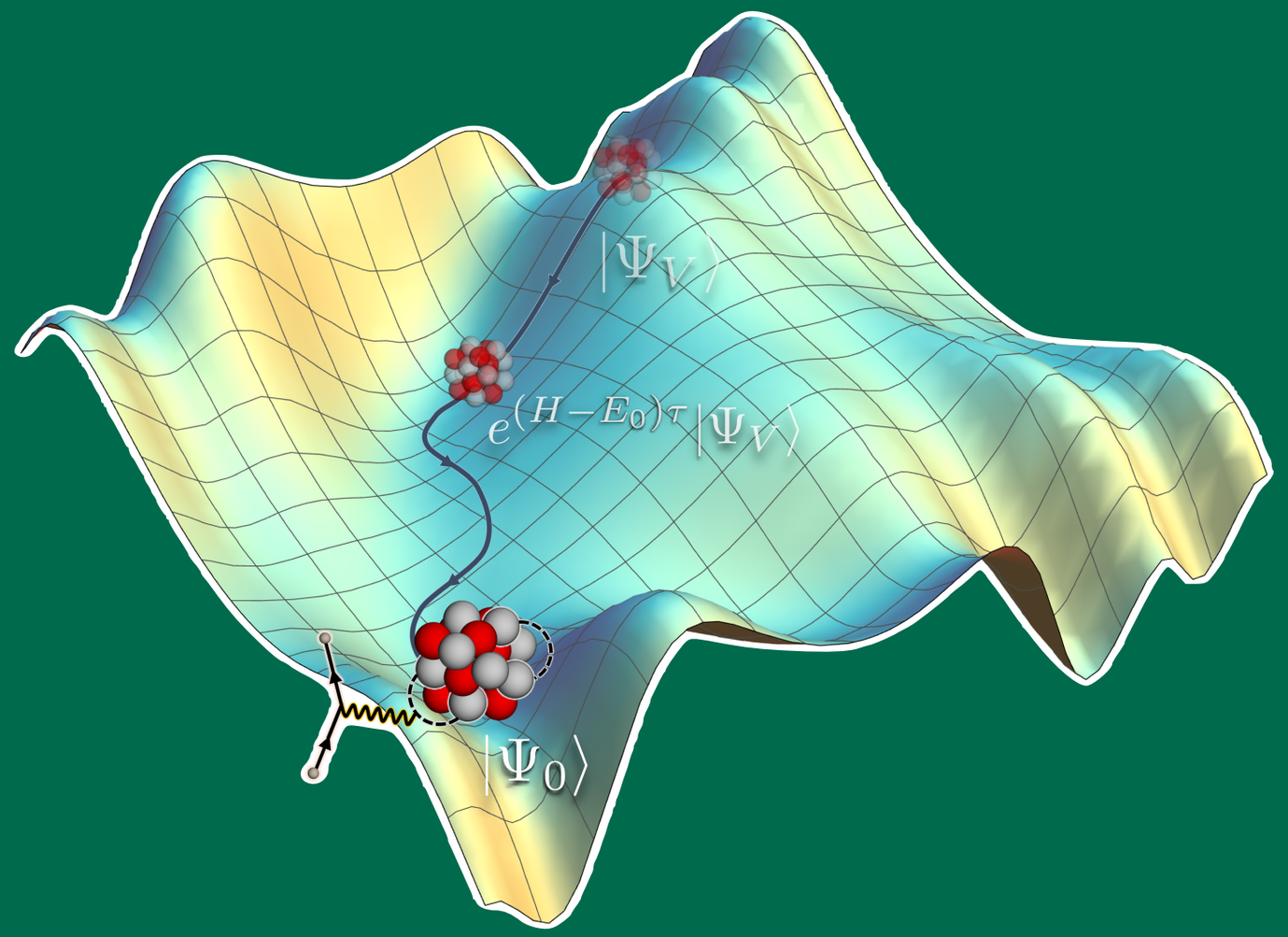
Single and double beta decays with quantum Monte Carlo methods

FRIB-TA: Future directions in nuclear beta decays at FRIB



September 8, 2025

Graham Chambers-Wall
Advisors: Saori Pastore & Maria Piarulli

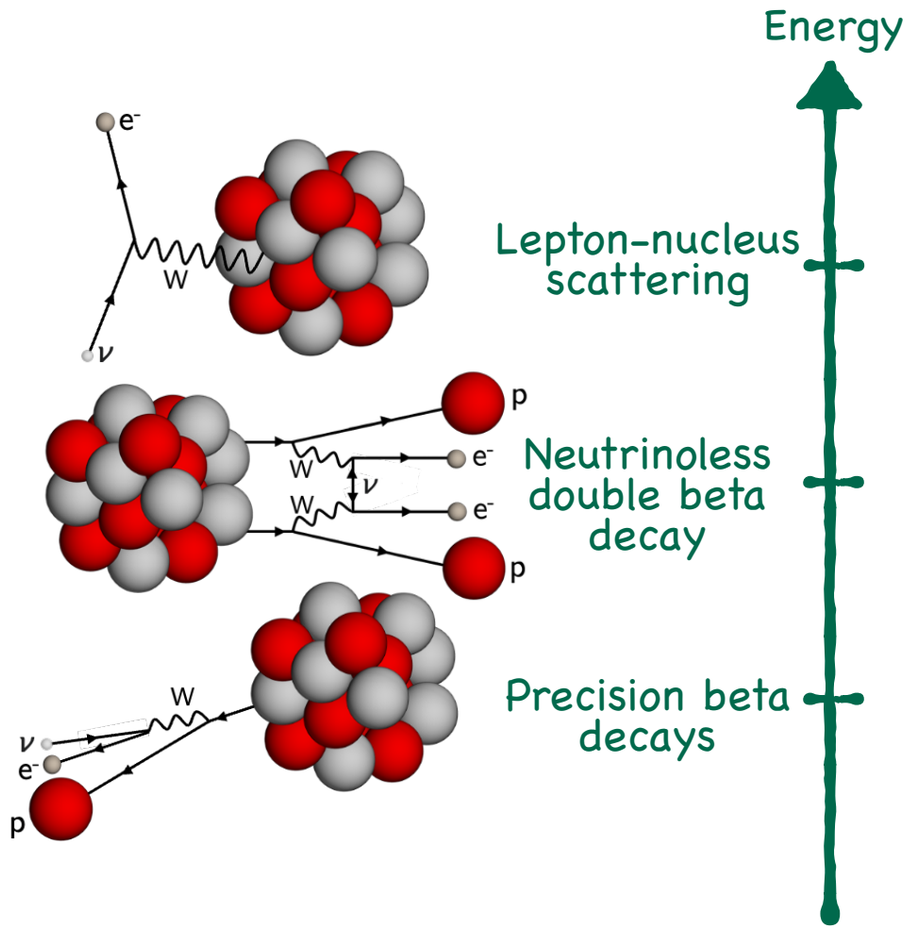
WashU

Nuclear physics for new physics

- Precision searches for new physics often requires input from nuclear theory across **range of energies**

- Handle on nuclear models necessary to disentangle known nuclear physics from possible BSM signatures

- Strategy: first validate models on **known observables**, then make **predictions for BSM searches**



Outline

1. **Microscopic description of nuclei**
    1. Many-body problem
    2. Quantum Monte Carlo
    3. Chiral effective field theory

2. **Single beta decays**
    1. Gamow-Teller matrix elements, role of two-nucleon currents
    2. Superallowed beta decay

3. **Neutrinoless double beta decay**
    1. 2N matrix elements
    2. 3N operators and their matrix elements in light nuclei

Nuclear many-body problem

Many-body hamiltonian

$$H = \sum_i t_i + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + \dots$$

$$\Psi(\boldsymbol{r}_1, \boldsymbol{r}_2, \dots, \boldsymbol{r}_A; s_1, s_2, \dots, s_A; t_1, t_2, \dots, t_A)$$

Spin-isospin vector in 3A dimensions with $2^A \times \binom{A}{Z}$ components

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Spin-isospin vector in 3A dimensions with $2^A \times \binom{A}{Z}$ components

^{4}He: 96
^{6}Li: 1,280
^{8}Be: 17,920
^{12}C: 3,784,704
…



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^{136}Xe: 29,047,067,511,683,262,845,258,801,231,860,593,413,323,900,569,008,453,449,667,034,231,949,394,496,716,800

Quantum Monte Carlo (QMC) methods

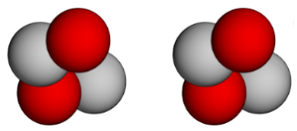
Variational Monte Carlo (VMC): minimize expectation value of Hamiltonian

$$E_V = \frac{\langle \Psi_V | H | \Psi_V \rangle}{\langle \Psi_V | \Psi_V \rangle} \geq E_0$$

$$|\Psi_V\rangle = \left[\mathcal{S} \prod_{i<j} \left(1 + U_{ij} + \sum_{k \neq i,j} \tilde{U}_{ijk} \right) \right] \left[\prod_{i<j} f_c(r_{ij}) \right] |\Phi_A(JMTT_3)\rangle$$

two- and three-body correlation operators

cluster structure



^{8}Be ~ 2α

Quantum Monte Carlo (QMC) methods

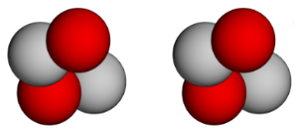
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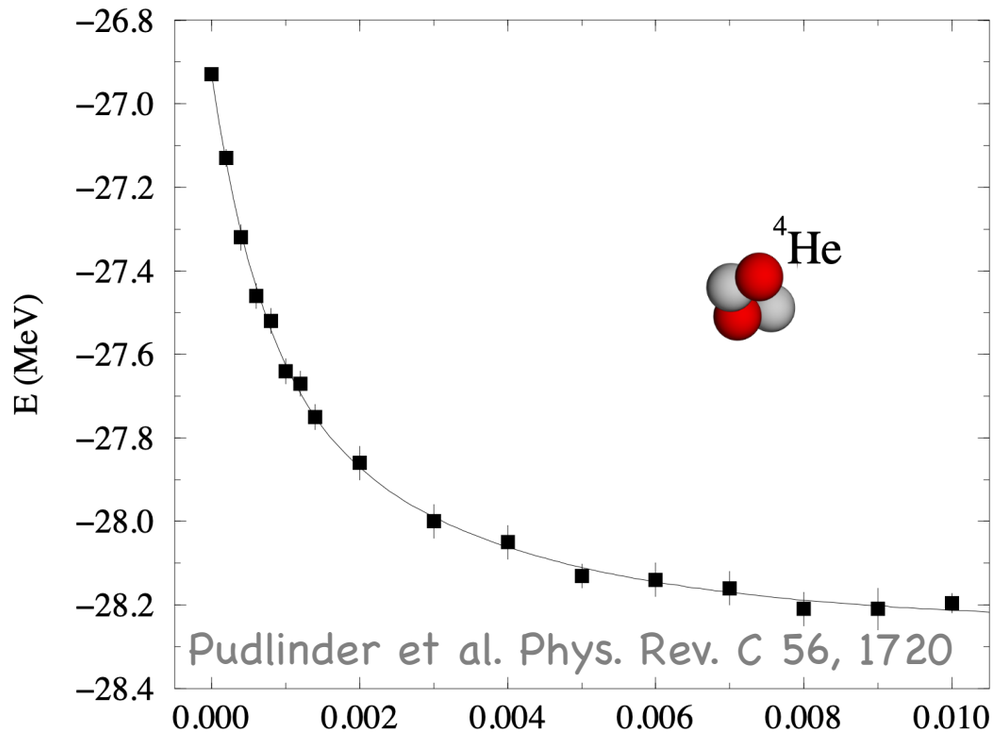


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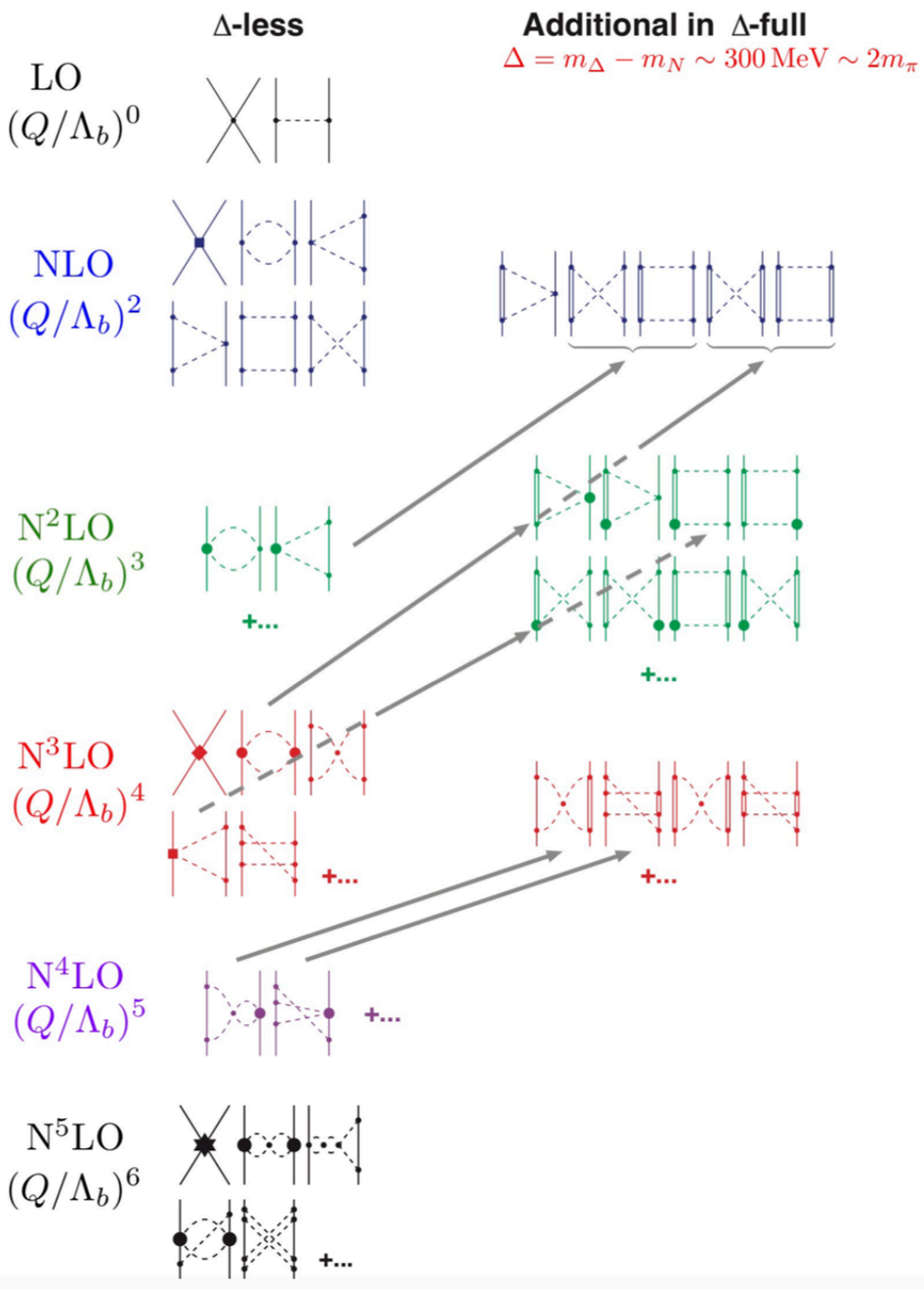
Green's Function Monte Carlo (GFMC): improve VMC solution by propagating in imaginary time to remove contaminations from excited states

$$\Psi(\tau) = e^{-(H-E_0)\tau} \Psi_V = \sum_n e^{-(E_n - E_0)\tau} a_n \psi_n$$

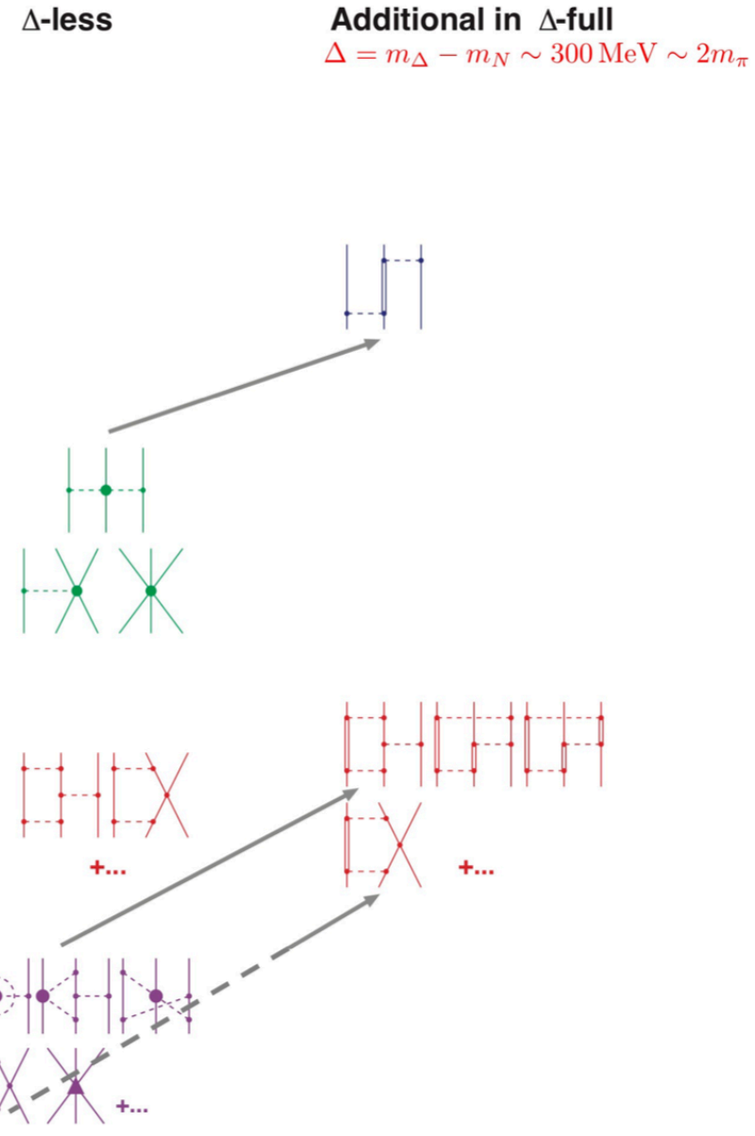
$$\lim_{\tau \to \infty} \Psi(\tau) = a_0 \psi_0$$



Chiral effective field theory



- Low-energy model of QCD based on approximate chiral symmetry
- Nucleons, pions, and deltas as degrees of freedom
- Provides hierarchy of many-nucleon forces and currents
- High-energy physics encoded in low-energy constants (LECs) (26 in 2N, 2 more in 3N forces)

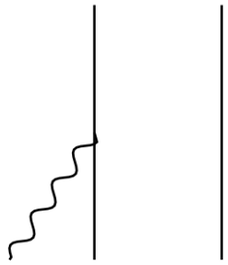


Machleidt and Entem Phys. Rep. **503** (2011)

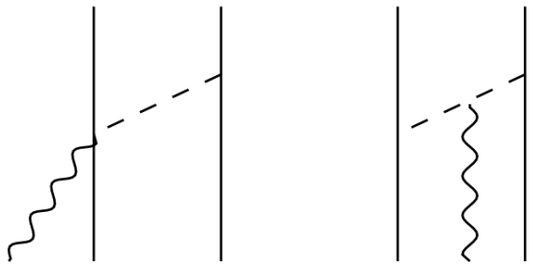
Piarulli and Tews Front. Phys. **30** (2020)

Electroweak currents

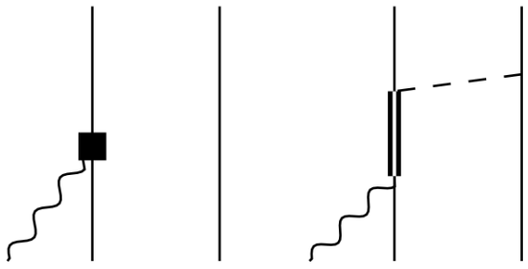
LO



NLO



N²LO



Need electroweak charge and current operators to calculate observables:

$$\rho = \sum_{i=1}^{A} \rho_i + \sum_{i<j} \rho_{ij} + \dots$$

$$\mathbf{j} = \sum_{i=1}^{A} \mathbf{j}_i + \sum_{i<j} \mathbf{j}_{ij} + \dots$$

External field (photon) interacts with a **single nucleon**  or a **correlated pair of nucleons** 

Pastore et al. PRC **80**, 034004 Piarulli et al. PRC **87**, 014006

Pastore et al. PRC **84**, 024001 Schiavilla et al. PRC **99**, 034005

Norfolk (NV2+3) interactions

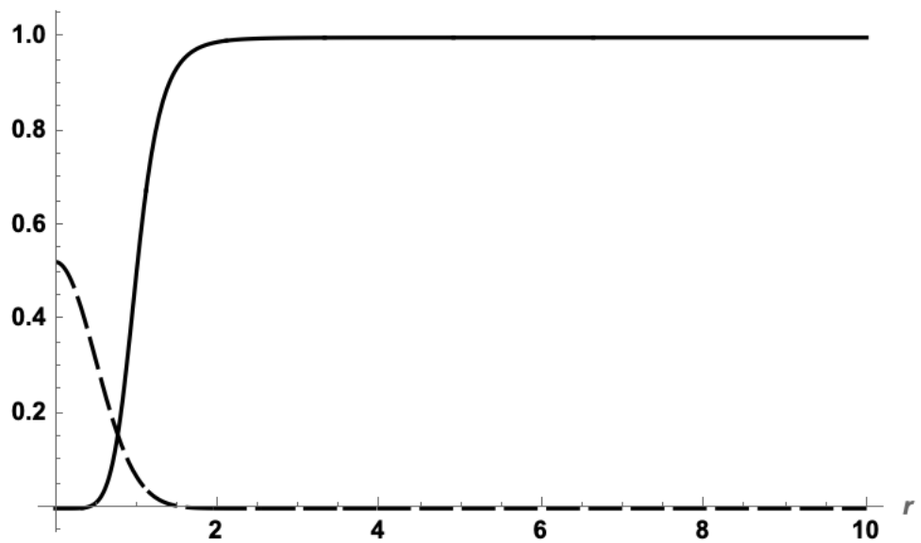
$$H = \sum_i t_i + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + \ldots$$

Compare different models (**Ia★**, **IIb★**, etc) with different LECs and regulators

I (**II**) fit to NN scattering data up to **125** (**200**) MeV

★ (no star) fit to triton ground state energy and β-decay (nd scattering)

a (**b**) use soft (hard) regulators **[R_L, R_S] = [1.2 fm, 0.8 fm]** (**[R_L, R_S] = [1.0 fm, 0.7 fm]**)



$$C_{R_L}(r) = 1 - \frac{1}{(r/R_L)^6 e^{(r-R_L)/a_L} + 1}$$

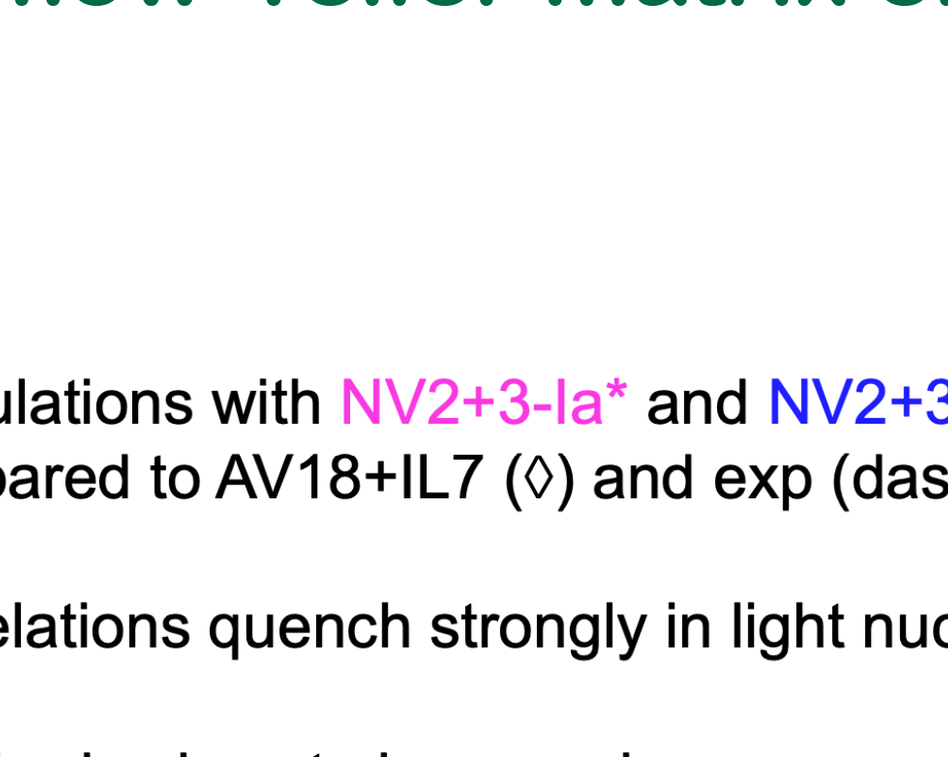
$$C_{R_S}(r) = \frac{1}{\pi^{3/2} R_S^3} e^{-(r/R_S)^2}$$

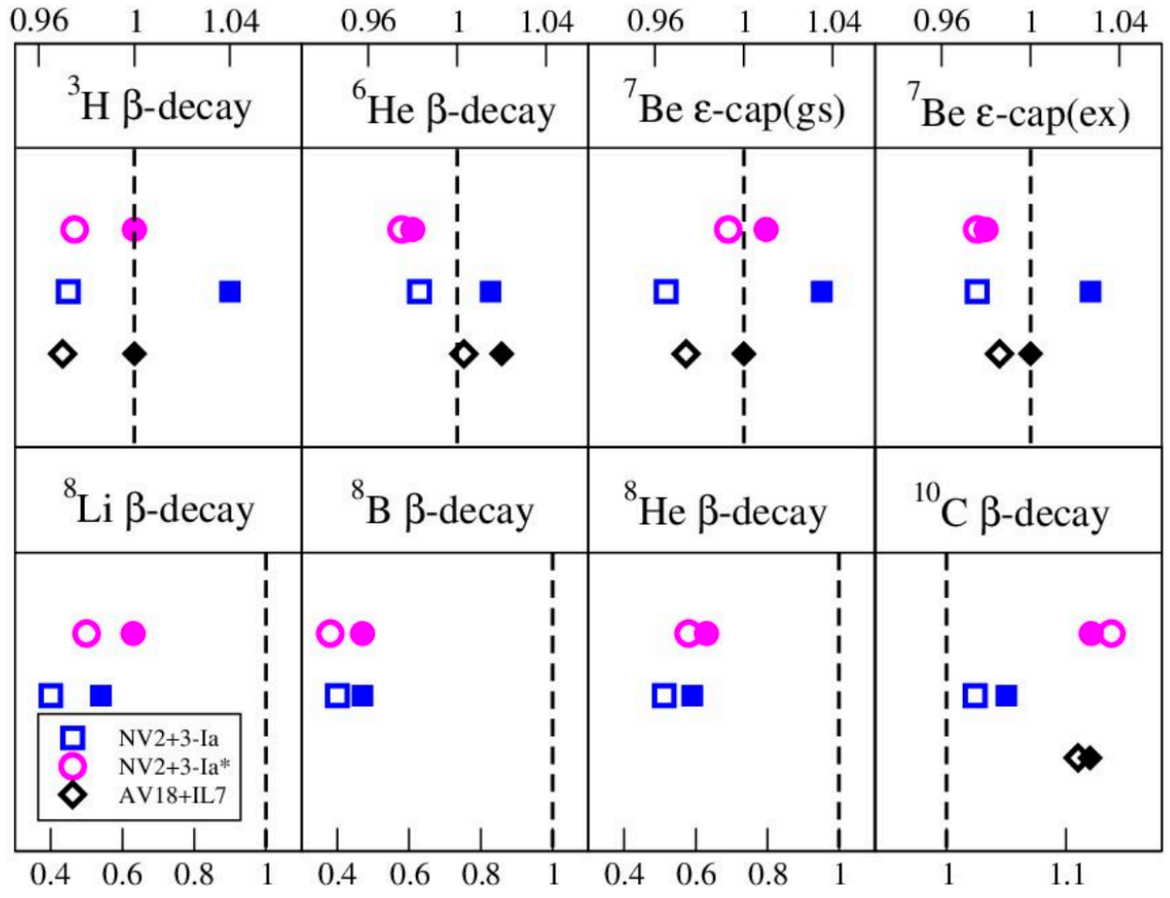
Gamow-Teller matrix elements

Calculations with NV2+3-Ia* and NV2+3-Ia compared to AV18+IL7 (◊) and exp (dashes)

Correlations quench strongly in light nuclei

Two-body almost always enhances



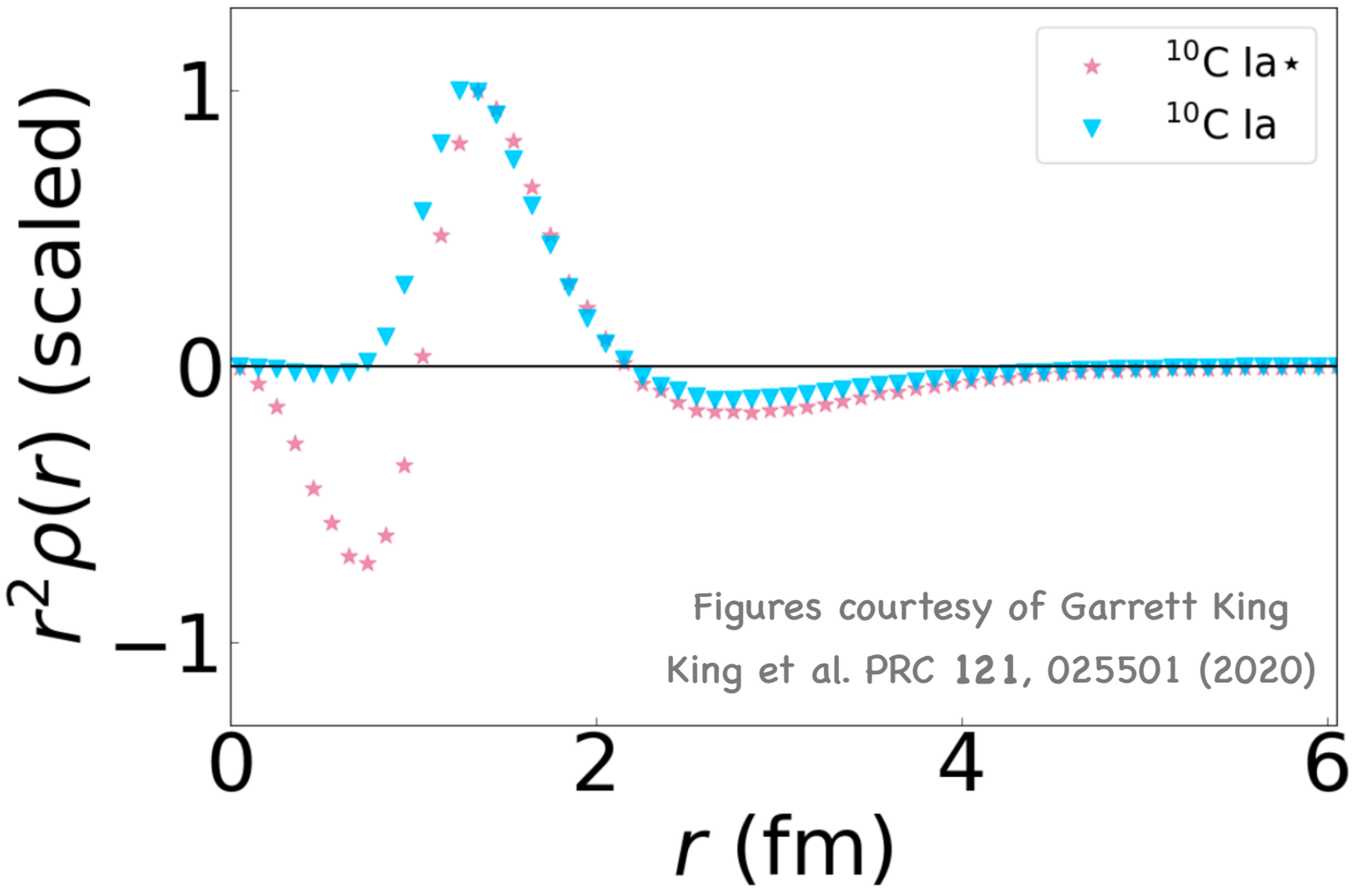


King et al. PRC **121**, 025501 (2020)

Figures courtesy of Garrett King

$$\text{GT RME} = \frac{\sqrt{2J+1}}{g_A} \frac{\langle J_F M | j^z_{\pm,5}(\boldsymbol{q} \to 0)) | J_i M \rangle}{\langle J_i M, 10 | J_f M \rangle}$$

Beta decay: two body densities

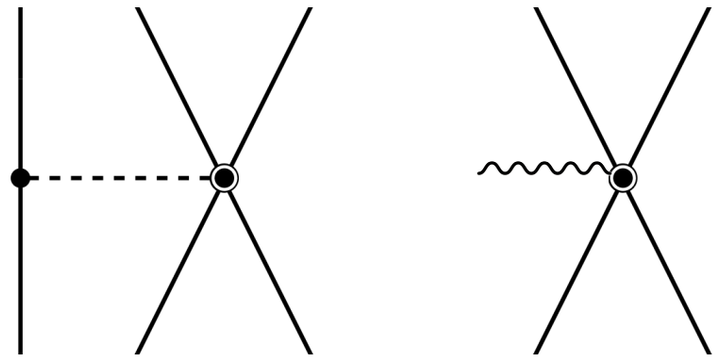


Different approaches to fitting give different **short-range** behavior

This affects the total **two-body matrix elements** for different models

$$M^{2\mathrm{b}}_{\mathrm{GT}} = \int dr\ 4\pi r^2 \rho^{2\mathrm{b}}_{\mathrm{GT}}(r)$$

Three-body LECs



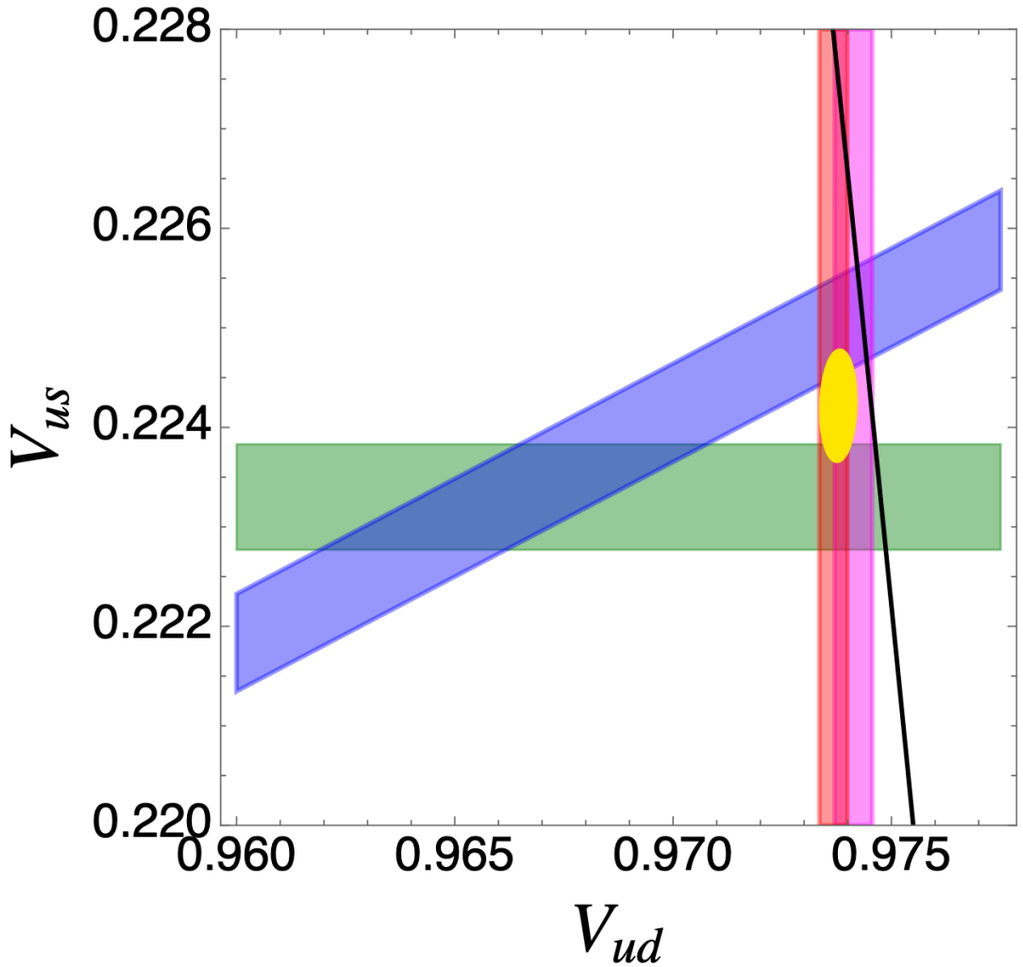
$$\boldsymbol{j}_{5,a}^{\mathrm{N3LO}}(\boldsymbol{q};\mathrm{CT}) = z_0 \mathcal{O}_{ij}(\boldsymbol{q})$$

$$z_0 \propto (c_D + \text{known LECs})$$

Model $\mathrm{NV2+3-Ia}$ fit c_D using strong interaction data only

Model $\mathrm{NV2+3-Ia}^\star$ fit c_D using both strong and weak data

Superallowed $0^+ \rightarrow 0^+$ beta decays



Cirigliano et al., PLB **838**, 137748

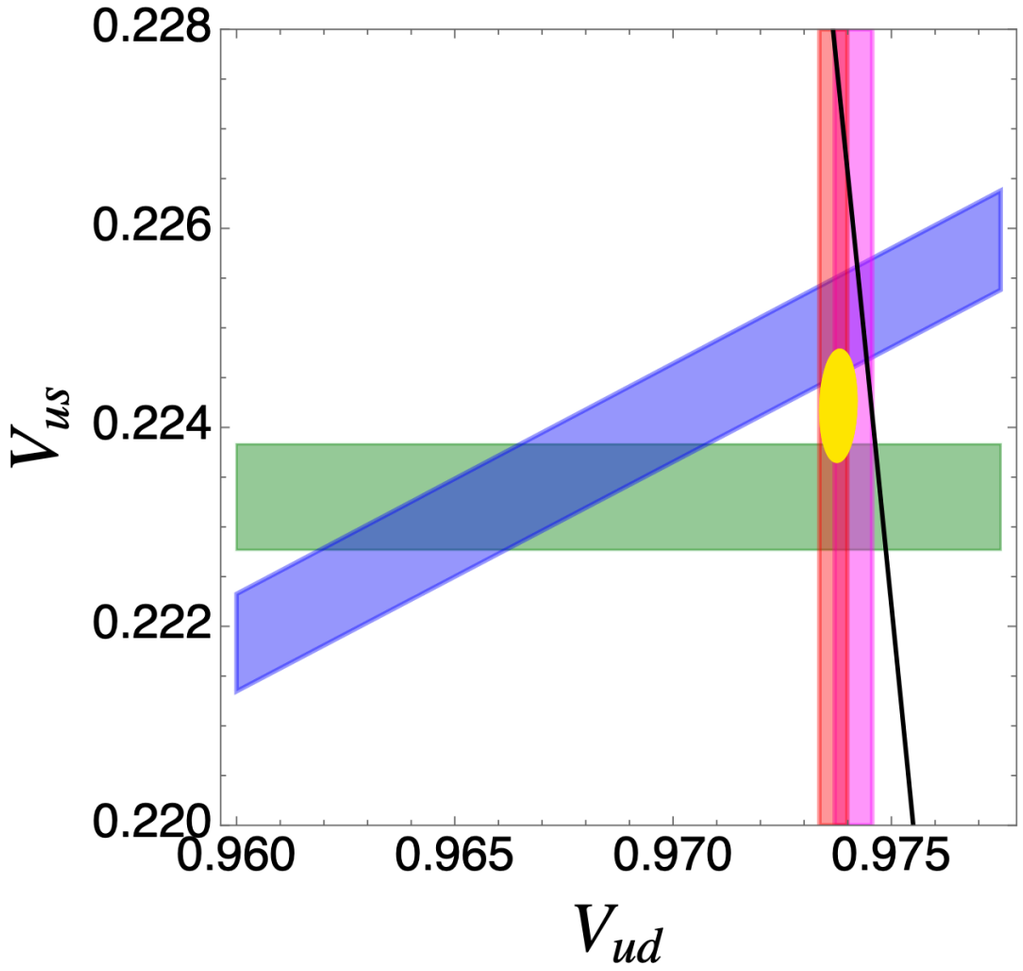
Competitive bounds to complement high energy physics searches

$$|V_{ud}|^2 + |V_{us}|^2 + \cancel{|V_{ub}|^2} = 1$$

Without need for radiative corrections:

$$ft = \frac{K}{2G_F^2 V_{ud}^2}$$

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Cirigliano et al., PLB **838**, 137748

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$$\mathcal{F}t = \frac{K}{2G_F^2 V_{ud}^2 (1 + \Delta_R^V)} = ft(1 + \delta_R')(1 + \boxed{\delta_{\rm NS}} - \delta_C)$$

nuclear structure correction

$^{10}C(0^+) \rightarrow {}^{10}B(0^+)$ beta decays

PHYSICAL REVIEW C **110**, 055502 (2024)

***Ab initio* electroweak corrections to superallowed β decays and their impact on V_{ud}**

Vincenzo Cirigliano,[1] Wouter Dekens,[1] Jordy de Vries,[2,3] Stefano Gandolfi,[4]
Martin Hoferichter,[5] and Emanuele Mereghetti[4]

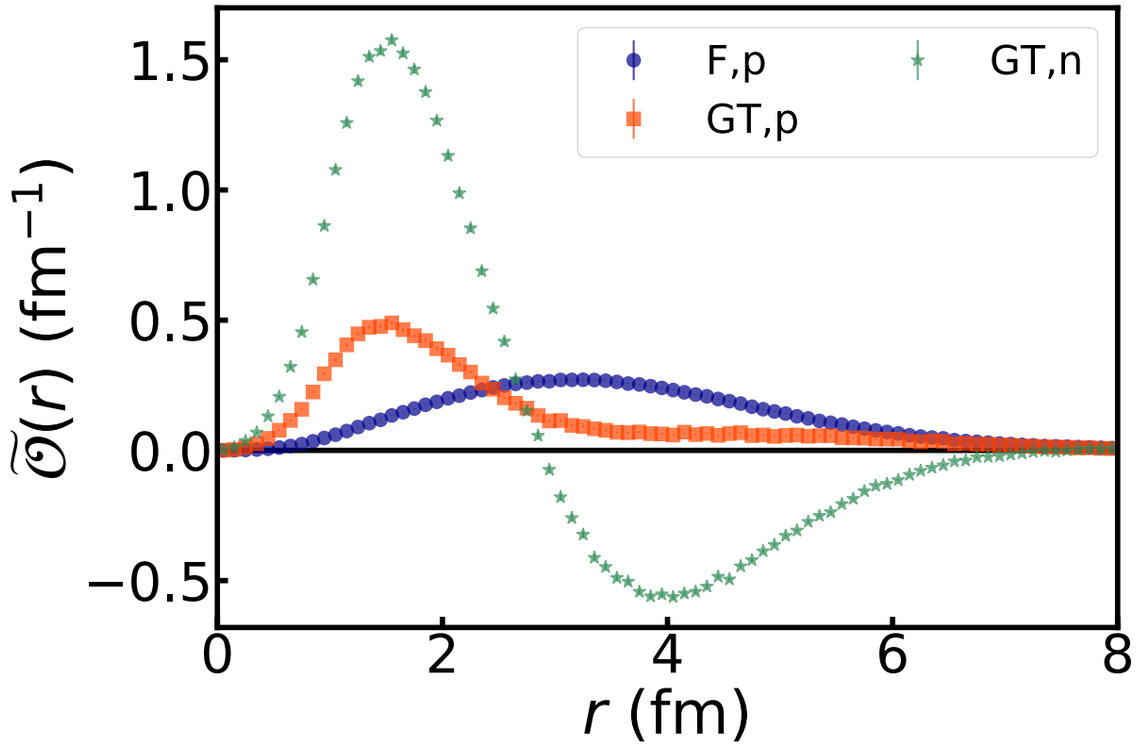
Evaluate densities of matrix elements:

$$M = \int dr\ f(r)\tilde{\mathcal{O}}(r)$$

GFMC: $\delta_{\rm NS} = -4.46(48) \times 10^{-3}$ to $-4.64(77) \times 10^{-3}$

Hardy and Towner: $\delta_{\rm NS} = -4.0(5) \times 10^{-3}$

Gennari et al PRL **134**, 012501: $\delta_{\rm NS} = -4.22(32) \times 10^{-3}$



Figures courtesy of Garrett King
King et al, in preparation

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PHYSICAL REVIEW C **110**, 055502 (2024)

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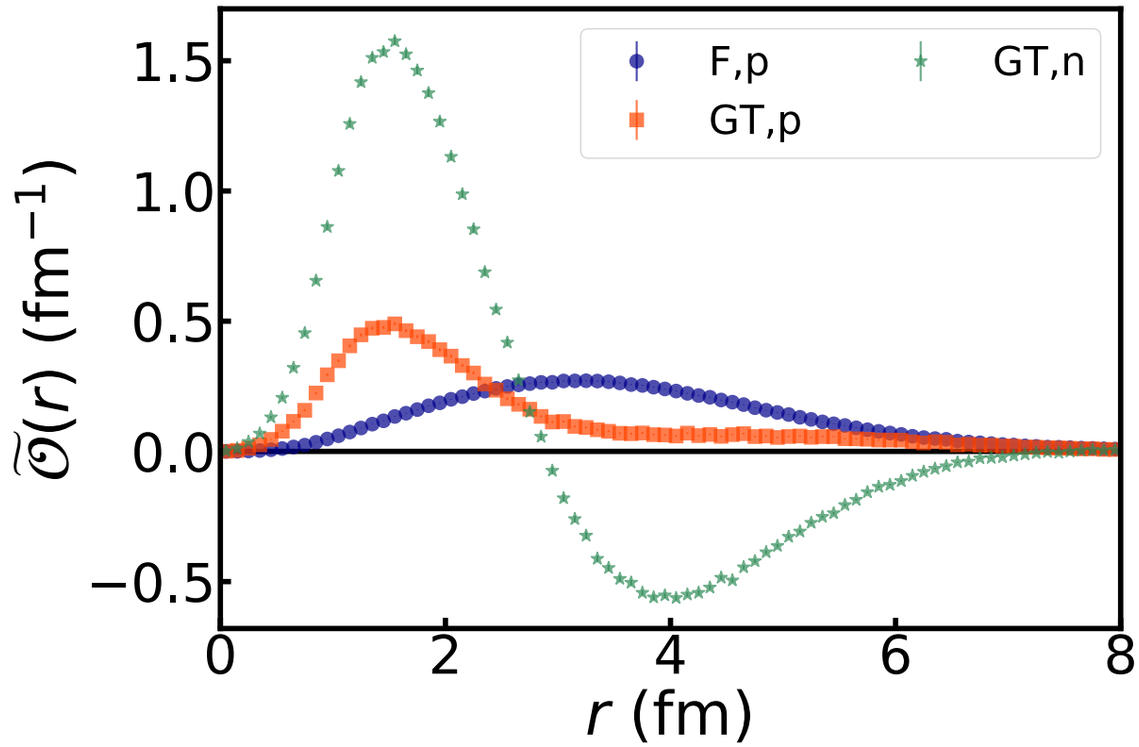
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Takeaway: Unknown LEC bulk of uncertainty
Estimate of uncertainty could be improved with UQ experts
Central values same, not large shift in V_{ud}

Neutrinoless double beta decay

Neutrinoless double beta decay

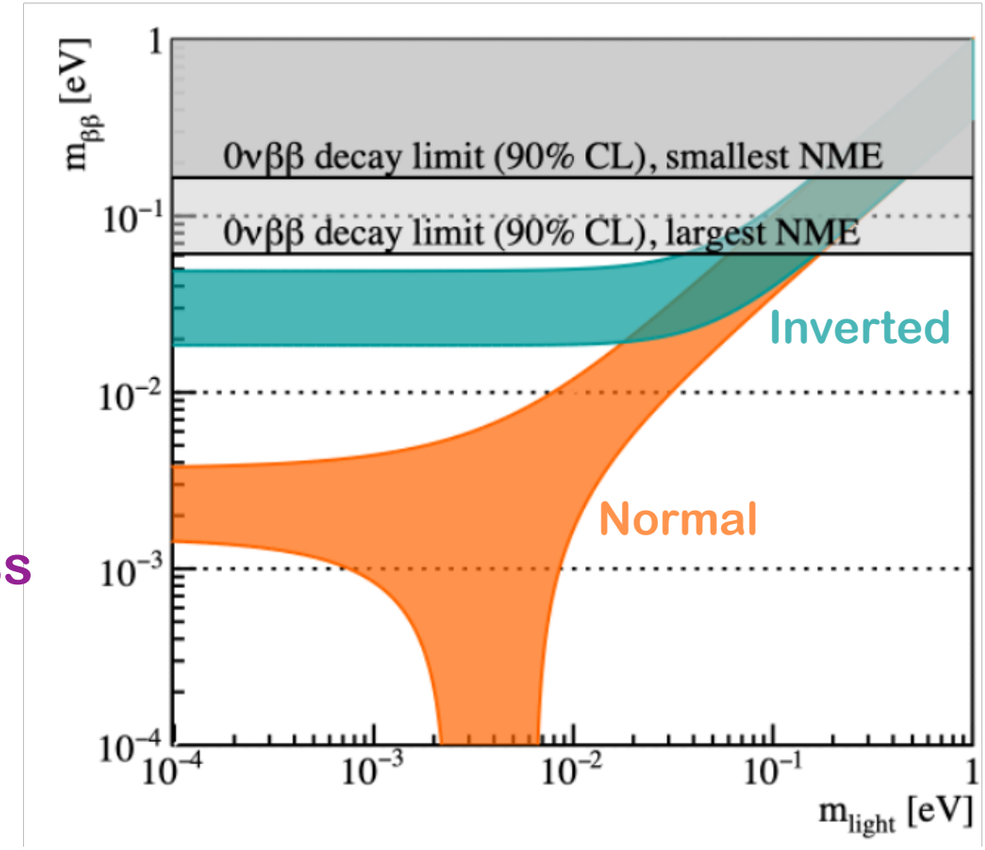
- Searches for $0\nu\beta\beta$ are currently one of the best probes into possible new (and high energy) physics

$$\left[T^{0\nu}_{1/2}\right]^{-1} = G^{0\nu} \left|\mathcal{M}^{0\nu}\right|^2 m^2_{\beta\beta}$$

- In standard mechanism, half-life depends on **nuclear matrix elements (NMEs)** and **effective Majorana mass**

$$\mathcal{M}^{0\nu} = \sum_{\alpha} \langle \Psi_f | \mathcal{O}^{0\nu}_{\alpha} | \Psi_i \rangle \qquad m_{\beta\beta} = \sum_{i} U^2_{ei} m_i$$

- NMEs are sum of **transition operators** acting on pairs, triplets, etc. of nucleons



Agostini et al. Rev. Mod. Phys. **95** (2023)

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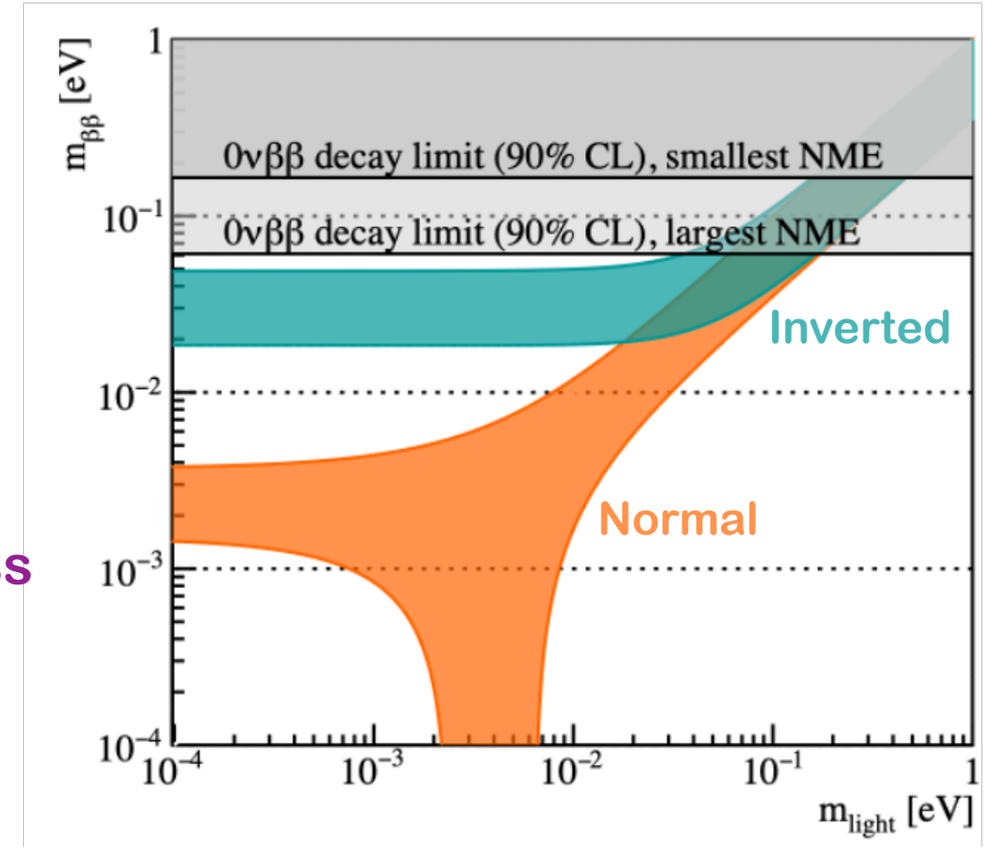
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$$V(\boldsymbol{r}_{ab}) \tau^+_a \tau^+_b \boldsymbol{\sigma}_a \cdot \boldsymbol{\sigma}_b$$

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Agostini et al. Rev. Mod. Phys. **95** (2023)

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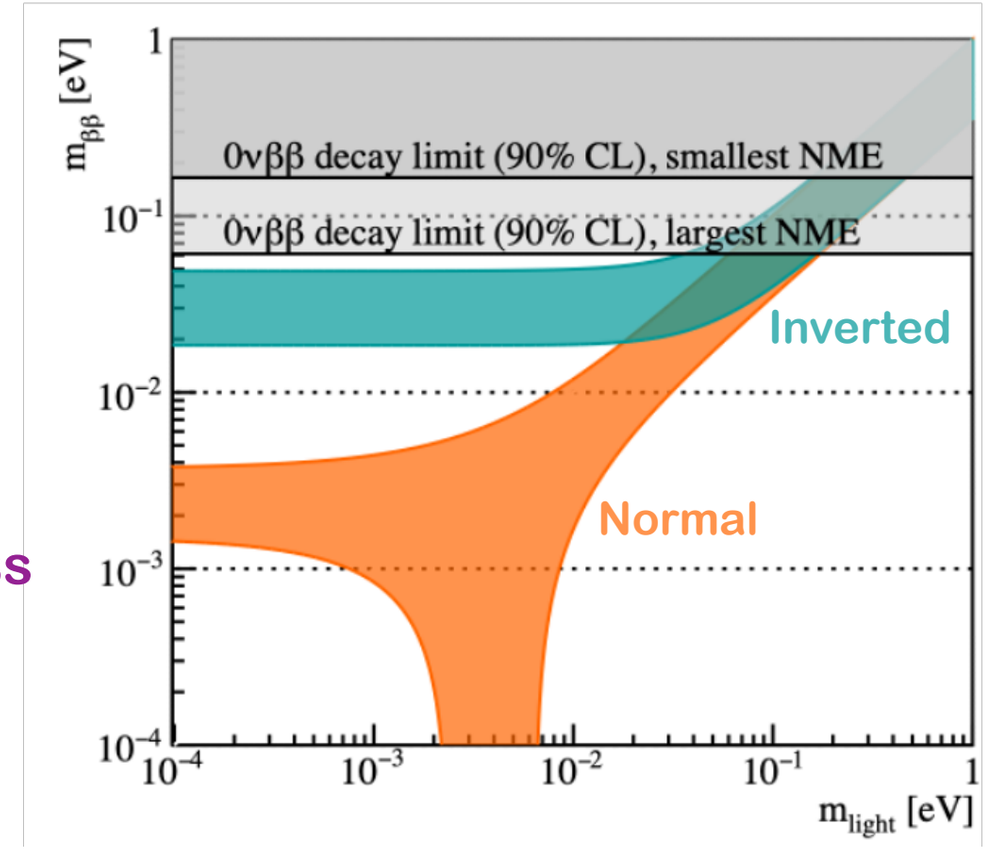
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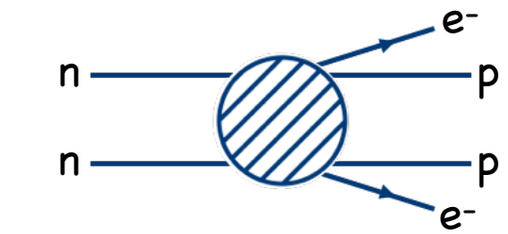
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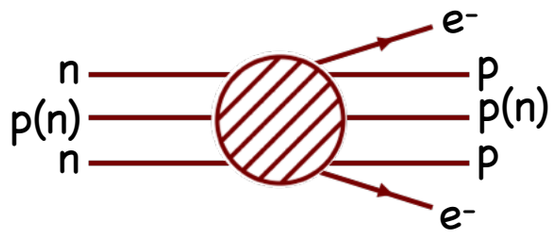
- Typically **only two-nucleon operators included** within Chiral EFT approach

- We want to **estimate the impact of three-nucleon operators** on NMEs



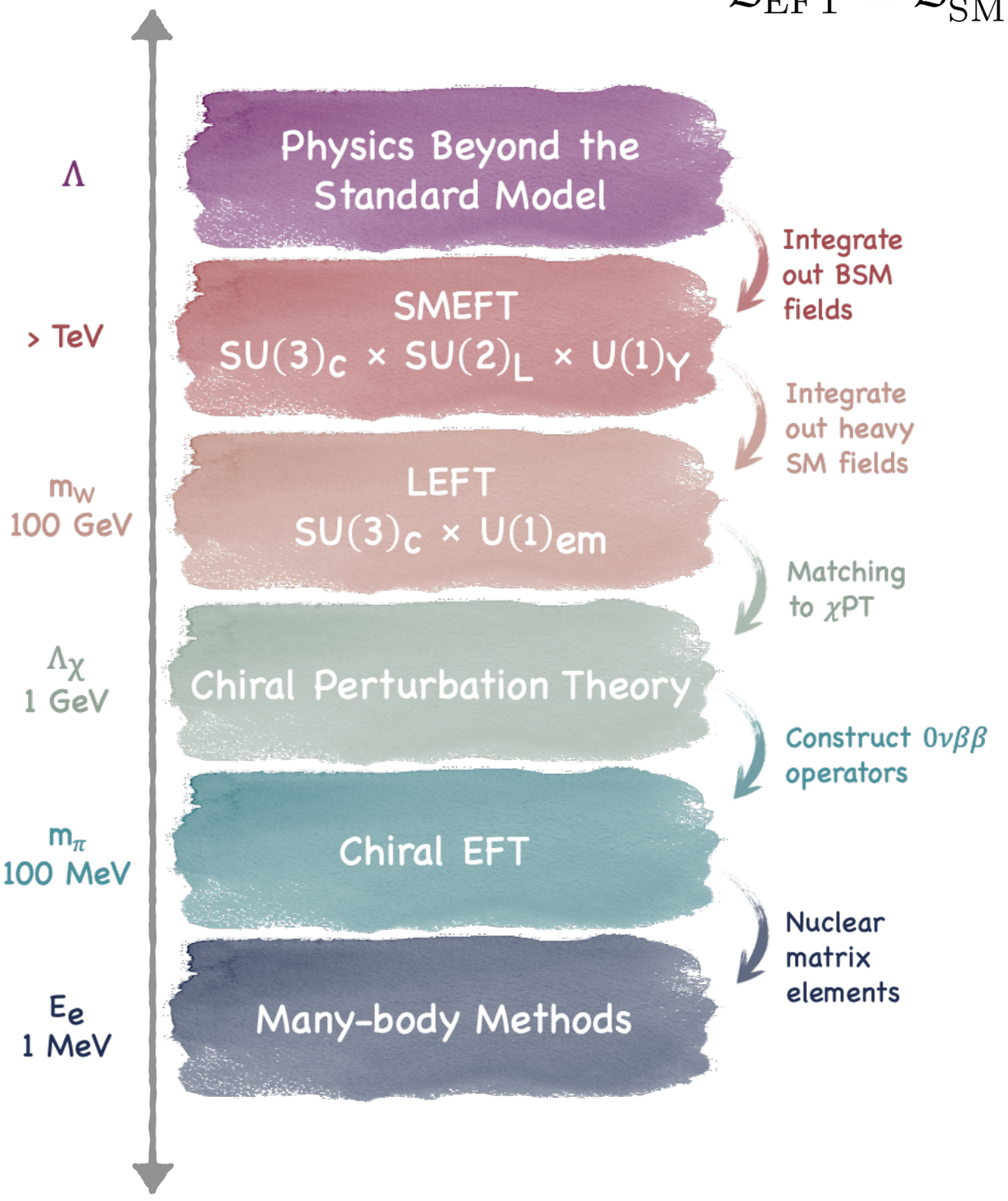
Agostini et al. Rev. Mod. Phys. **95** (2023)





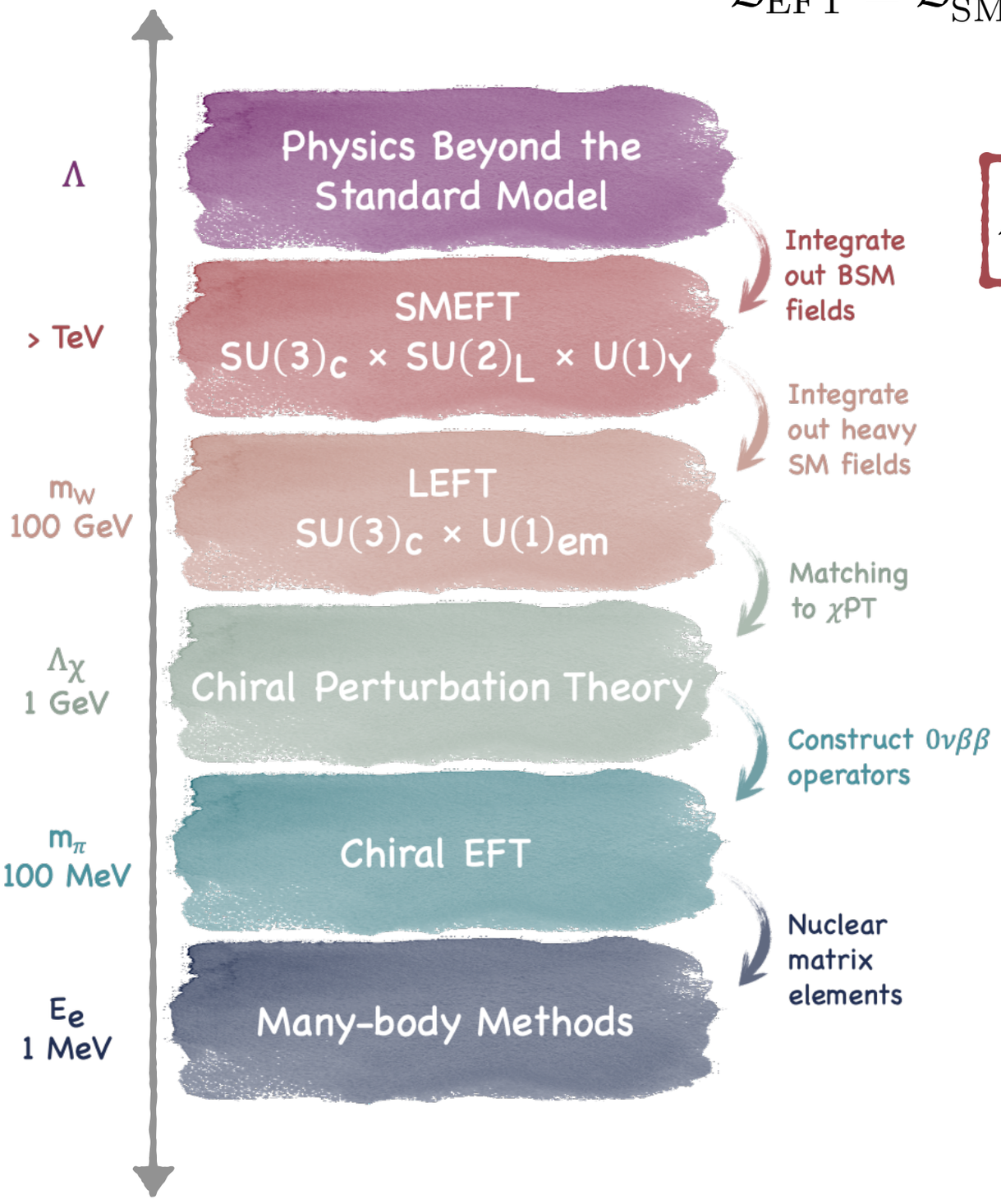
EFT approach from top to bottom

$$\mathcal{L}_{\rm EFT} = \mathcal{L}^{(4)}_{\rm SM} + \frac{C^{(5)}}{\Lambda}\mathcal{O}^{(5)} + \sum_i \frac{C^{(7)}_i}{\Lambda^3}\mathcal{O}^{(7)} + \sum_i \frac{C^{(9)}_i}{\Lambda^5}\mathcal{O}^{(9)}_i + \dots$$



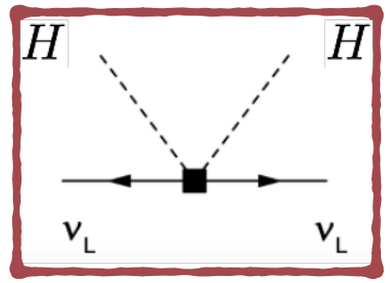
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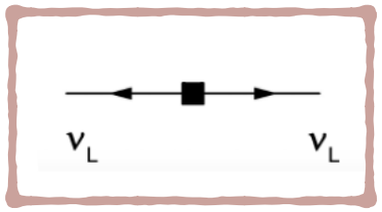
$$\mathcal{L}^{(5)} = \frac{c_5}{\Lambda}\left(L^T C\tilde{H}\right)\left(\tilde{H}^T L\right)$$

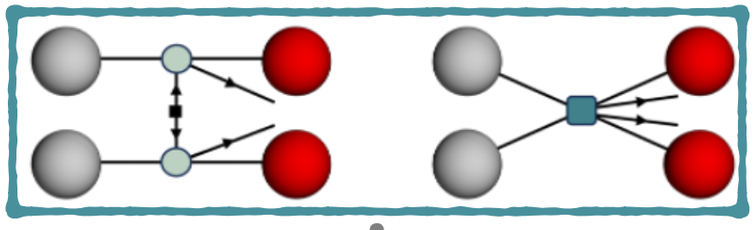
Weinberg dim-5 operator

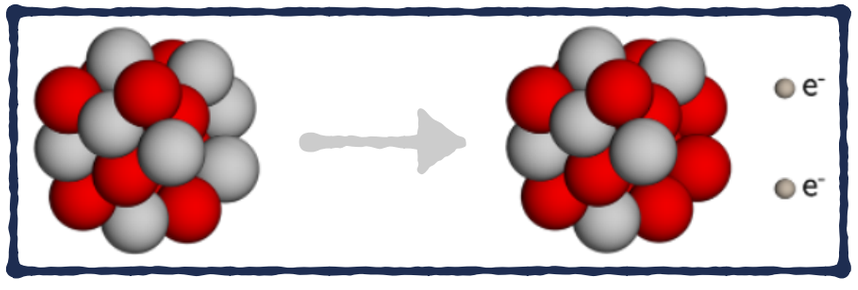


$$\mathcal{L}^{(3)} = c_5\frac{v^2}{\Lambda}\nu^T C\nu$$

dim-3 Majorana mass term



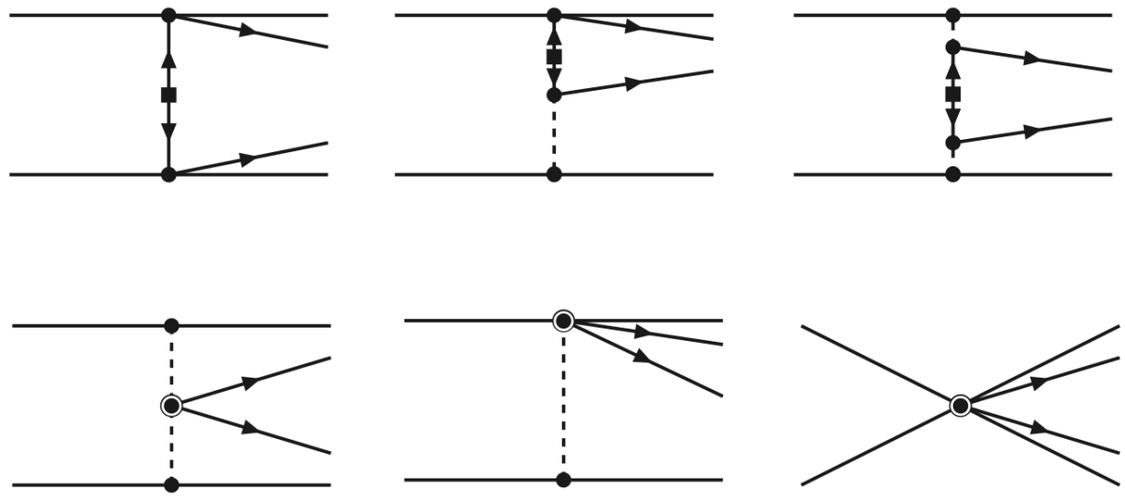




Two-nucleon potentials

LO

$\mathcal{O}(G_F^2 Q^0/\Lambda_\chi^0)$



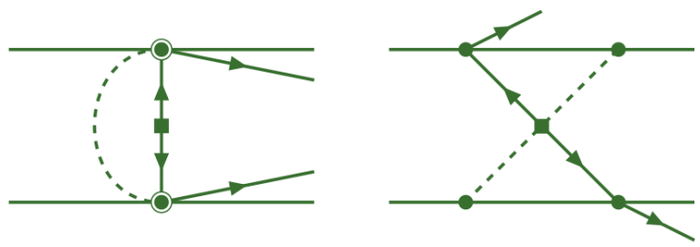
NLO

$\mathcal{O}(G_F^2 Q/\Lambda_\chi)$

✗

N²LO

$\mathcal{O}(G_F^2 Q^2/\Lambda_\chi^2)$

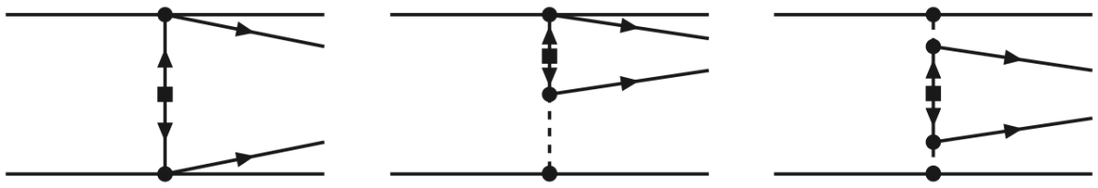


+ ...

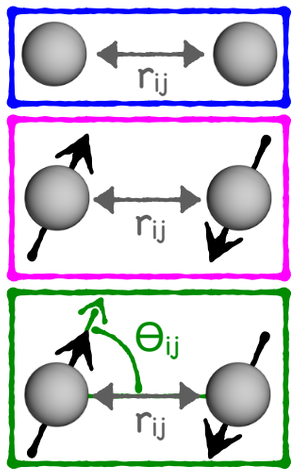
Two-nucleon potentials

LO

$$\mathcal{O}(G_F^2 Q^0/\Lambda_\chi^0)$$



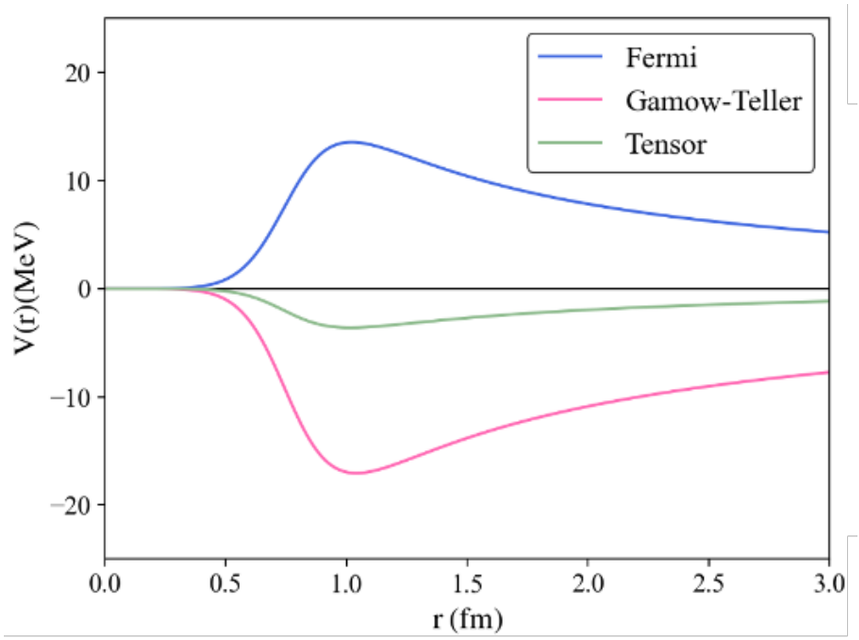
operators



$$\mathcal{O}_F = \sum_{a<b} \tau_a^+ \tau_b^+$$

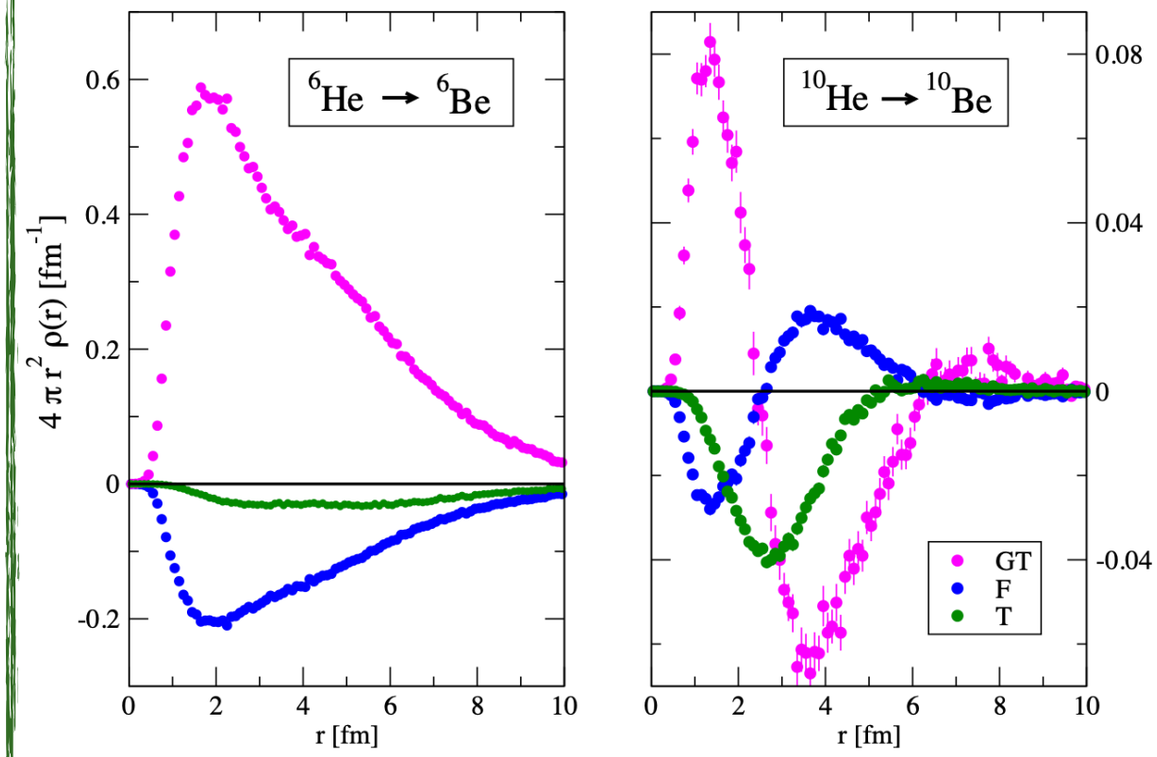
$$\mathcal{O}_{GT} = \sum_{a<b} \tau_a^+ \tau_b^+ \boldsymbol{\sigma}_a \cdot \boldsymbol{\sigma}_b$$

$$\mathcal{O}_T = \sum_{a<b} \tau_a^+ \tau_b^+ S_{ab}(\hat{r})$$



densities

$$4\pi r^2 \rho_i(r) = \langle \Psi_f | \sum_{a<b} \mathcal{O}_i \delta(r_{ab} - r) | \Psi_i \rangle$$



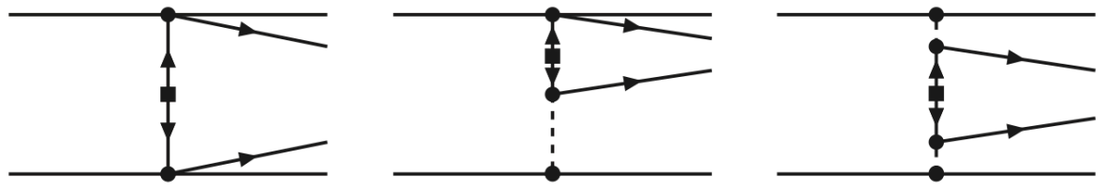
$$M_i = 4\pi R_A \int dr V_i(r)(4\pi r^2 \rho_i(r))$$

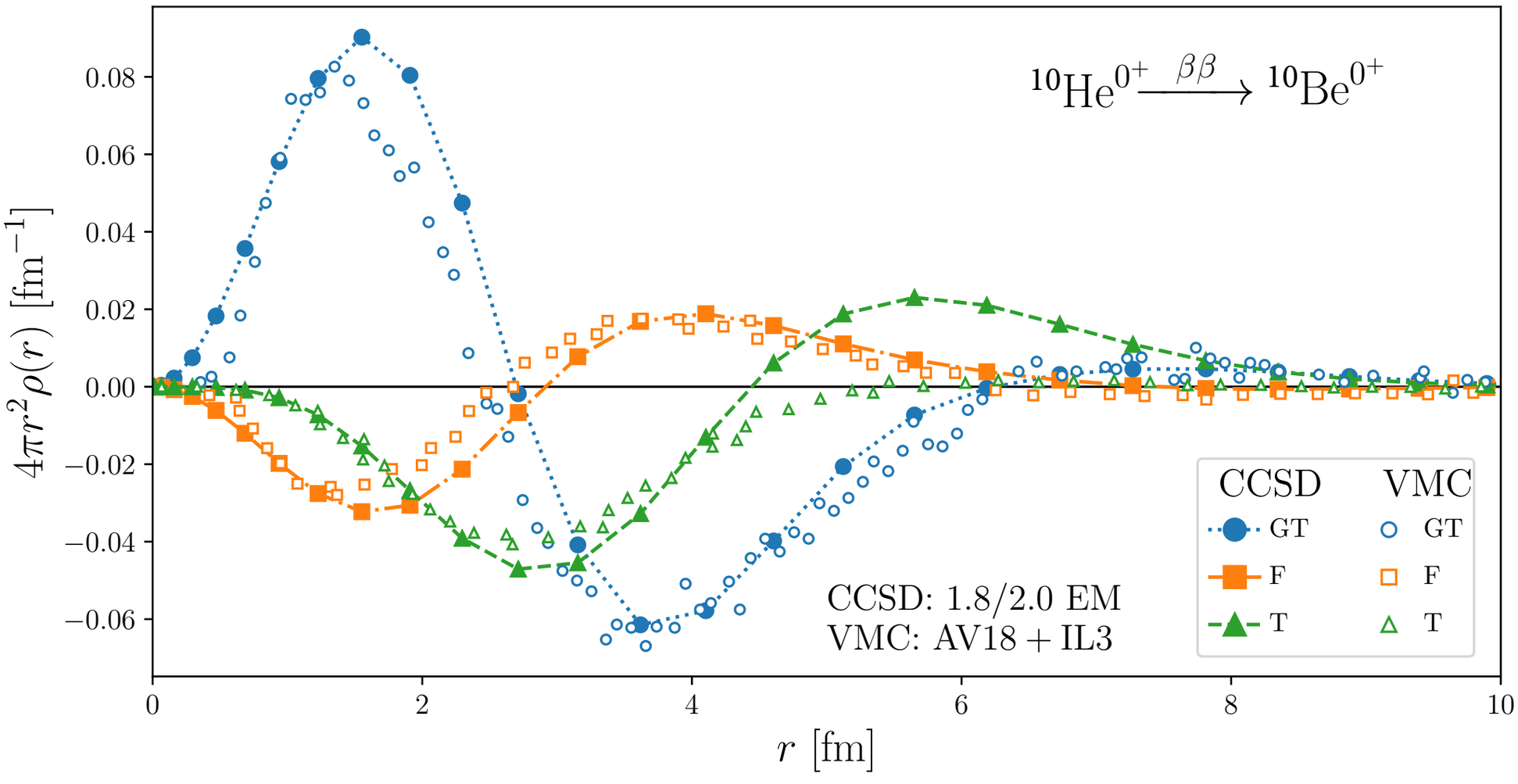
Cirigliano et al. PRL **120** (2018) Cirigliano et al. PRC **97** (2018) Pastore et al. PRC **97** (2018)

Benchmark calculations

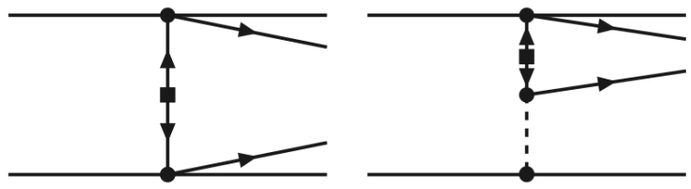
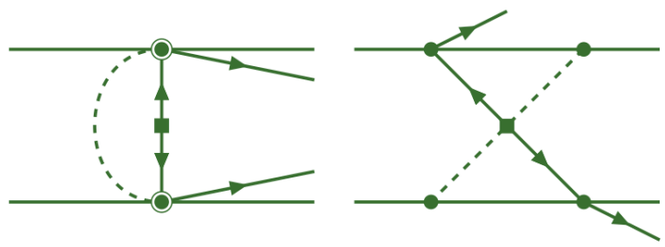
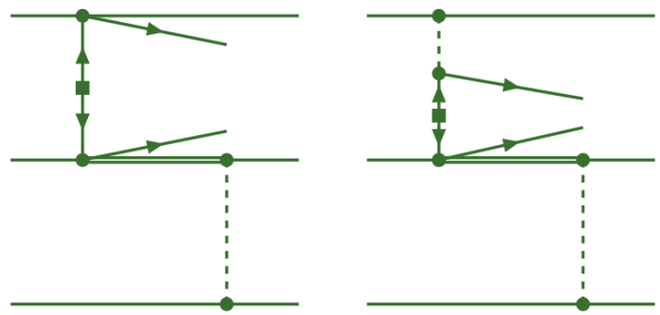
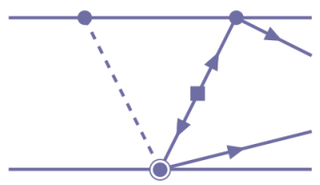
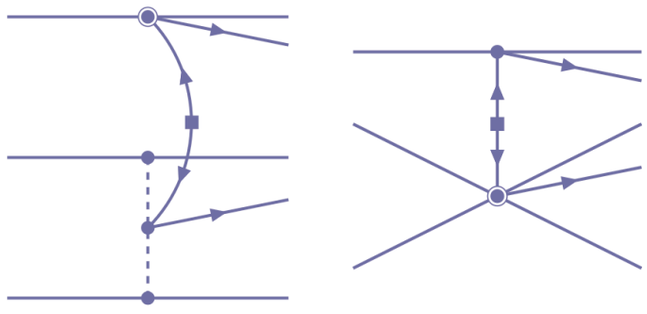
LO

$$\mathcal{O}(G_F^2 Q^0/\Lambda_\chi^0)$$





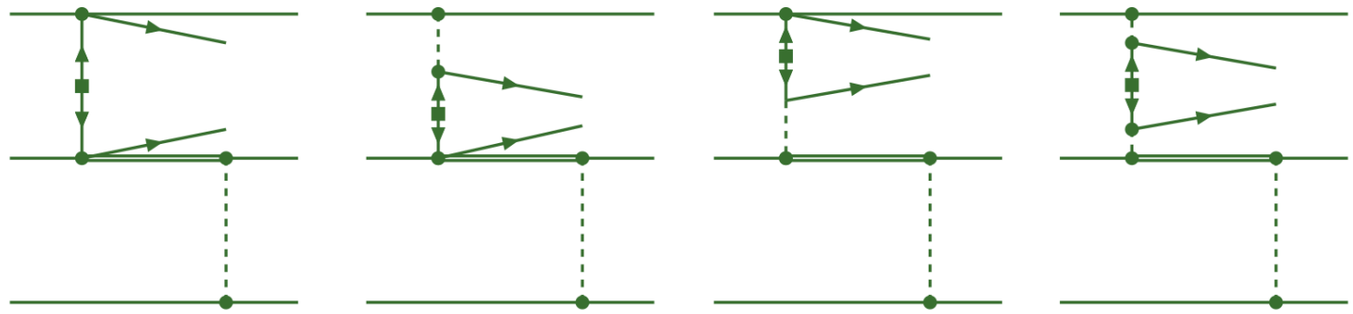
See Sam Novario's talk next week Wed @ 9:45 am

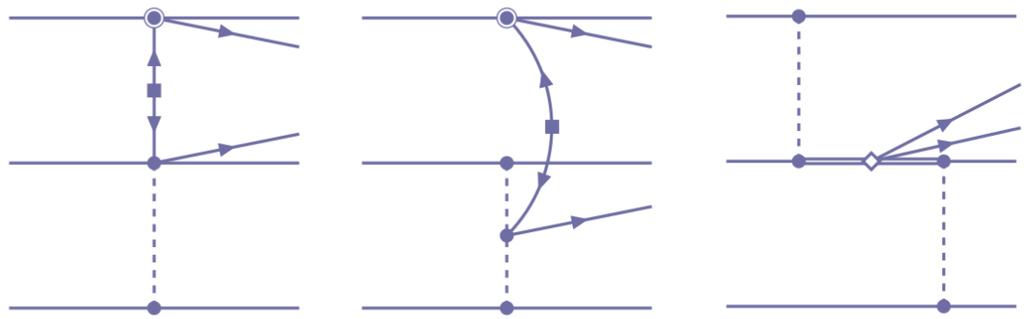
	2N	3N
LO $\mathcal{O}(G_F^2 Q^0/\Lambda_\chi^0)$	 + ...	✗
NLO $\mathcal{O}(G_F^2 Q/\Lambda_\chi)$	✗	✗
N²LO $\mathcal{O}(G_F^2 Q^2/\Lambda_\chi^2)$	 + ...	 + ...
N³LO $\mathcal{O}(G_F^3 Q^2/\Lambda_\chi^3)$	 + ...	 + ...

Three-nucleon potentials

N²LO

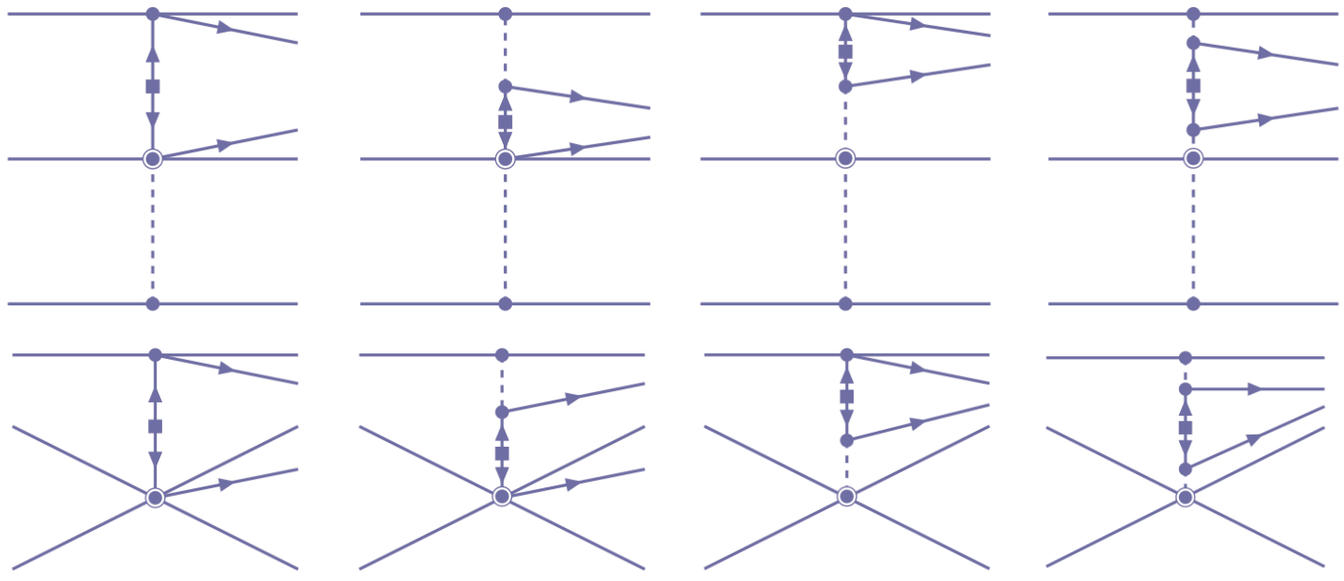
$\mathcal{O}(G_F^2 Q^2/\Lambda_\chi^2)$





N³LO

$\mathcal{O}(G_F^3 Q^2/\Lambda_\chi^3)$



Three-nucleon potentials structure

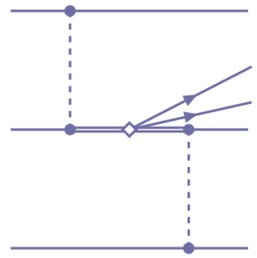
- Additional spin and isospin structure

$$\{(\boldsymbol{\sigma}_j \cdot \hat{\boldsymbol{r}}_{ij})(\boldsymbol{\sigma}_k \cdot \hat{\boldsymbol{r}}_{ij}), (\boldsymbol{\sigma}_j \cdot \hat{\boldsymbol{r}}_{ij})(\boldsymbol{\sigma}_k \cdot \hat{\boldsymbol{r}}_{ik})(\hat{\boldsymbol{r}}_{ij} \cdot \hat{\boldsymbol{r}}_{ik}), \ldots\} \otimes \tau_i^+ \tau_j^+$$

$$\{\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \times \boldsymbol{\sigma}_k, (\boldsymbol{\sigma}_j \cdot \hat{\boldsymbol{r}}_{ij})(\boldsymbol{\sigma}_k \cdot \boldsymbol{\sigma}_i \times \hat{\boldsymbol{r}}_{ij}), \ldots\} \otimes \tau_i^+ [\boldsymbol{\tau}_j \times \boldsymbol{\tau}_k]^+$$

- Additional LECs (**known** and **unknown**) $h_A, \kappa_1, c_1, c_3, c_4, c_D$
- Momentum- and coordinate-space potentials

$f_4^\Delta + f_5^\Delta$



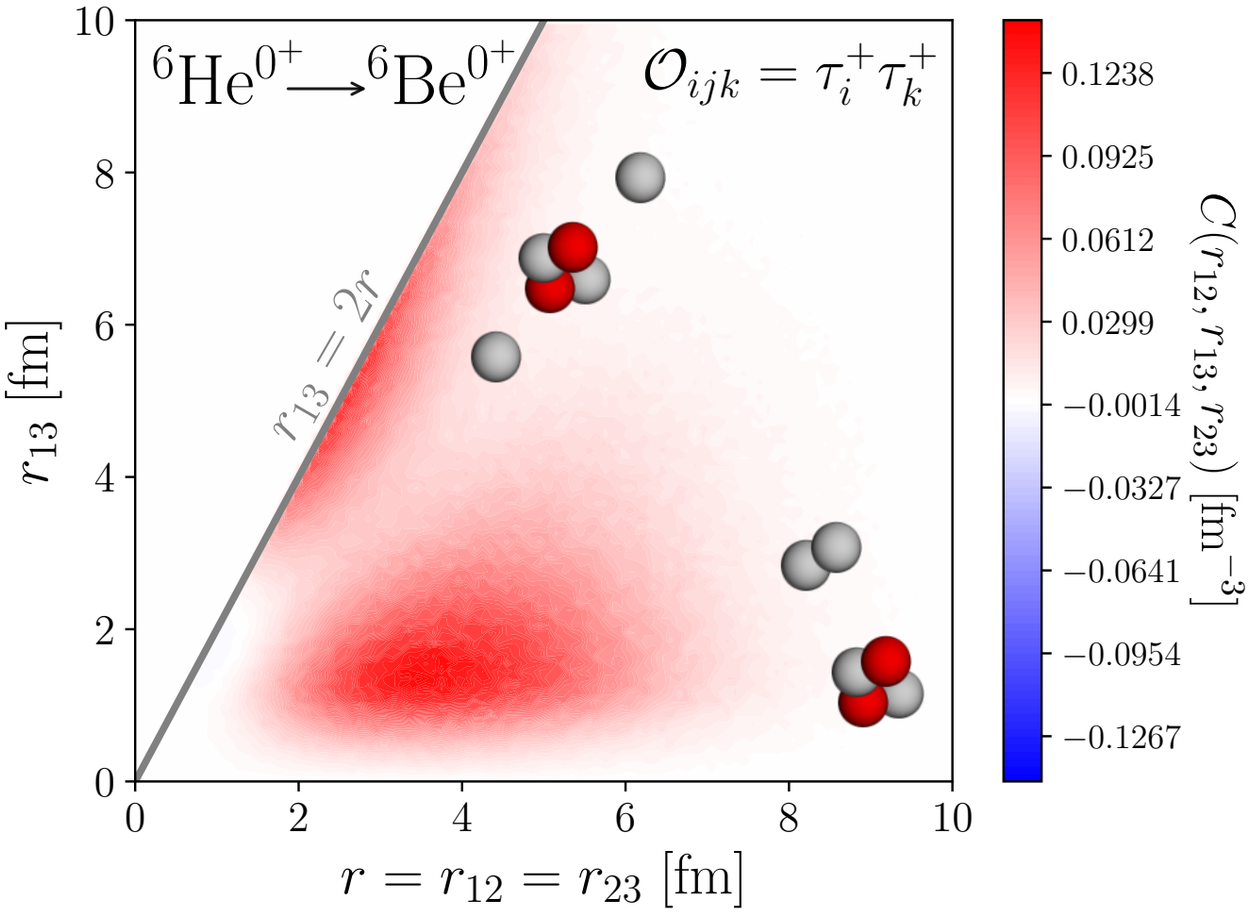
$$m_{\Delta^{++}} - m_{\Delta^+} - m_{\Delta^0} + m_{\Delta^-} =$$

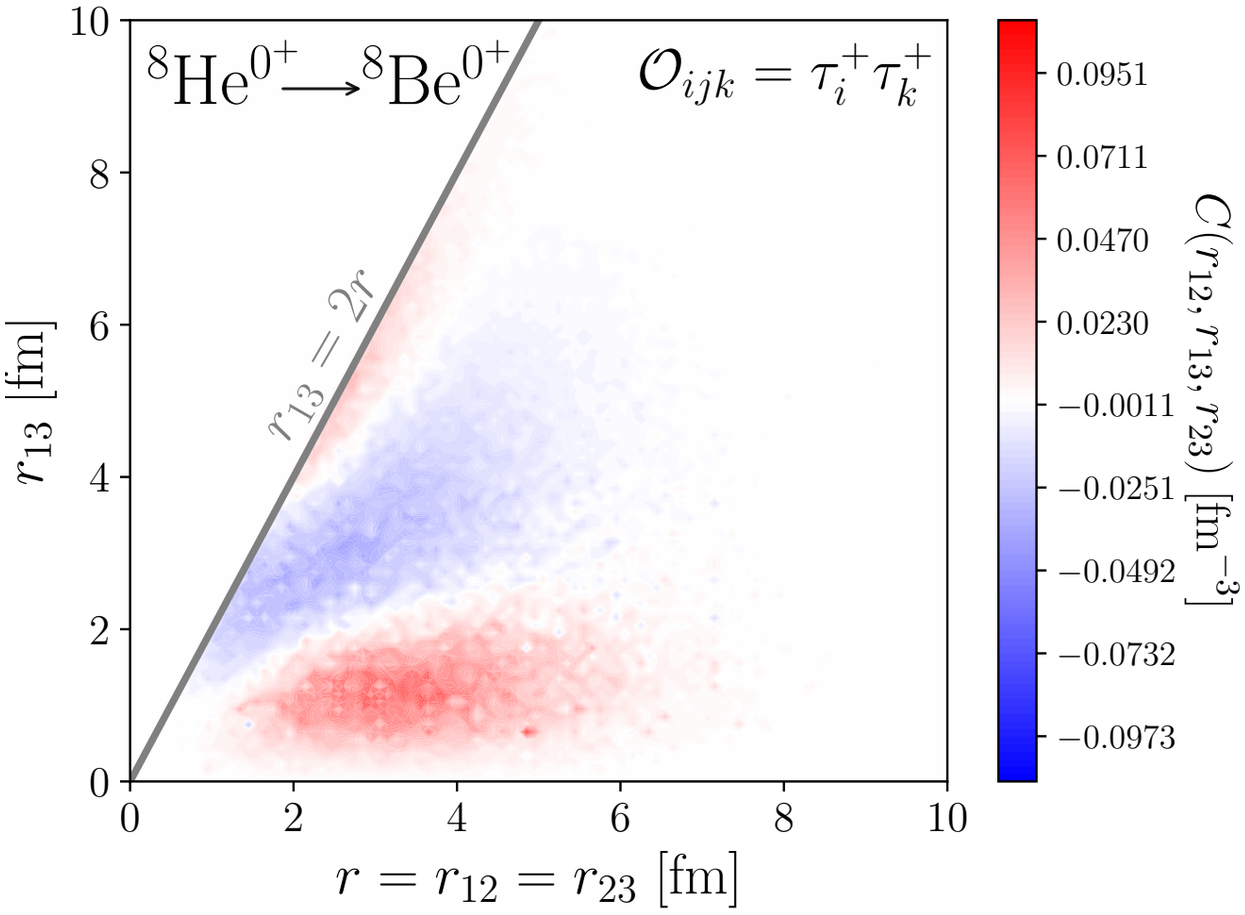
$$\delta m_\Delta^{(2)} = -\frac{4}{3} e^2 F_\pi^2 f_4^\Delta$$

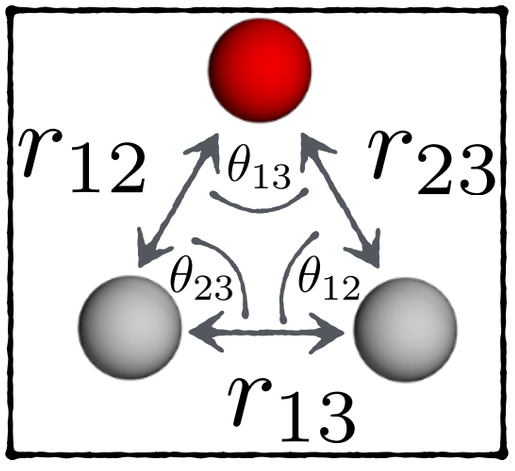
$$\delta m_\Delta^{(2)} = -1.7 \pm 2.7 \text{ MeV}$$

E. Epelbaum et al. Nuc. Phys. A. **806** (2008)

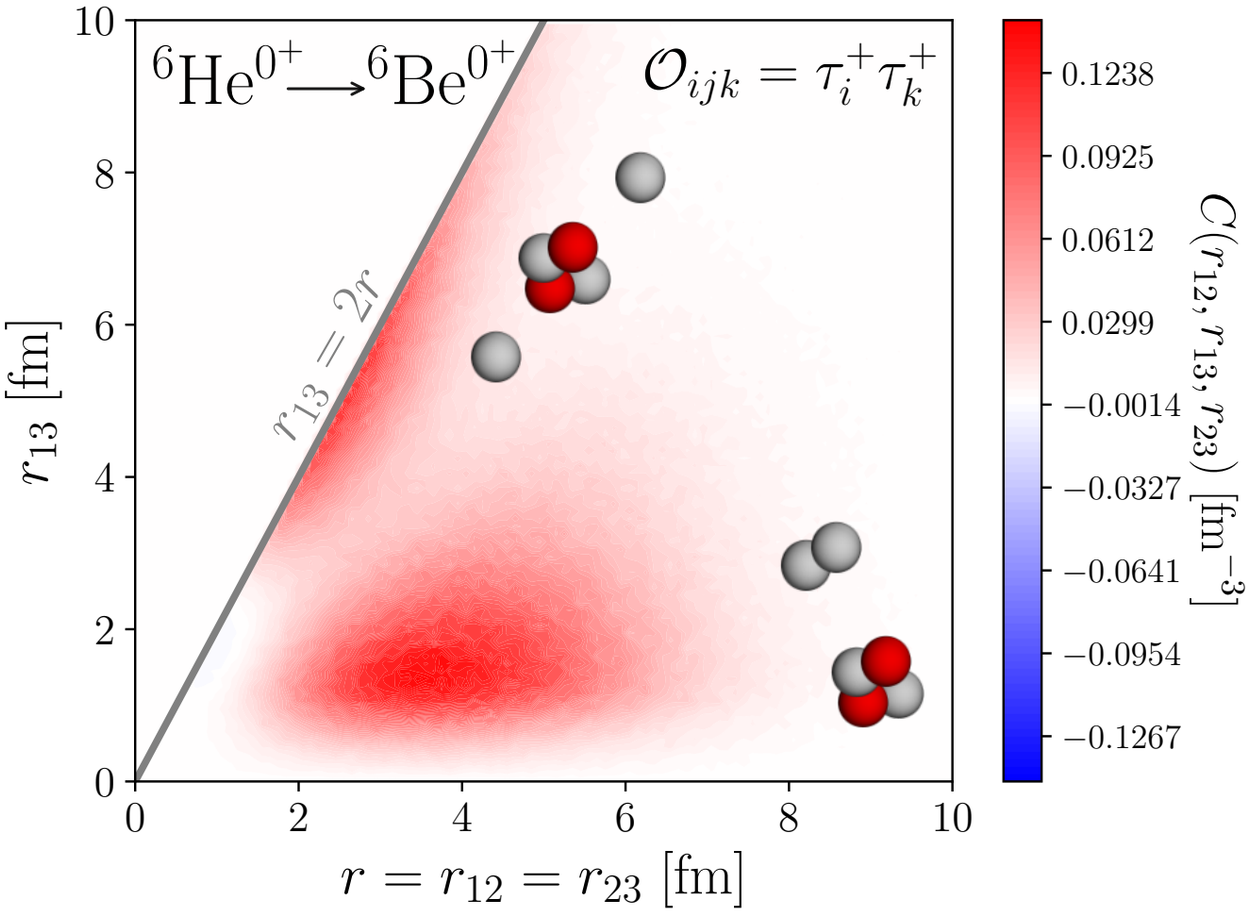
Isospin densities

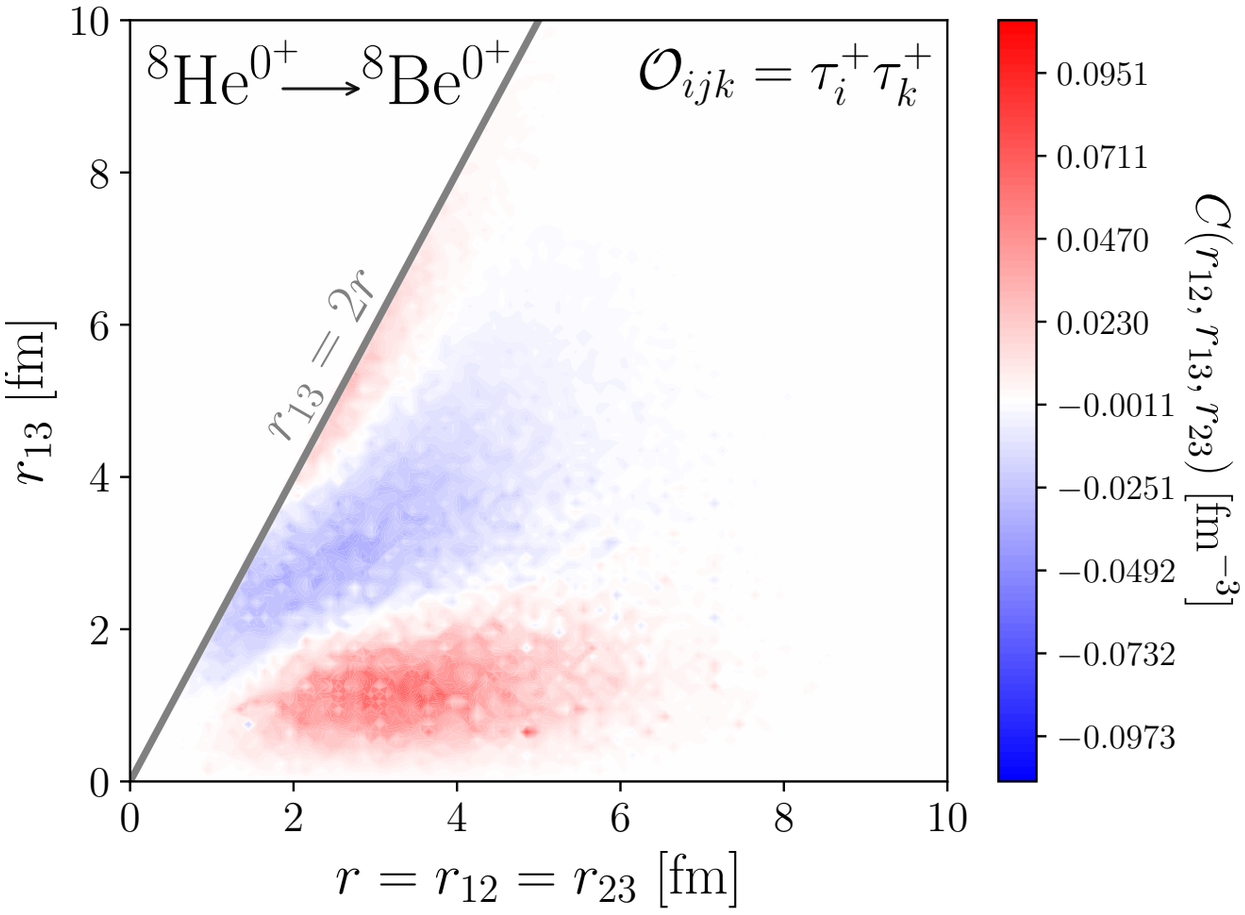


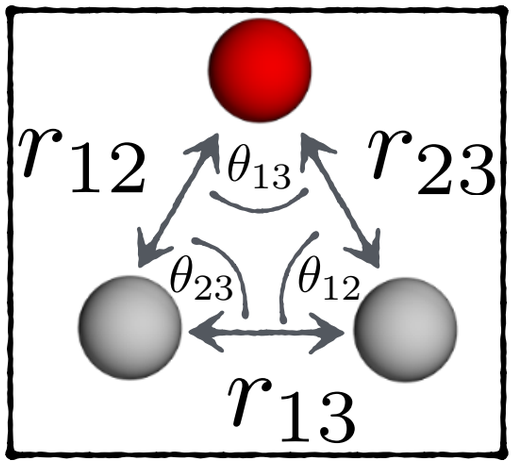




Isospin densities



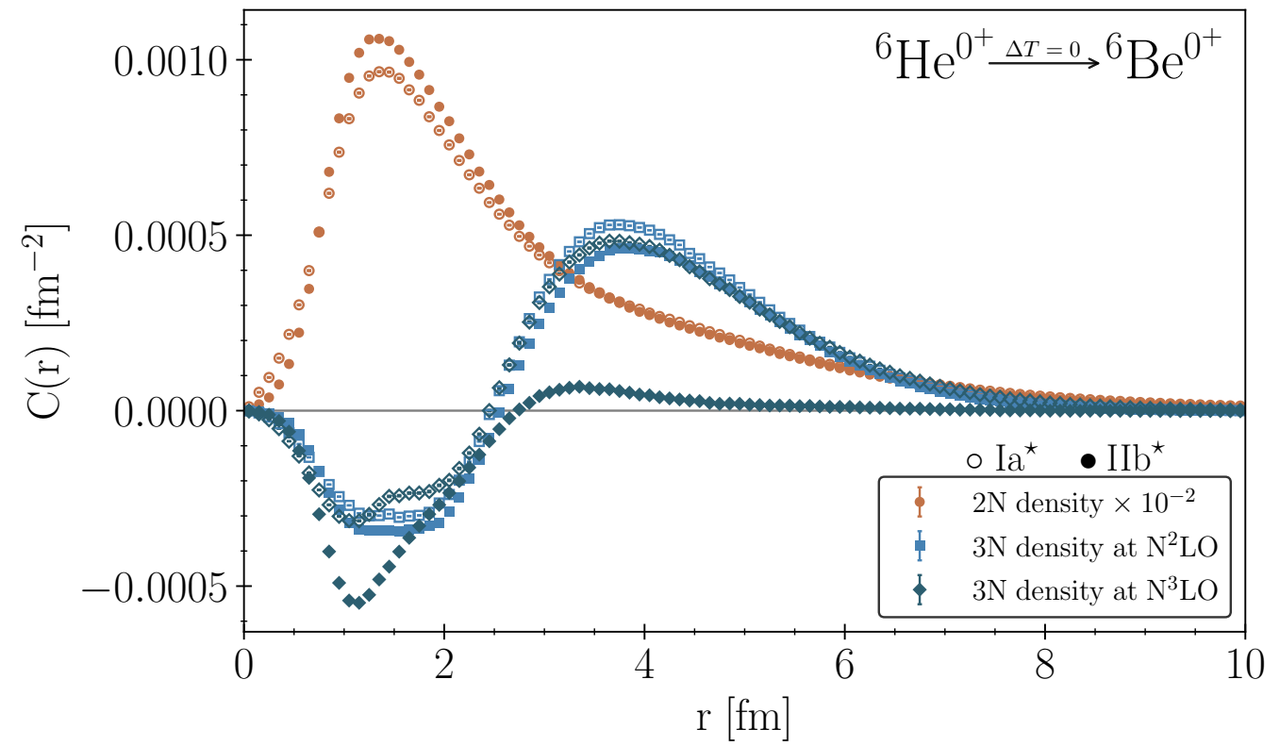


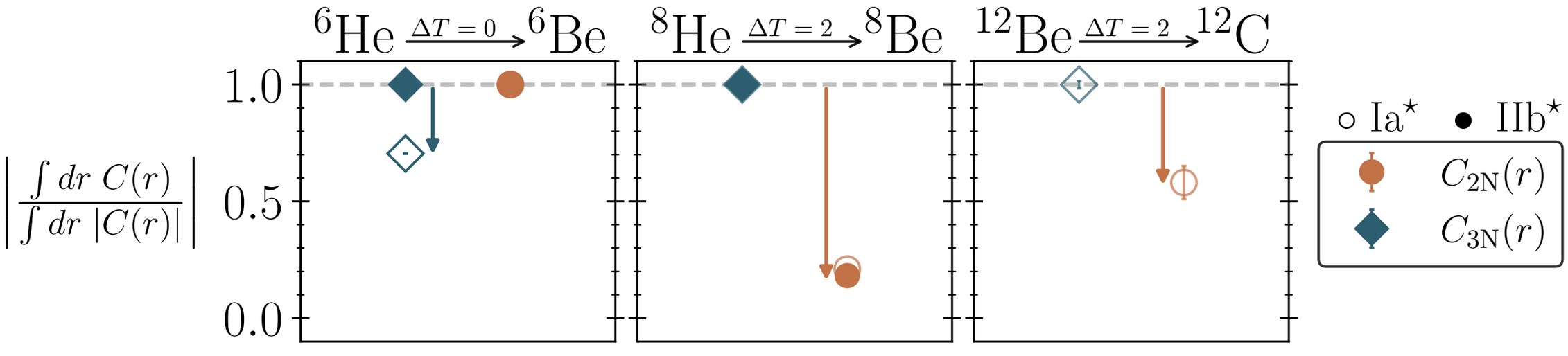


$$C(r_{13}) = \int dr_{12} \int dr_{23}\, C(r_{12}, r_{13}, r_{23})$$

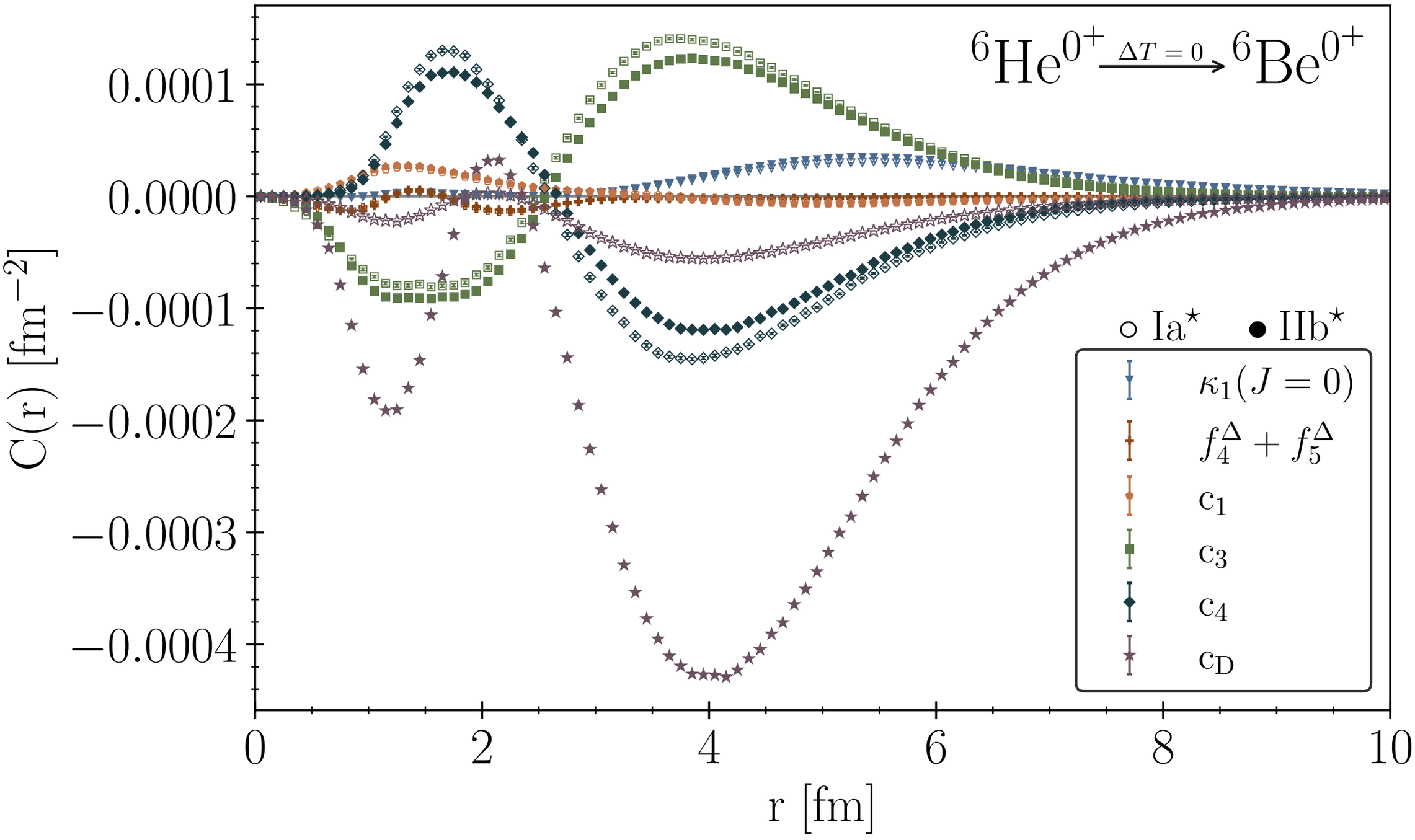
$$M_{3N} = \int dr_{13}\, C(r_{13})$$

2N and 3N densities

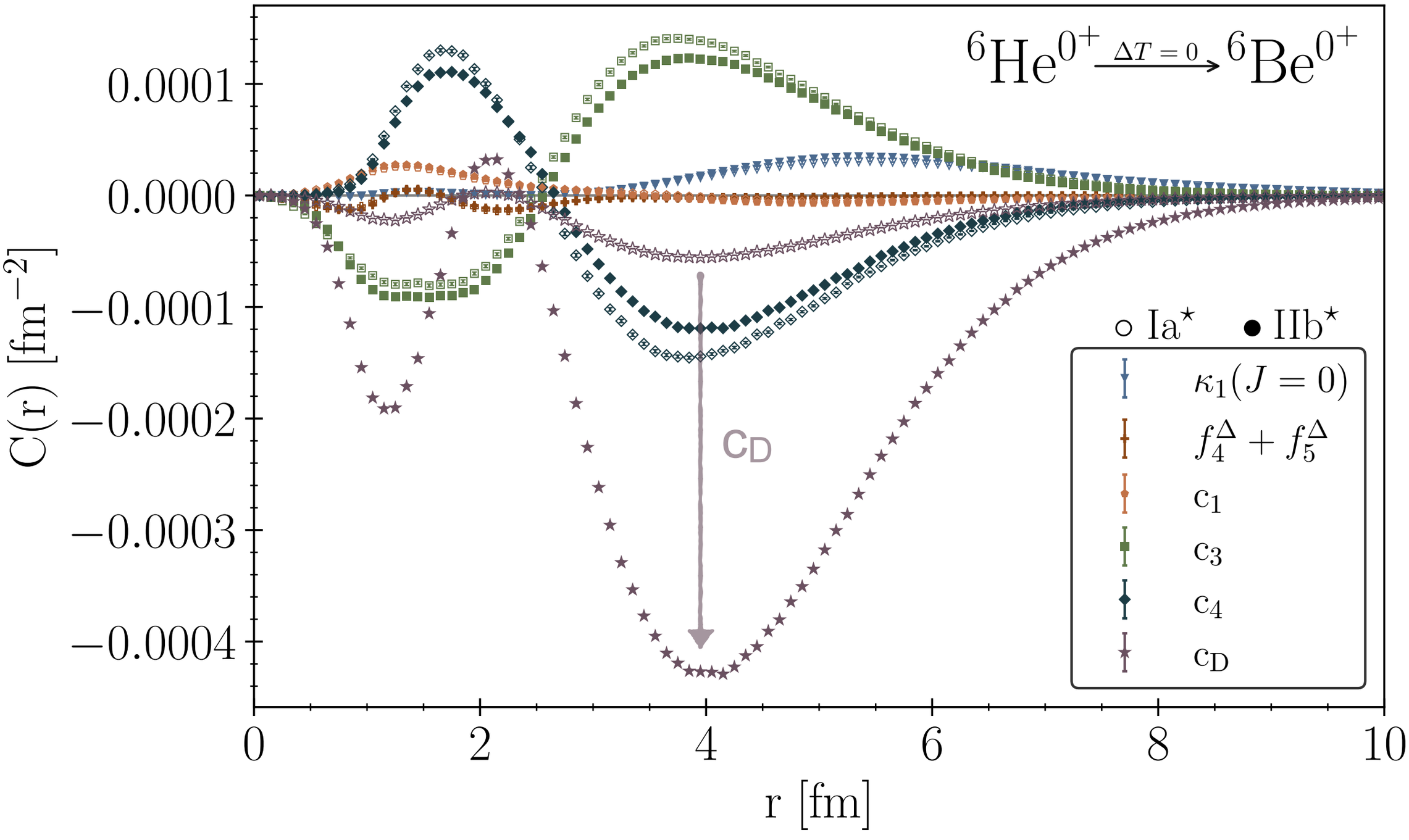




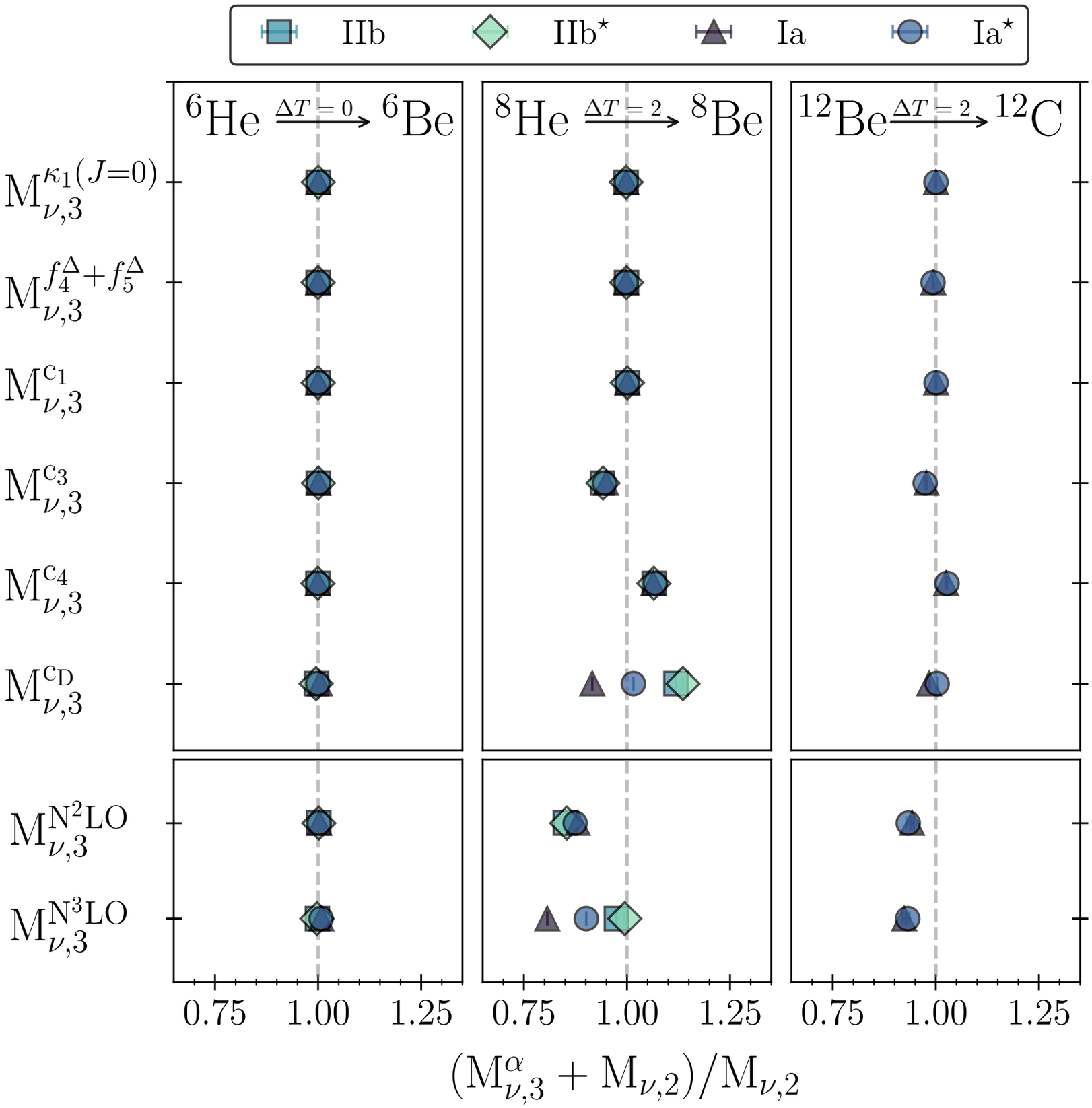
N³LO 3N densities for ⁶He → ⁶Be



N³LO 3N densities for ^{6}He → ^{6}Be

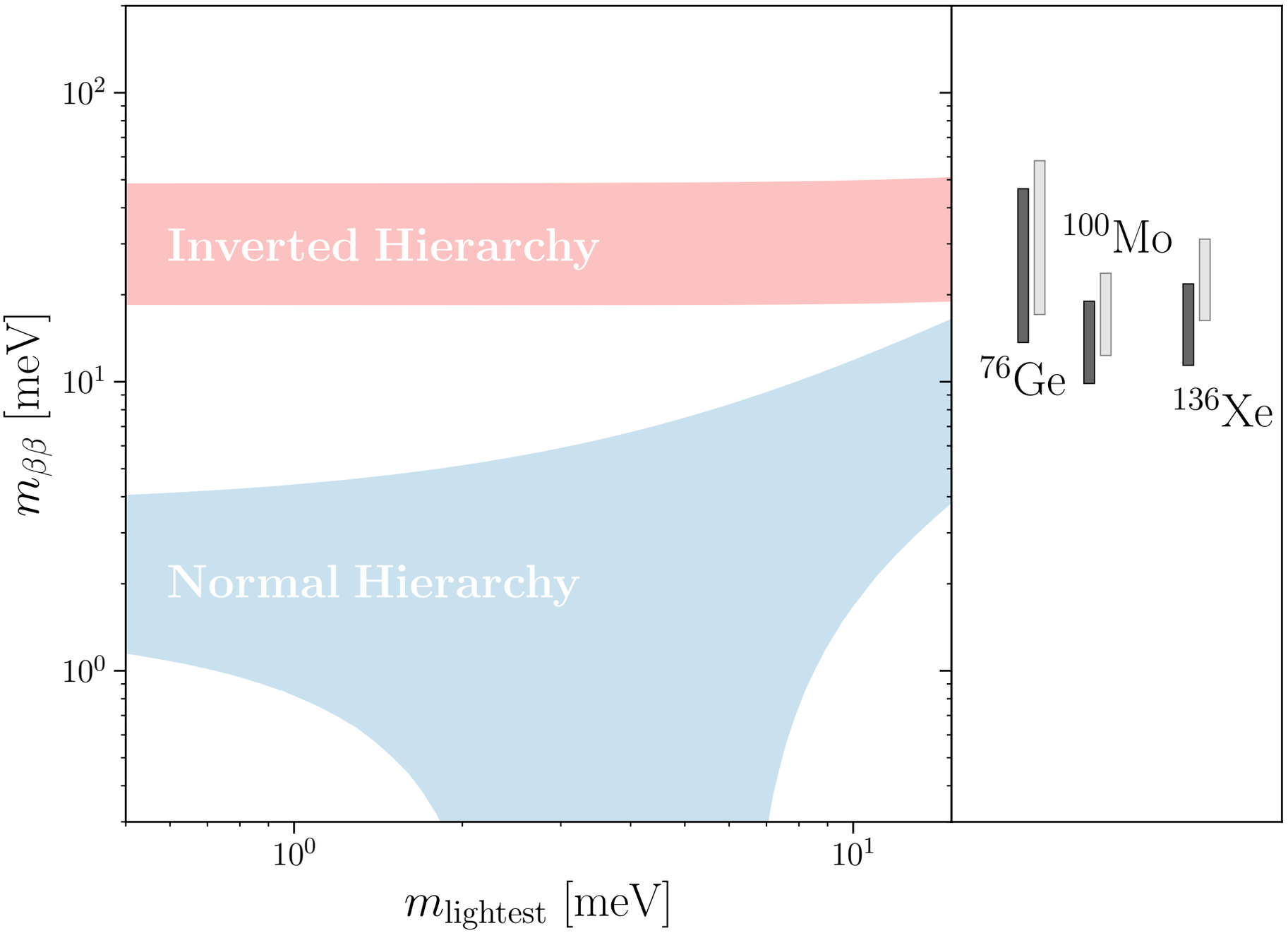


VMC three-nucleon matrix elements



	Ia (Ia*)	IIb (IIb*)
c_D	3.666 (–0.635)	–4.480 (–5.25)
c_E	–1.638 (–0.090)	–0.412 (0.05)
z_0	0.090 (1.035)	2.806 (3.059)

Physics implications: naive extrapolation



IMSRG NME

Belley et al. PRL **132**, 182502

Belley et al. arXiv.2307.15156

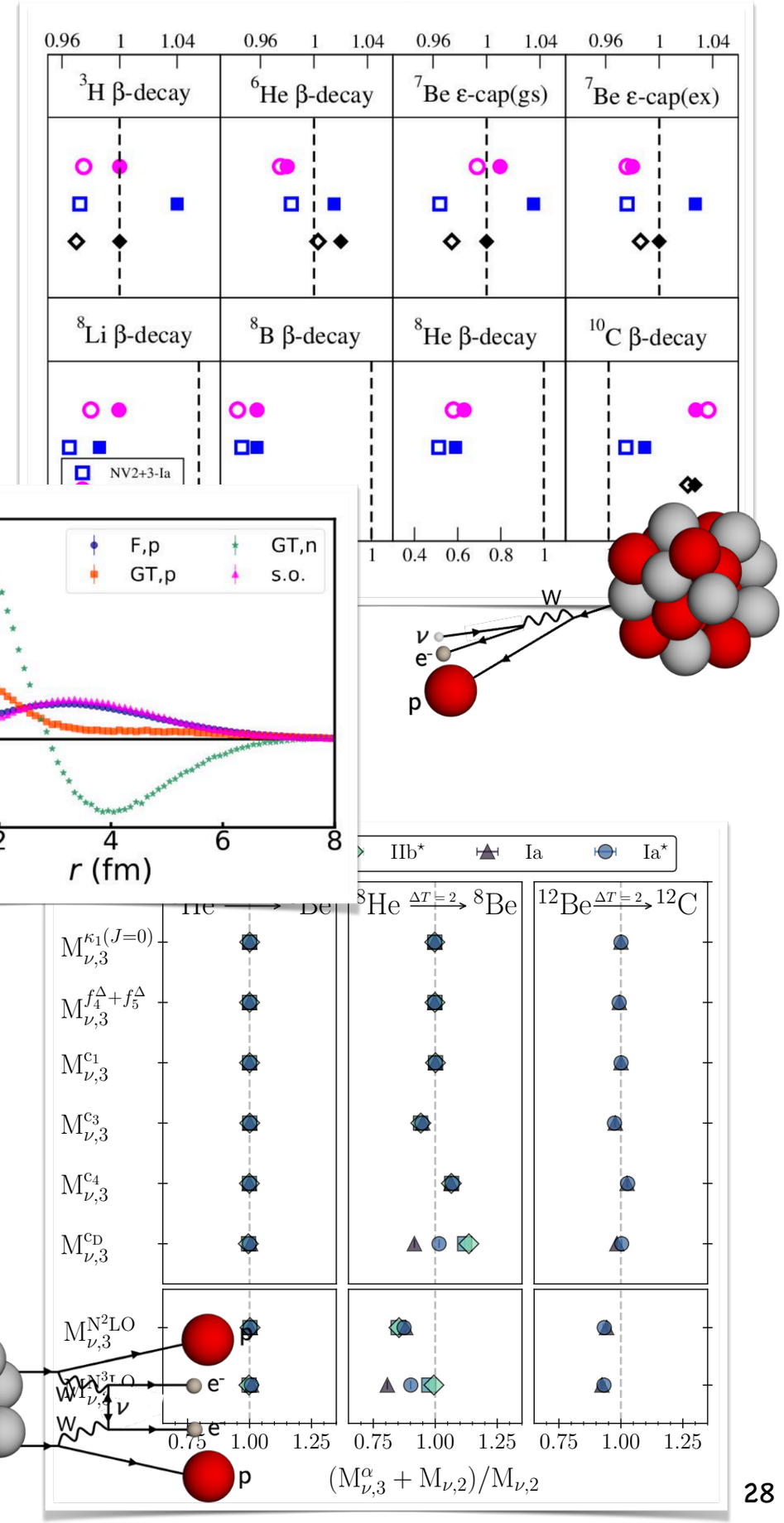
SM NME

Coraggio et al. PRC **105**, 034312

$T^{0\nu}_{1/2} \sim 10^{28}$ yr

Conclusions

- Many-body method and chiral EFT is useful tool for studying beta decays
- Uncertainties in LECs plague uncertainties in predictions
- Single and double beta decay useful in new physics search
- Importance of 3N operators in neutrinoless double beta decay calculations
- In future, higher order corrections to superallowed (3N) and neutrinoless (2N at N3LO) decays



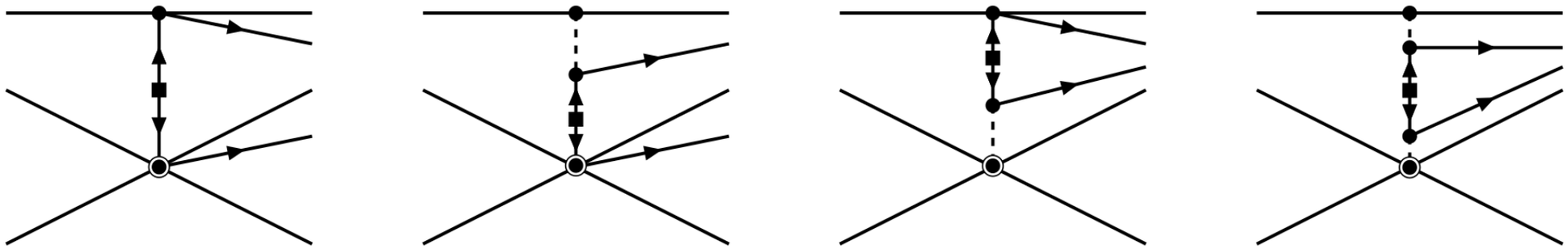
Thank you!

WUSTL: Pastore, Piarulli, Novario, Weiss
LANL: King, Mereghetti, Carlson, Gandolfi
ANL: Wiringa
JLab/ODU: Andreoli, Gnech, Schiavilla

Backup

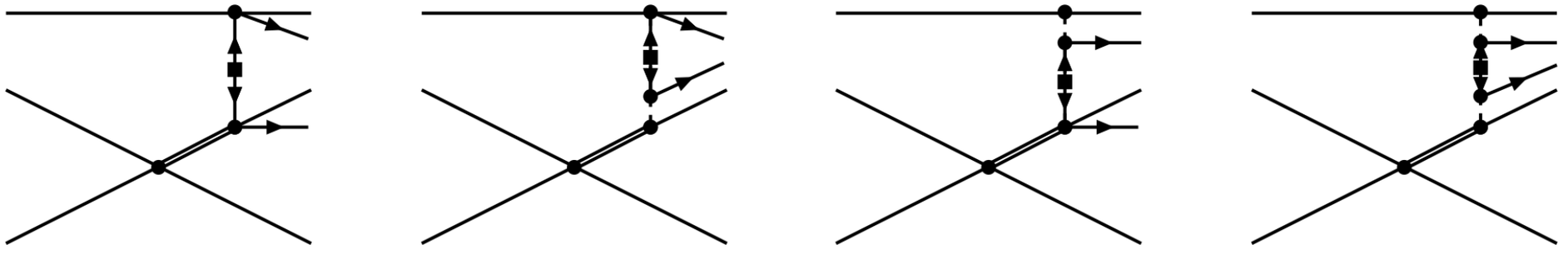
D_T diagrams

c_D

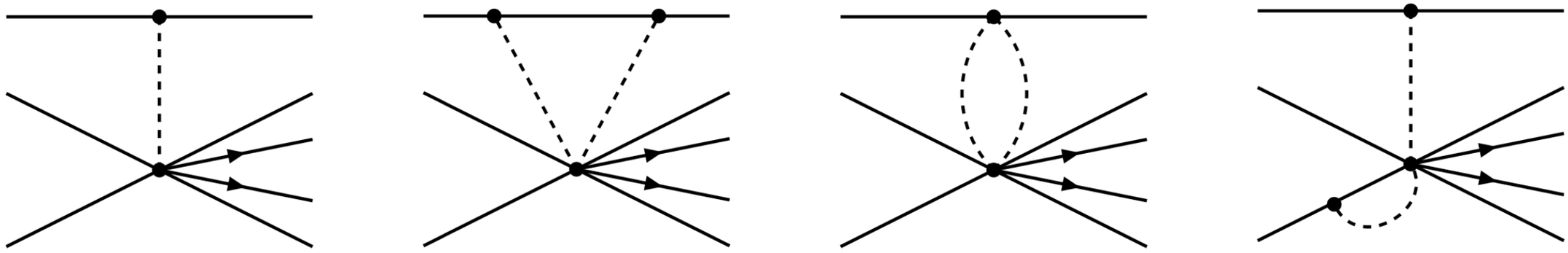




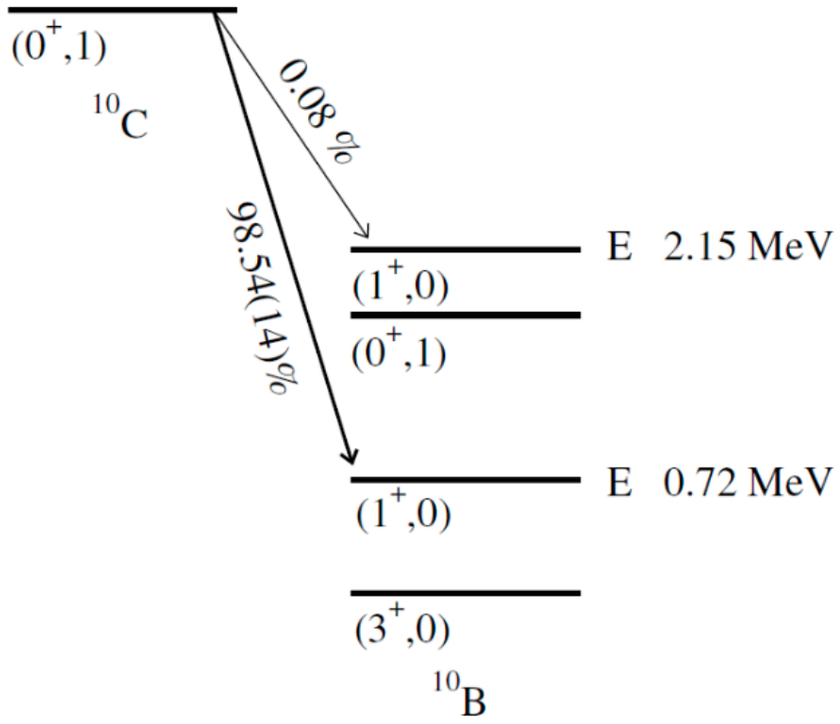
D_T



g_ν^{NN} in the three-nucleon sector



^{10}C to ^{10}B GT beta decay



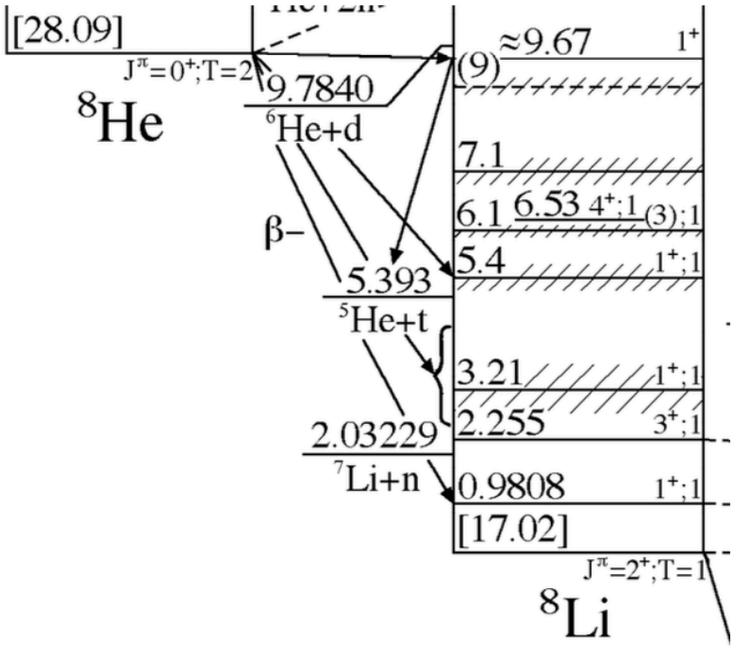
https://nucldata.tunl.duke.edu/

Two states of the same quantum numbers nearby

The result depends strongly on the *LS* mixing of the *p*-shell

Particularly sensitive to the $^3S_1$ and $^3D_1$ mixing because *S* to *S* produces a larger m.e. and ^{10}C is predominantly *S* wave

^{8}He to ^{8}Li GT beta decay



https://nucldata.tunl.duke.edu/

Three (1^{+};1) states within a few MeV

Different dominant spatial symmetries → sensitivity to the precise mixing of small components in the wave function

Improving the mixing of the small components in the (1^{+};1) states is crucial to getting an improved m.e.