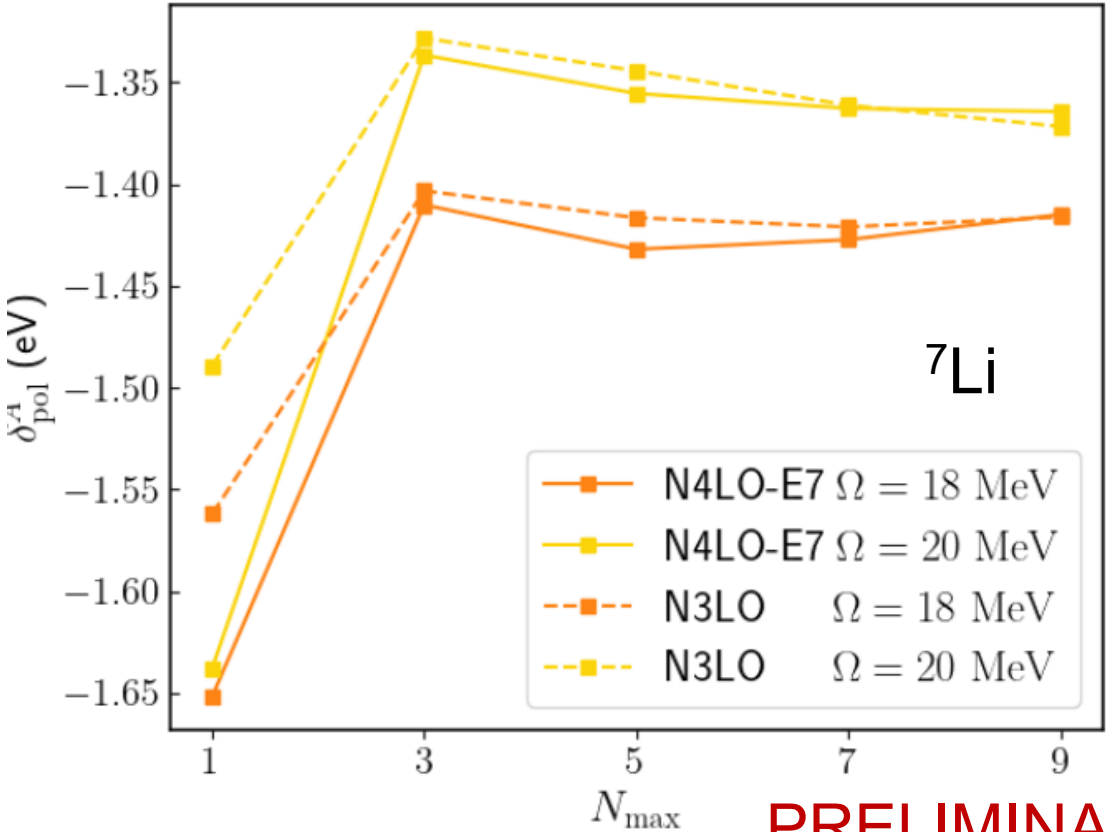
$$\delta E = \delta_{\text{QED}} + \mathcal{C} r_c^2 + \delta_{\text{NS}}$$



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Quartet collaboration meeting on muonic x-rays (2025)



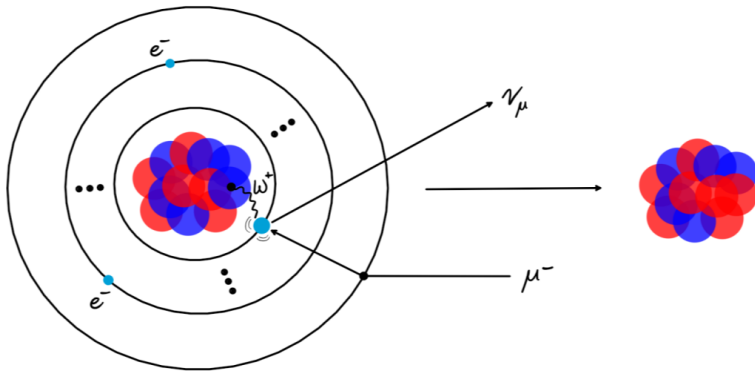
PRELIMINARY

Ordinary Muon Capture

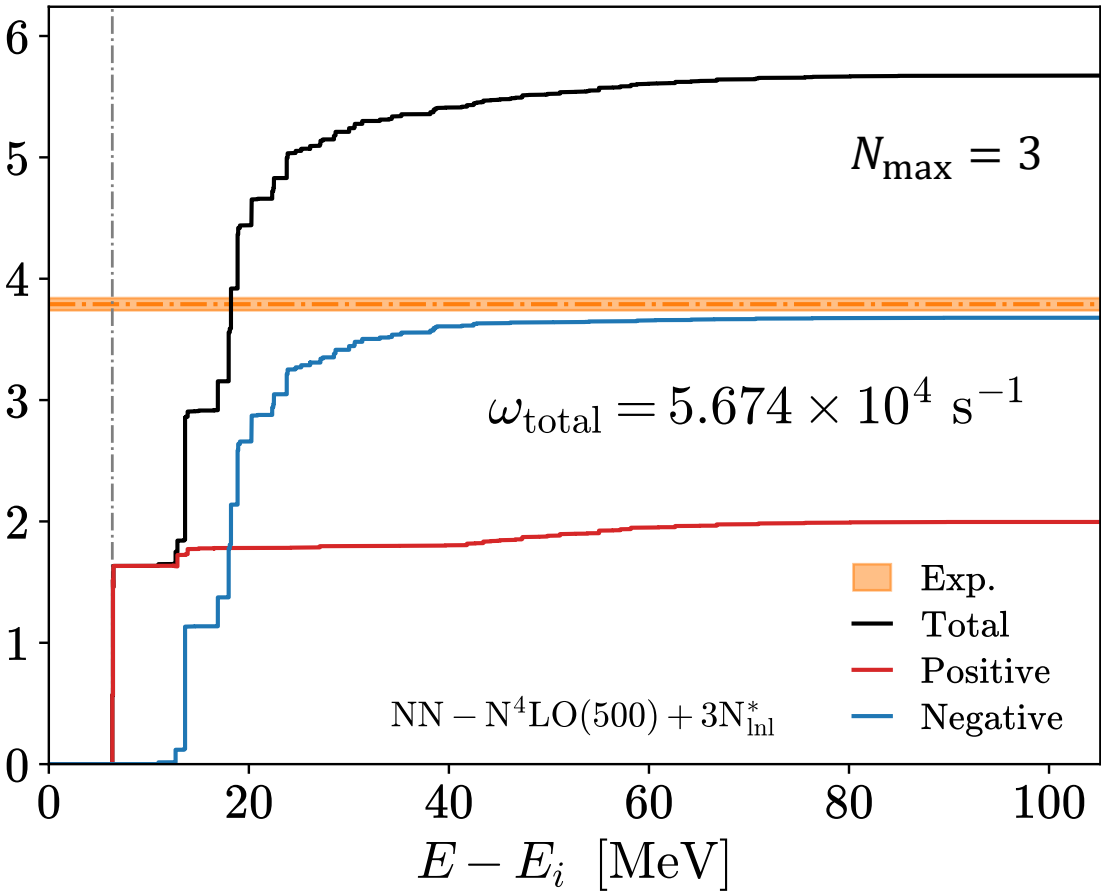
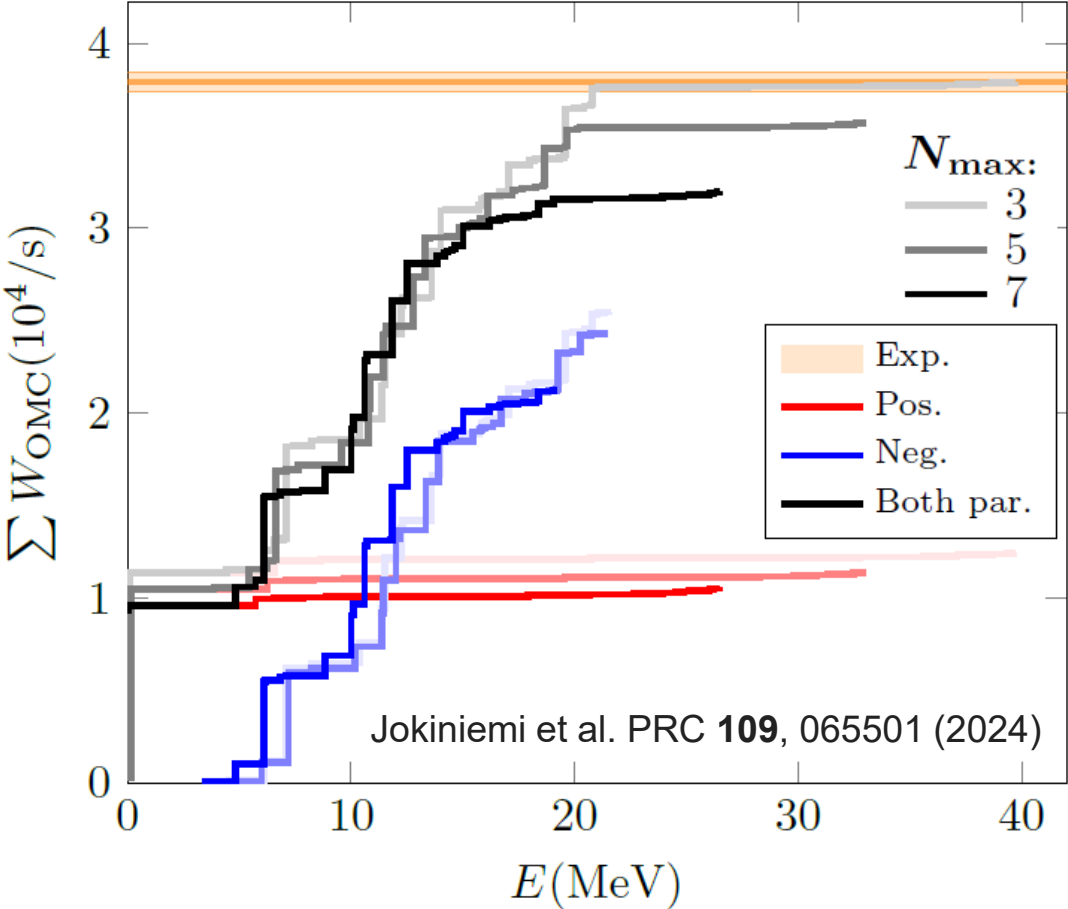
Why? Neutrino-less double beta decay!

1. Identical currents and coupling constants
2. Similar phase-space (different from electron-induced beta decay)

A measurable process which probes physics relevant to **neutrinoless double beta decay**



Comparison to OMC Literature for $^{12}\text{C} \rightarrow ^{12}\text{B}$

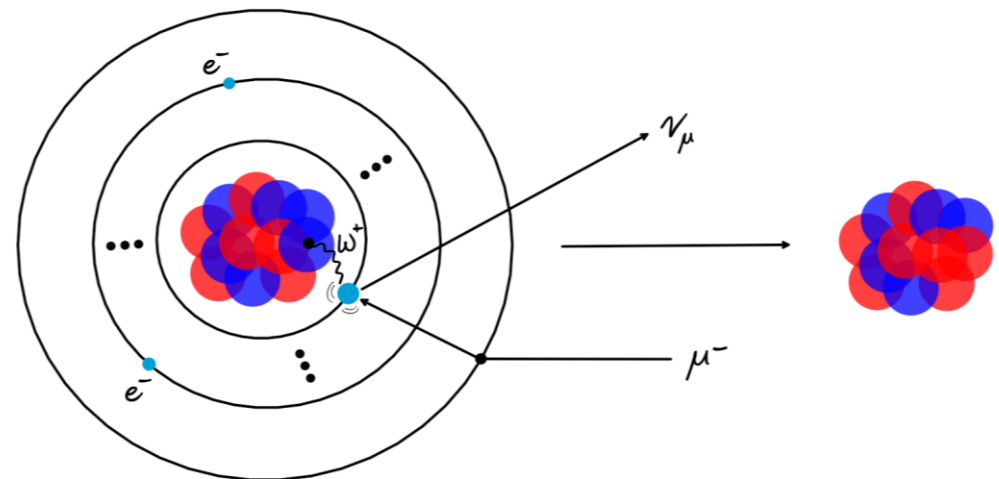
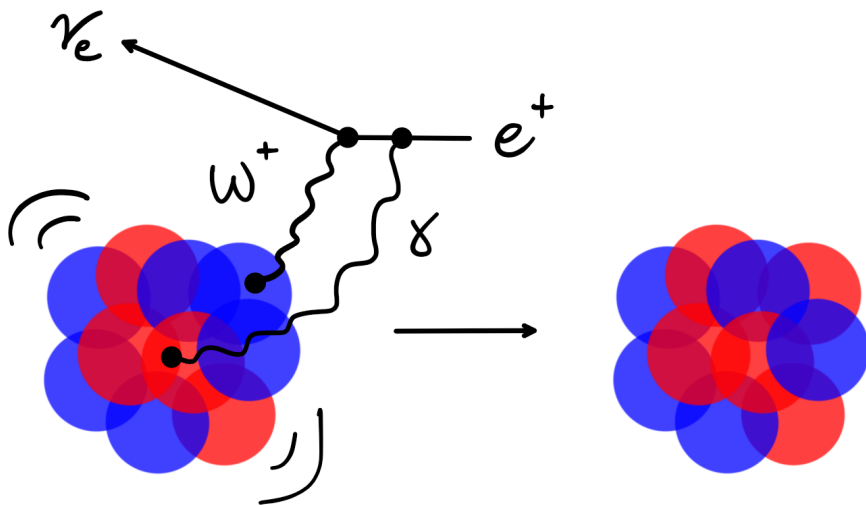


DANGEROUSLY PRELIMINARY

The future of electroweak theory in light nuclei

40

- **Improved precision for electroweak radiative corrections in nuclei** with the Lanczos Strengths Method coupled to the ab initio NCSM
- Systematic improvements available, e.g., consistent currents in chiral EFT
- Limited by matching of high-energy QCD to low-energy nuclear theory



Thank you
Merci

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One-loop radiative correction

42

$$\mathcal{M} = \mathcal{M}_{\text{tree}} + \underline{\delta\mathcal{M}_{\text{one-loop}}} + \cdots$$

$$\begin{aligned} \delta\mathcal{M}_{\text{one-loop}} = & -i\sqrt{2} G_F e^2 L_\lambda(k_f, k_i) \int \frac{d^4 q}{(2\pi)^4} R_W(q) R_e(q) R_\gamma(q) \\ & \times \underline{\left[\epsilon^{\mu\nu\alpha\lambda} q_\alpha T_{\mu\nu}(p_f, p_i; q) \right]} \end{aligned}$$

Wick rotation and electron energy expansion

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$$\square_{\gamma W}^b(E_e) = \underbrace{(\square_{\gamma W}^b)_{\text{Wick}}(E_e)}_{\text{blue line}} + \underbrace{(\square_{\gamma W}^b)_{\text{Res},e}(E_e)}_{\text{red line}} + \underbrace{(\square_{\gamma W}^b)_{\text{Res},T_3}(E_e)}_{\text{red line}}$$

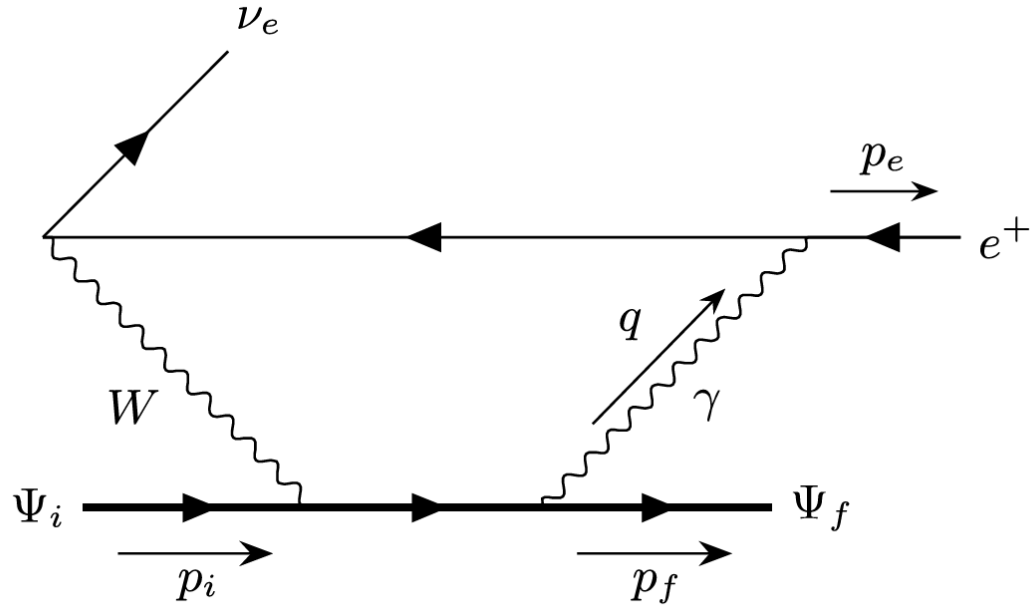
Wick rotated box diagram and electron residue
contribution are regular as $E_e \rightarrow 0$

Nuclear residue contribution is singular

$$\square_{\gamma W}^b(E_e) = \Xi_0 + E_e \Xi_1 + (\square_{\gamma W}^b)_{\text{Res},T_3}(E_e) + \mathcal{O}(E_e^2)$$

Deriving the non-relativistic Compton amplitude

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$$J(\vec{r}) = \int \frac{d^3r}{(2\pi)^3} e^{i\vec{q} \cdot \vec{r}} J(\vec{q})$$

$$J^\mu(t, \vec{x}) = e^{-iHt} J^\mu(0, \vec{x}) e^{iHt}$$

$$T^{\mu\nu}(p_f, p_i; q) = -\frac{i}{2} \langle \Phi_f; p_f | J_{\text{em}}^\mu(-\vec{q})(z_f - H)^{-1} J_W^{\dagger\nu}(\vec{q}) | \Phi_i; p_i \rangle$$

$$- \frac{i}{2} \langle \Phi_f; p_f | J_W^{\dagger\nu}(-\vec{q})(z_i - H)^{-1} J_{\text{em}}^\mu(\vec{q}) | \Phi_i; p_i \rangle$$

Deriving the non-relativistic Compton amplitude

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$$M_{JM}(q) := \int d^3r \mathcal{M}_{JM}(q, \vec{r}) \rho(\vec{r})$$

$$L_{JM}(q) := \int d^3r \frac{i}{q} \left(\vec{\nabla} \mathcal{M}_{JM}(q, \vec{r}) \right) \cdot \vec{J}(\vec{r})$$

$$T_{JM}^{\text{el}}(q) := \int d^3r \frac{1}{q} \left(\vec{\nabla} \times \vec{\mathcal{M}}_{JJ}^M(q, \vec{r}) \right) \cdot \vec{J}(\vec{r})$$

$$T_{JM}^{\text{mag}}(q) := \int d^3r \vec{\mathcal{M}}_{JJ}^M(q, \vec{r}) \cdot \vec{J}(\vec{r})$$