Threshold effects on proton-induced reactions in light nuclei

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Workshop on Atomic and Nuclear Quantum Effects Near Threshold
31 May - 2 June 2023

CRB supported in part by the U.S. D.O.E. through grants No. DE-FG02-88ER40387 and DE-NA0004065

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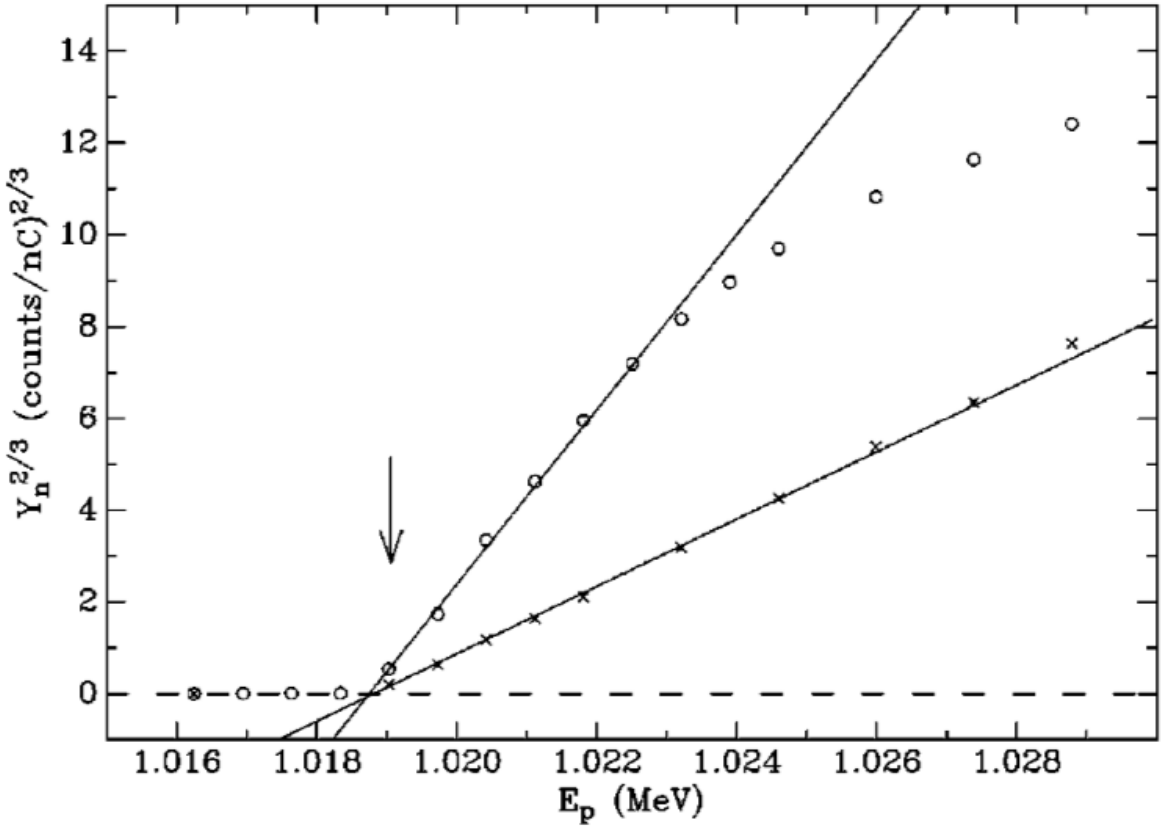
Outline

- Phenomenological R-matrix
- Isospin and mirror symmetry
- Threshold states in ^{11}B

Phenomenological R-matrix

- The phenomenological R-matrix theory of Wigner, Lane, Thomas *et al.* is a "go-to" method for describing near-threshold resonances, interference effects, etc...
- Long-ranged Coulomb interacts and coupling to the the continuum are explicitly taken into account.
- It does not, however, explicitly model many-body dynamical effects.

Example of threshold physics: $^3\mathrm{H}(p,n)^3\mathrm{He}$



- $\sigma_{nx} \propto E_n^{\ell-1/2} \rightarrow$ "$1/v$" law, for $\ell = 0$.
- Detailed balance $\rightarrow \sigma_{xn} \propto E_n^{\ell+1/2}$.
- Yield: $Y \propto \int \sigma_{xn}\, dE_n$ because of target thickness.
- $Y^{2/3}$ should extrapolate linearly to the threshold energy, for $\ell = 0$.

PROC. PHYS. SOC., 1964, VOL. 84

A model for nuclear threshold levels

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MS. received 30*th June* 1964

Abstract. The observation of some levels in light nuclei closer to thresholds than one might expect from average level densities suggests the investigation of models which might produce such an effect. One model based on the *R*-matrix theory of nuclear reactions is shown to predict that near thresholds of channels with small barriers there should be an increased density of levels with large reduced widths. The experimental evidence for such an effect is meagre.

What is the idea here?

- Energy levels have a boundary condition requirement outside the nuclear surface: exponential decay (bound states) or some version of an outgoing wave (unbound states).

- Near a threshold, this condition is strongly non-linear in energy and leads an "attraction" of levels to the threshold. The effect may also be termed "excitation energy compression."

- The effect is strongest for low orbital angular momentum and when the wavefunction has a large component in the two-body channel which has a nearby threshold.

- There are many examples of great practical significance:
  fusion [^{3}H$(d,n)^4$He, ^{3}He$(d,p)^4$He],
  neutron detection [^{3}He$(n,p)^3$H, ^{6}Li$(n,t)^4$He, ^{10}B$(n,\alpha)^7$Li],
  and astrophysics [^{12}C$(\alpha,\gamma)^{16}$O,...].

Further Remarks

- It is easy to lose sight of this compression if one is just doing phenomenology.
- The Thomas-Ehrman shift has the same physical origin.
- This effect is also closely related to the change of basis between the Wigner-Lane-Thomas boundary condition (constant logarithmic derivative) and the energy level boundary condition.
- There may also be additional dynamical enhancements.
- When estimating nuclear reaction rates via Monte Carlo methods, caution should be employed, as this effect non-statistical.

Isospin and Mirror Symmetry

- The R-matrix approach is very convenient for implementing isospin and mirror symmetry.
- In astrophysical applications, information from the mirror nucleus may be very be very useful for estimating reaction rates.
- See also:

  "Spectroscopic factors, overlaps, and isospin symmetry from an R-matrix point of view"

  CRB, Phys. Rev. C **102**, 034328 http://doi.org/10.1103/PhysRevC.102.034328

Definitions

- Physical quantities:
  - E_R – level energy
  - Γ_c – partial width
  - C_c – Asymptotic Normalization Constant (ANC)
- Single-particle quantities:
  - $\Gamma_{c,\mathrm{sp}}$ – single-particle partial width
  - $C_{c,\mathrm{sp}}$ – single-particle ANC

    $$u(\text{large } r) = C_{c,\mathrm{sp}} W_c(r) \quad \text{if } \int_0^\infty u^2\, dr = 1$$
  - $\gamma_{c,\mathrm{sp}}$ – single-particle reduced width

    $$\gamma = u(a) \left(\frac{\hbar^2}{2\mu a \int_0^a u^2\, dr} \right)^{1/2}$$

Spectroscopic Factors

- Traditional spectroscopic factor $\mathcal{S}_c = \mathcal{A}_c^2$, all space normalization:
  - $\Gamma_c = \mathcal{S}_c \Gamma_{c,\mathrm{sp}}$
  - $C_c^2 = \mathcal{S}_c C_{c,\mathrm{sp}}^2$ or $C_c = \mathcal{A}_c C_{c,\mathrm{sp}}$
- Internal spectroscopic factor $\mathbb{S}_c = \mathbb{A}_c^2$, internal normalization:
  - $\gamma_c = \mathbb{A}_c \gamma_{c,\mathrm{sp}}$
- Interestingly, the internal spectroscopic factor definition is the one originally introduced by Macfarlane and French (1960).
- Note also that the spectroscopic factors calculated using a traditional shell model with harmonic oscillator basis may perhaps be closer to the internal spectroscopic factor.

R-Matrix Theory is a Natural Tool

- The reduced-width amplitude is the projection of the many-body wave function onto a particular channel configuration at the channel radius.
- Continuum coupling (long-ranged Coulomb and angular momentum barriers) is properly taken into account.
- Relation to the width or ANC:

$$\Gamma = 2\mathcal{P}(a)\,\frac{\gamma^2}{1+\gamma^2\left(\frac{\partial\hat{\mathcal{S}}}{\partial E}\right)_a}$$

$$C^2 = \frac{2\mu a}{\hbar^2\mathcal{W}^2(a)}\,\frac{\gamma^2}{1+\gamma^2\left(\frac{\partial\hat{\mathcal{S}}}{\partial E}\right)_a}$$

- The tail of the phenomenological nuclear potential can be taken into account in the penetration factors

  $\rightarrow$ unification of "spectroscopic factor / single-particle w.f." and the "R-matrix" descriptions.

Isospin

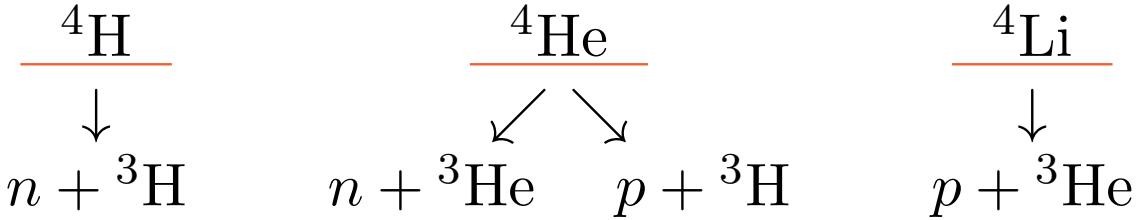
Suppose that multiplet of isospin T_B couples to a two-body decay channel with isospins T_A and T_a. Then across the multiplet, the coupling $B \to A + a$ may described by

- $\mathcal{A}_{c'} = \langle T_A T_{A3}, T_a T_{a3} | T_B T_{B3} \rangle \tilde{\mathcal{A}}_c$ – traditional
- $\mathbb{A}_{c'} = \langle T_A T_{A3}, T_a T_{a3} | T_B T_{B3} \rangle \tilde{\mathbb{A}}_c$ – internal
- $\gamma_{c'} = \langle T_A T_{A3}, T_a T_{a3} | T_B T_{B3} \rangle \tilde{\gamma}_c$ – reduced width amplitude

where $\tilde{\mathcal{A}}_c$, $\tilde{\mathbb{A}}_c$, and $\tilde{\gamma}_c$ are common across the multiplet.

For example:

$T = 1$

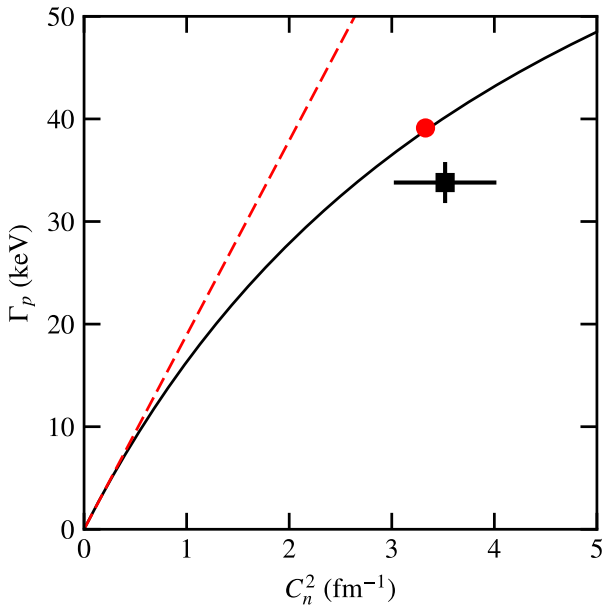


ANCs and/or widths can be related across the mulitplet!

But each of the formulas means something slightly different...

First excited $1/2^+$ states of ^{13}C and ^{13}N

- How are C_n^2 in ^{13}C and Γ_p in ^{13}N related?
- Equal (traditional) spectroscopic factors and the method of Timofeyuk and collaborators predict a *linear* relationship.
- The equal reduced width amplitude prediction is non-linear.



■ experiment

— R-matrix

- - - Timofeyuk *et al.*
PRL **91**, 232501 (2003)

● pure single-particle

Multiple levels with the same J^π

- The outgoing wave boundary condition satisfied by the bound or resonant states breaks isospin.
- The constant boundary condition of Wigner, Lane, Thomas,... (WLT) basis does not.
- $\rightarrow$ apply isospin in the WLT basis. This includes an energy shift and changes in charges.

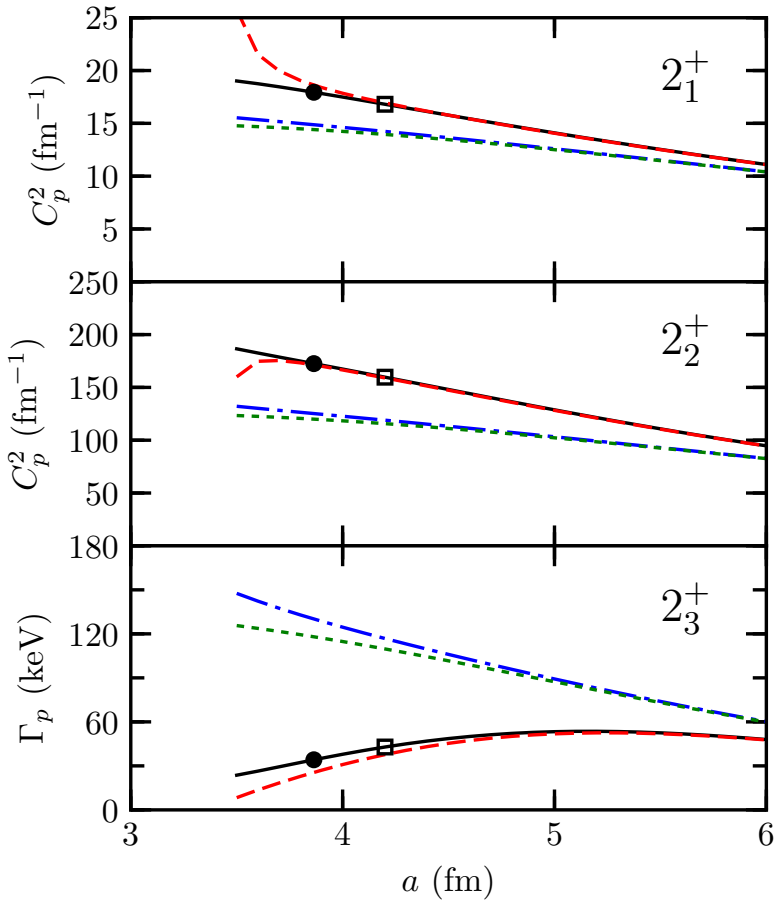
Multiple levels with the same J^π, continued

- These basis changes involve linear (and sometimes non-linear) algebra.
- The relative signs of the reduced-width amplitudes are required. These may be taken from a shell-model calculation or treated as an uncertainty.
- This leads to a mixing of the physical states across the multiplet.
- This also provides the multi-level generalization of the Thomas-Ehrman shift.

2^+ states of ^{18}O and ^{18}Ne

- The proton ANC of the 2_2^+ state of ^{18}Ne is important for determining the $^{17}\mathrm{F}(p,\gamma)^{18}\mathrm{Ne}$ reaction rate in the hot CNO cycle.
- There is also a striking (apparent) isospin violation for the 2_3^+ states of ^{18}O – ^{18}Ne that is well-established experimentally.

predictions in ^{18}Ne from measured ^{18}O ANCs:



blue, green: naive mirror symmetry

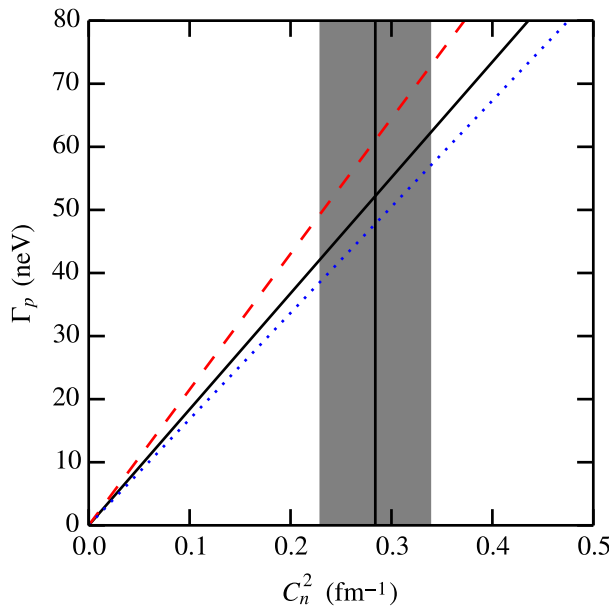
red, black: multi-level mirror symmetry

$\ell = 0$ mirror states in ^{27}Al and ^{27}Si

- The Γ_p of the 7590-keV $9/2^+$ state of ^{27}Si ($E_R = 127$ keV) is essential for understanding the rate of the $^{26}\mathrm{Al}(p,\gamma)^{27}\mathrm{Si}$ reaction rate in AGB and Wolf-Rayet stars.

- The mirror state in ^{27}Al has been recently studied via $^{26}\mathrm{Al}(d,p)$: S.D. Pain *et al.*, PRL **114**, 212501 (2015) and V. Margerin *et al.*, PRL **115**, 062701 (2015).

- This is *not* a single-particle state: $S \approx 0.01$.



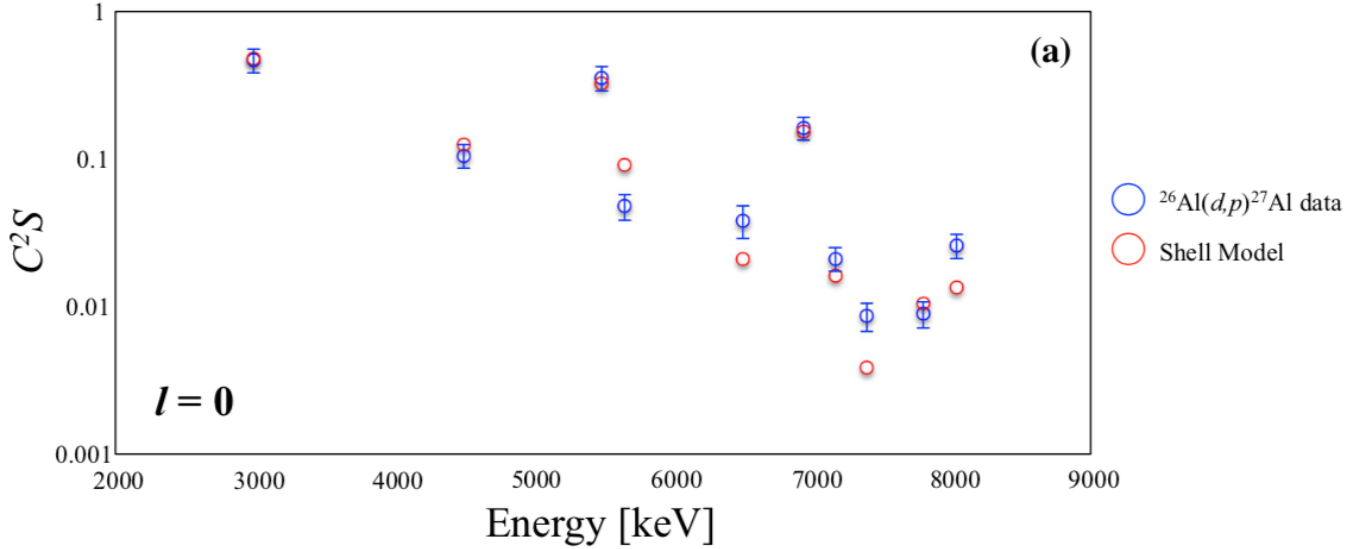
experiment

R-matrix

equal $\mathbb{S}$, Timofeyuk *et al.*

equal $\mathcal{S}$

What about multi-level effects?



^{27}Al states, Lotay *et al.*, Eur. Phys. J. A 56, 3 (2020)

- Re-implement USDB shell model calculation from this paper.
- Result: the reduced width amplitude deduced from the mirror reaction is reduced by about 20%. The resonance strength is thus reduced by about 40%.
- The effect arises from mixing with the lowest-lying 9/2+ state, which dominates the spectroscopic strength in this J^π.
- The change is in the same direction, but about a factor of two larger than that reported by Pain *et al.* using the Gamow shell model.

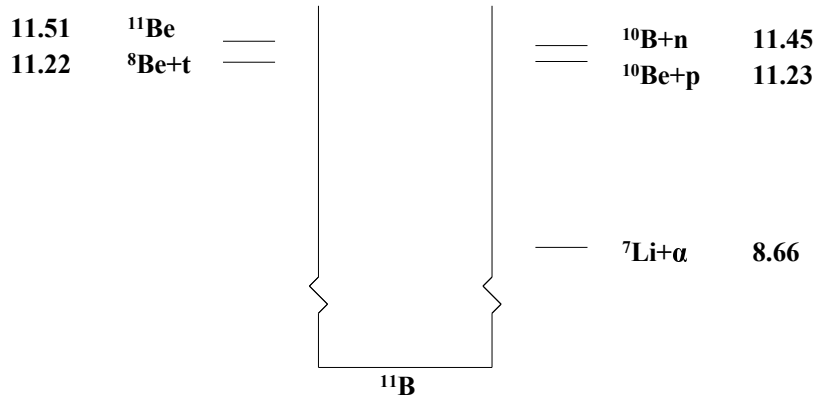
Summary

- The R-matrix approach to mirror / isospin symmetry is very powerful:
  - the continuum is properly accounted for (external isospin symmetry breaking)
  - extensible to multiple levels
- Many applications are possible:
  - predicting widths or ANCs from mirror levels
  - calculating the continuum correction to the traditional shell model

Experimental study of threshold states in ^{11}B

Motivations

- $^{11}\mathrm{Be}(\beta^-) \rightarrow {}^{10}\mathrm{Be} + p$?
- Astrophysics: $p + {}^{10}\mathrm{Be}$ reactions in CCSN may effect the nucleosynthesis of Be and B; BBN and the first stars.
- Physics of near-threshold states.
- Isobaric analog states.



Thresholds in $^{11}\mathrm{B}$

Summary of previous experimental work

- $^{11}\mathrm{Be}(\beta^-) \rightarrow {}^{10}\mathrm{Be} + p$ at ISOLDE via AMS. Riisager *et al.*, Eur. Phys. J. A 56:100 (2020), and papers therein.
- $^{11}\mathrm{Be}(\beta^-) \rightarrow {}^{10}\mathrm{Be} + p$ at TRIUMF via proton detection. Ayyad *et al.* Phys. Rev. Lett. 123, 082501 (2019).
- $^{10}\mathrm{Be}(p,p)$ at NSCL via inverse kinematics. Ayyad *et al.*, Phys. Rev. Lett., 129, 012501 (2022).
- $^{10}\mathrm{Be}(d,n)^{11}\mathrm{B}^*$ at FSU via inverse kinematics and detecting charged-particle breakup of $^{11}\mathrm{B}^*$. Lopez-Saavedra *et al.*, Phys. Rev. Lett. 129, 012502 (2022).

The latter two papers agree on the excitation energy (≈ 11.42 MeV) and proton spectroscopic (≈ 0.3) factor of a new state in ^{11}B. It must be a $1/2^+$ state.

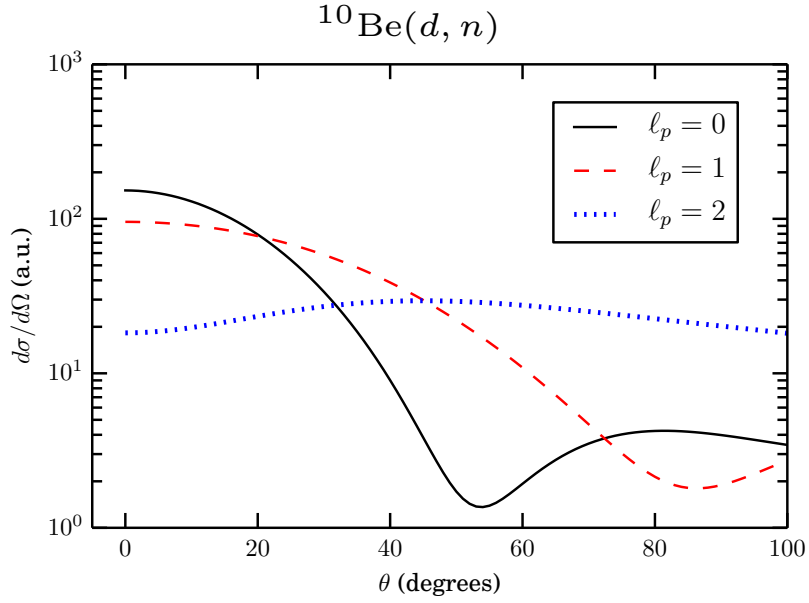
Summary of theoretical work

- Shell model in continuum: Okołowicz *et al.*, Phys. Rev. Lett. 124, 042502 (2020).
- Shell model: Volya, Europhys. Lett., 130, 12001 (2020).
- Halo EFT: Elkamhawy *et al.*, Phys. Lett. B 821, 136610 (2021).
- Shell model, no core, with continuum: Atkinson *et al.*, Phys. Rev. C 105, 054316 (2022).
- Shell model in continuum: Okołowicz *et al.*, J. Phys. G: Nucl. Part. Phys. 49, 10LT01 (2022).

My summary: Understanding the $^{11}\mathrm{Be}(\beta^-) \rightarrow {}^{10}\mathrm{Be} + p$ reported the initial ISOLDE experiments is difficult. The existence of a $1/2^+$ state near the $^{10}\mathrm{Be}+p$ threshold with a large $^{10}\mathrm{Be}+p$ spectroscopic factor is highly plausible.

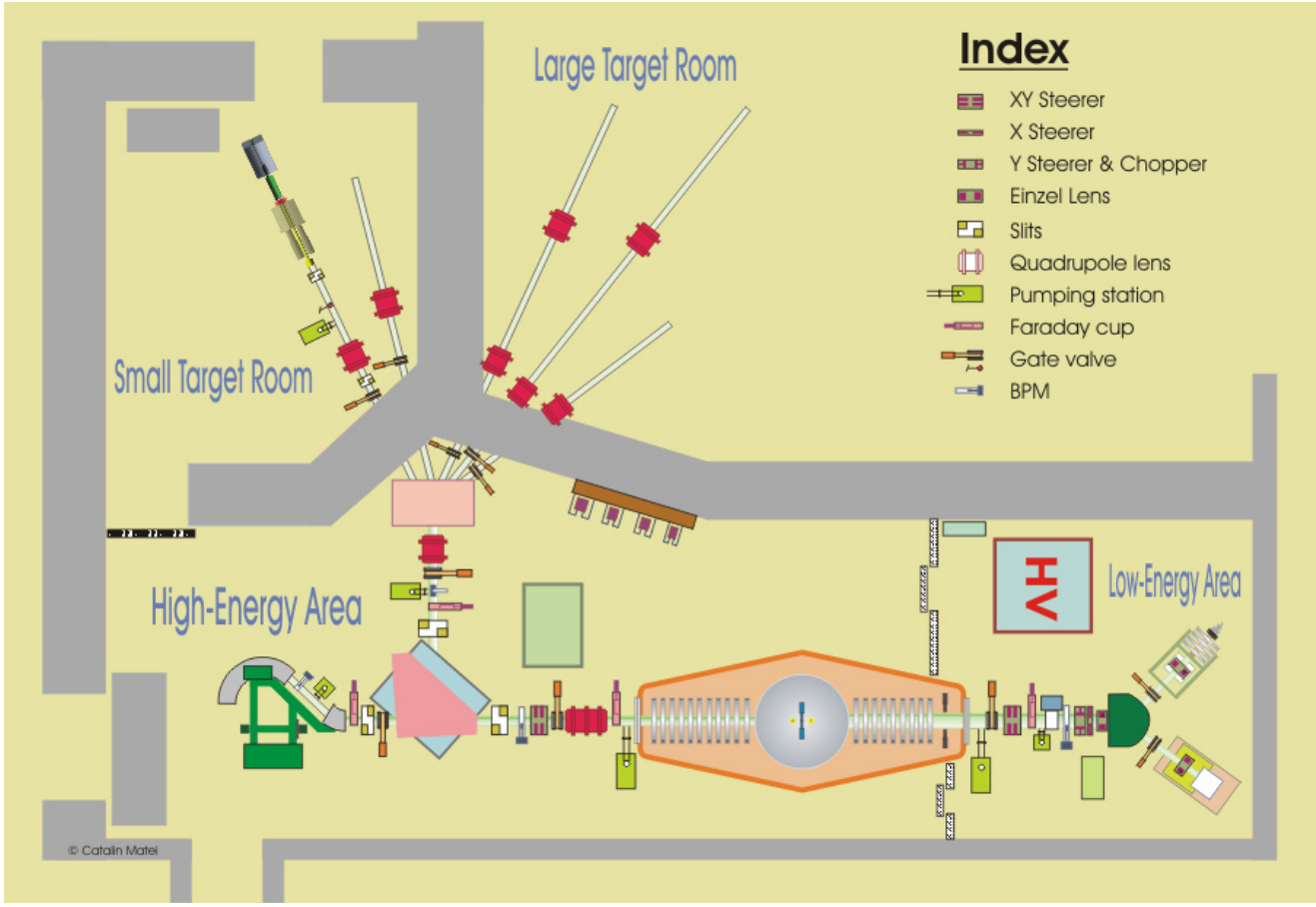
Our approach

- Measure $^{10}\mathrm{Be}(d, n)$ at forward angles in normal kinematics.
- The proton transfer cross section [in the Distorted Wave Born Approximation (DWBA)] is proportional to the proton spectroscopic factor.
- The angular distribution provides information about ℓ_p in the final state.
- Sensitivity to a $1/2^+$ state is optimized at $\theta_n = 0^\circ$.
- We have performed measurements at $E_d = 4$ and 7 MeV.
- $E_d = 4$ MeV optimizes the neutron energy resolution near $E_x = 11$ MeV.



$E_d = 4$ MeV DWBA calculations

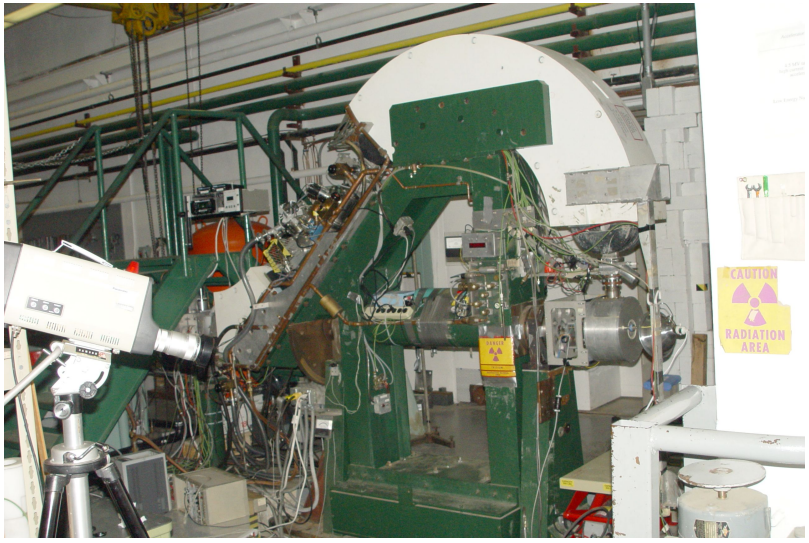
Edwards Accelerator Laboratory



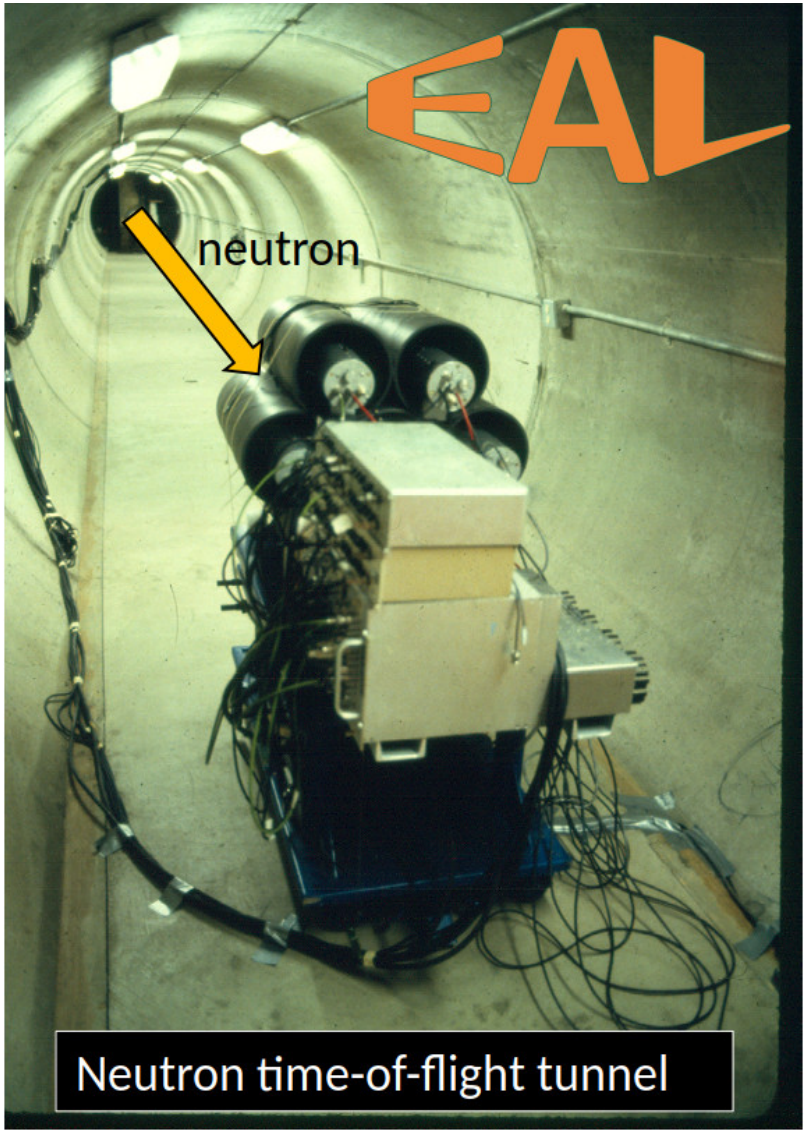
- 4.5-MV Tandem Accelerator – Just celebrated 50 years!
- Pelletron charging system, upgraded to Alphatross He ion source
- Unique beam swinger and 30-m TOF tunnel
- Specializations: TOF techniques, neutrons
- http://inpp.ohio.edu/~oual/

Association for Research
at University Nuclear Accelerators

Beam swinger and time-of-flight tunnel



swinger magnet



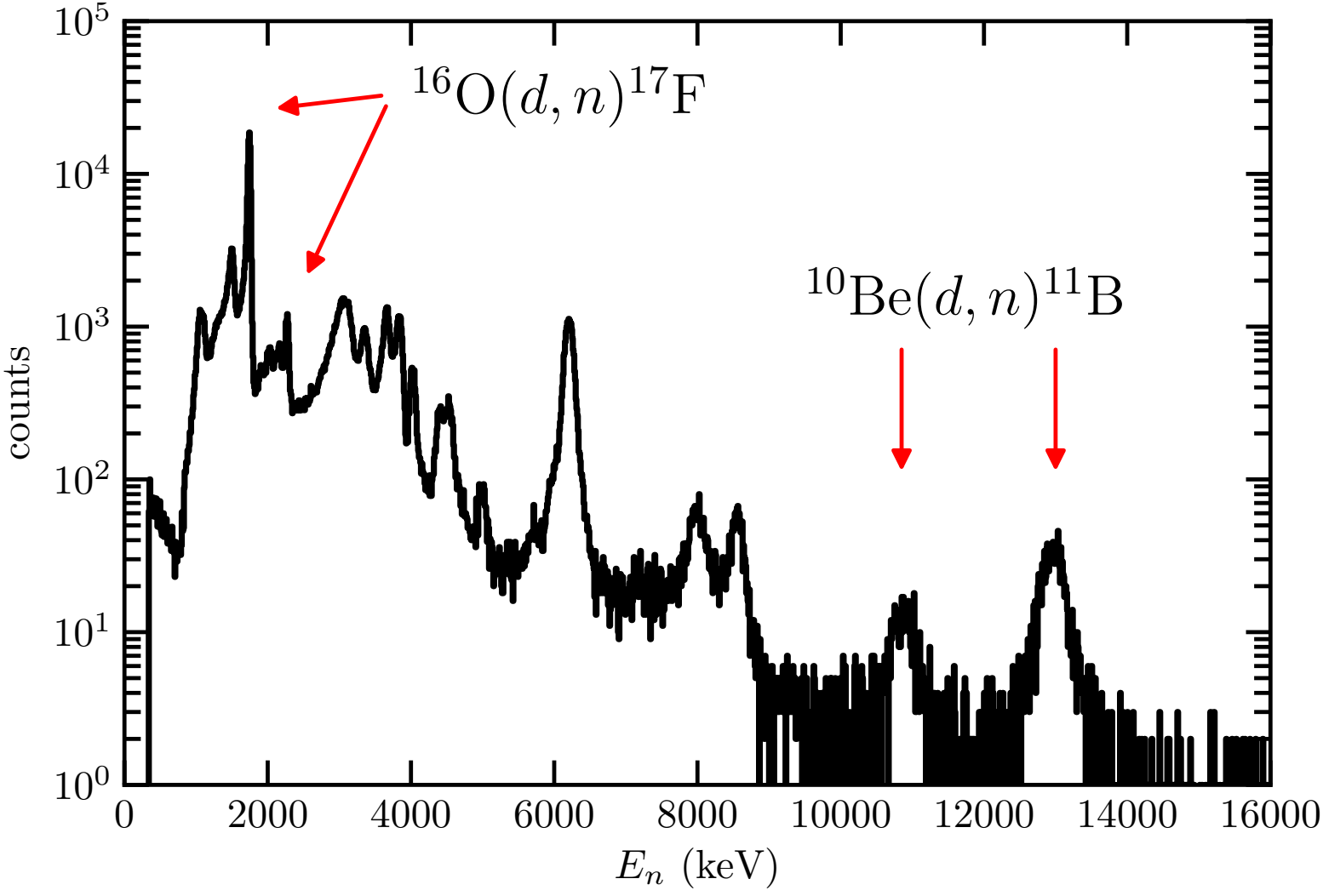
^{10}Be target

- We utilized a 85-μg/cm^2 BeO target, enriched to 94% ^{10}Be ($t_{1/2} = 1.4 \times 10^6$ years).
- The BeO is a 3.6-mm-diameter deposit on 4.9 mg/cm^2 Pt foil.
- The beam energy loss is ≈ 10 keV in our measurements.
- This target is one of three ^{10}Be targets in our laboratory that were produced by David Goosman at BNL in the early 1970s, see D. R. Goosman, Nucl. Instrum. and Methods 116, 445 (1974).

Neutron time-of-flight parameters

- Utilize a 5” diameter, 2” thickness Saint Gobain liquid scintillator organic at flight path of 5 m.
- The FWHM timing resolution was 1.2-1.4 ns.
- The time between a beam pickoff signal and the detected neutron is measured with a time $\rightarrow$ amplitude converter.
- The ns/channel is calibrated using a random time spectrum and a precision delay generator.
- The time offset is determined from the γ-ray peak produced at the target.
- The uncertainty in the measured neutron energy near $E_n = 1.5$ MeV is 8 keV.

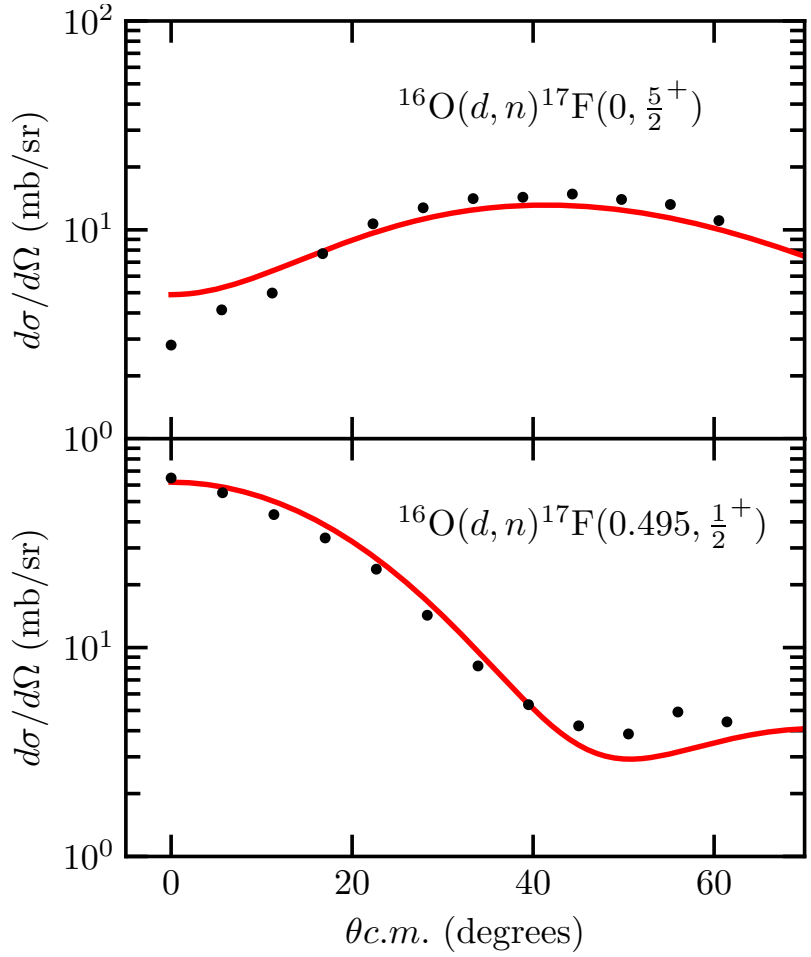
Neutron energy spectrum at $E_d = 4$ MeV, $\theta_n = 0°$



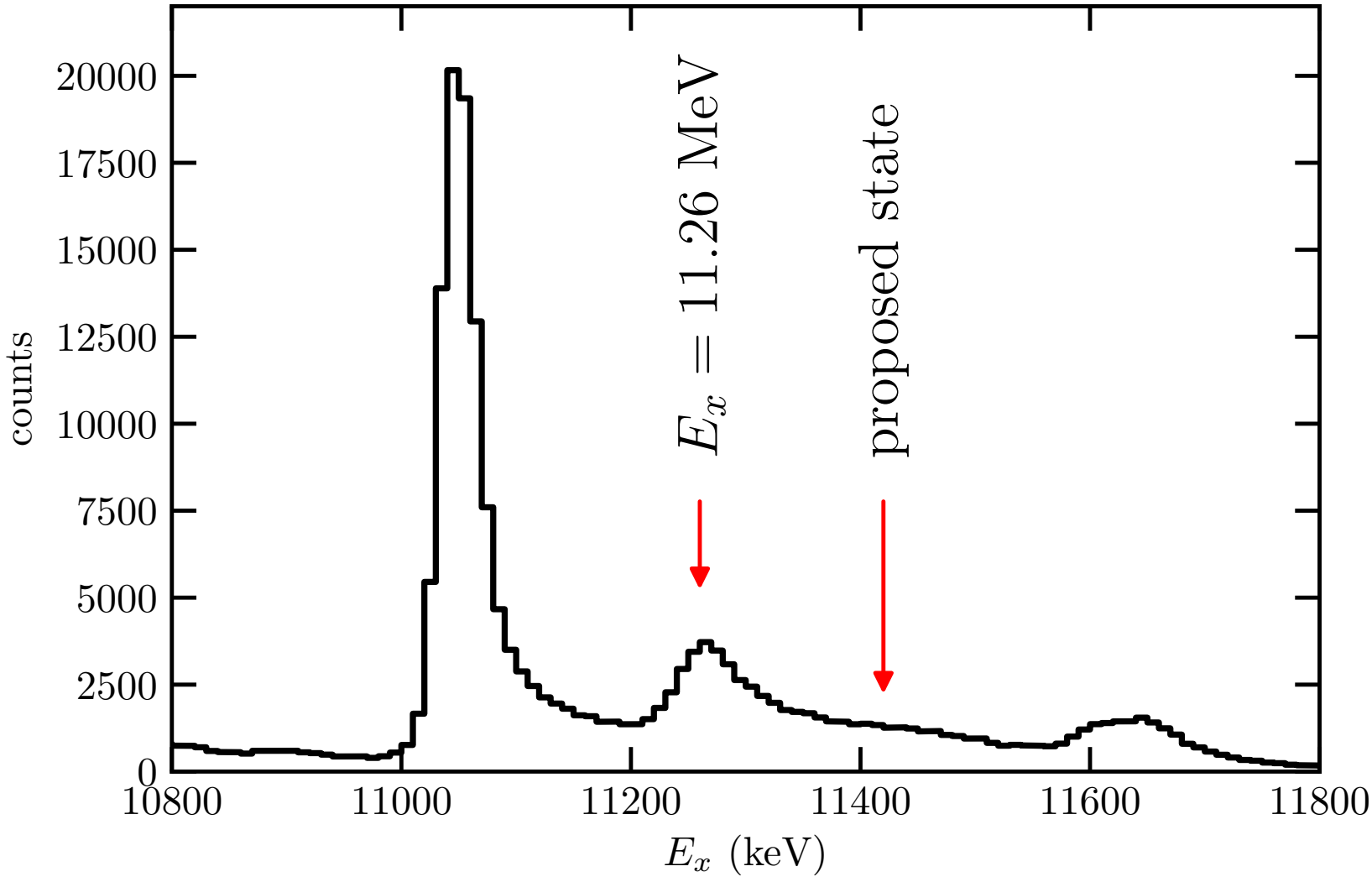
- Background reactions identified: ^{16}O(d,n), ^{9}Be(d,n), and ^{12}C(d,n) – from the collimators and beamstop as well as the target.

$^{16}\mathrm{O}(d,n)^{17}\mathrm{F}$ cross sections at $E_d = 4$ MeV

- The ground and first excited states of ^{17}F are well known to be nearly perfect single-particle (proton) states.
- This is exactly what we are looking for the ^{11}B!
- The ground state and first excited state are $\ell_p = 2$ and 0, respectively.

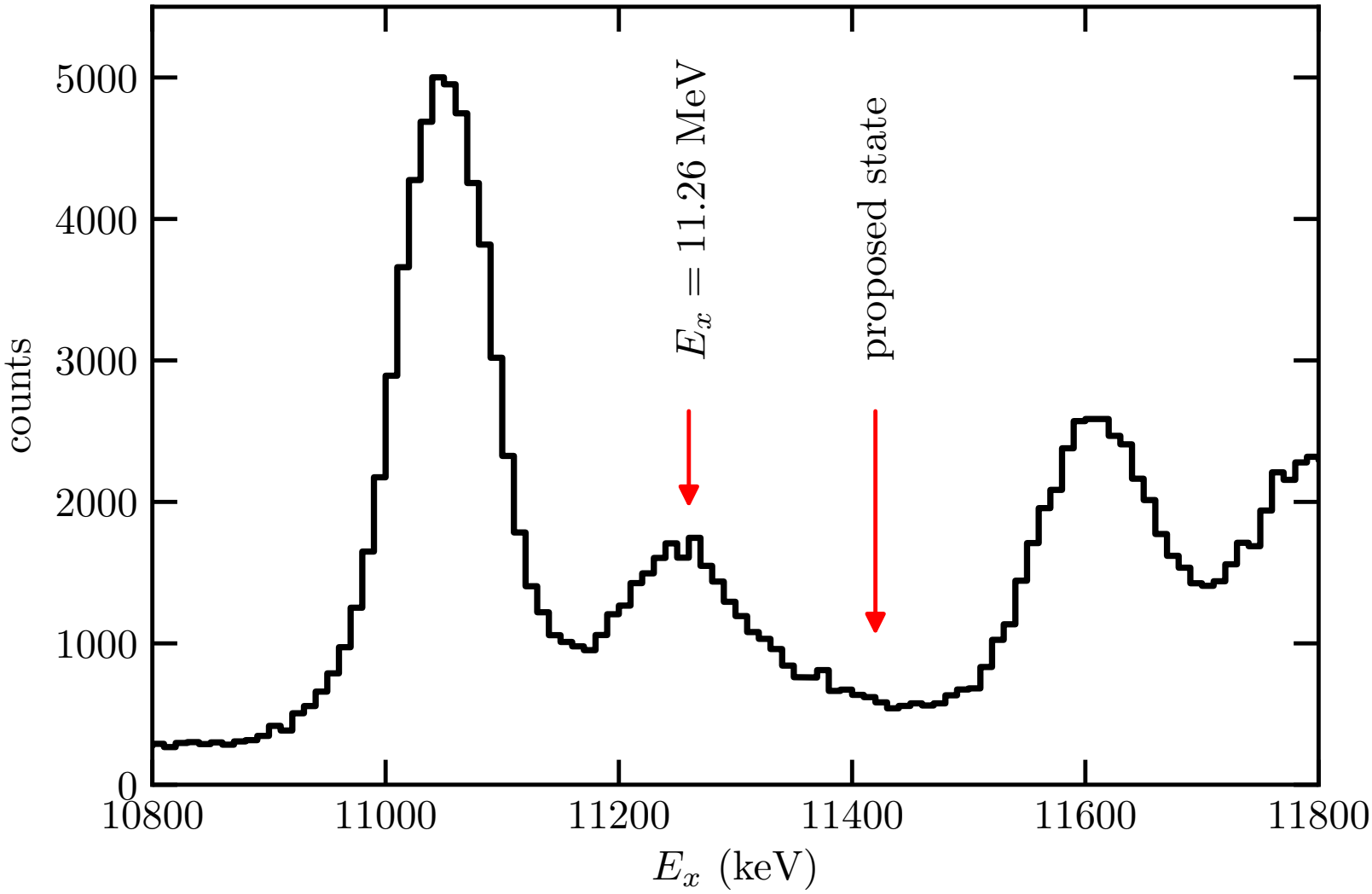


E_x spectrum at $E_d = 4$ MeV, $\theta_n = 0°$



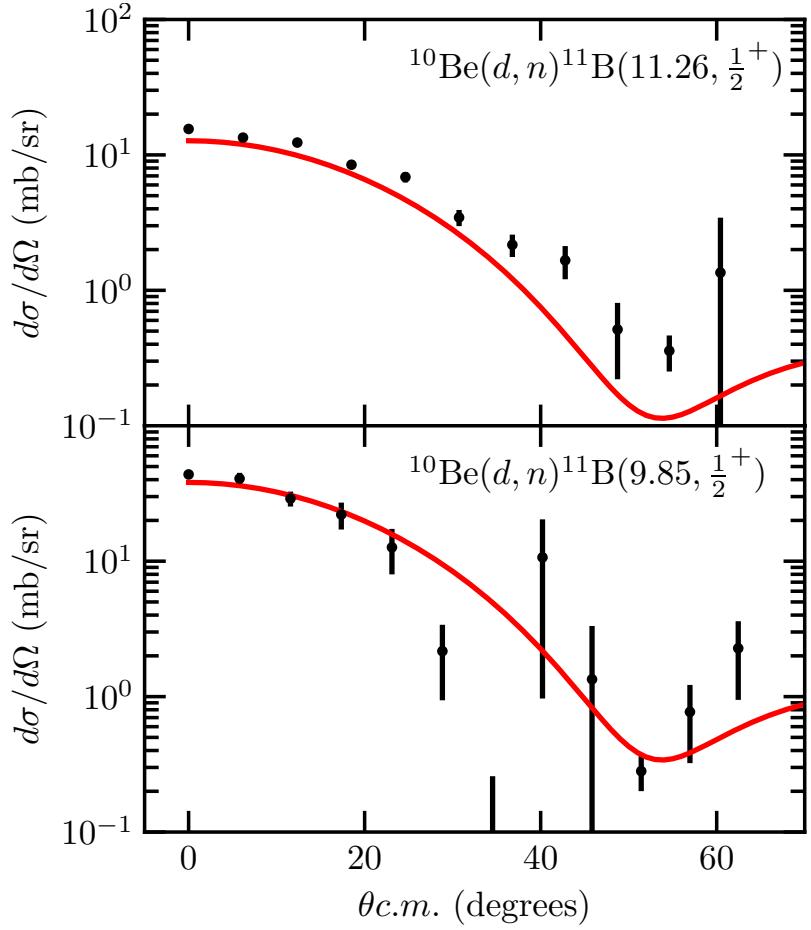
- The resolution in E_x is about 40 keV here.

E_x spectrum at $E_d = 7$ MeV, $\theta_n = 0°$



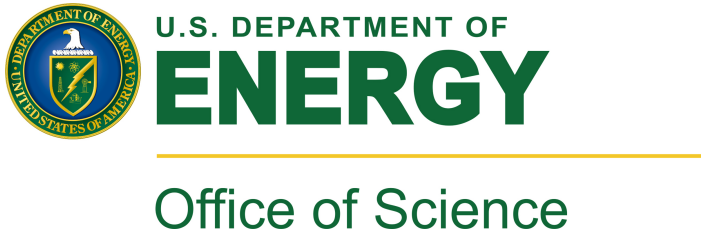
$^{10}\mathrm{Be}(d,n)^{11}\mathrm{B}$ cross sections at $E_d = 4$ MeV

- States at $E_x = 9.85$ and 11.26 MeV in ^{11}B are strongly populated.
- Both appears to be $\ell_p = 0$, $J^\pi = 1/2^+$ states.
- In the case of the 9.85-MeV state, the TUNL (2012) compilation lists it as a $3/2^+$ state.



Summary

- I would like to thank my collaborators: Y. Jones-Alberty, T. N. Massey, K. Brandenburg, B. Chauhan, J. Derkin, G. Hamad, M. Jeswald, Z. Meisel, S. N. Paneru, M. Saxena, N. Singh, D. Soltesz, S. K Subedi, A. V Voinov, and J. Warren.
- We *do not* see evidence for a state at $E_x = 11.42$ MeV.
- We do observe $1/2^+$ states at 9.85 and 11.26 MeV which have significant proton spectroscopic factors.
- This suggests the β-delayed α spectrum from ^{11}Be arises mainly from $1/2^+$ states in ^{11}B.

Thank you for your attention.

And thanks again to my collaborators and funding sources for making this all possible!