
Extrapolating to zero energy in two-body scattering and capture reactions

Daniel Phillips
Ohio University



with Xilin Zhang, Ken Nollett,
Carl Brune, Mahesh Poudel,
and Andrius Burnelis



OHIO
UNIVERSITY

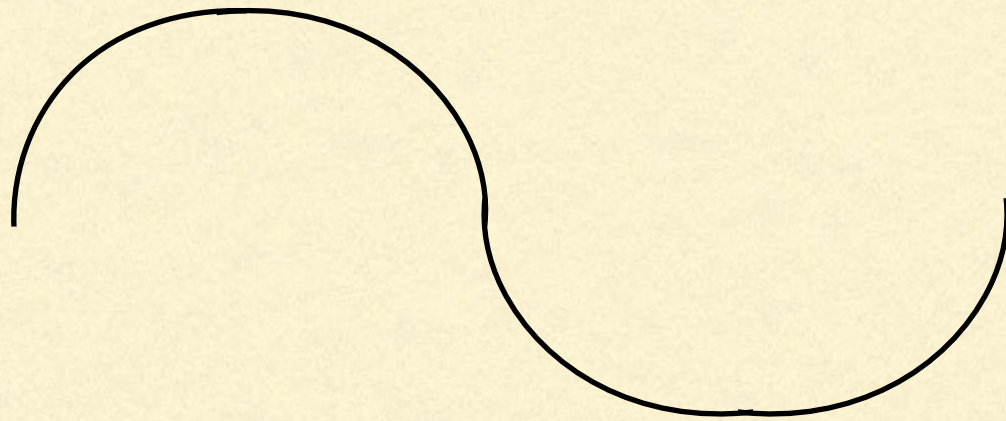
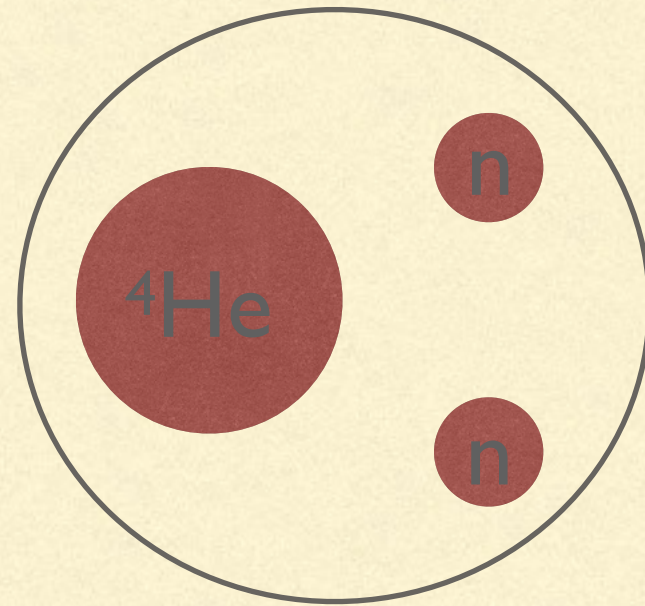
**RESEARCH SUPPORTED BY THE DOE OFFICE OF SCIENCE, THE
SSAP, AND THE NSF OAC**

Outline

- Part 1: EFT for threshold capture reactions
 - What is Halo EFT? How does it model ${}^3\text{He} + {}^4\text{He} \rightarrow {}^7\text{Be} + \gamma$?
 - Halo EFT evaluation of $S(E)$ below $E_{\text{cm}}=2$ MeV
 - Intermezzo: What's all the fuss about Bayesian methods?
 - Part 2: Halo EFT for ${}^3\text{He}$ - ${}^4\text{He}$ scattering
 - Fit to the SONIK data
 - Vacuum polarization effects in ${}^3\text{He}$ - ${}^4\text{He}$ scattering
 - Part 3: $2n$ halos are a universal threshold phenomenon
-

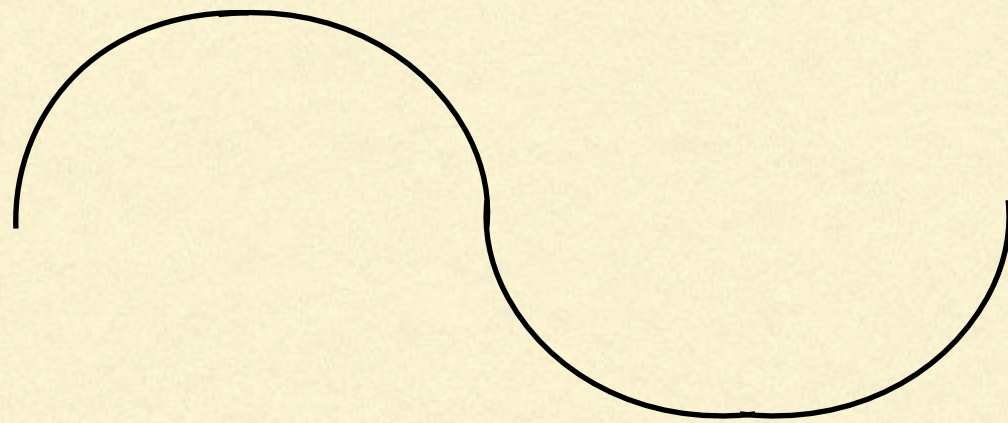
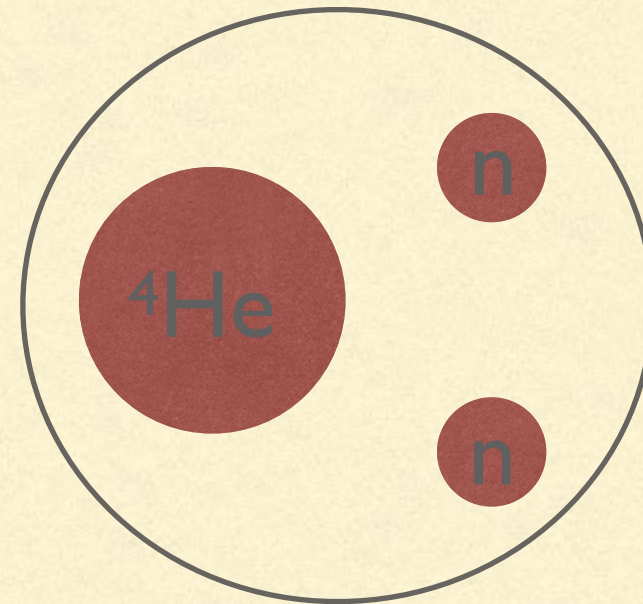
Halo EFT

$$\lambda \gg R_{\text{core}}; \lambda \lesssim R_{\text{halo}}$$



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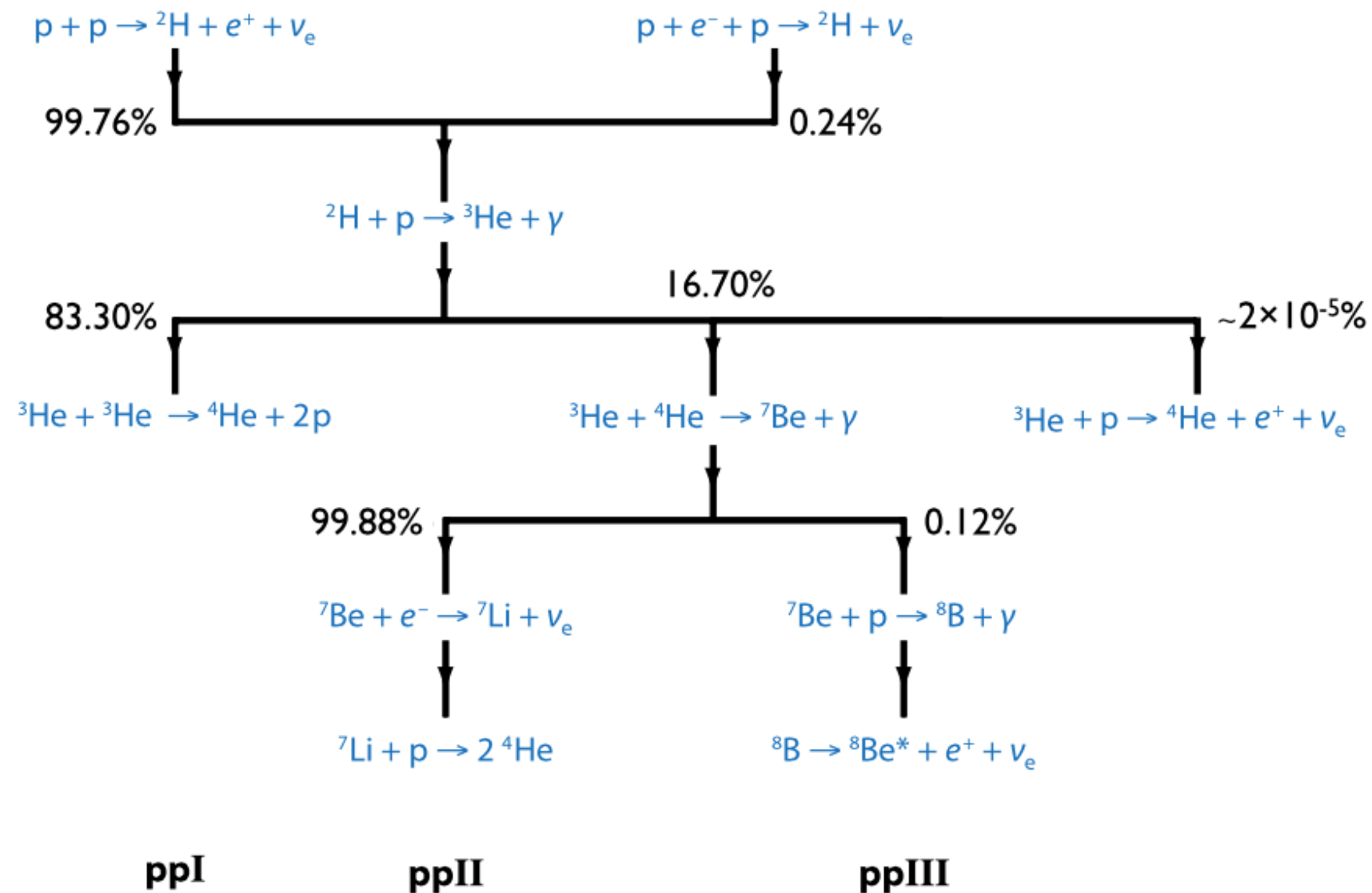


- Define $R_{\text{halo}} = \langle r^2 \rangle^{1/2}$. Seek EFT expansion in $R_{\text{core}}/R_{\text{halo}}$. Valid for $\lambda \lesssim R_{\text{halo}}$
- Typically $R \equiv R_{\text{core}} \sim 2$ fm. And since $\langle r^2 \rangle$ is related to the neutron separation energy we are looking for systems with neutron separation energies less than 1 MeV
- By this definition the deuteron is the lightest halo nucleus, and the pionless EFT for few-nucleon systems is a specific case of halo EFT

Why is $^3\text{He}(^4\text{He},\gamma)$ important?

Adelberger et al., Rev. Mod. Phys. 83, 195 (2011)

- Accurate knowledge of $^3\text{He}(^4\text{He},\gamma)$ needed to reliably predict amount of ^7Be in the Sun
- Therefore key for prediction of ^8B solar neutrino flux
- BBN implications, but I will not discuss those here



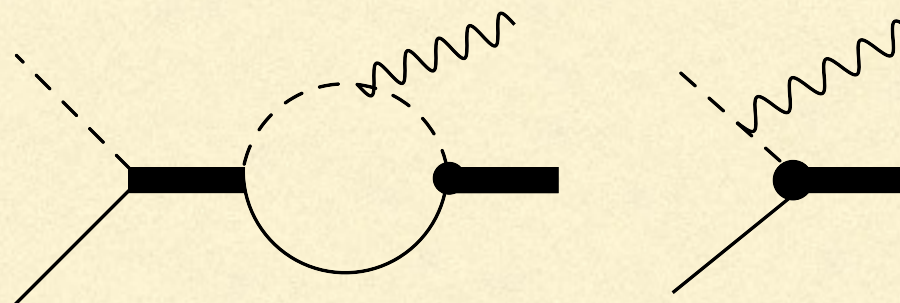
p-wave bound states and capture thereto

Hammer & DP, NPA (2011)

- At LO p-wave In halo described solely by its ANC and binding energy

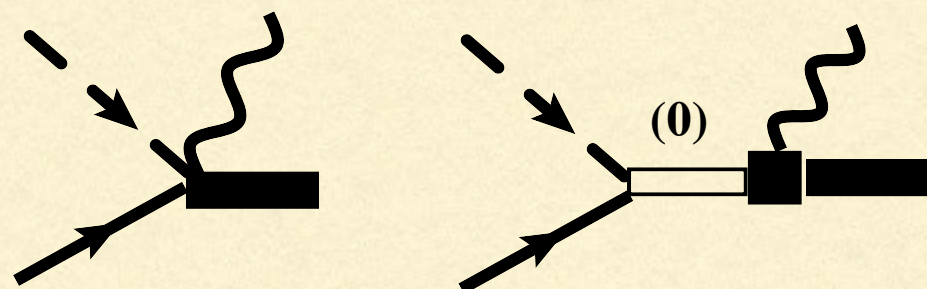
$$u_1(r) = A_1 \exp(-\gamma_1 r) \left(1 + \frac{1}{\gamma_1 r} \right) \quad \gamma_1 = \sqrt{2m_R B}$$

- Capture to the p-wave state proceeds via the one-body E1 operator:
“external direct capture”



$$E1 \propto \int dr u_1(r) r (\cos(kr) + \sin(kr) \cot \delta); k \cot \delta \text{ from ERE}$$

- NLO: piece of the amplitude representing capture at short distances, represented by a contact operator $\Rightarrow$ there is an LEC that must be fit



${}^3\text{He} + {}^4\text{He} \rightarrow {}^7\text{Be} + \gamma_{\text{EI}}$ at LO in Halo EFT

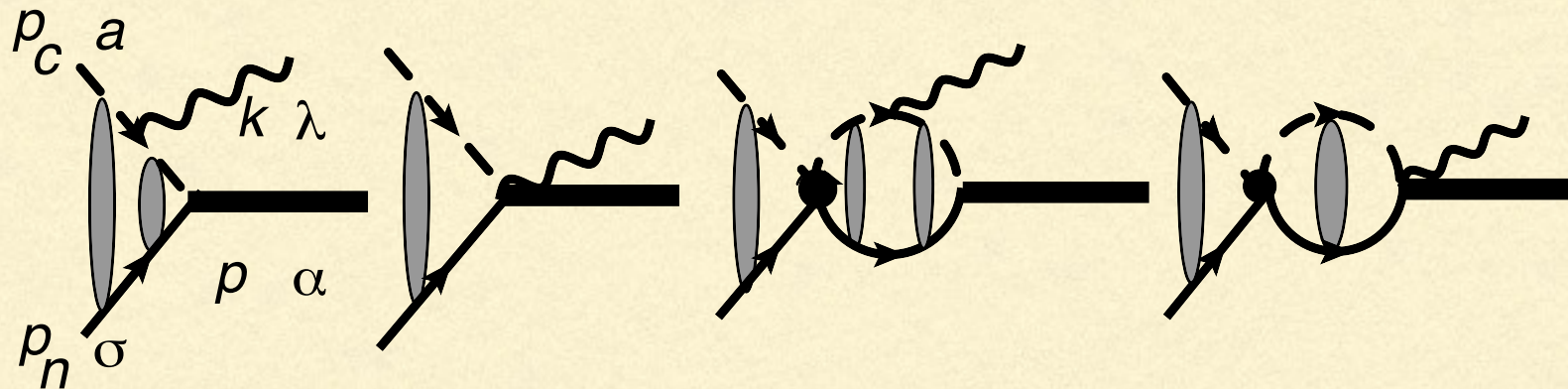
Zhang, Nollett, DP, JPG (2019), cf. Rupak, Higa, Vaghani, EPJA (2018)

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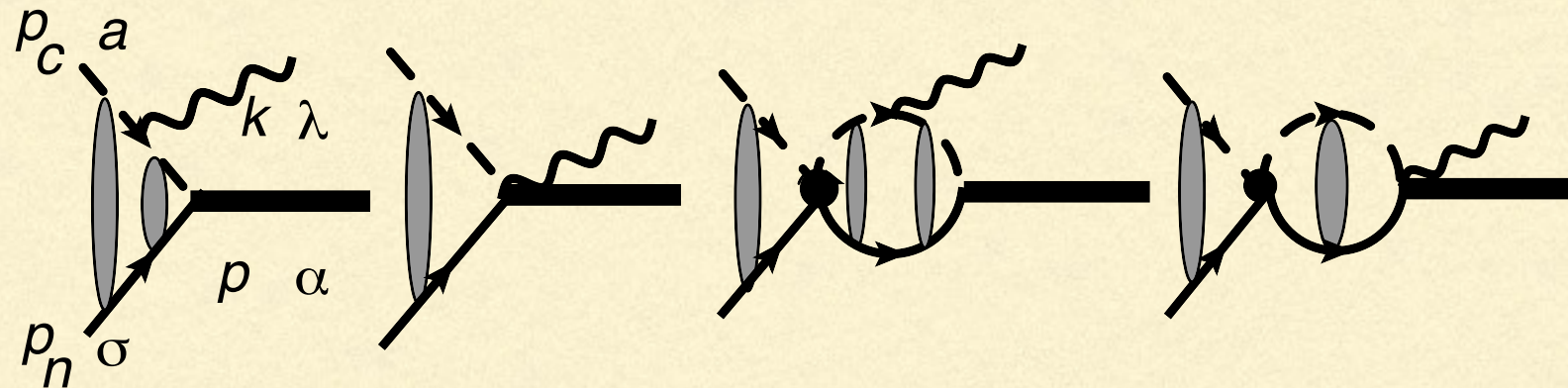
- In this system $R_{\text{core}} \sim 1.5$ fm, $R_{\text{halo}} \sim 3$ fm
- Also need to include Coulomb interactions non-perturbatively:
 $k_C = Q_c Q_n \alpha_{\text{EM}} M_R = 17$ MeV; $a \sim 10$ s of fm, both $\sim R_{\text{halo}}$



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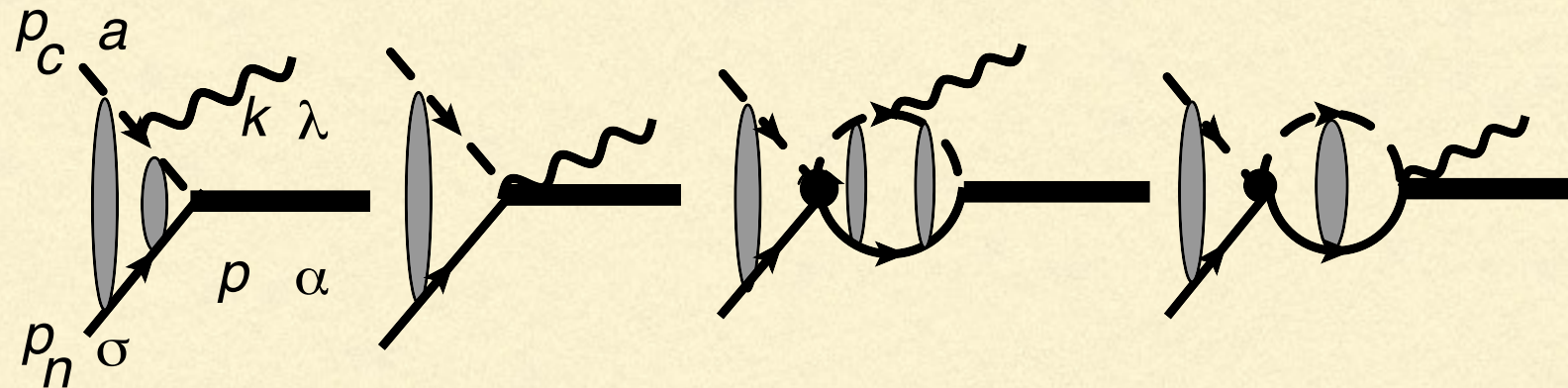


- Scattering wave functions are linear combinations of Coulomb wave functions F_0 and G_0 . Bound state wave function = the appropriate Whittaker function.

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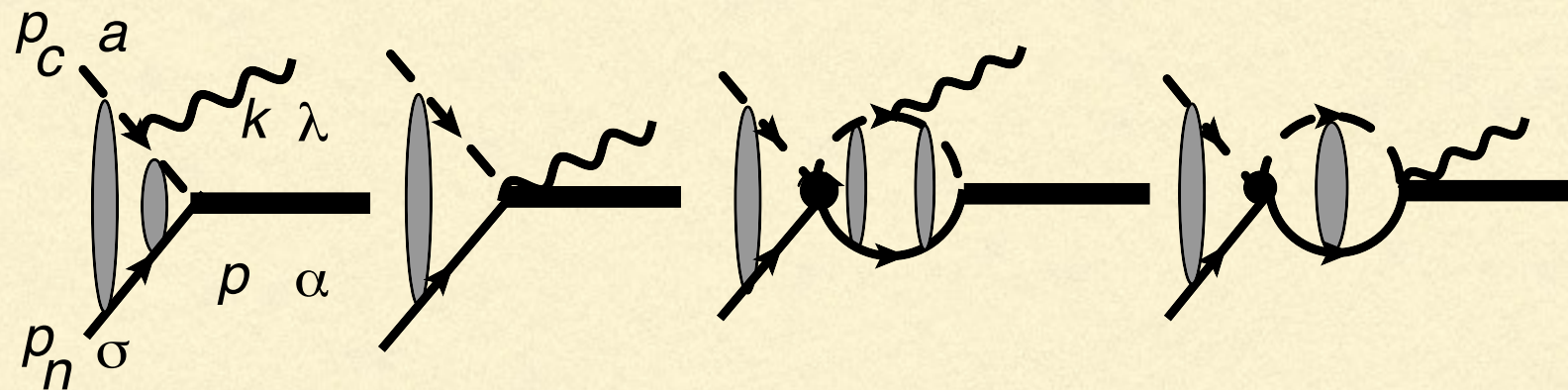
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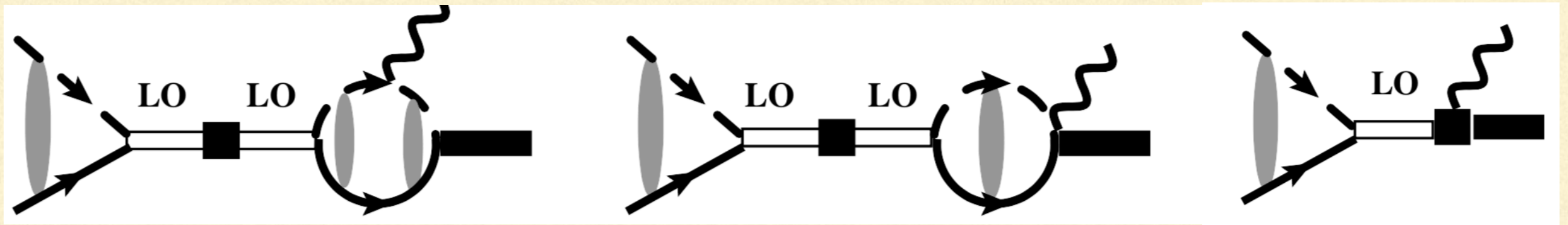
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- Can also predict capture to the excited $1/2^-$ in ${}^7\text{Be}$

Three parameters at
leading order

Additional ingredients at NLO

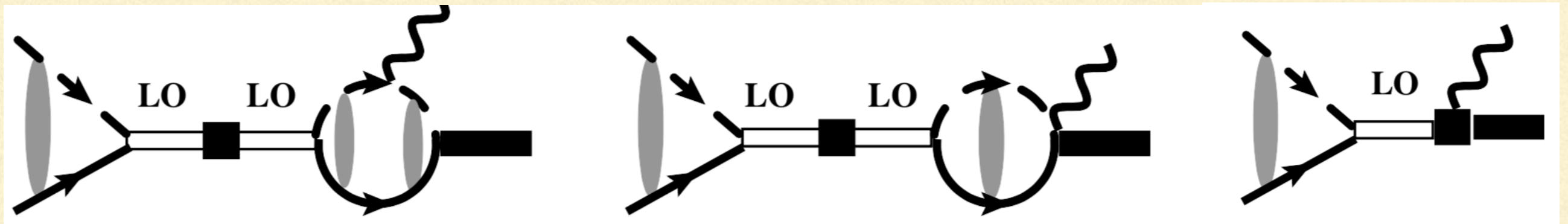


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Three more parameters at NLO

- Effective range (can add shape parameter which enters at N³LO)
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- Can also consider contact interaction for D-wave capture, $\bar{L}_D$ (enters at N⁴LO)

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 - Weizmann
 - Luna (L)
 - Erna
 - Notre Dame
 - Atomki
 - Plus 34 branching-ratio data
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 - Specify CMEs
 - S-factor: by set
 - Branching ratio: none
-

$^3\text{He}(^4\text{He},\gamma)$ results

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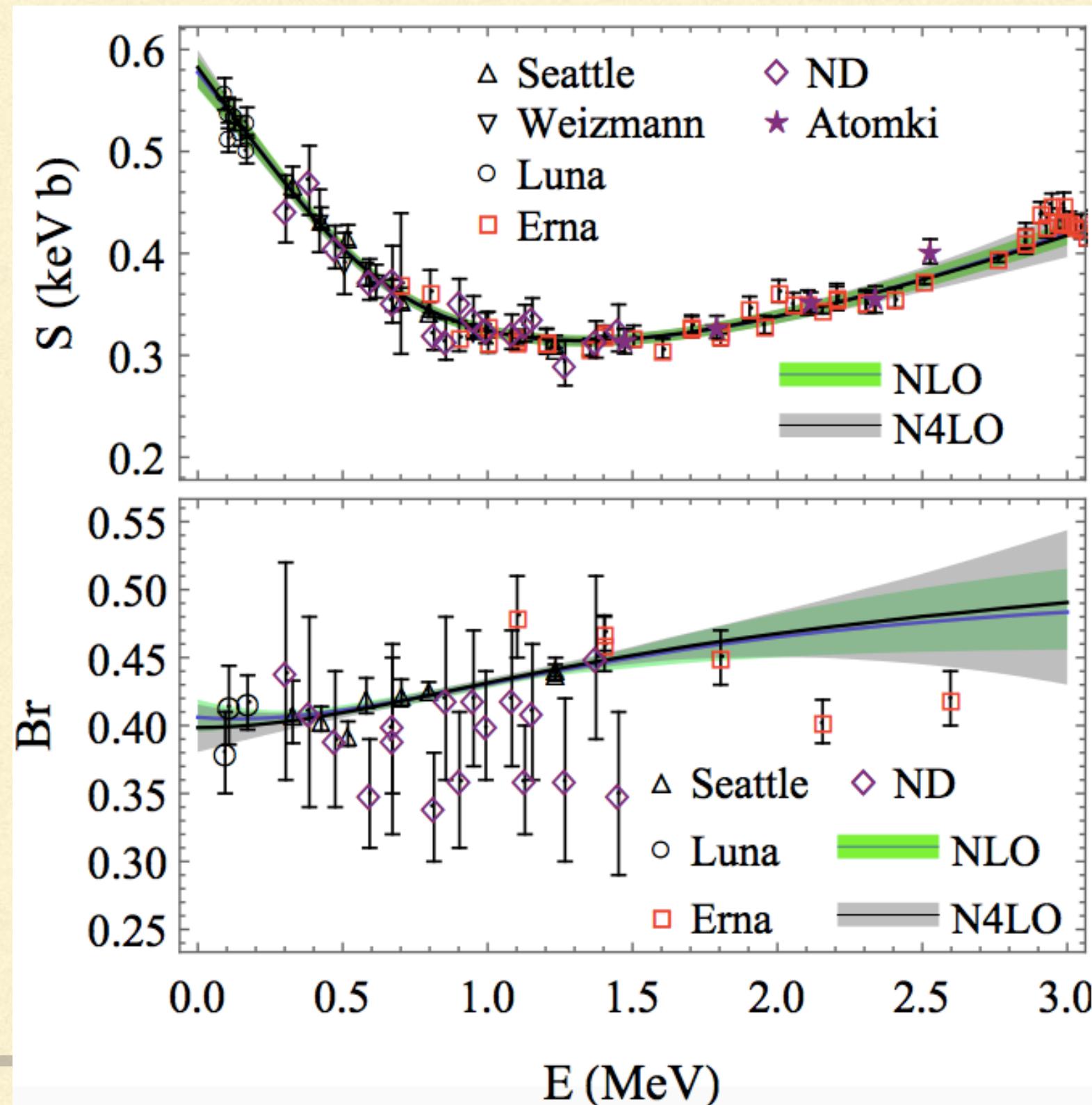
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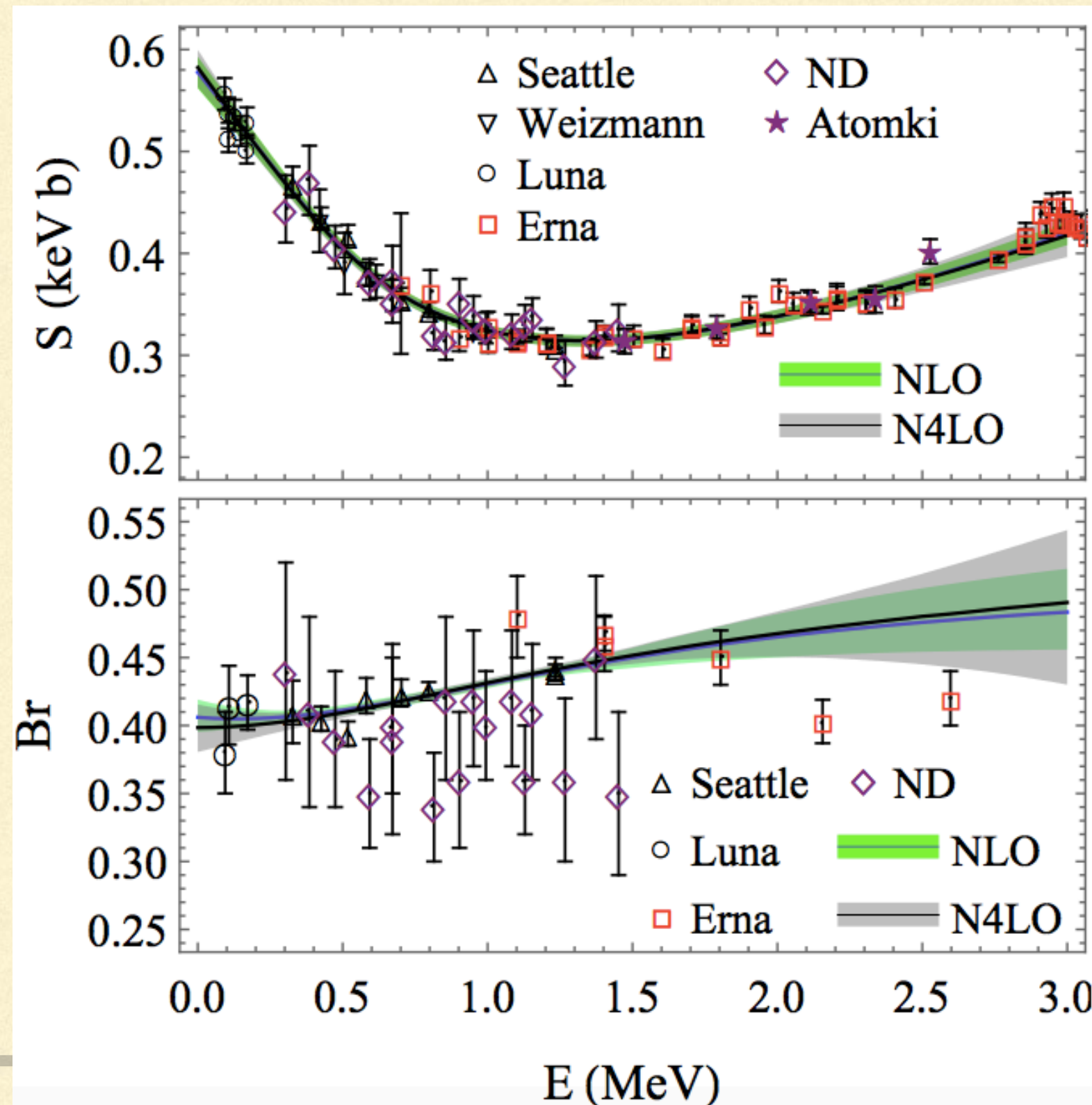
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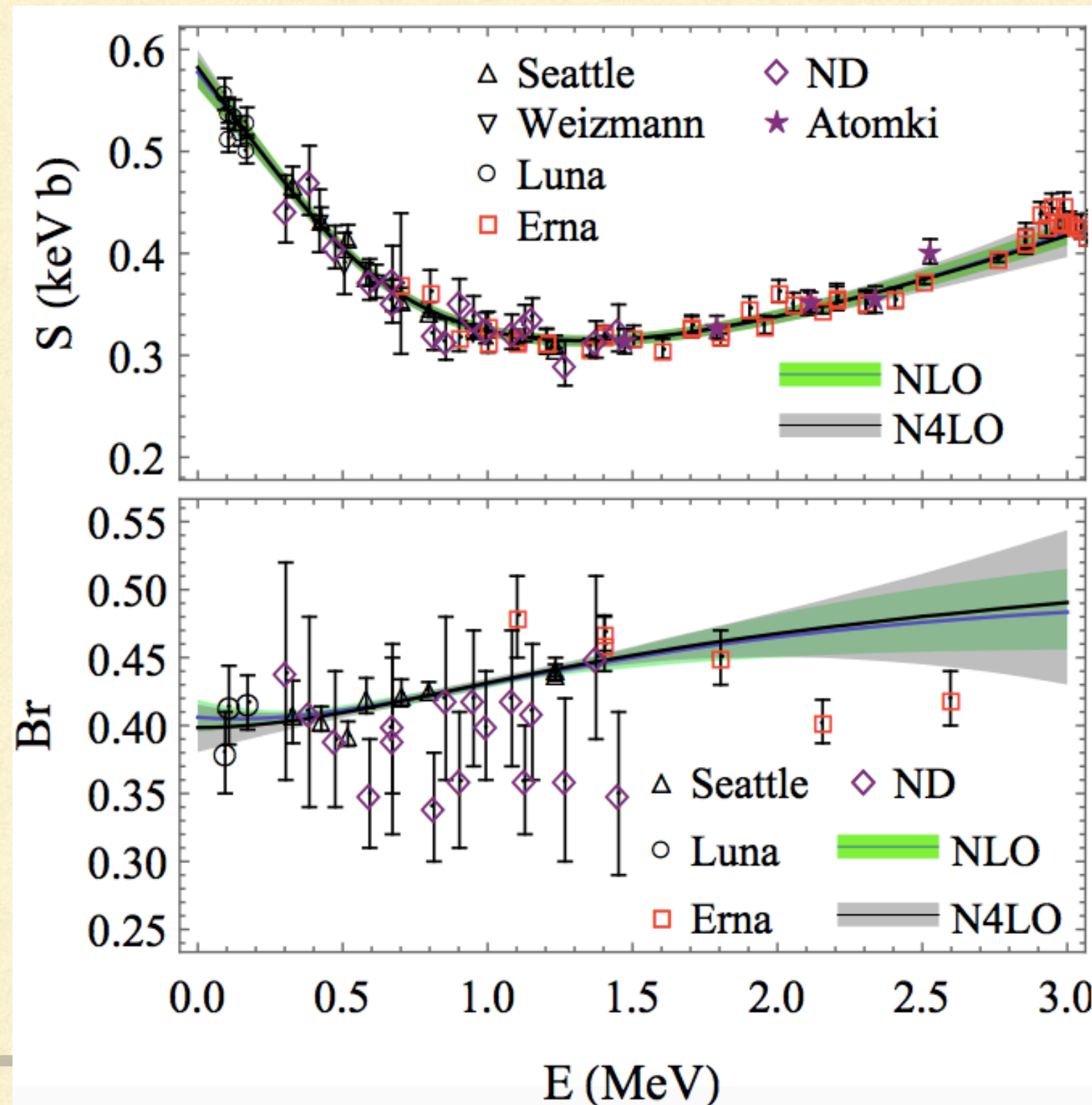
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- Bayesian evidence ratio $\cong 6$ for NLO cf. N⁴LO

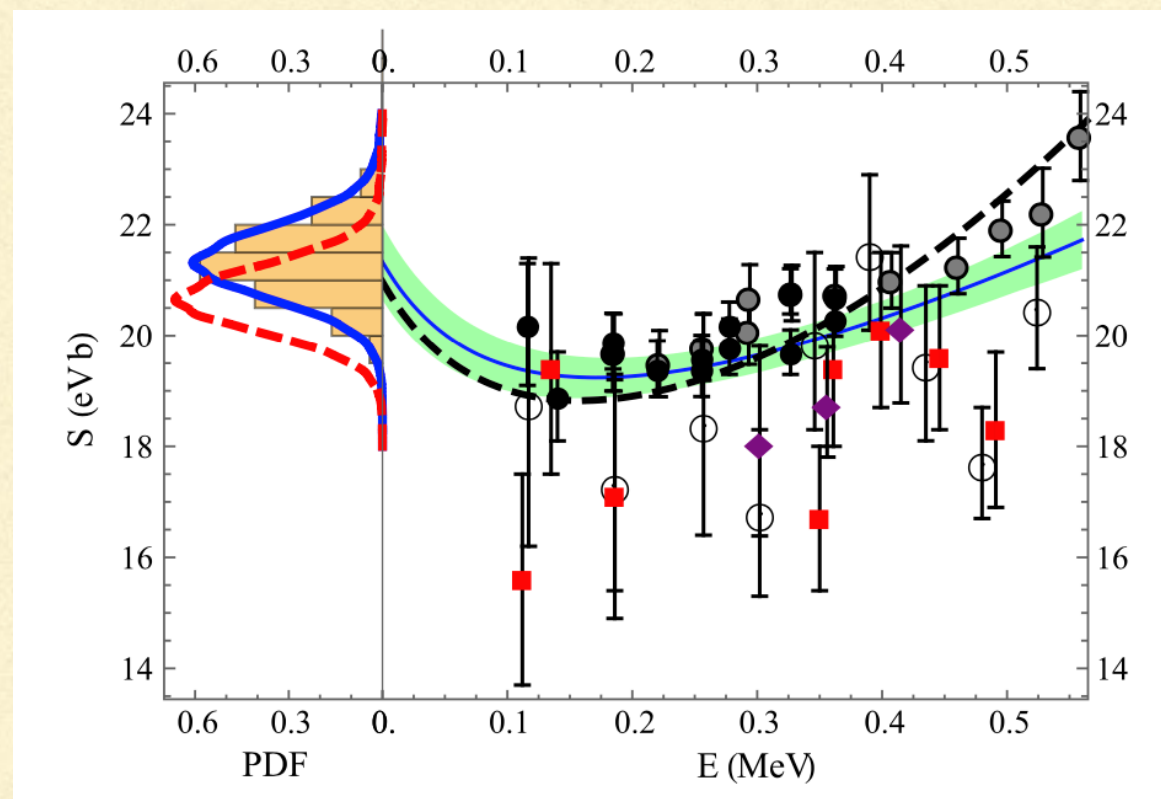


Bayesian extrapolation of solar-fusion cross sections

- Several forefront analyses of solar-fusion cross sections adopted a Bayesian approach to the extrapolation of experimental data to solar energies
- Done with EFT; R-matrix; polynomial; ab initio parameterizations of $S(E)$

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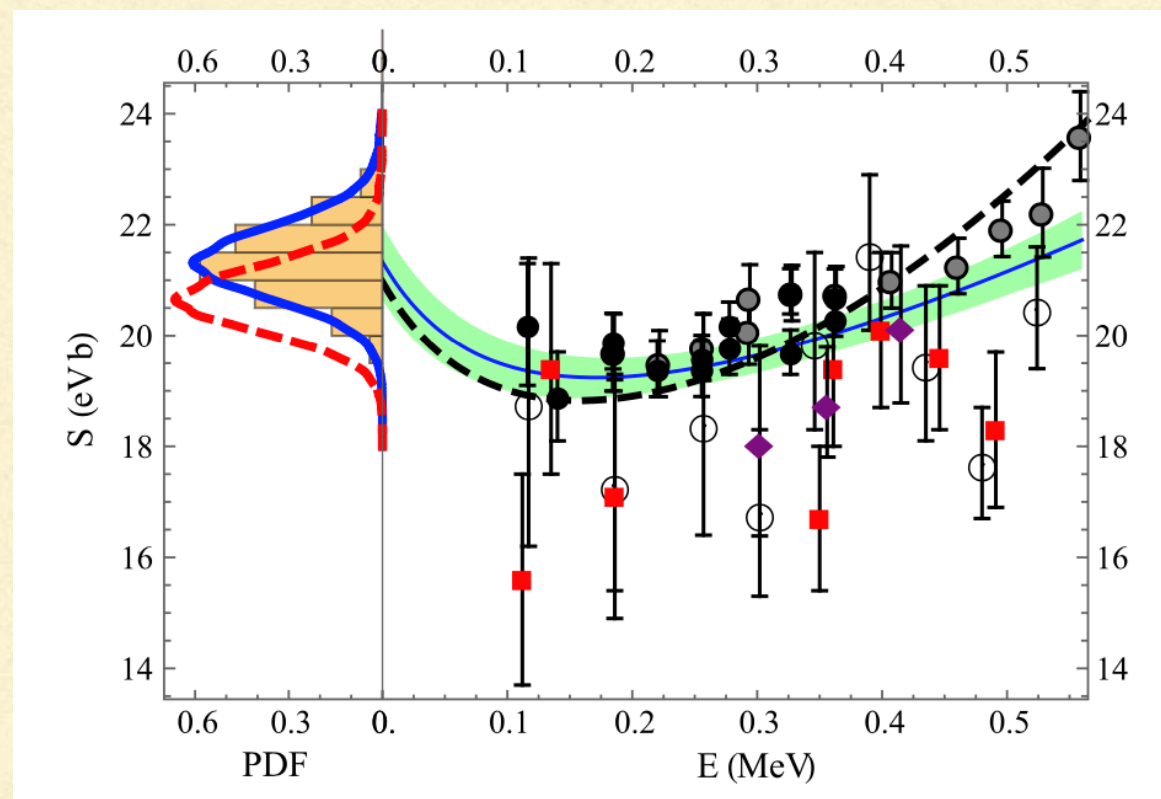
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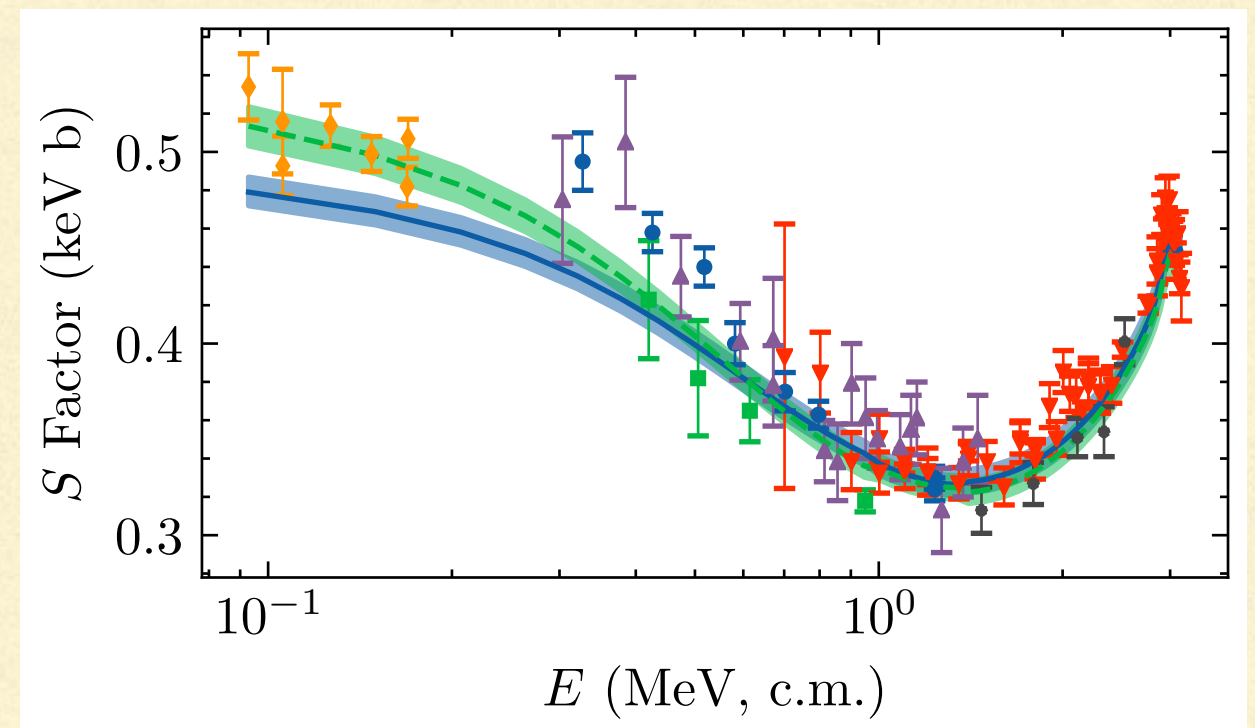
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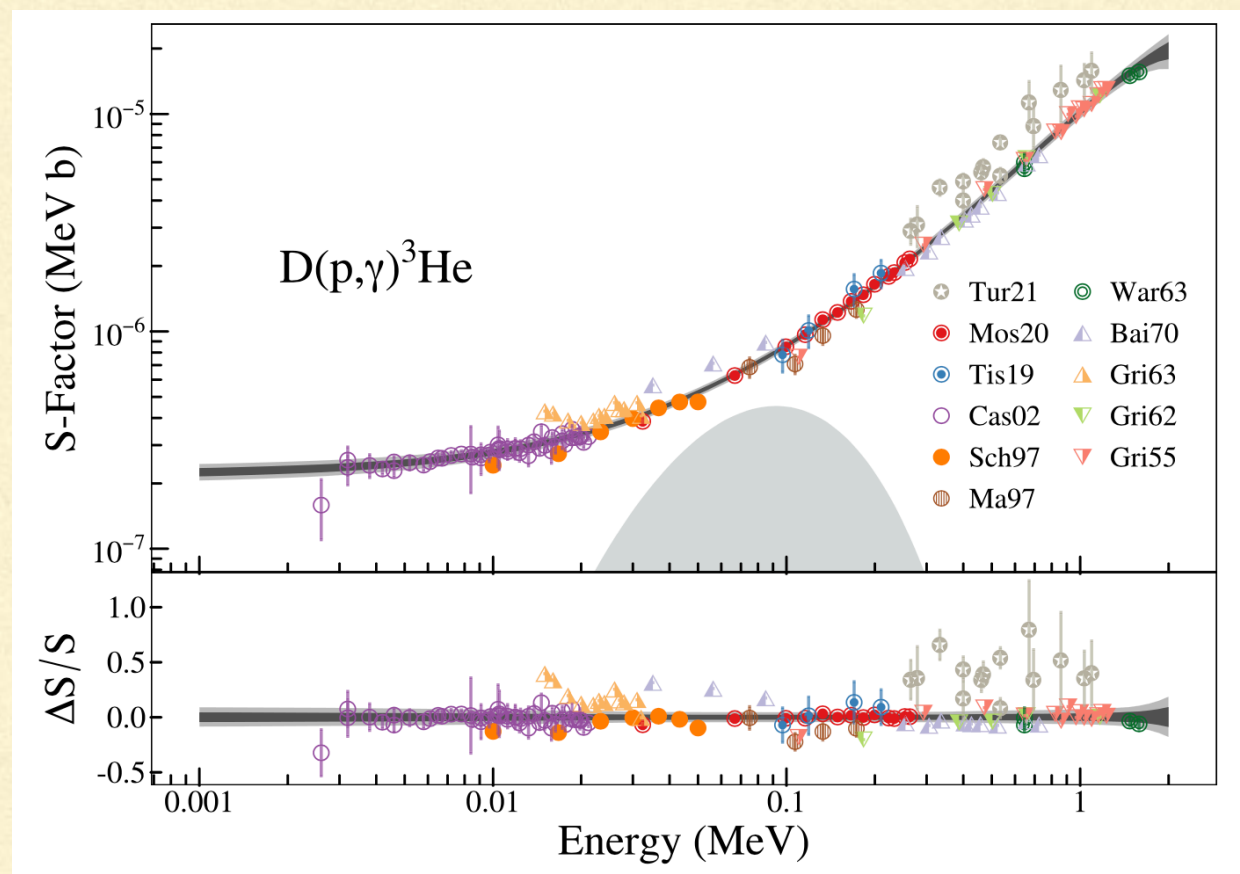
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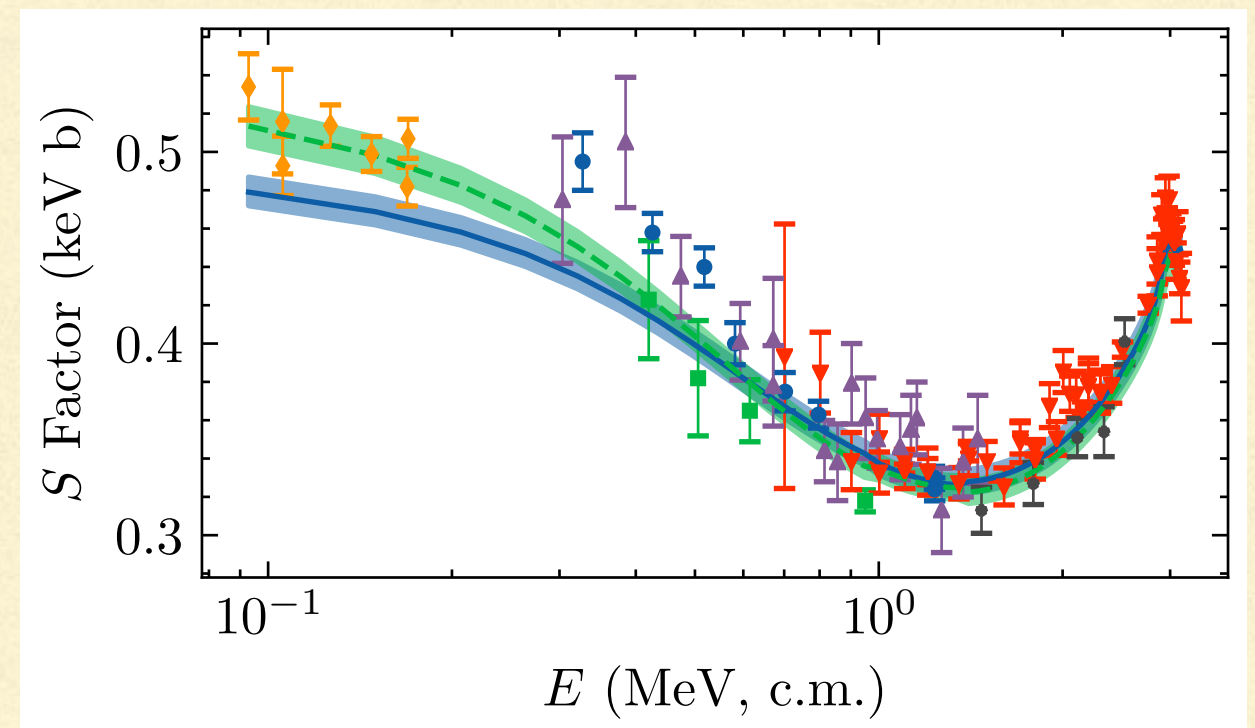
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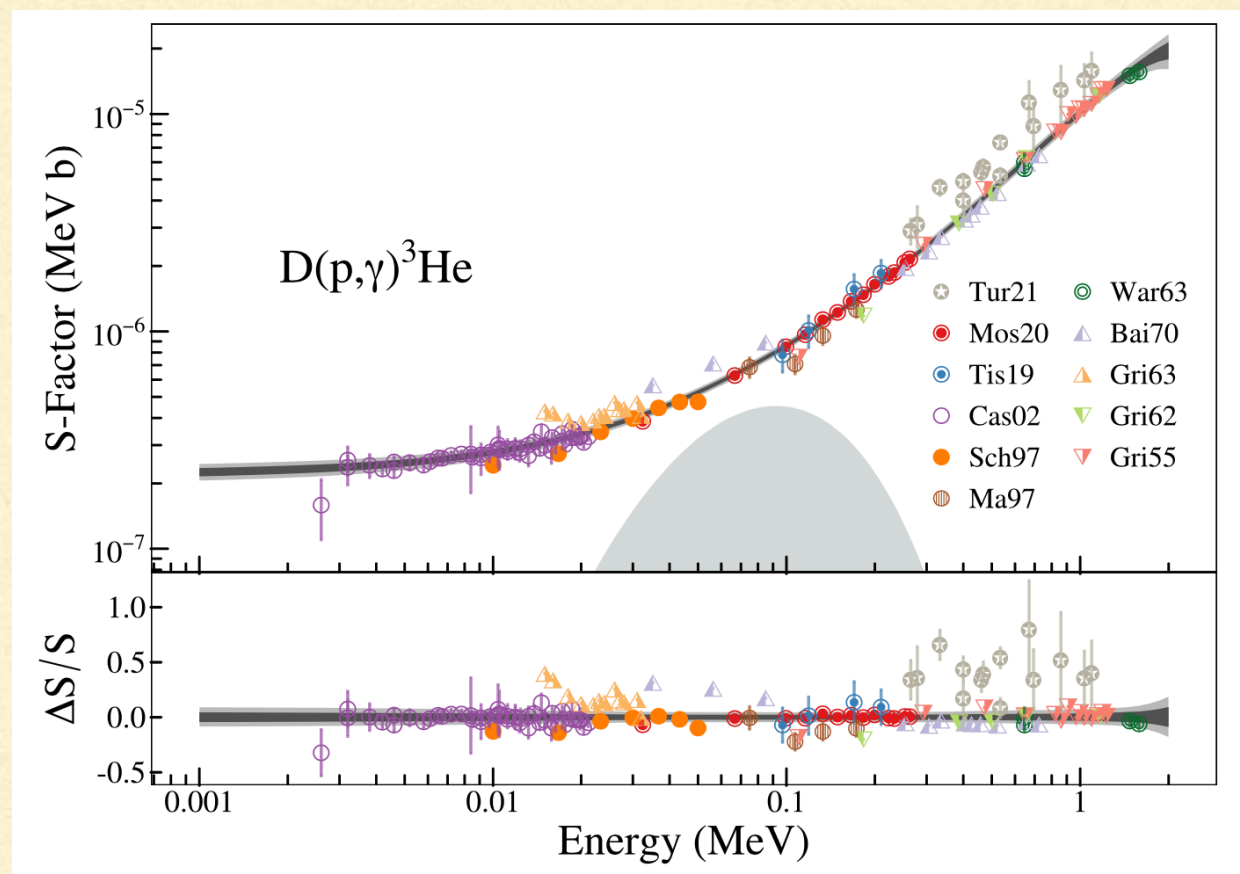
Moscato et al., *Astrophys.J.* 923 (2021) 49



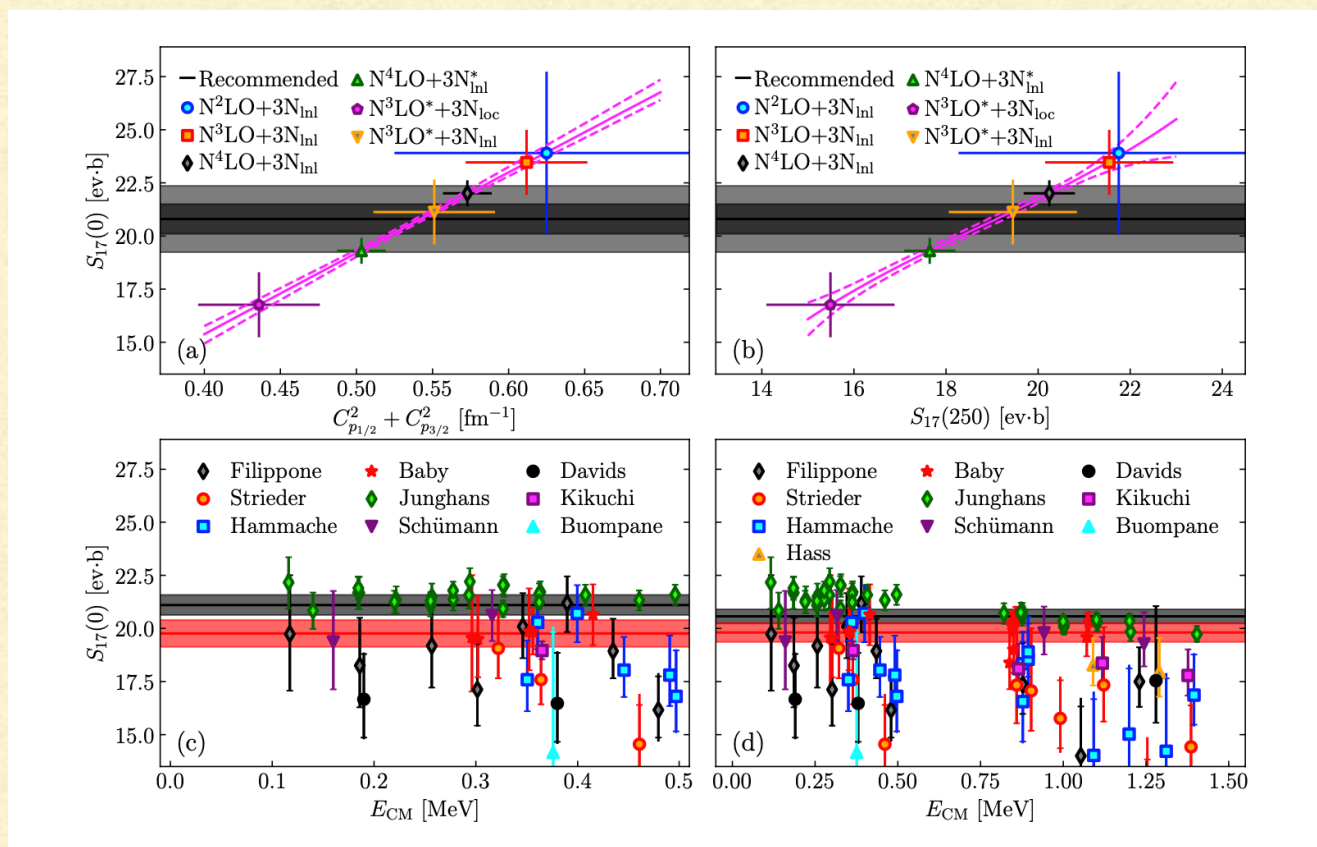
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Moscato et al., *Astrophys.J.* 923 (2021) 49



Kravvaris et al., arXiv:2202.11759

The Bayes-ics

Thomas Bayes (1701?-1761)



<http://www.bayesian-inference.com>

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↑
Posterior

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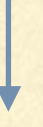
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Model evidence

Typically evaluated by MCMC sampling

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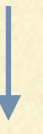
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Posterior



Model evidence

Typically evaluated by MCMC sampling

Marginalization: $\text{pr}(x|\text{data}, I) = \int dy \text{pr}(x, y|\text{data}, I)$

Allows us to integrate out “nuisance” parameters, evaluate derived quantities

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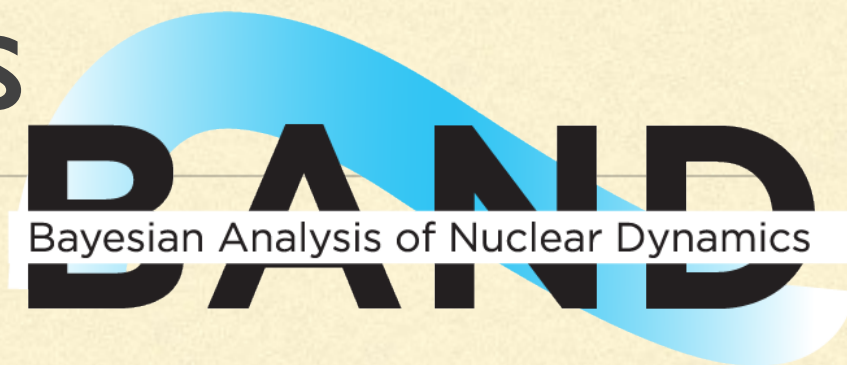
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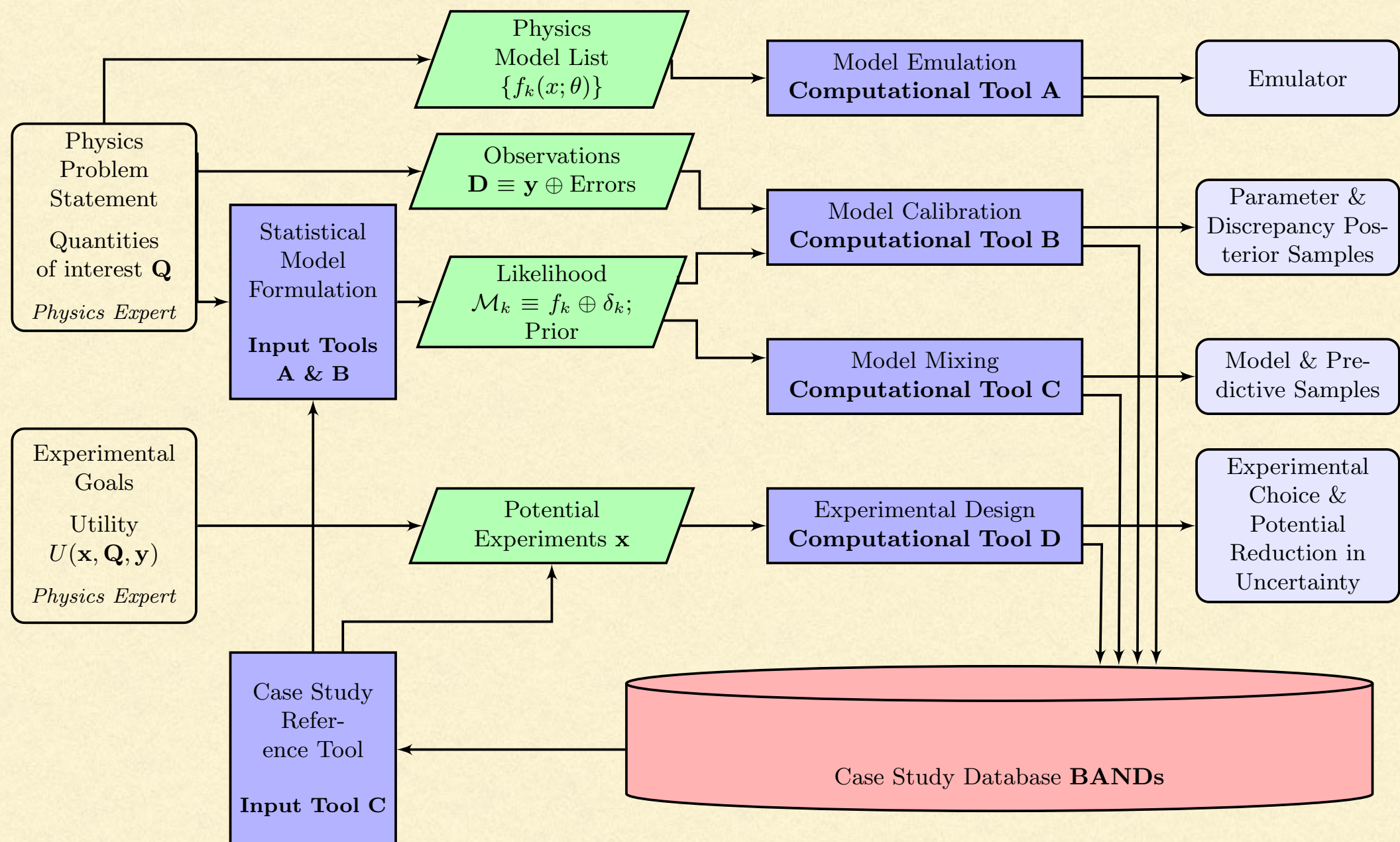
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 - Not just experimental imperfections either! Theory imperfections can be accounted for too
-

For all your Bayesian needs

Cyberinfrastructure
Framework funded by
NSF's OAC



<https://bandframework.github.io/>



EFT treatment of $^3\text{He} + ^4\text{He}$ scattering

Poudel, Phillips, JPG (2022)

- Analyze SONIK data, Barnard data, and Boykin et al. A_y data

- Using Halo EFT to N3LO, $\mathcal{O}(Q^3)$

$$Q = \frac{(p, q)}{\Lambda}$$

- $1/2^+$: a_0, r_0, P_0

- $1/2^-, 3/2^-$: a_I, r_I, P_I ($\Leftrightarrow E_{7\text{Be}}, \text{ANC}, P_I$)

$$\Lambda = 200 \text{ MeV}$$

- $7/2^-$: Resonance at $E_{\text{cm}} = 2.98 \text{ MeV}$ with fitted Γ (R-matrix form)

- Likelihood: includes theory uncertainty based on convergence pattern of EFT expansion.

$$\mathcal{L} \propto \exp \left(-\frac{1}{2} \vec{r}^T (\Sigma^{\text{th}} + \Sigma^{\text{exp}}) \vec{r} \right) \quad \Sigma_{\alpha\beta}^{\text{th}} = (\Delta y)_{\alpha} (\Delta y)_{\beta}$$

$$r_{j\alpha} = y_{j\alpha} - f_{\alpha} \mu(x_{j\alpha}; \theta_{\text{EFT}})$$

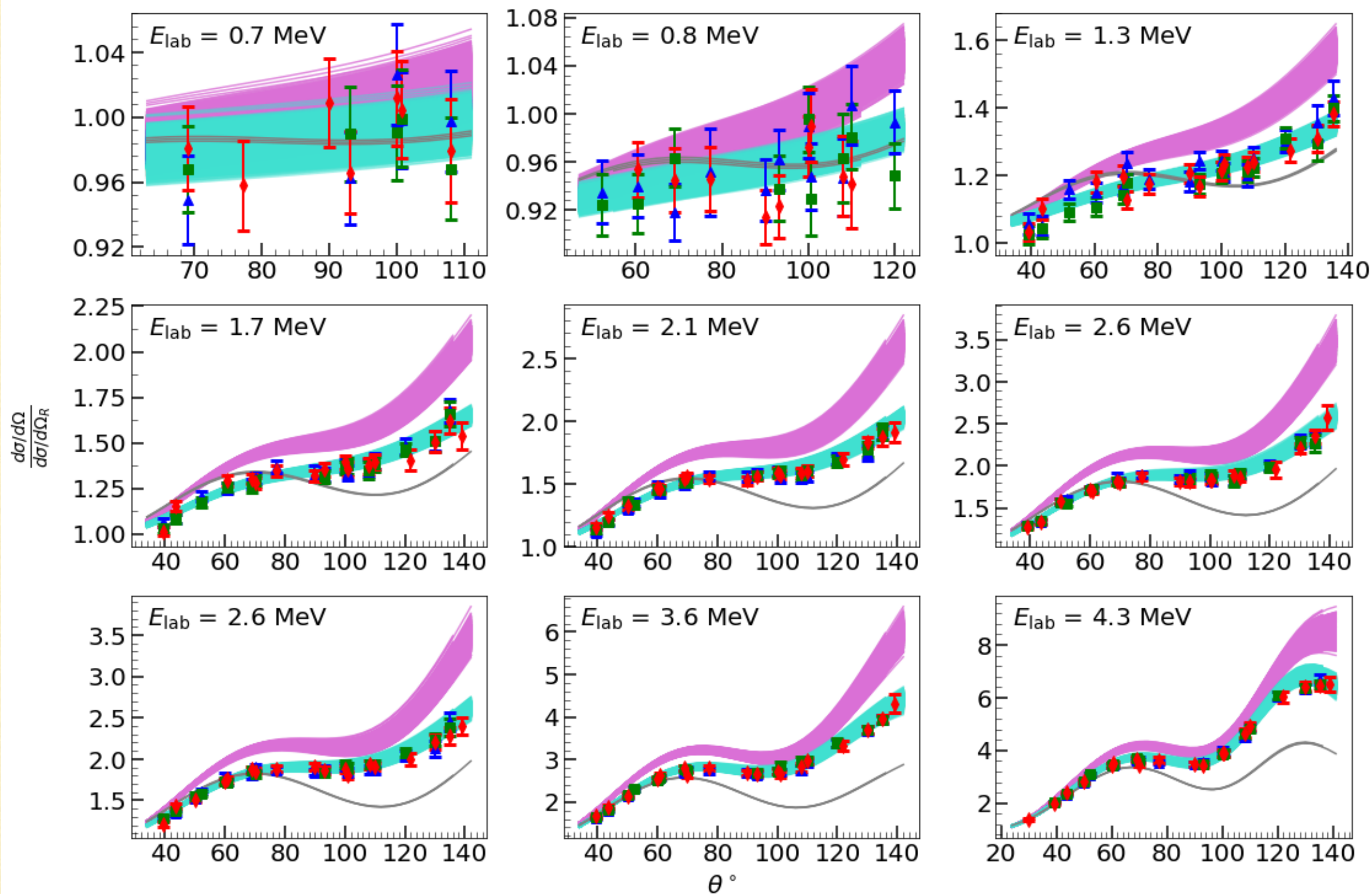
$$\Delta y_{\alpha} = y_{\text{ref}} \bar{c} Q_{\alpha}^{\nu+1}$$

Pick data sets

- Scattering data
 - SONIK*: 451 from 0.385 to 2.7 MeV
 - Barnard: 646 from 1.49 to 2.7 MeV
 - Boykin et al.: 9 A_γ data from 2.1 to 2.7 MeV
- Specify CMEs
 - SONIK: by energy
 - Barnard: 5%

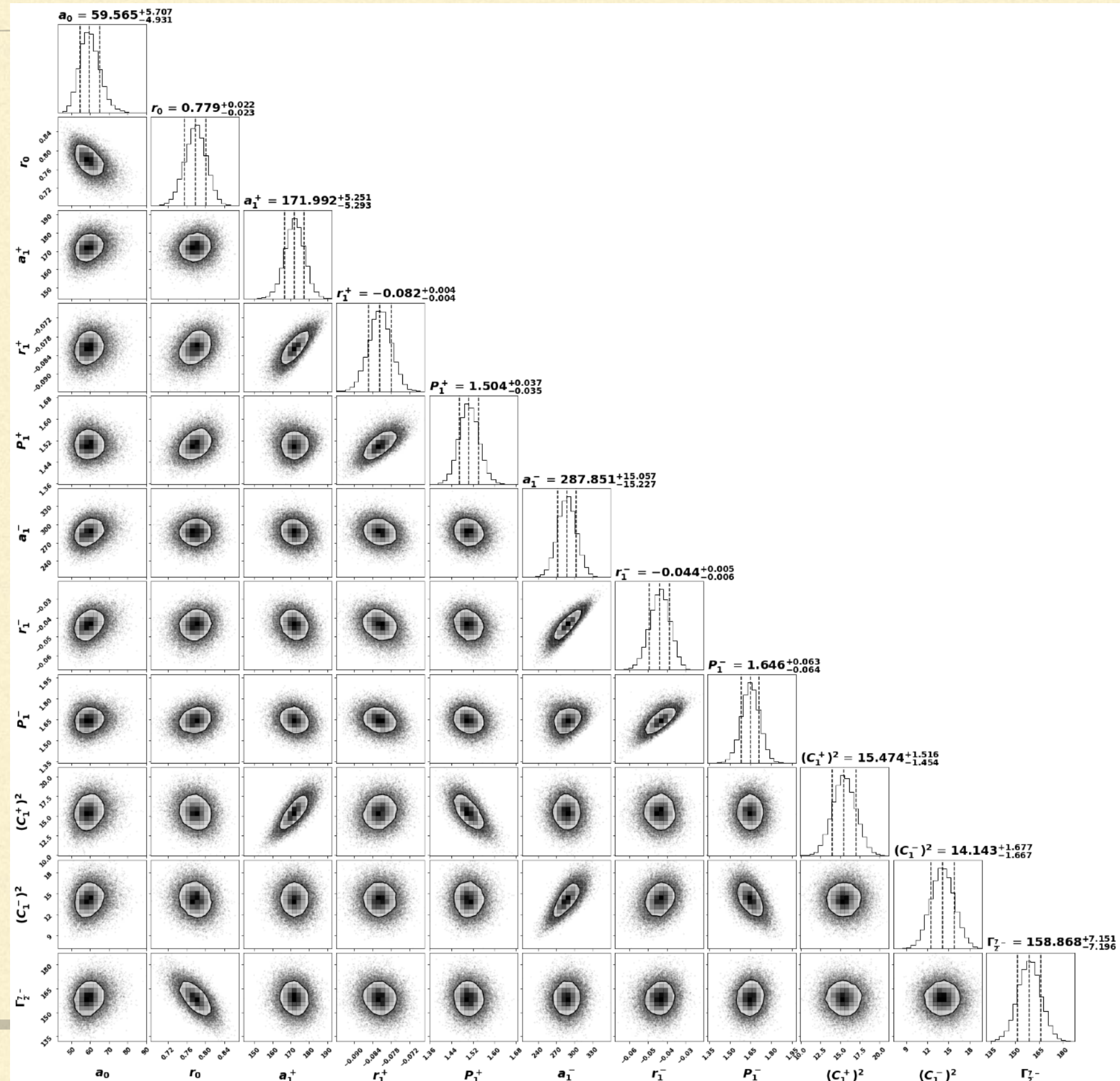
*Paneru et al., submitted to Phys. Rev. C; Paneru, Ohio University Ph.D. thesis, 2020

SONIK data, with truncation uncertainty



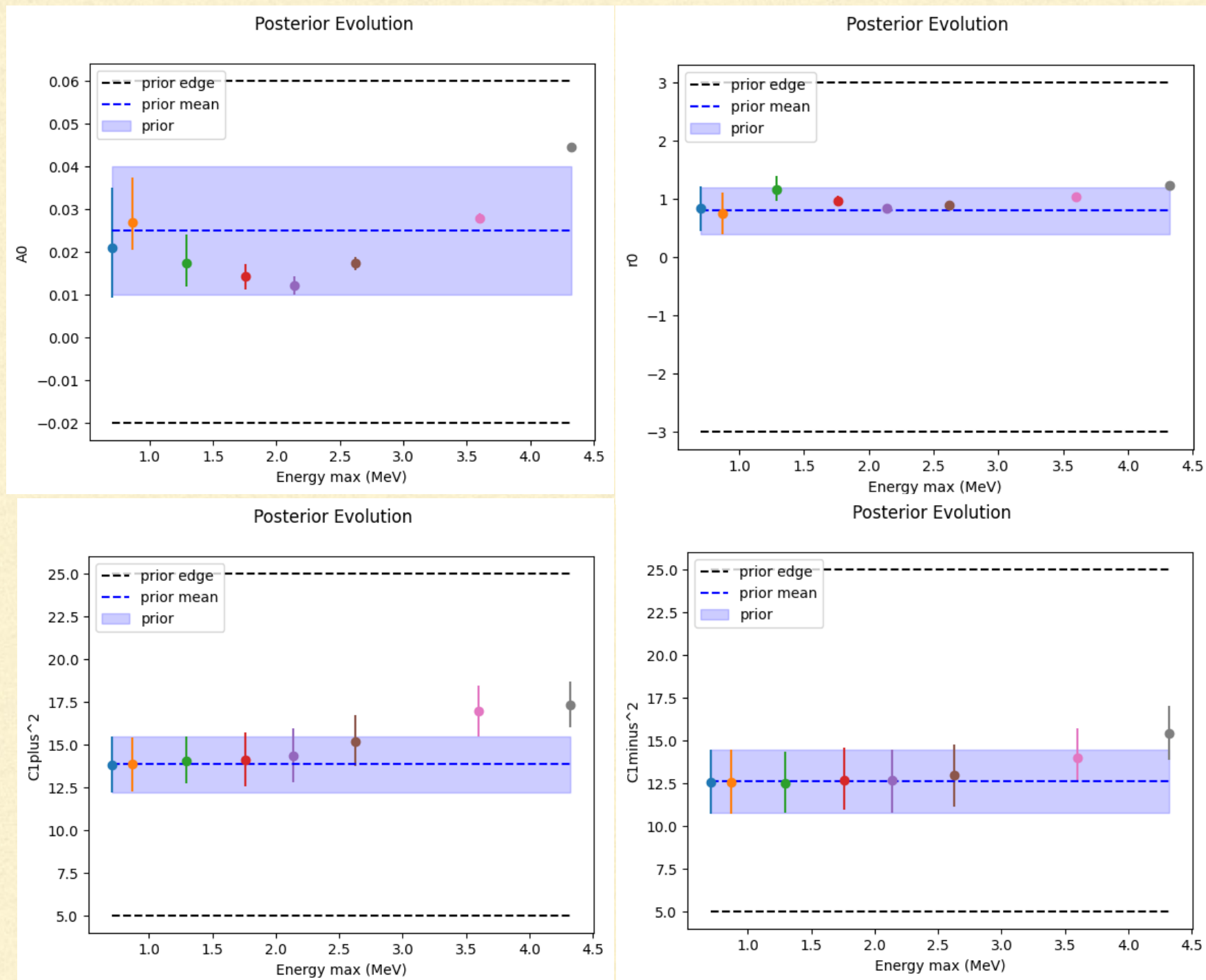
ERT parameters from scattering data

- Imposed prior on ANCs from capture data, so not *solely* from scattering data
- Consistent values:
 $C_1^{+2} = 15.5 \pm 1.5$ fm;
 $C_1^{-2} = 14.1 \pm 1.7$ fm
- $a_0 = 60^{+6}_{-5}$ fm cf.
 $a_0 = 50^{+7}_{-6}$ fm from capture and lower number from R-matrix



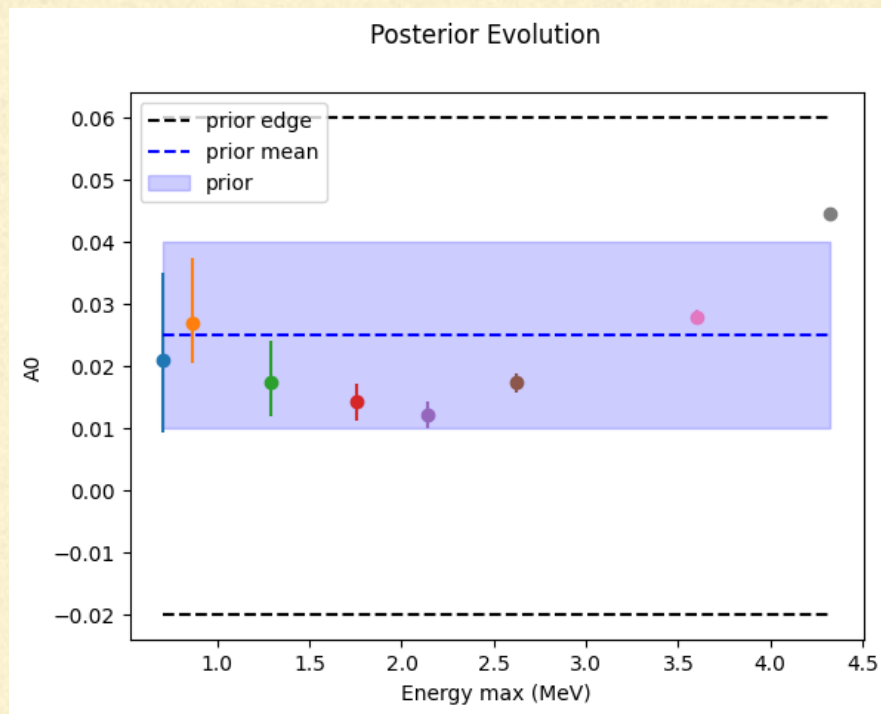
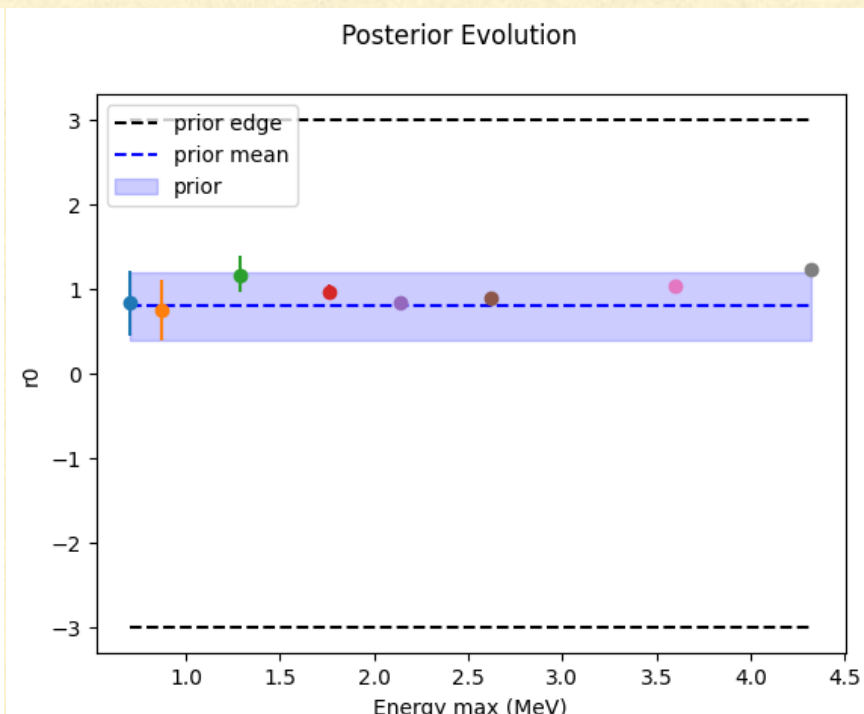
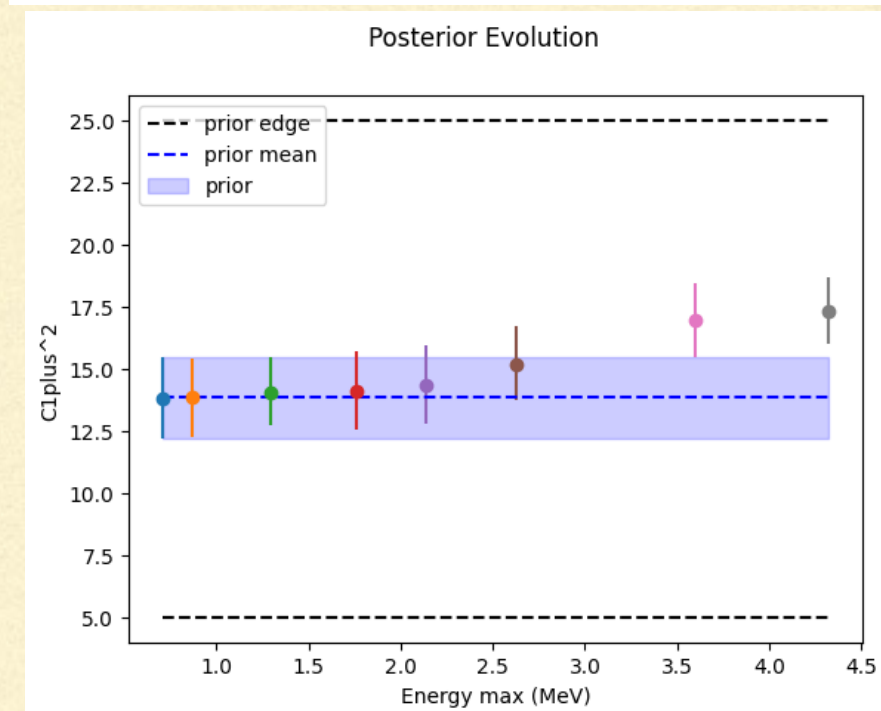
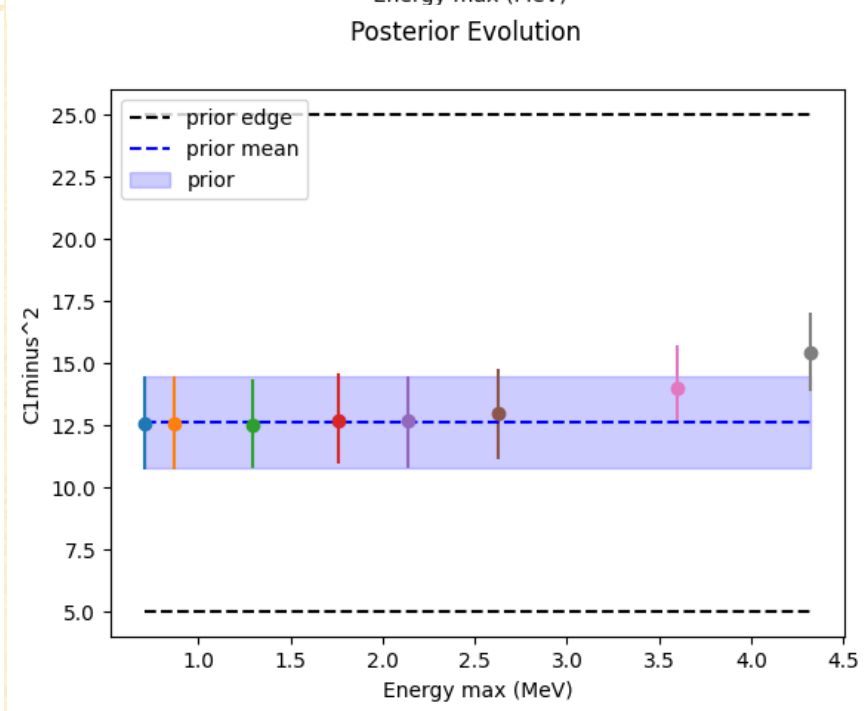
Should extrapolations to $E=0$ use only low-energy data?

Burnelis, Poudel, Brune, Phillips



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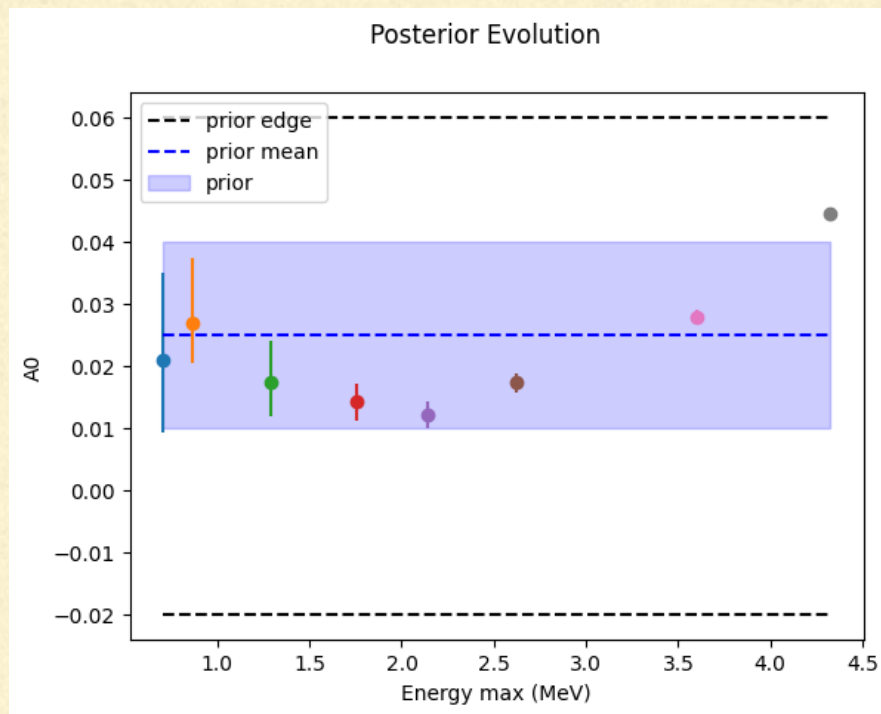
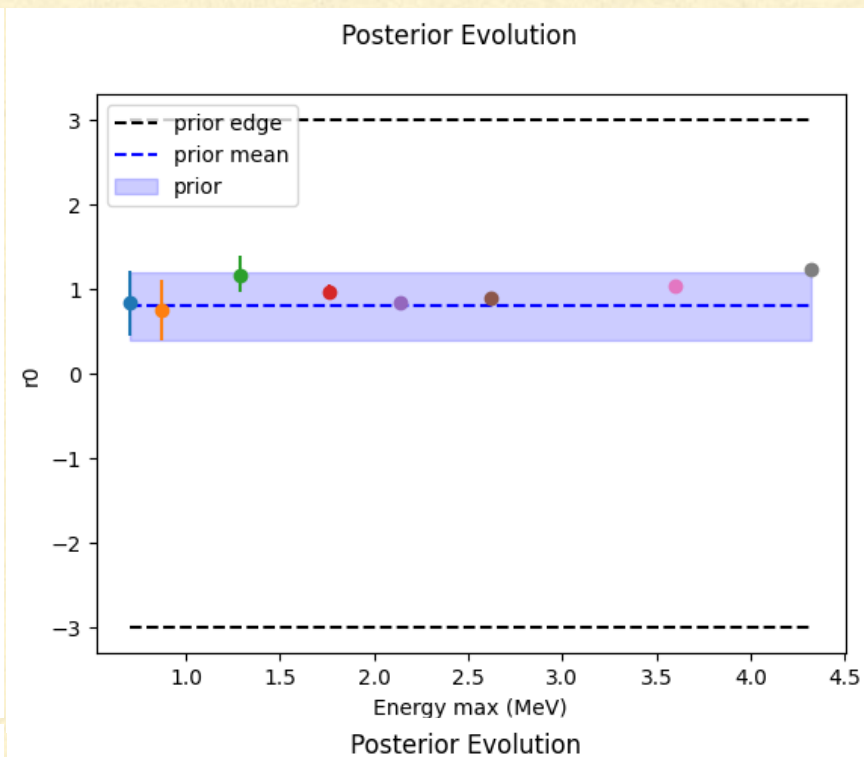
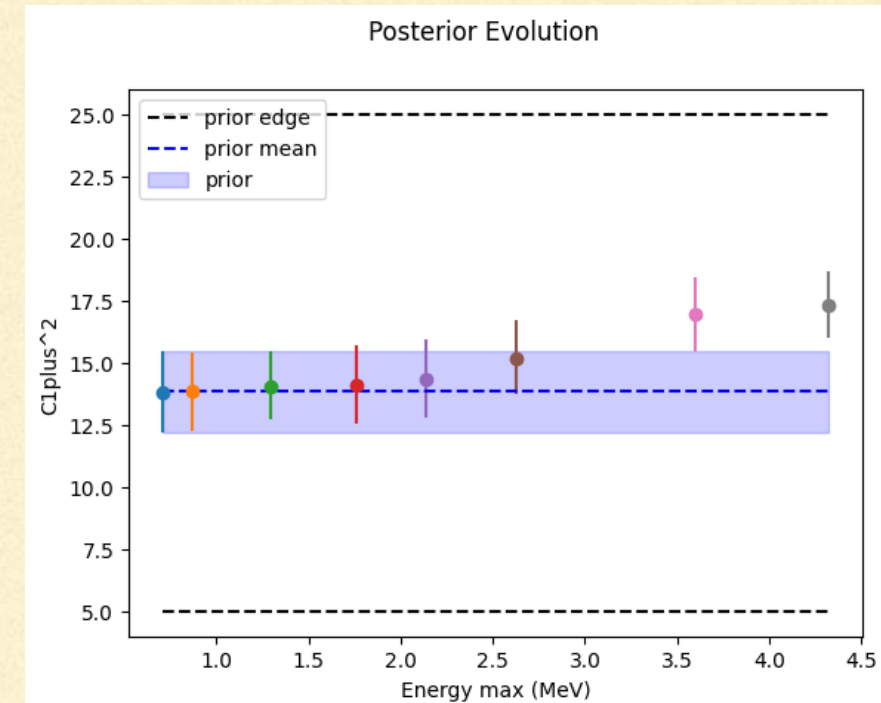
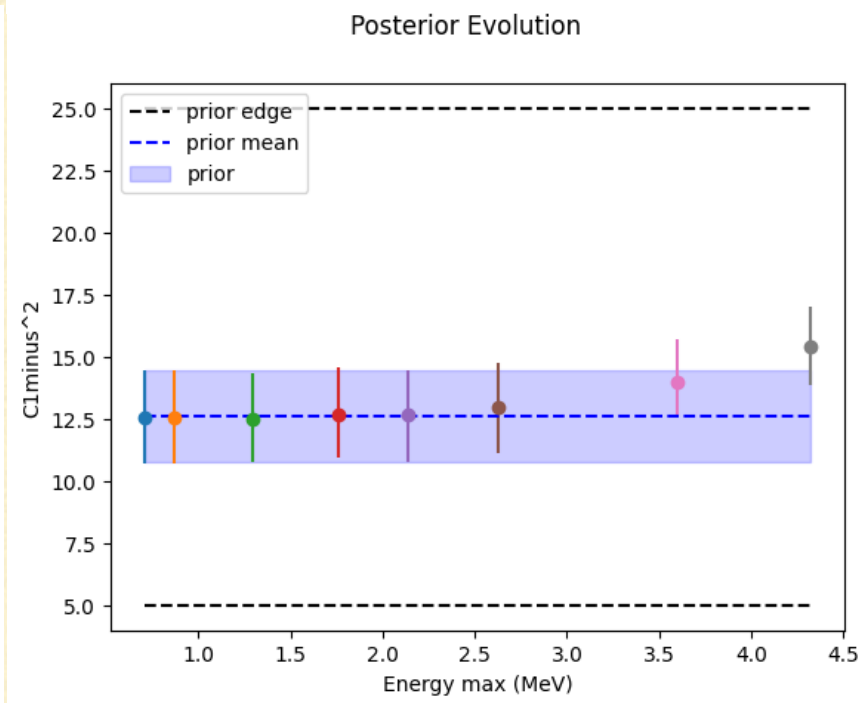
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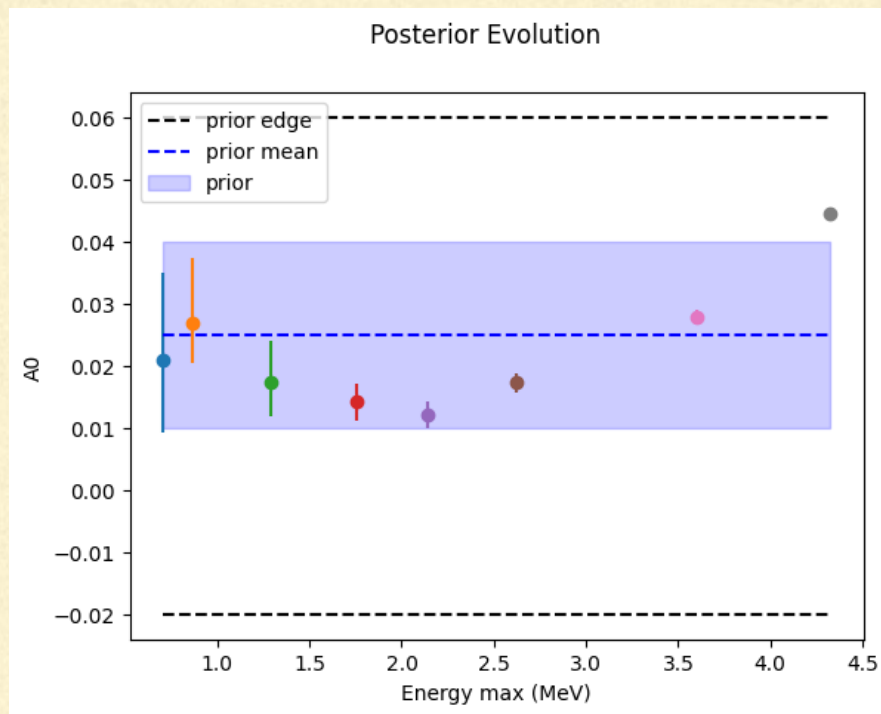
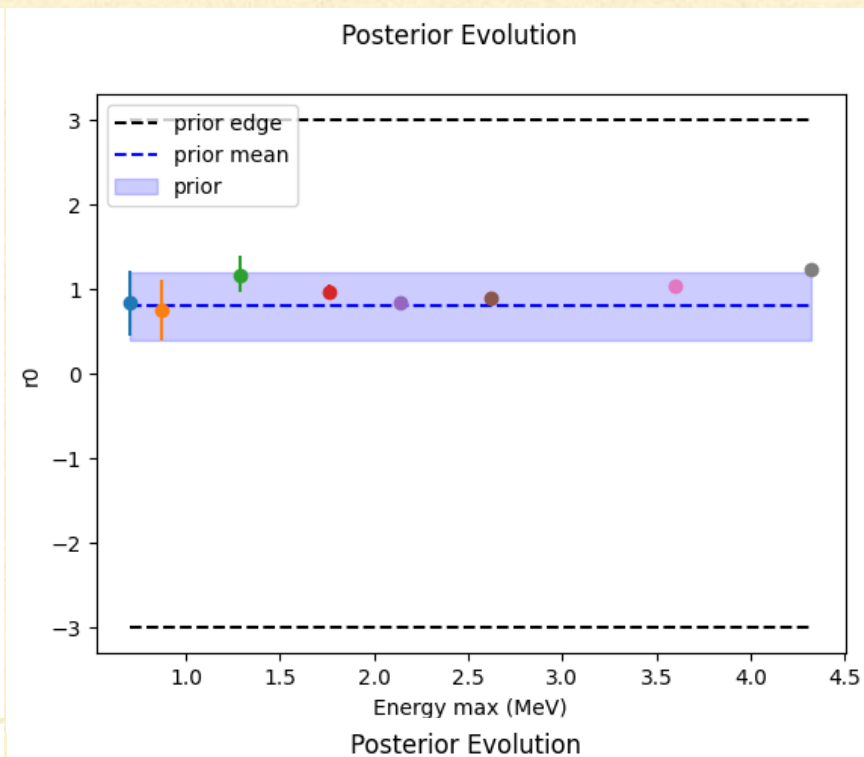
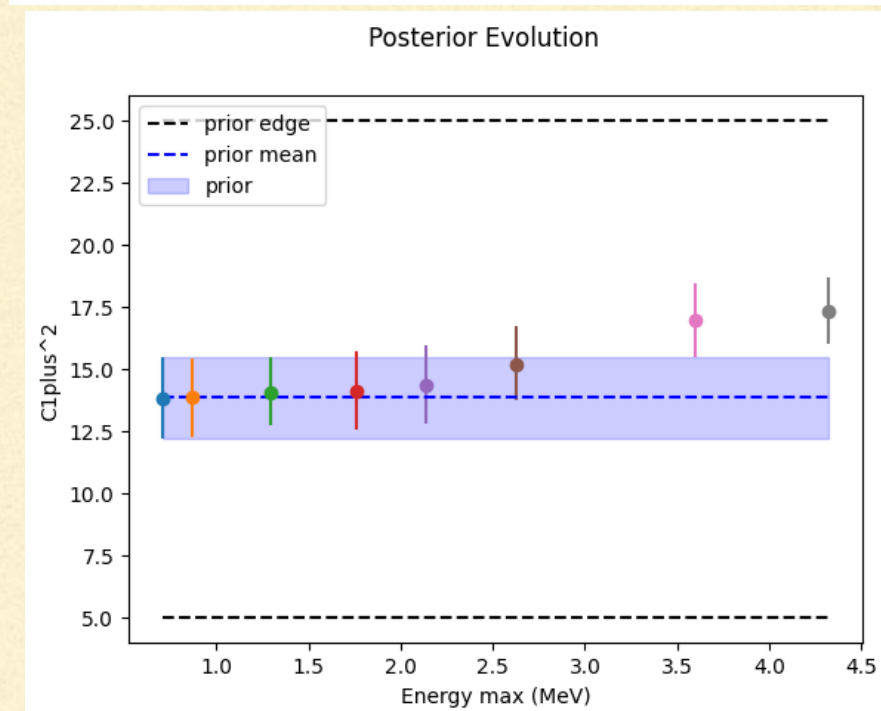
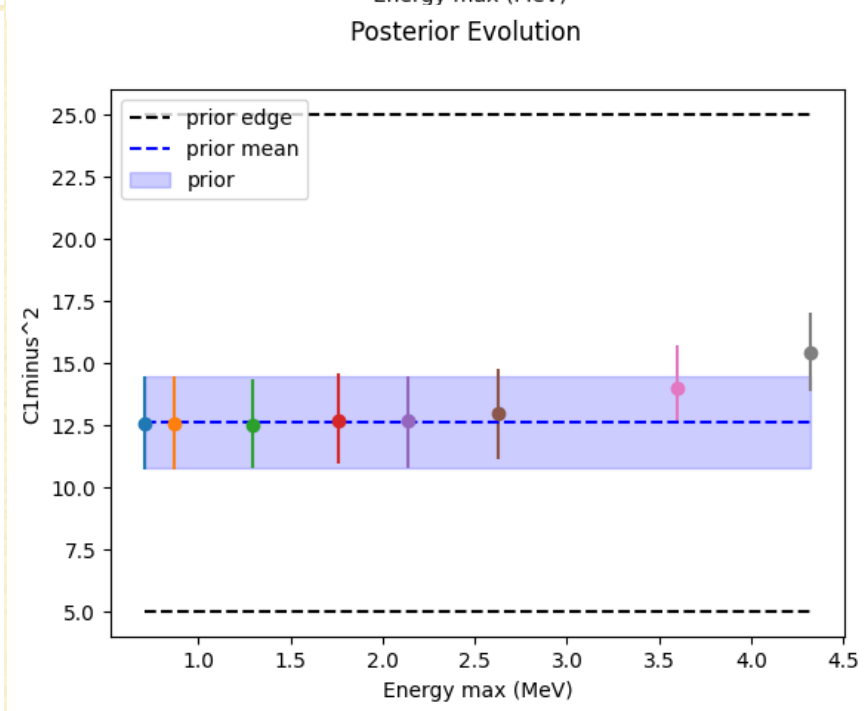
Burnelis, Poudel, Brune, Phillips



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Burnelis, Poudel, Brune, Phillips



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- But not F-waves
- Note that result **can** be pulled outside 1-sigma range of prior
- But, in this analysis at least, result is not stable with E_{max}

Summary up to now

- EFT organizes the low-energy amplitude of peripheral reactions in powers of $k R_{\text{core}}$ and $R_{\text{core}}/R_{\text{halo}}$
 - Parametric uncertainties in EFT analyses can be quantified by MCMC sampling of the Bayesian posterior and evaluating derived quantities
 - In such EFT analyses the truncation uncertainty can be quantified and its impact on extrapolations, evaluations, and parameters assessed
 - But the EFT approach so far is limited to rather simple reactions
 - ${}^7\text{Be}(p,\gamma)$ also evaluated using the same approach (also to NLO)
 - Reduced uncertainty in both reactions compared to “Solar Fusion II”
 - Higa, Rupak, Vaghani have computed capture reactions in EFT too: approach differs in some details, but overall conclusions are similar
-

Higher-order EM corrections?

- An accurate fit to the numerous low-energy proton-proton scattering data requires the inclusion of several EM effects beyond the Coulomb potential
 - Coulomb $\rightarrow$ Relativistic Coulomb; (irreducible) two-photon exchange; magnetic-moment interaction; vacuum polarization
 - Vacuum polarization has the biggest impact: it improves the total χ^2 to pp data below 30 MeV by $\simeq 100$, a 10 standard-deviation effect
Bergervoet et al. (1988)
 - These effects are also included in modern analyses of pp fusion
Schiavilla, Stoks, et al. (1998); Park et al. (2003); Marcucci et al. (2016); Acharya et al. (2023)
 - Including vacuum polarization in a semi-classical calculation of fusion changes the height of the Coulomb barrier $\Rightarrow$ affects fusion cross section
 - Estimated to alter ${}^3\text{He}(\alpha, \gamma)$ reaction rate at solar energies by 1.6%
Kamionkowski & Bahcall (1994)
-

Vacuum polarization: formalism

Poudel, Brune, Phillips, in preparation

- Provided VP potential can be treated as a perturbation, its presence can be accounted for via an additive term in the scattering amplitude:

$$f_{VP} = \frac{1}{2ip} \sum_L (2L + 1) P_L(\cos \theta) e^{2i(\sigma_L + \delta_L)} 2i\beta_L$$

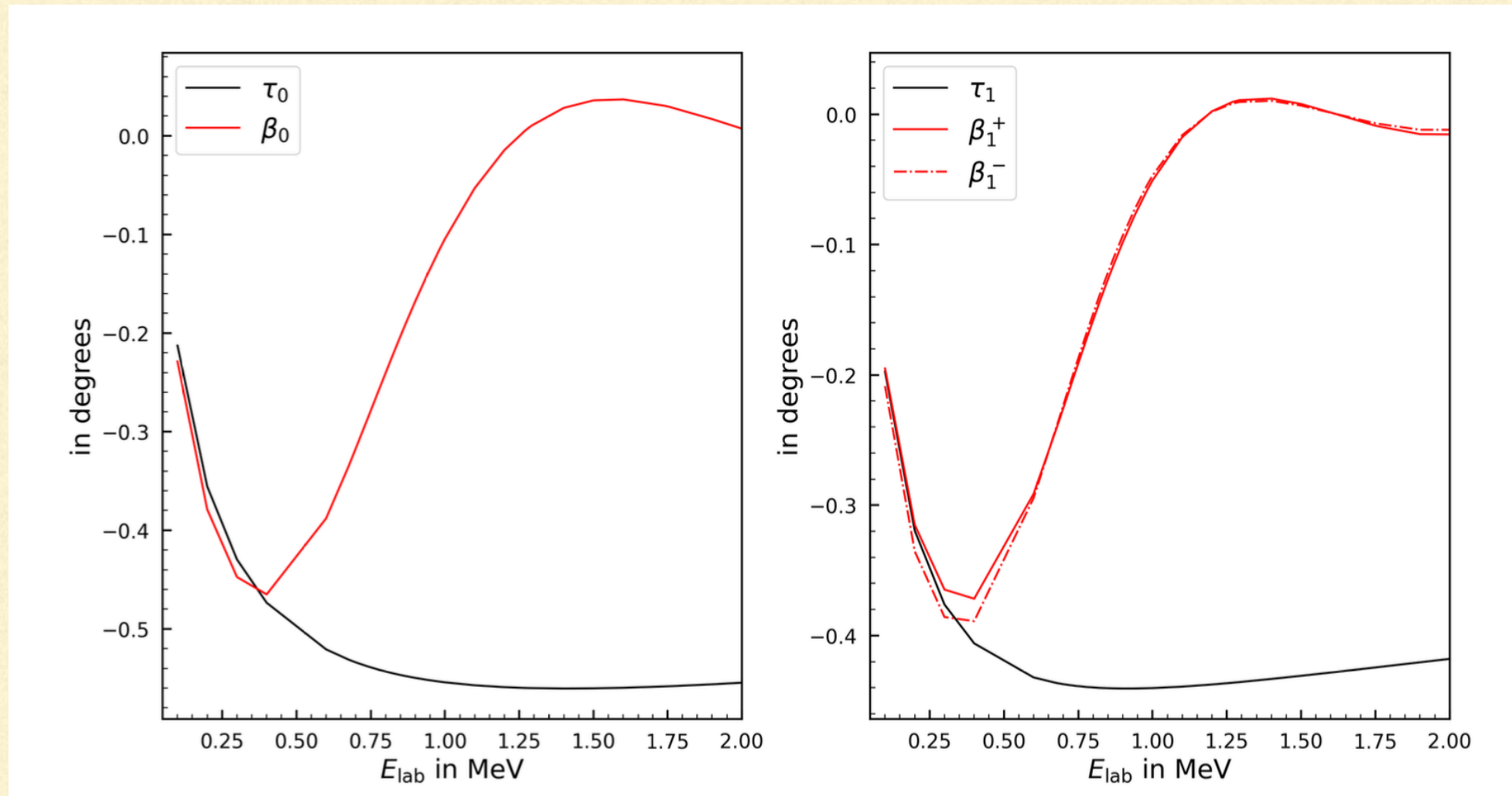
- β_L : VP phase shift in background of Coulomb and strong potential

$$\beta_L = -\frac{2m_R}{p} \int_0^\infty V_{VP}(r) (\cos \delta_L F_L(pr; \eta) + \sin \delta_L G_L(pr; \eta))^2$$

- UV divergent, but that part absorbed into polynomial behavior and so is indistinguishable from effective-range expansion for $e^{i\delta_L} \sin \delta_L$

- Additional term in ERE: $\Delta K_L \propto p^{2L} \int_0^\infty V_{VP}(r) C_L^2 u_L^2(r) - \text{divergent part}$

Vacuum polarization: results



- Phase shifts are small and go to zero for E_{lab} above about 1.25 MeV

- As $E \rightarrow 0$, $\beta_L \rightarrow \tau_L \equiv -\frac{2m_R}{p} \int_0^\infty dr V_{VP}(r) F_L^2(pr; \eta)$

Phase shift of VP
relative to
Coulomb alone

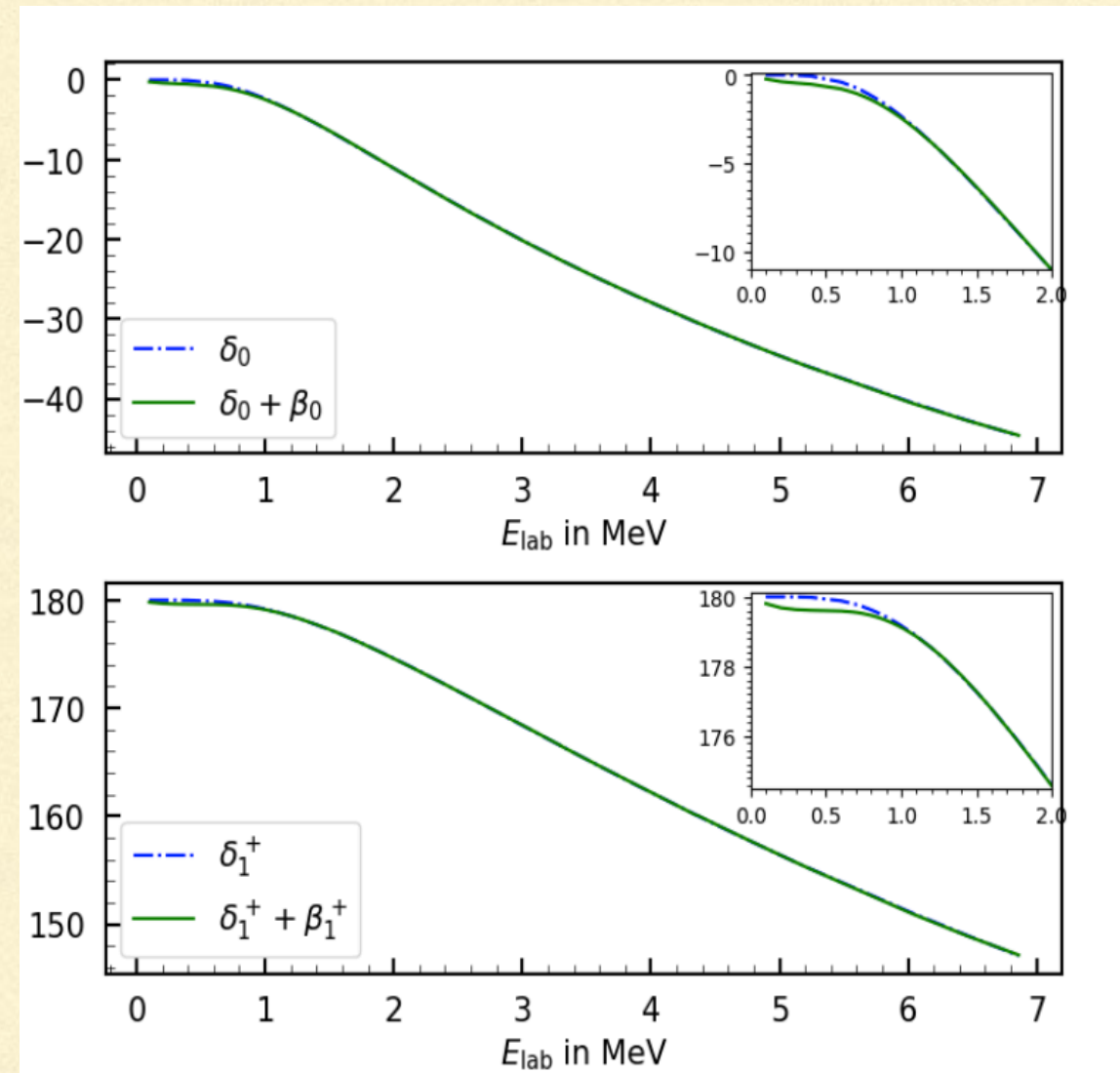
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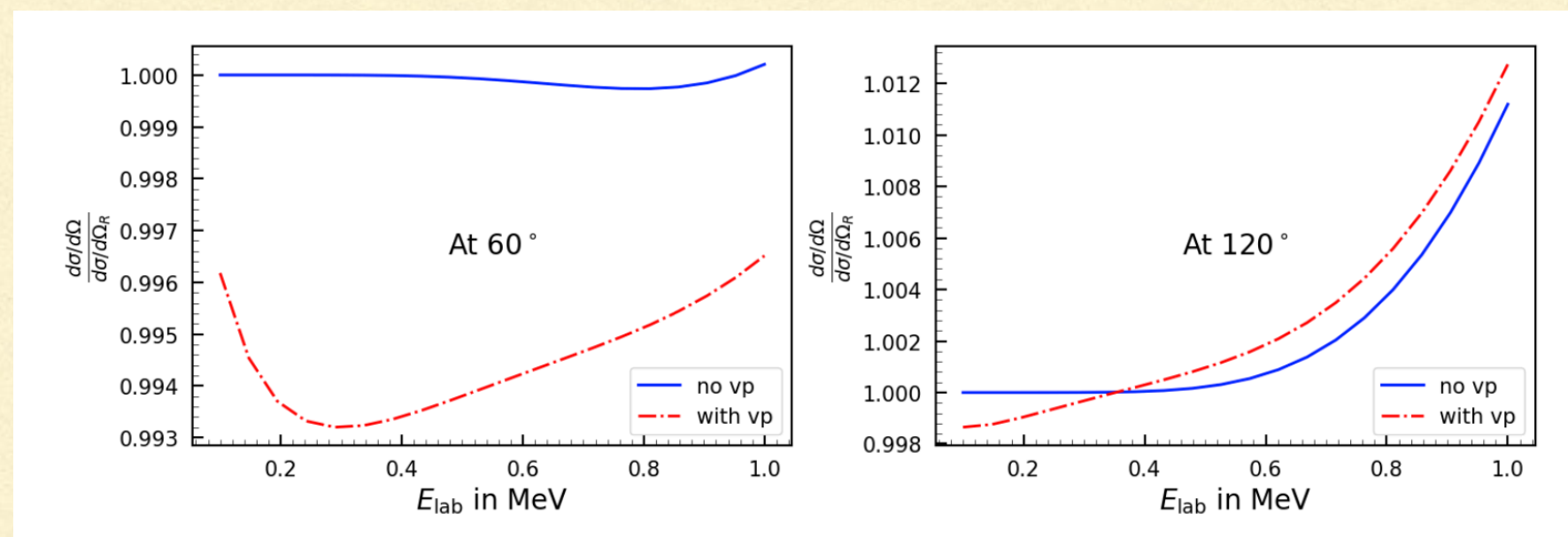
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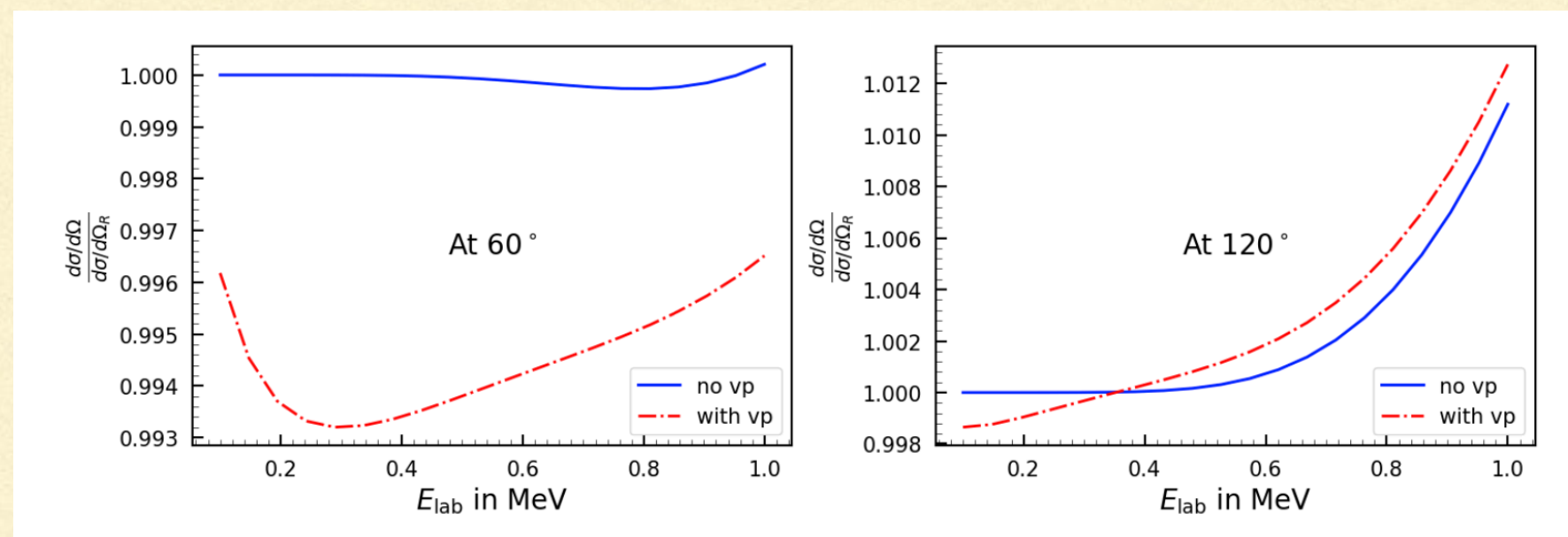
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- Assessment of effect for capture in same formalism still to be done



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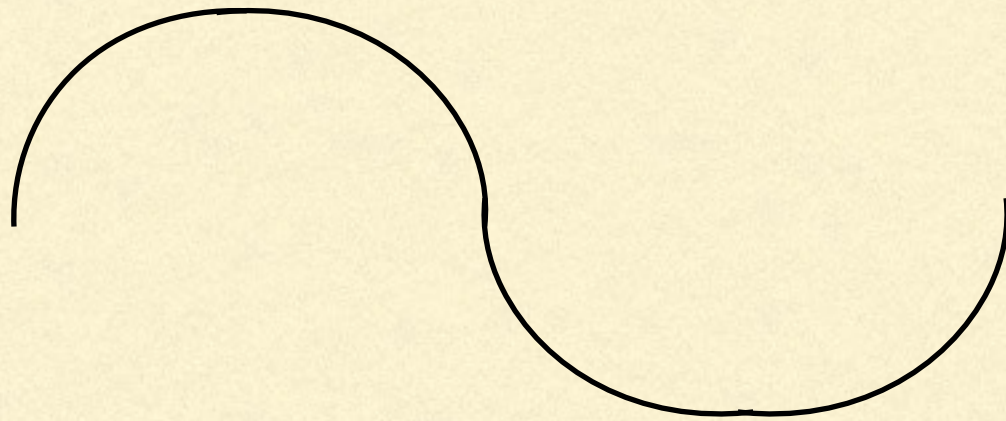
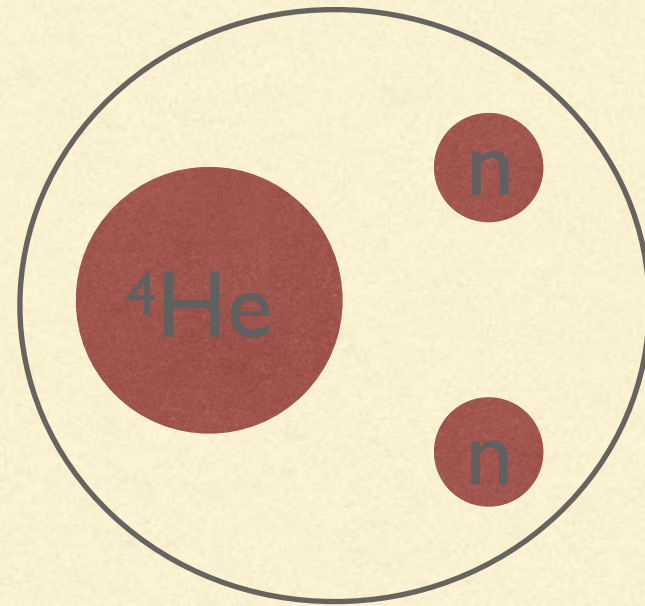
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 - “Why waste your labour on what does not satisfy?”
-

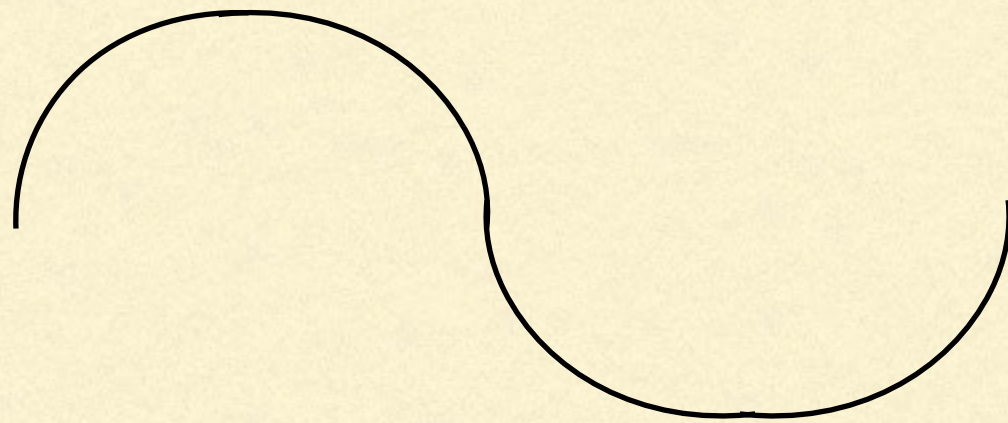
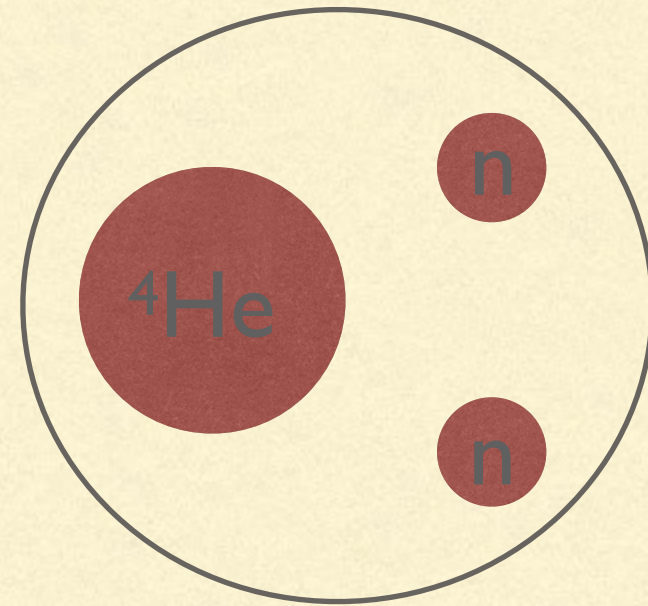
Halo EFT

$$\lambda \gg R_{\text{core}}; \lambda \lesssim R_{\text{halo}}$$



Halo EFT

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- Define $R_{\text{halo}} = \langle r^2 \rangle^{1/2}$. Seek EFT expansion in $R_{\text{core}}/R_{\text{halo}}$. Valid for $\lambda \lesssim R_{\text{halo}}$
- Typically $R \equiv R_{\text{core}} \sim 2$ fm. And since $\langle r^2 \rangle$ is related to the neutron separation energy we are looking for systems with neutron separation energies less than 1 MeV
- By this definition the deuteron is the lightest halo nucleus, and the pionless EFT for few-nucleon systems is a specific case of halo EFT

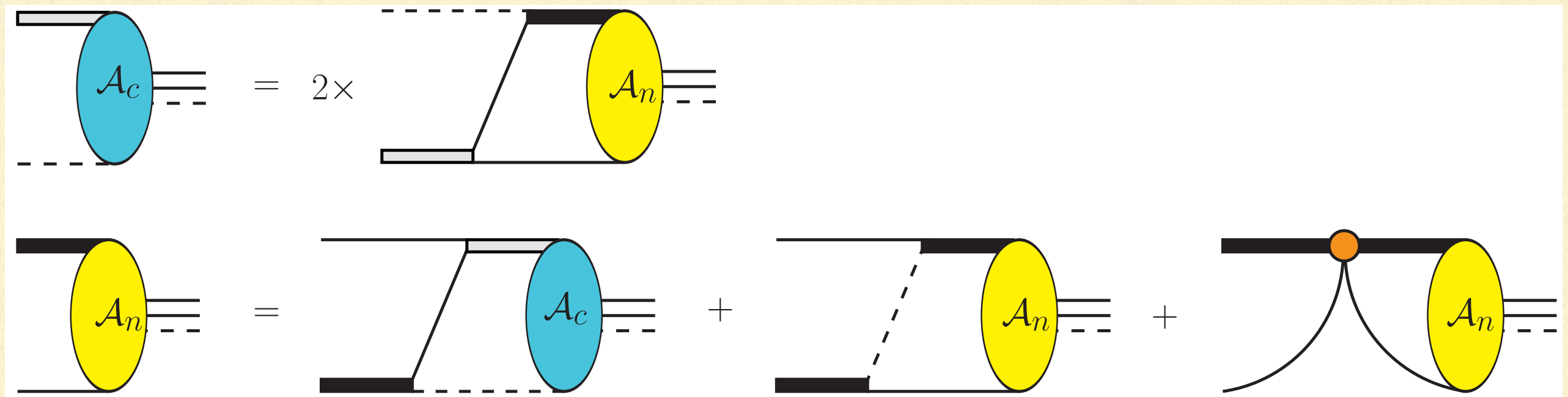
Equations for s-wave $2n$ halo

Canham, Hammer (2008)

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Canham, Hammer (2008)

- Core- n and n - n contact interactions at leading order: solve 3B problem

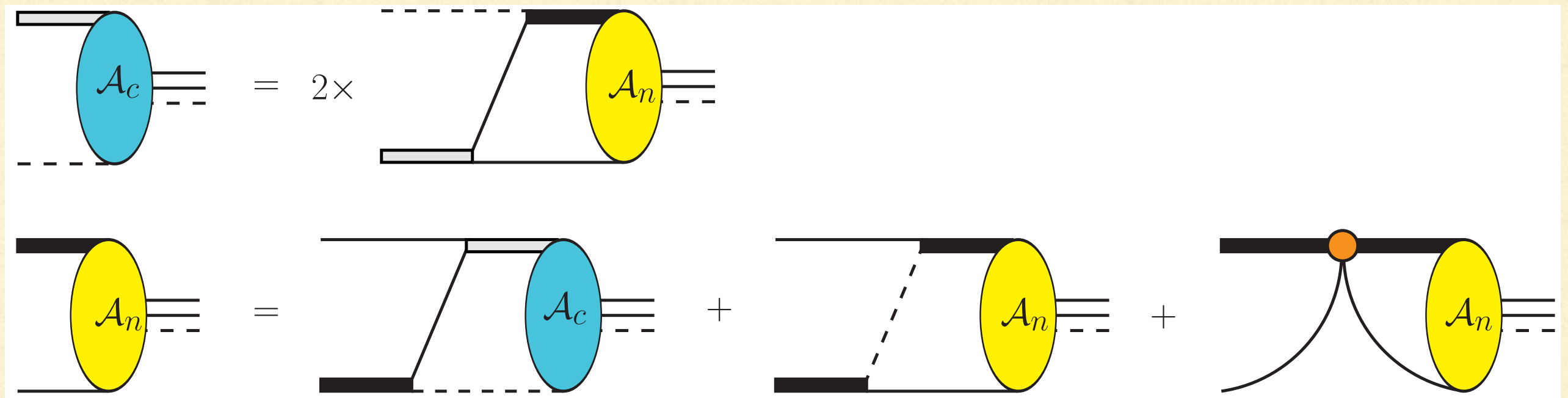


- (cn)- n contact interaction to stabilize three-body system

Equations for s-wave 2n halo

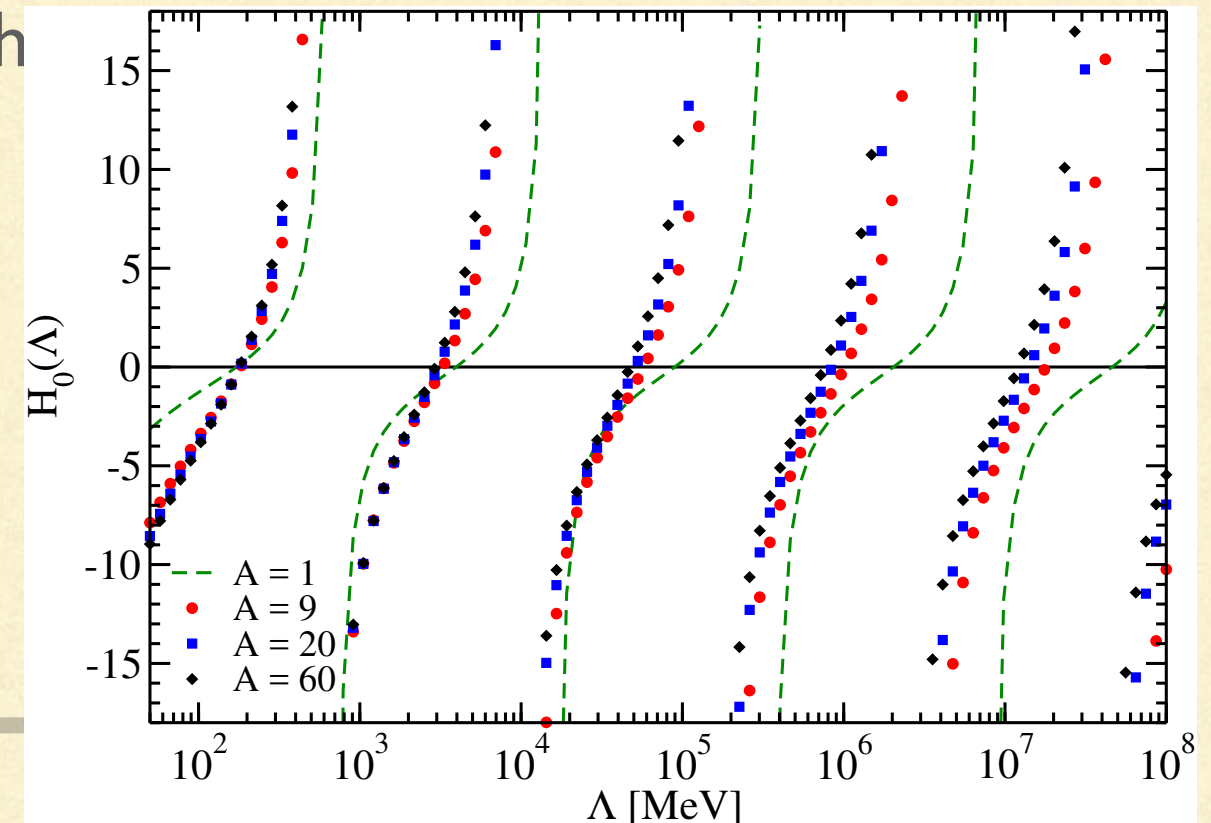
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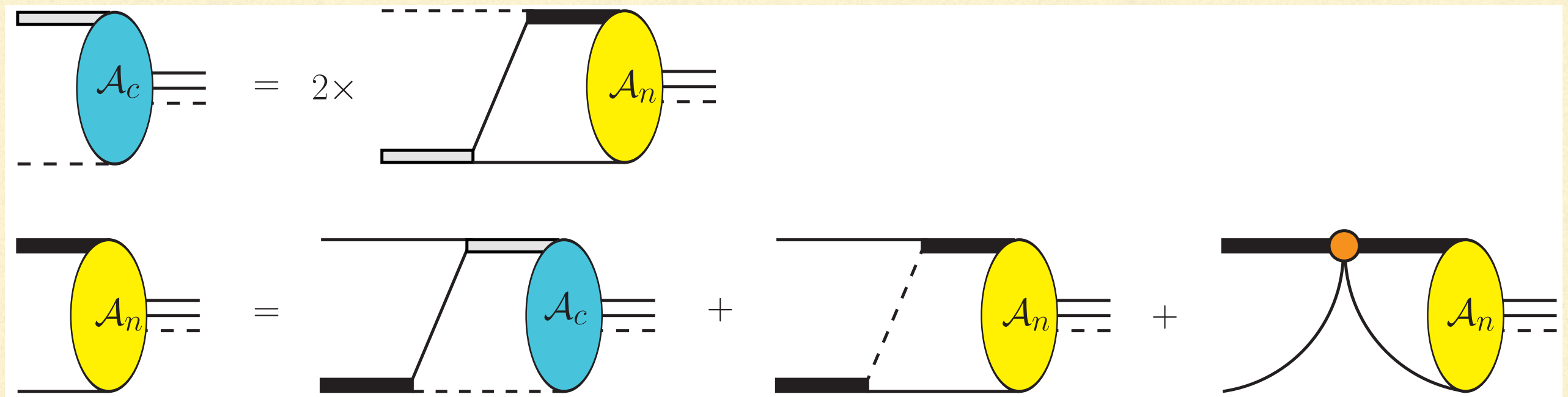
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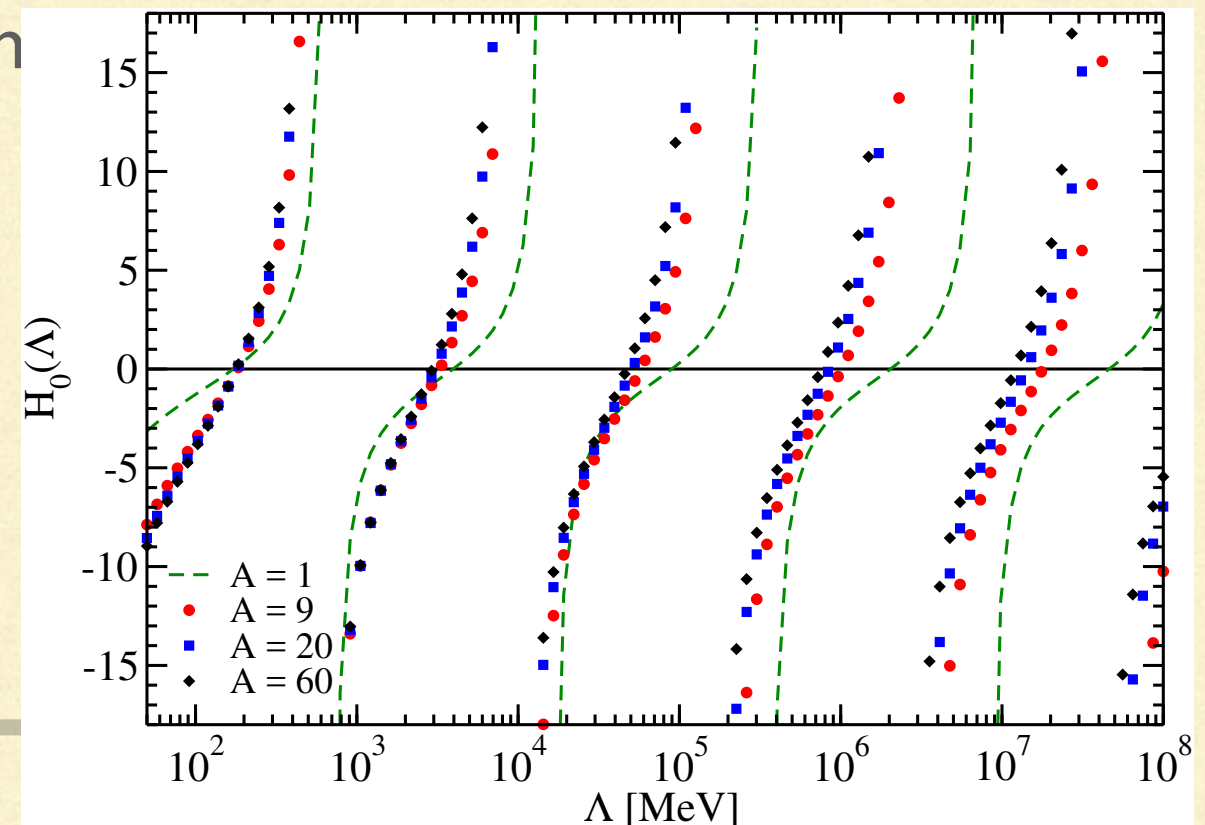


- (cn)-n contact interaction to stabilize the

- Efimov-Thomas effects

- Inputs: $E_{nn} = 1/(m a_{nn}^2)$, E_{nc} , S_{2n} ($=B$)

- Output: everything; up to $O(R_{\text{core}}/R_{\text{halo}})$

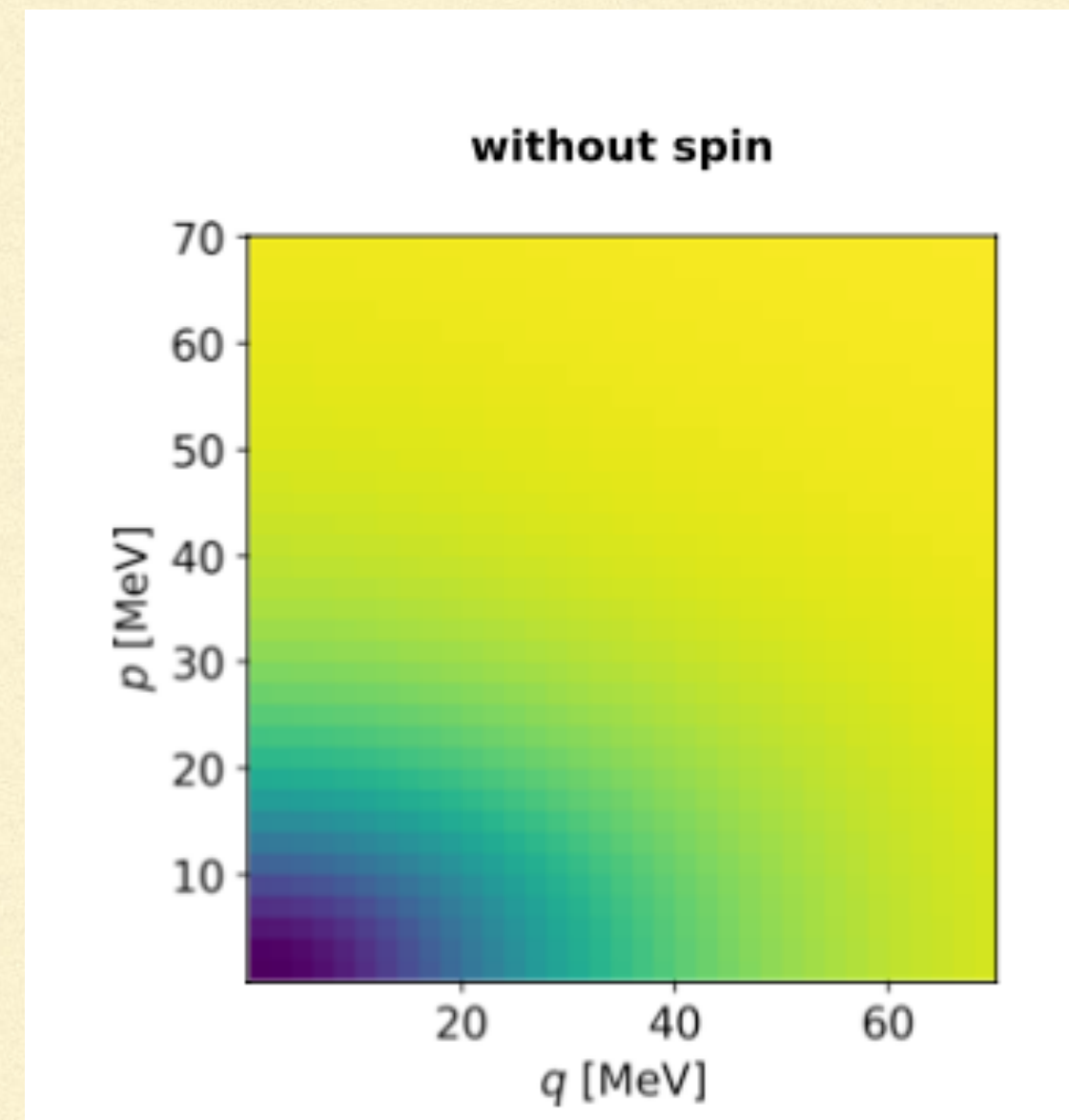


^{11}Li as a $2n$ halo

Acharya, Göbel, Hammer, DP, PRC (2023)

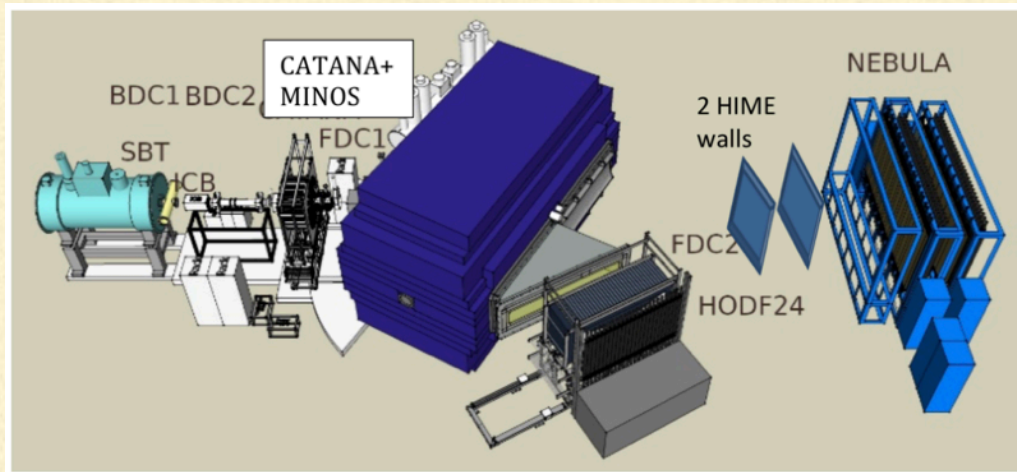
- $a_{nn} = -18.7$ fm, $E_{nc} = 0.026$ MeV
- $S_{2n} = 369$ keV
- Calculations done with a cutoff of 470 MeV, but results checked for a cutoff of 700 MeV
- Here results with a spin-0 core, but we also examined case of spin-3/2 core
- Results identical if spin-1 and spin-2 nc interactions have equal strength
- Reasonable agreement (LO accuracy) with Coulomb dissociation data for ^{11}Li , once nn FSI and perturbative nc FSI included

^{11}Li momentum distribution



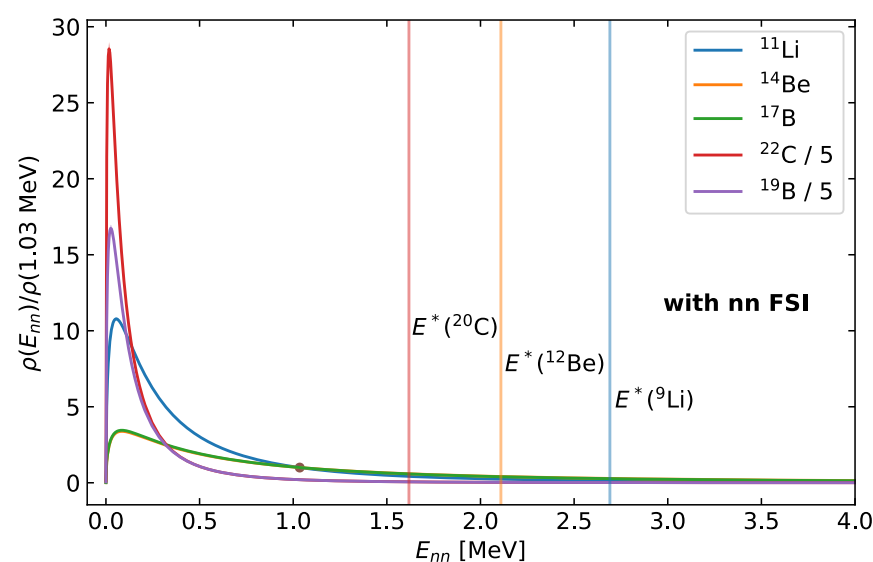
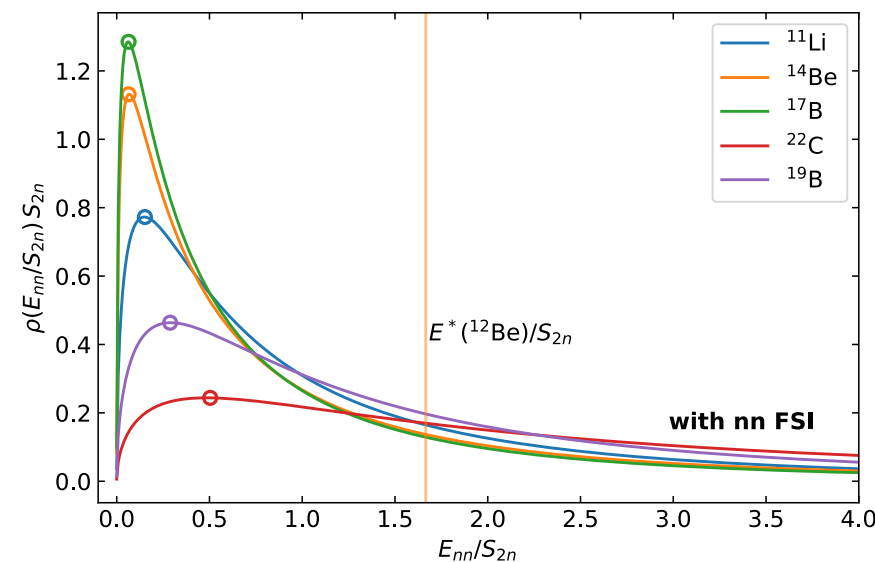
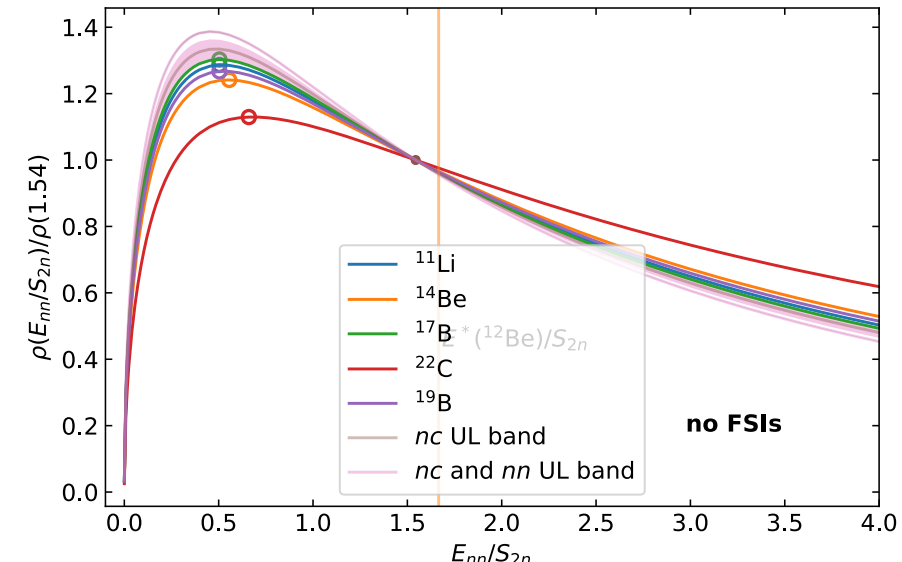
Universal 2n spectra in Borromean halos

Göbel, Aumann, et al., PRC 104 (2021) 024001; Göbel et al., in preparation



Opportunity to measure 2n spectra accurately via quasi-free knockout of core on, e.g., proton target, in inverse kinematics

- Measure the nn energy spectrum for a variety of two-neutron halo nuclei and show they are related by universality
- Calculation: 3B solution of Halo EFT + inclusion of FSI via Møller operator



Applications to $1n$, $2n$, and $1p$ halos

- Coulomb dissociation of ^{11}Be , breakup of ^{11}Be on a C target, $^{10}\text{Be}(d,p)$, delayed proton emission in ^{11}Be beta decay
Hammer, DP (2011); Capel, Hammer, DP (2018); Schmidt, Hammer, Platter (2018); Yang et al. (2018)
- $^{14}\text{C}(d,p)$, Coulomb breakup of ^{15}C , $^{14}\text{C}(n,\gamma)$, one-neutron knockout on ^{15}C
Moschini, Yang, Capel (2019); Hebborn, Capel (2021)
- ^{17}C as a d-wave $1n$ halo
Braun, Hammer, Platter (2018)
- ^3F as a $1n$ halo
Elkamhawy, Hammer (2022)
- Radius- S_{2n} correlation in ^{22}C ; electric radii of $2n$ halos
Acharya, Ji, DP (2013); Canham, Hammer (2008, 2010); Hagen, Hammer., Platter (2013); Vanasse (2017)
- $^{16}\text{O}(p,\gamma)^{17}\text{F}^*$; properties of $1/2^+$ level
Ryberg, Forssén, Hammer, Platter (2014, 2016)
- $^7\text{Be}(p,\gamma)$, radius of ^8B
Ryberg, Forssén, Hammer, Platter (2014, 2016); Zhang, Nollett, DP (2014, 2016, 2018); Higa, Premarathna, Rupak (2020)
- Prediction of possible Efimov state in ^{62}Ca
Hagen, Hagen, Hammer, Platter (2013)
- ^6He structure; $^6\text{He}(p.p'\alpha)nn$ for nn scattering length
Ji, Elster, DP (2014); Ryberg, Forssén, Platter (2017); Göbel et al. (2019, 2021)

Summary, second half

- Halo EFT provides a systematic way to treat weakly-bound nuclei
Theoretical uncertainties assessed as $(R_{\text{core}}/R_{\text{halo}})^{n+1}$
 - “Higher-order” electromagnetic effects can be computed in an organized way
 - Three-body systems: at leading order ${}^6\text{Li}$ is generated by s-wave interactions between the nn and $n\alpha$ pairs. S-wave three-body force adjusted to ${}^6\text{Li}$ S_{2n}
 - Other s-wave $2n$ halos can be calculated using same formalism
 - Predict neutron-pair momentum distribution from QF knockout of core: result images halo momentum distribution + nn FSI
 - Similar approach, but with perturbative nc interactions in final state, permits computation of Coulomb dissociation; final results in reasonable agreement with experimental data
 - Universal dynamics for $2n$ s-wave halos: EFT for 3B phenomena near threshold
-

Backup Slides

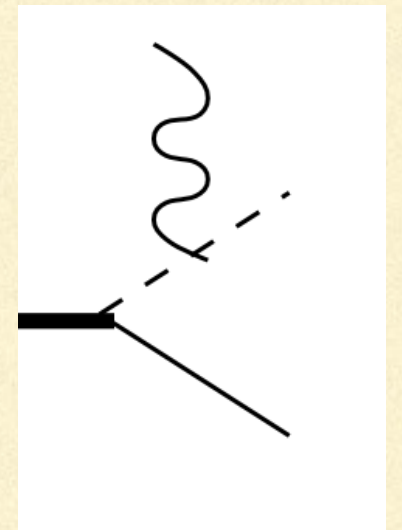
Predicting dissociation

$$\mathcal{M} = \frac{eZg_0 2m_R}{\gamma_0^2 + \left(\mathbf{p} - \frac{\mathbf{k}}{A}\right)^2}$$

$$\gamma_0 = \sqrt{2m_R S_{1n}}$$

$$p = \sqrt{2m_R E}$$

$$E1 \propto \int_0^\infty dr j_1(pr) r u_0(r); \quad u_0(r) = A_0 e^{-\gamma_0 r}$$



Chen, Savage (1999)

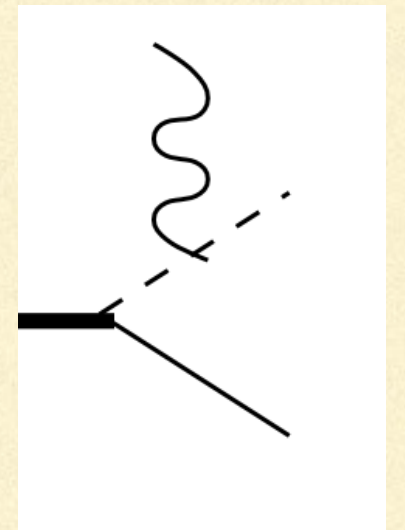
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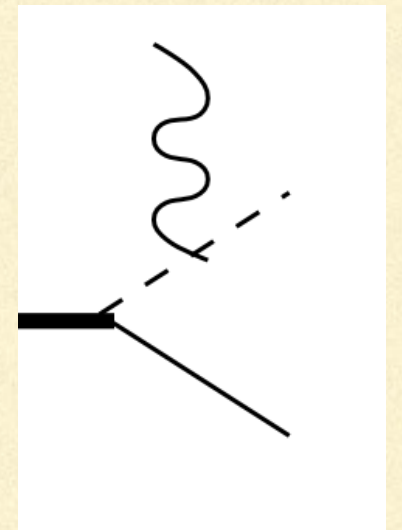
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Chen, Savage (1999)

$$\frac{dB(E1)}{e^2 dE} = \frac{6m_R}{\pi^2} \frac{Z^2}{A^2} A_0^2 \frac{p^3}{(\gamma_0^2 + p^2)^2}$$

Bertulani, Typel, Baur

Universal E1 strength formula for S-wave halos

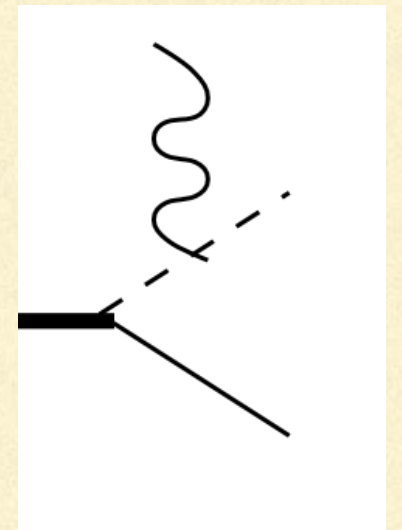
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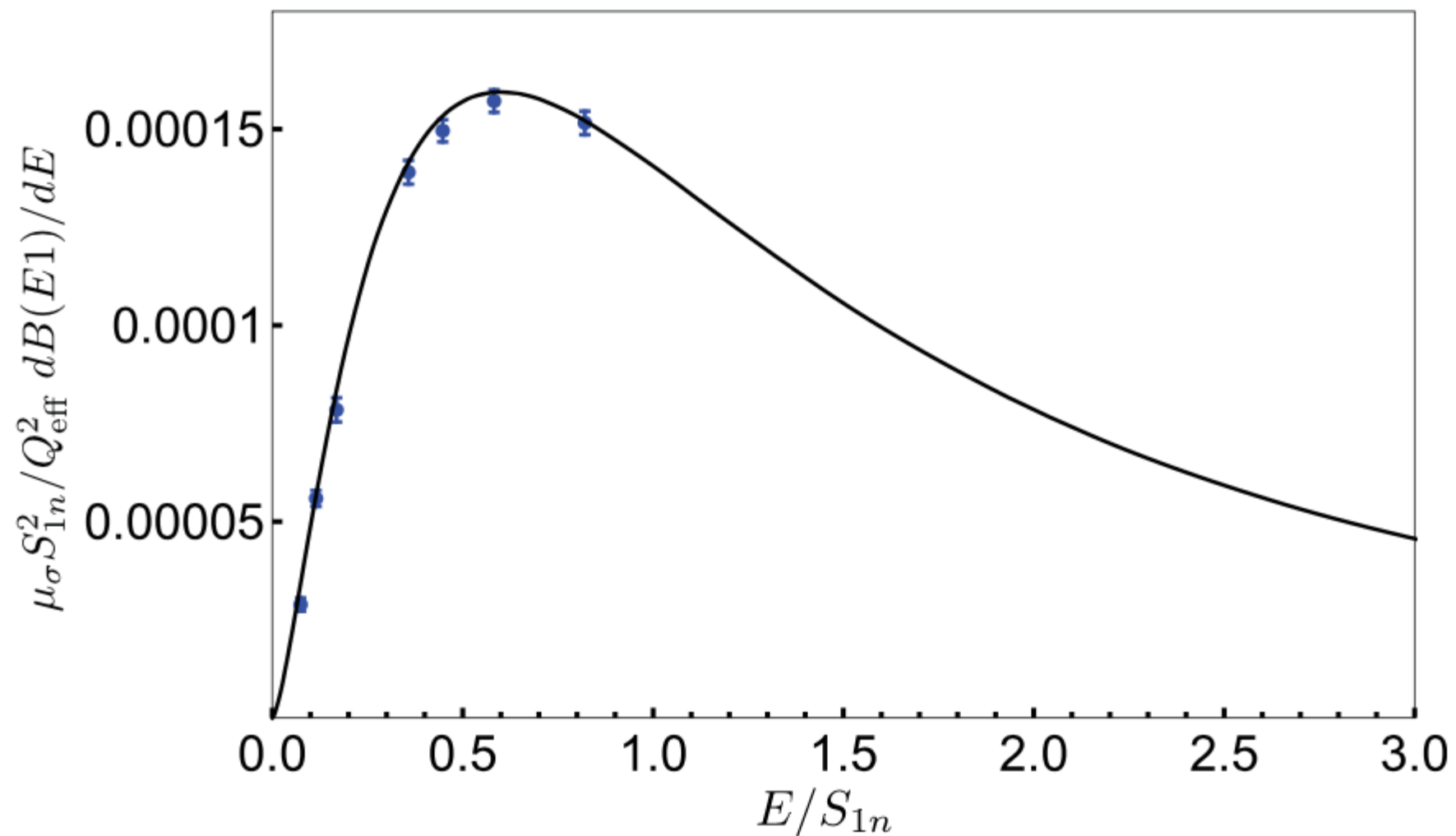
Bertulani, Typel, Baur

Universal E1 strength formula for S-wave halos

- Final-state interactions suppressed by $(R_{\text{core}}/R_{\text{halo}})^3$

- Short-distance piece of E1 m.e.: $L_{E1} \sigma^\dagger \mathbf{E} \cdot (n \overleftrightarrow{\nabla} c) + \text{h.c.} \sim \left(\frac{R_{\text{core}}}{R_{\text{halo}}}\right)^4$

Results

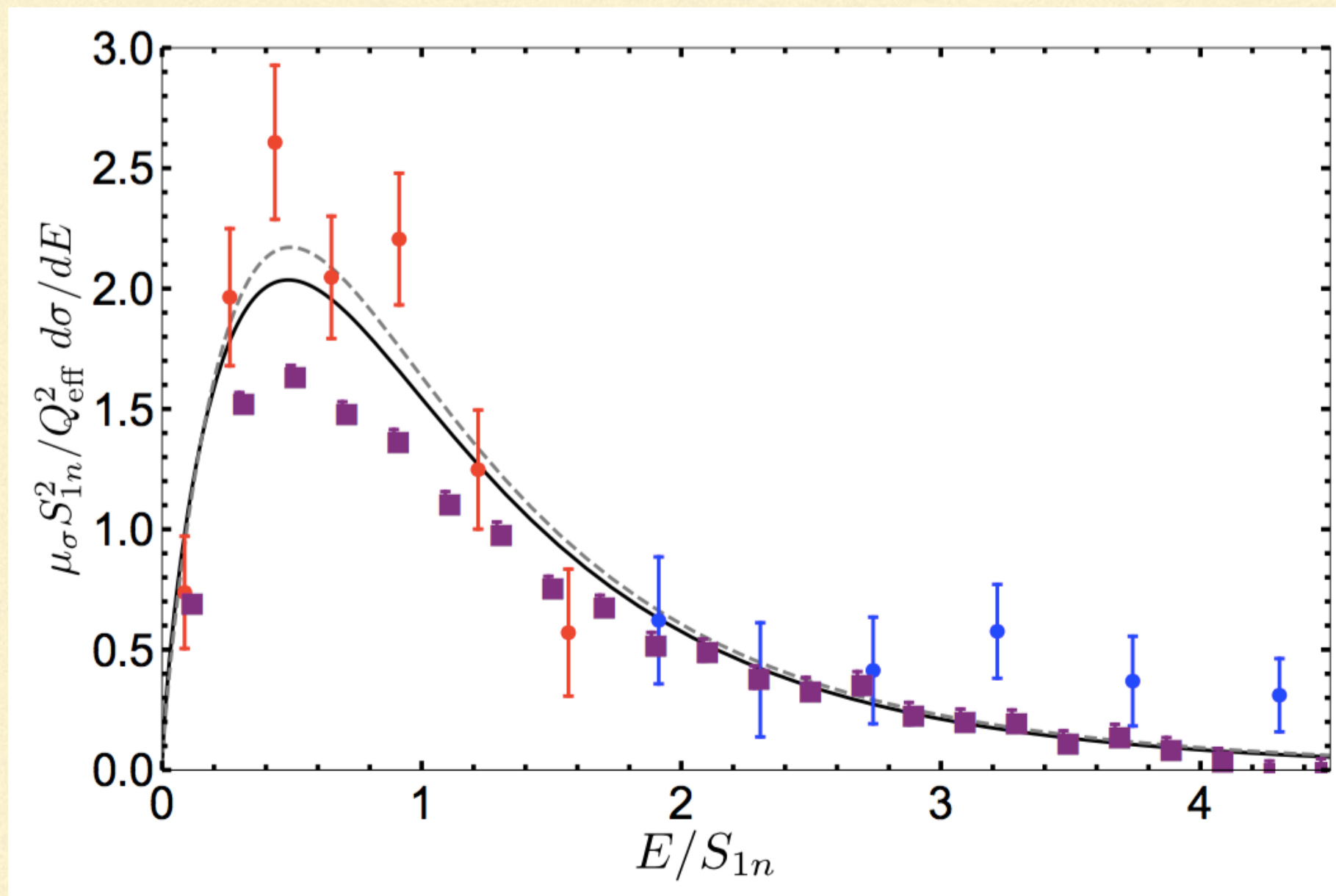


Deuteron

Data: Tornow et al. (2003)
Analysis: Chen, Savage (1999);
Hammer, Ji, DP (2017)

Results

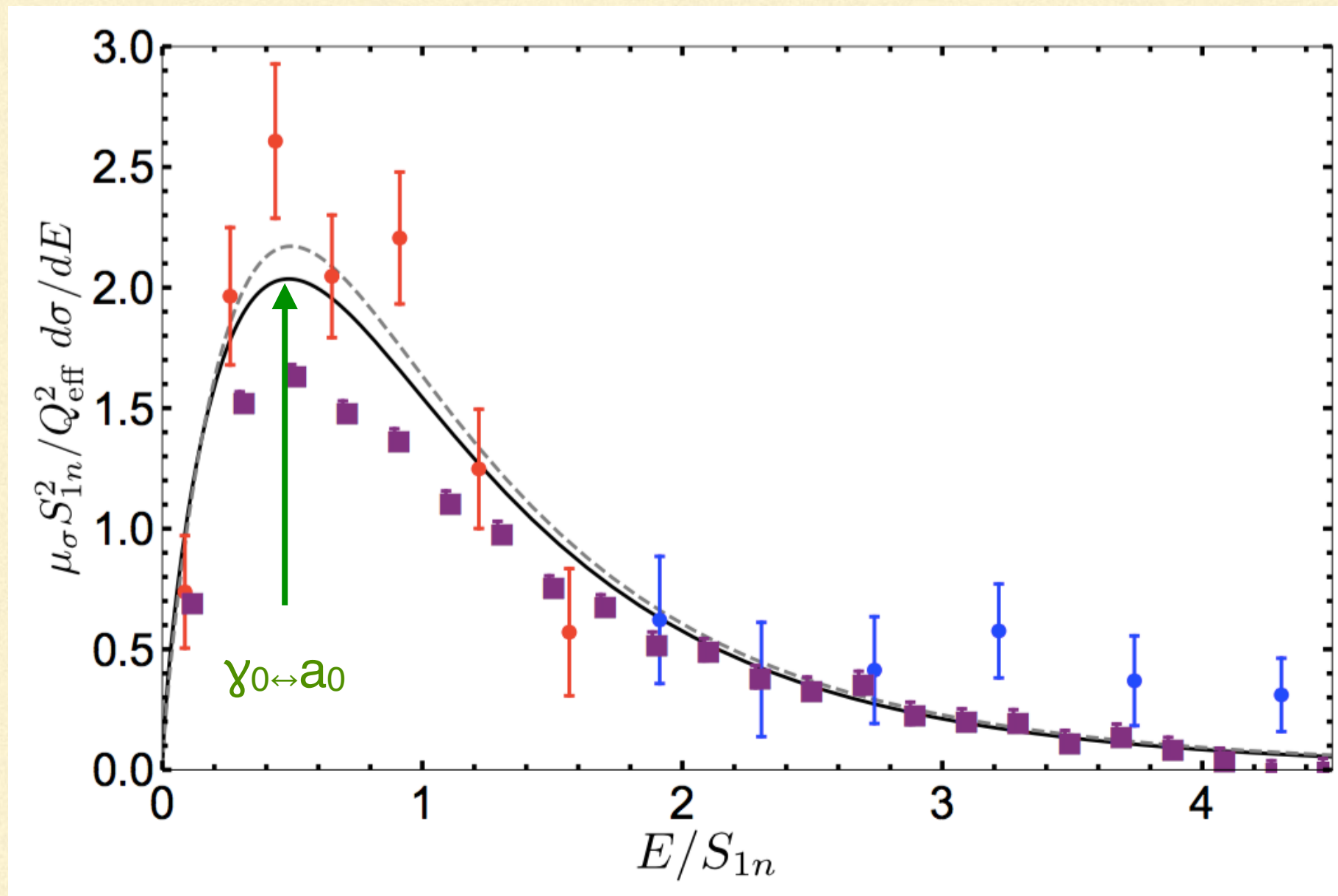
Results



^{19}C

Data: Nakamura et al. (1999, 2003);
Fukuda et al. (2004)
Analysis: Acharya, DP (2013);
Hammer, Ji, DP (2017)

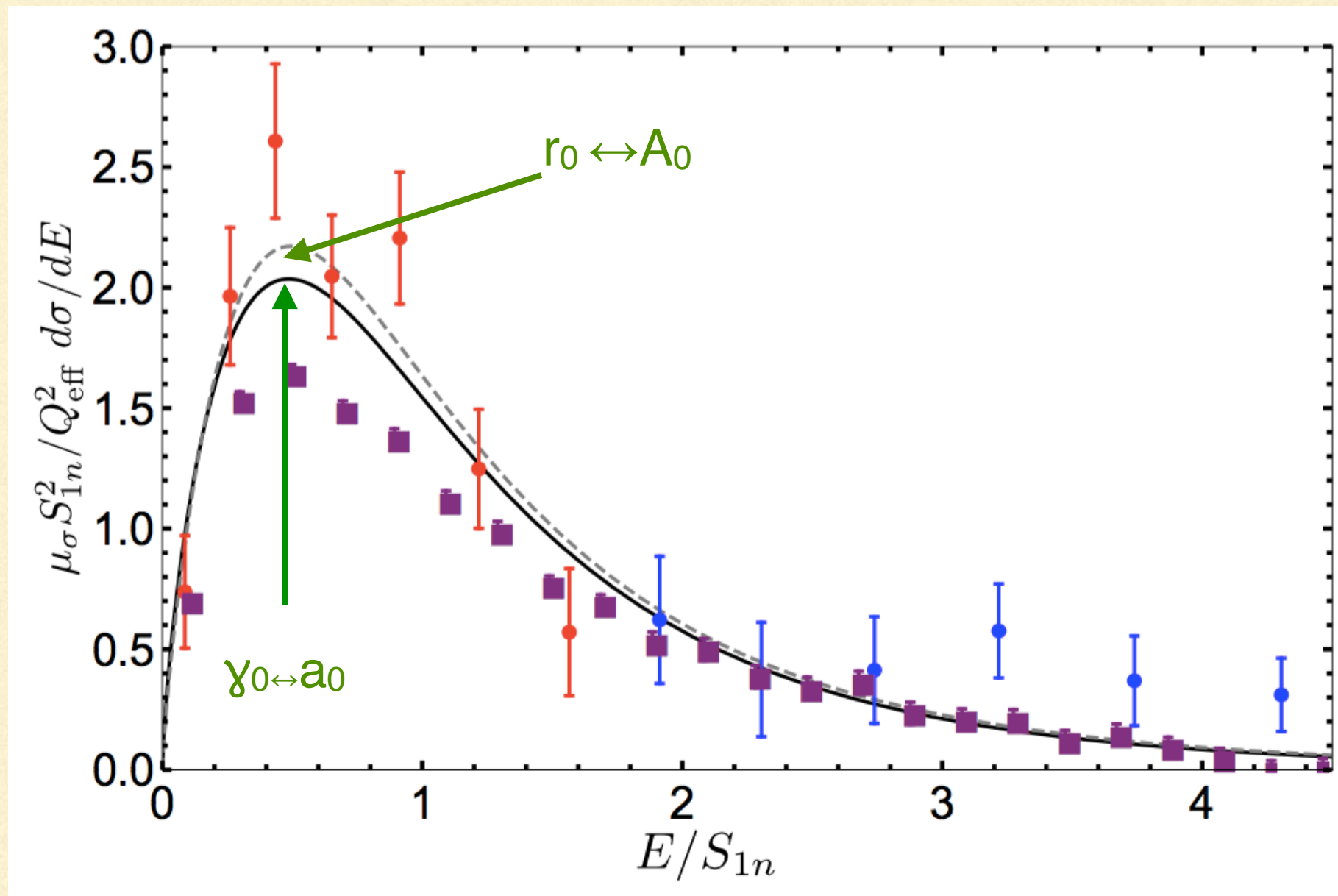
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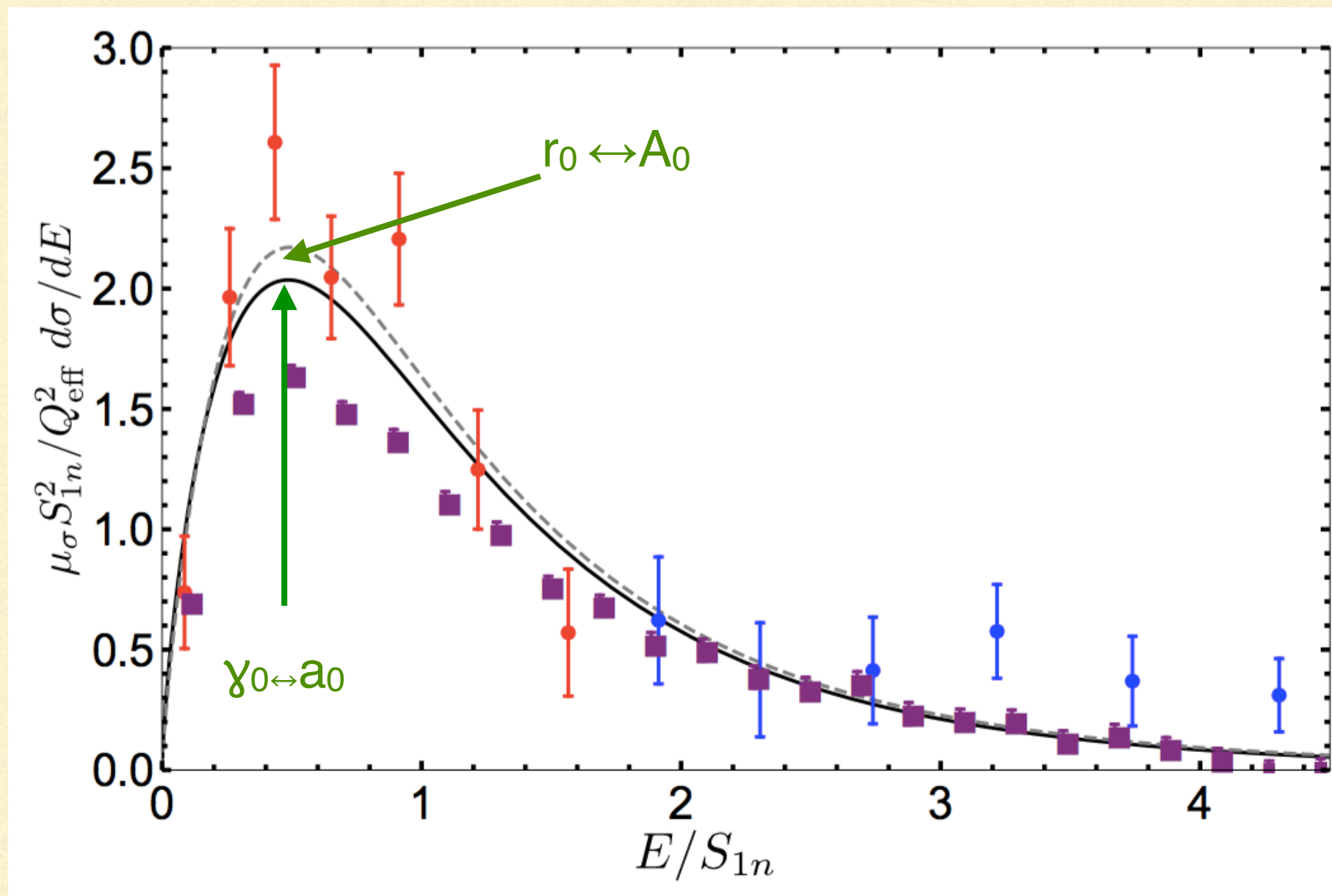
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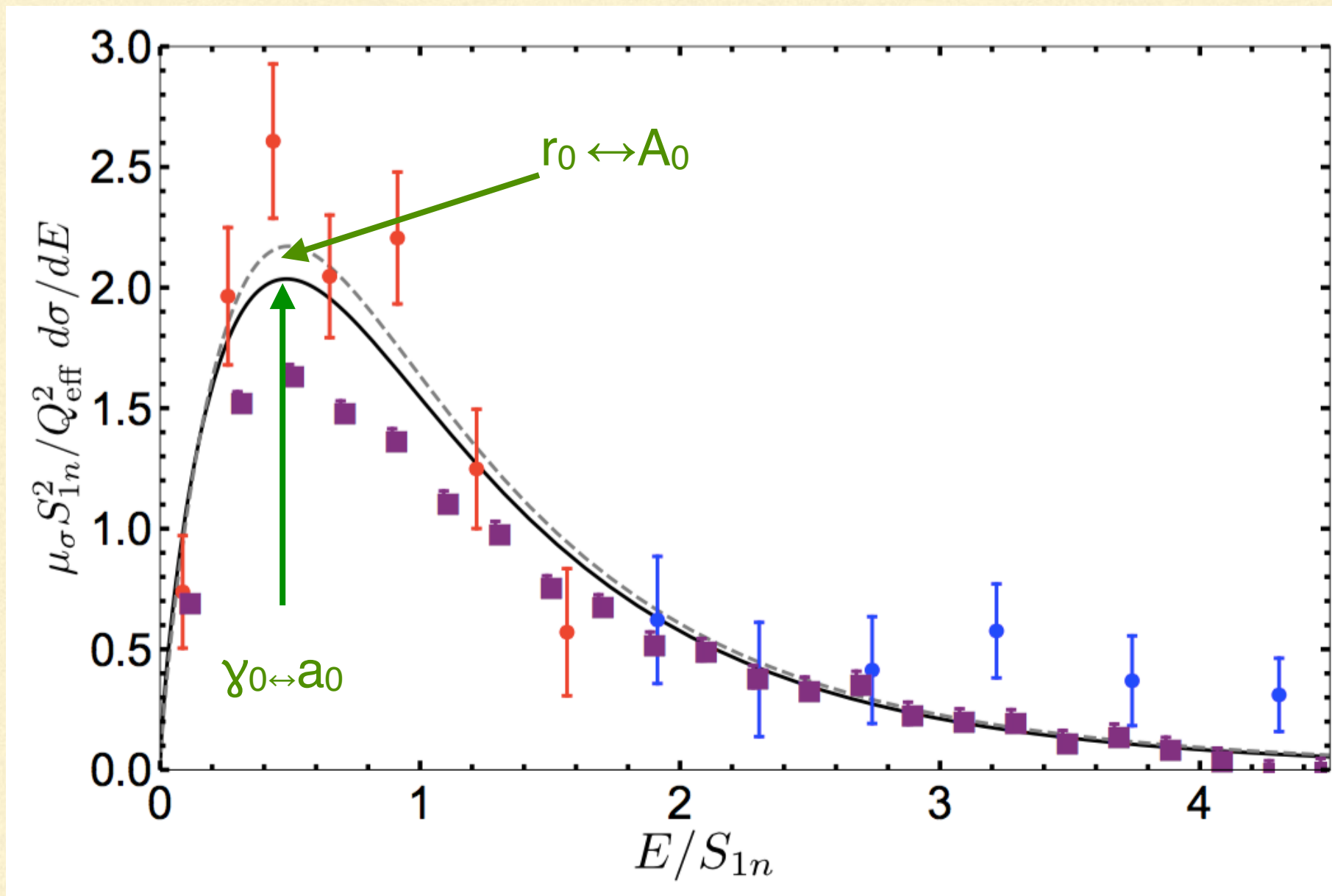


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Determine S-wave ^{18}C -n scattering
parameters $\Leftrightarrow$ ^{19}C ANC from dissociation data.

Results



^{19}C

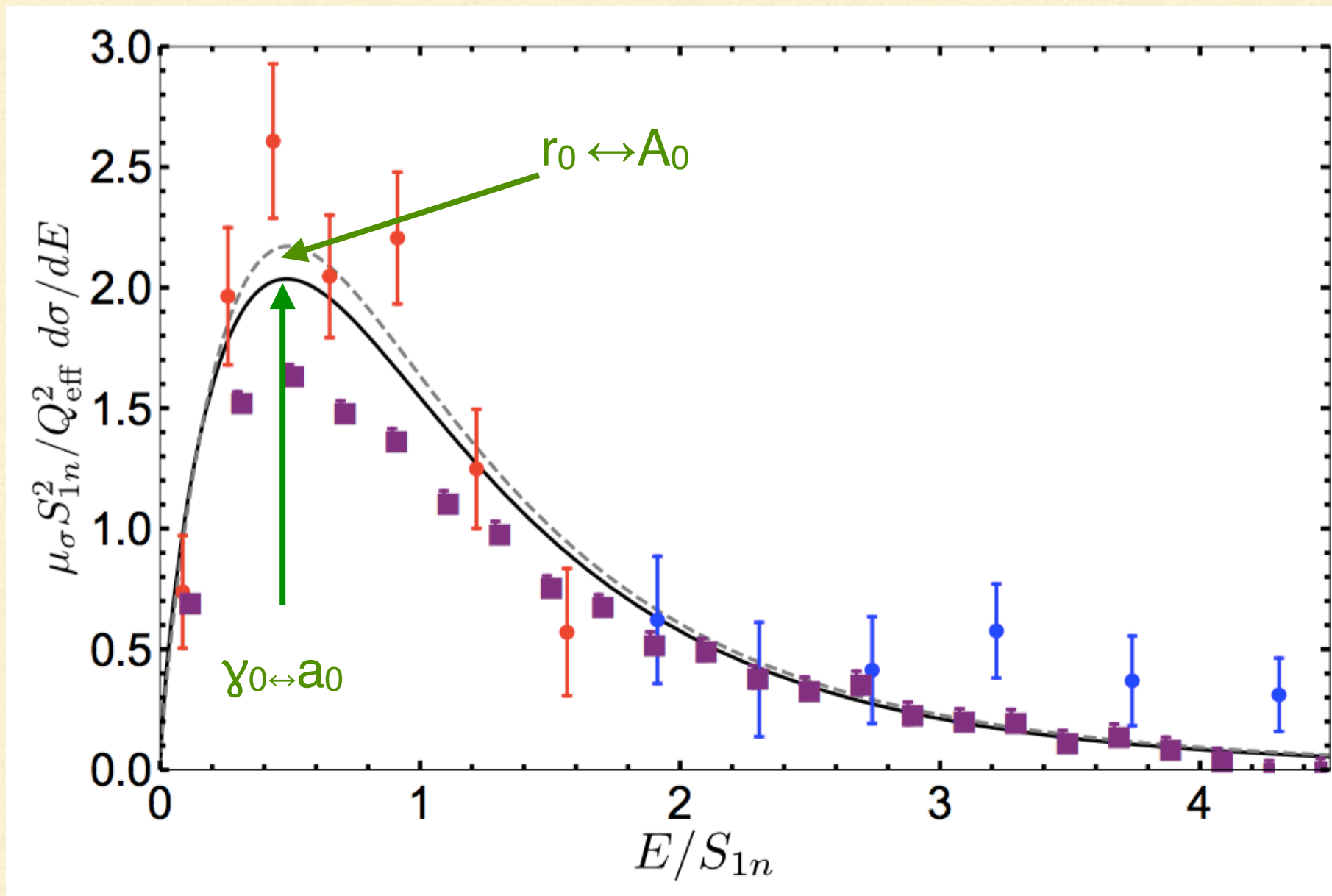
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■ For ^{19}C :

$$a = (7.75 \pm 0.35(\text{stat.}) \pm 0.3(\text{EFT})) \text{ fm};$$

$$r_0 = (2.6_{-0.9}^{+0.6}(\text{stat.}) \pm 0.1(\text{EFT})) \text{ fm}.$$

Results



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Data: Nakamura et al. (1999, 2003);
Fukuda et al. (2004)
Analysis: Acharya, DP (2013);
Hammer, Ji, DP (2017)

Combine Halo EFT with reaction
theory: Capel, Hammer, Hebborn,
Moschini, Yang, DP, et al.

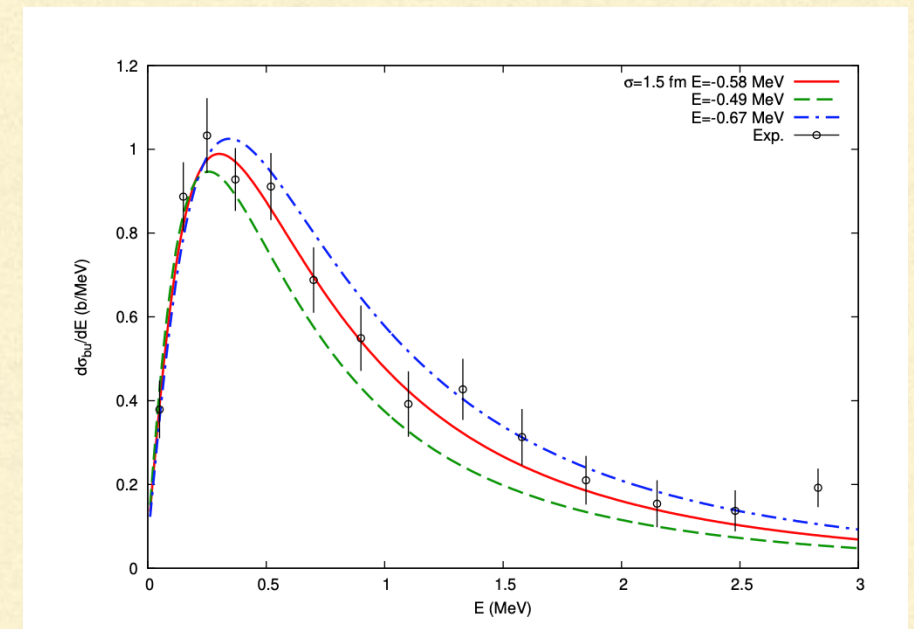
■ For ^{19}C :

$$a = (7.75 \pm 0.35(\text{stat.}) \pm 0.3(\text{EFT})) \text{ fm};$$

$$r_0 = (2.6_{-0.9}^{+0.6}(\text{stat.}) \pm 0.1(\text{EFT})) \text{ fm}.$$

Example: reactions on In halos

- ^{11}Be : breakup on ^{12}C and ^{208}Pb target, one-nucleon knockout, (d,p) Hammer, DP (2011); Capel, Hammer, DP (2018); Hebborn, Capel (2021); Schmidt, Hammer, Platter (2018)
- Coulomb breakup of ^{15}C , $^{14}\text{C}(n,\gamma)$, $^4\text{C}(d,p)$, one-nucleon knockout Moschini, Yang, Capel (2019); Hebborn, Capel (2021)
- ^{19}C : Coulomb dissociation on ^{208}Pb target Capel, DP, + TALENT students (2023)
 - Further experimental opportunities: investigation of low-lying spectrum of ^{19}C through breakup on a light target; refinement of ^{208}Pb data; transfer measurements, e.g., (d,p), especially at low beam energy or forward angles, to constrain ANC. Knockout measurements with higher precision.
 - *Ab initio* theory opportunity: computation of ANCs, pattern of low-lying resonances



SREFT for a two-body system

Kaplan, Savage, Wise; van Kolck;
Birse, Richardson, McGovern

$$-\frac{\hbar^2}{2m_R}\nabla^2\psi + V(\mathbf{r})\psi(\mathbf{r}) = E\psi(\mathbf{r})$$

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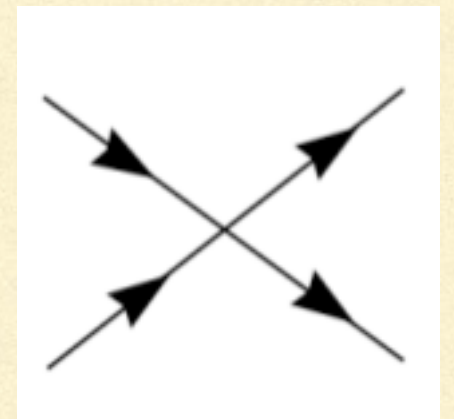
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$$-iC_0$$

SREFT for a two-body system

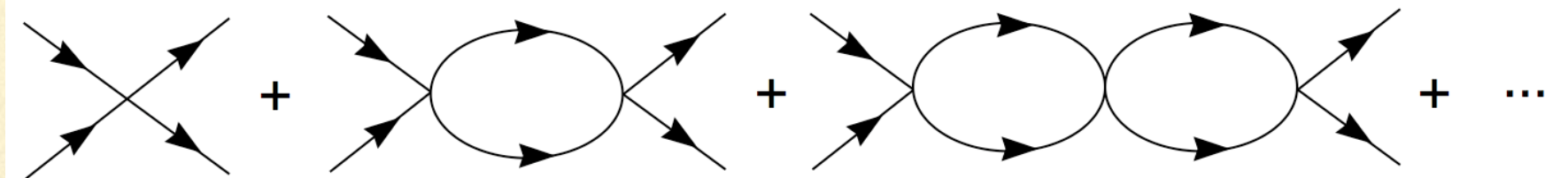
Kaplan, Savage, Wise; van Kolck;
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Quantum corrections essential

$$t_0^{2B} = \frac{2\pi a}{m_R} \frac{1}{1 + iak}$$



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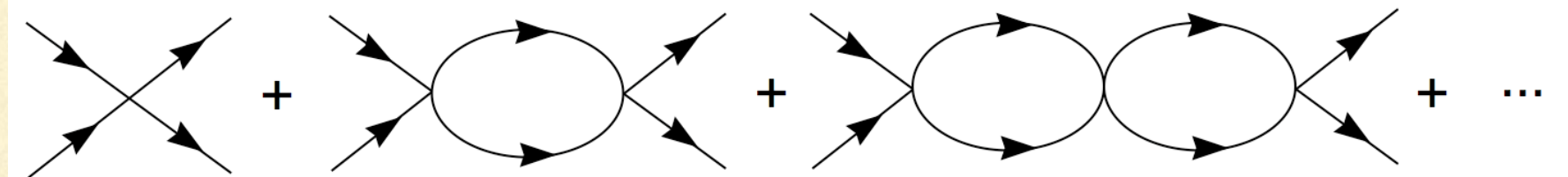
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First corrections are of relative order kR , R/a : “higher order”

Leading in systematic EFT expansion $\Rightarrow$ Estimate theory uncertainty

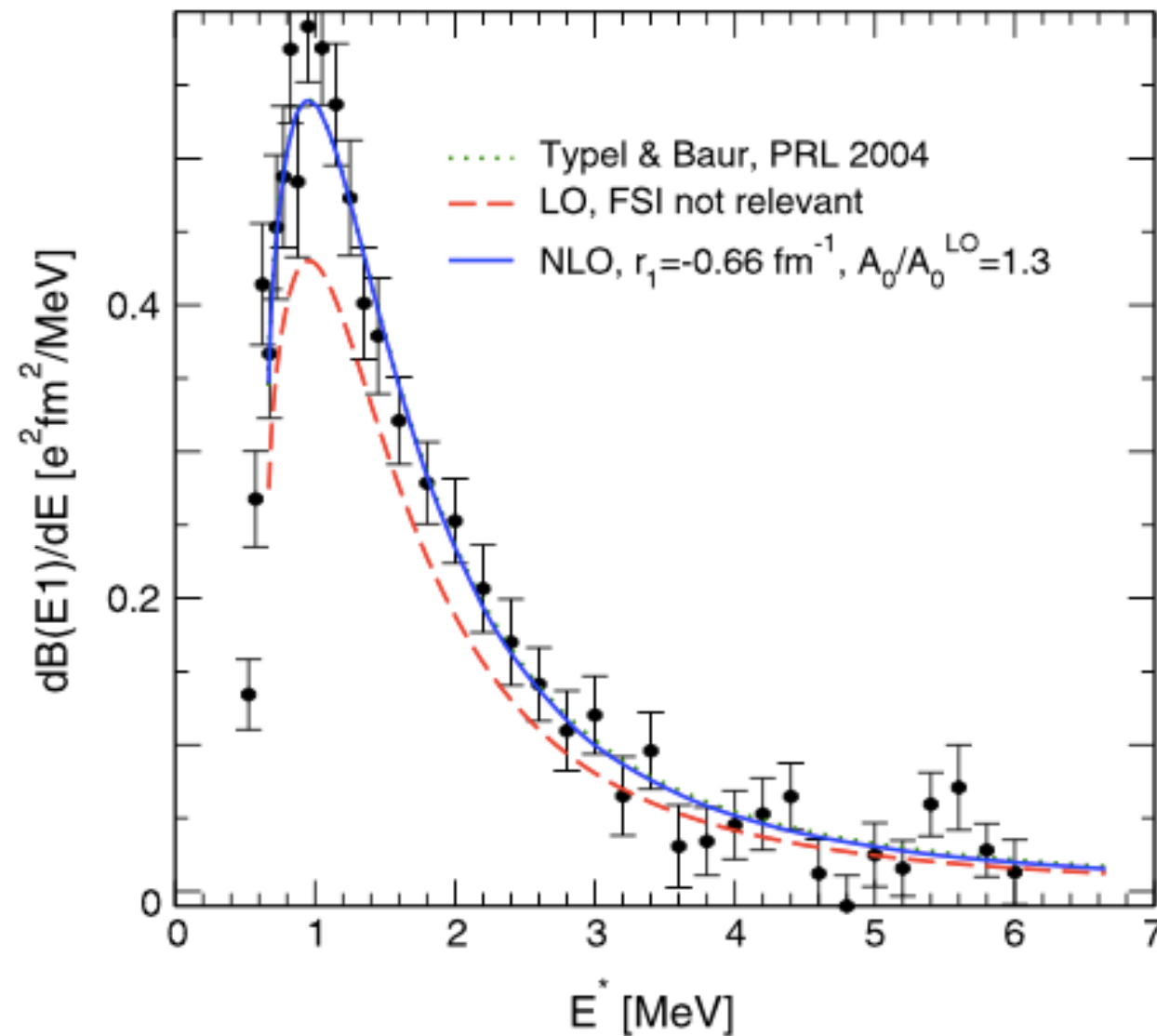
Coulomb dissociation of ^{11}Be : result

Data: Palit et al., 2003

Analysis: Hammer, Phillips. NPA, 2011

- Reasonable convergence
- Information on value of r_0 through fitting of A_0 :

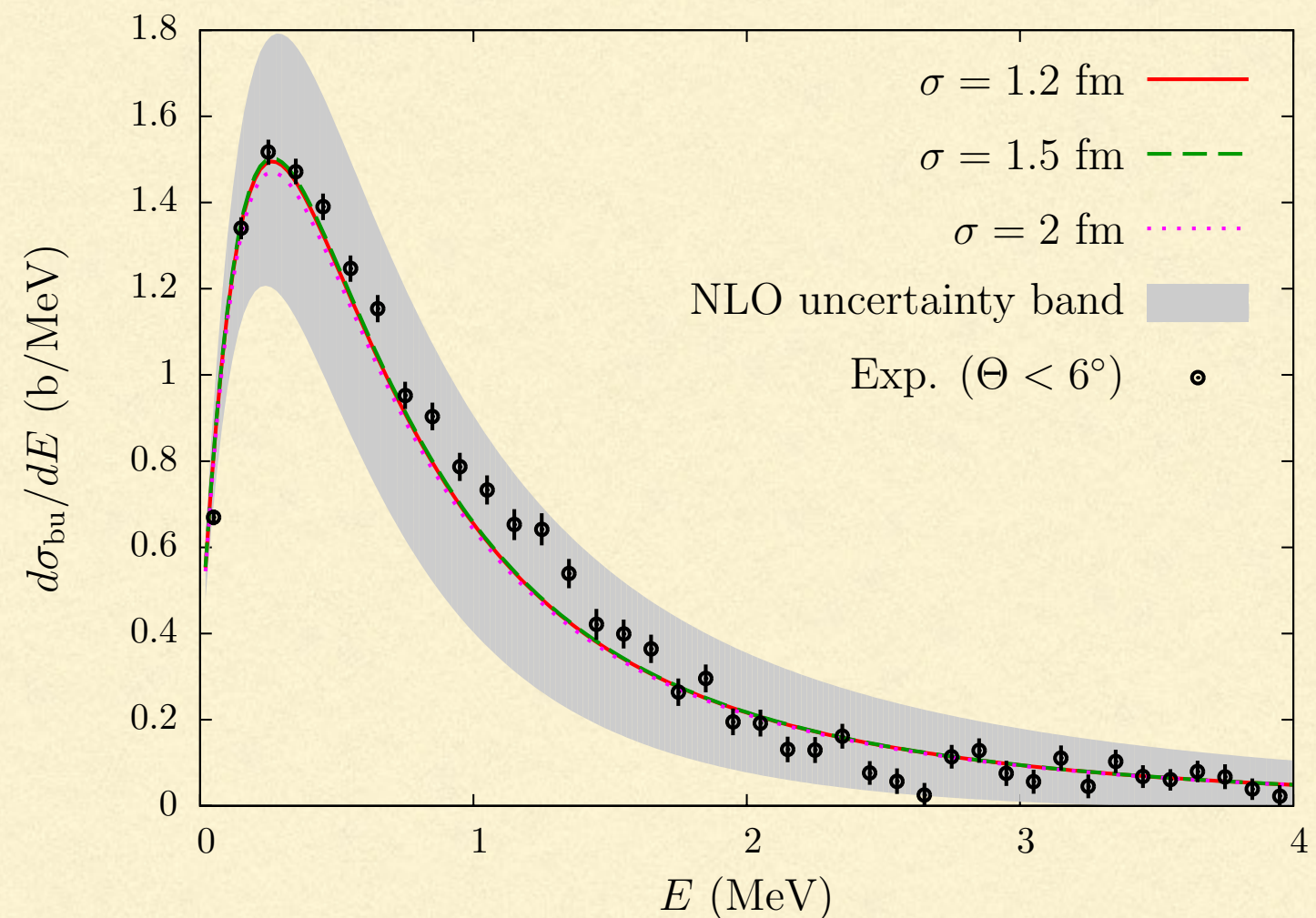
$$r_0 = 2.7 \text{ fm}$$



Ab initio → Halo EFT → Reaction theory

- ^{11}Be is a halo nucleus: last neutron only bound to ^{10}Be by 503 keV. Has a p-wave halo state with $S_{1n}=184$ keV.
- Model Coulomb dissociation of ^{11}Be via sophisticated “Dynamical Eikonal Approximation”: includes nuclear and Coulomb ^{208}Pb - ^{10}Be -n potentials
- Use Halo EFT to identify important ^{10}Be -n inputs for reaction-theory calculation: s- and p-wave phase shifts
- Take those from *ab initio* calculation of Calci et al. based on modern nuclear forces and NCSMC (PRL **117**, 242501)

Capel, DP, Hammer, Phys. Rev. C 98, 034610 (2018)
Data: Fukuda et al., Phys. Rev. C 70, 054606 (2004).



No dependence on interior of
 ^{10}Be -n potential used

Building the pdf

- χ^2 needs to include cross-section and branching-ratio data

$$\chi^2 \equiv \sum_J^{N_{\text{exp}}} \left\{ \sum_{j=1}^{N_{s,J}} \frac{\left[(1 - \xi_J) S(\vec{g}; E_{Jj}) - D_{Jj} \right]^2}{\sigma_{Jj}^2} + \frac{\xi_J^2}{\sigma_{c,J}^2} \right\} + \sum_{l=1}^{N_{br}} \frac{\left[Br(\vec{g}; E_l) - \tilde{D}_l \right]^2}{\sigma_{br,l}^2}$$

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- Mild Bayesian priors:
 - Independent gaussian priors for ξ_i , centered at zero and with width=CME
 - Other EFT parameters, a , r , L , and two ANCs assigned flat priors, corresponding to natural ranges
- Probability $e^{-\chi^2/2}$ sampled using Markov Chain Monte Carlo

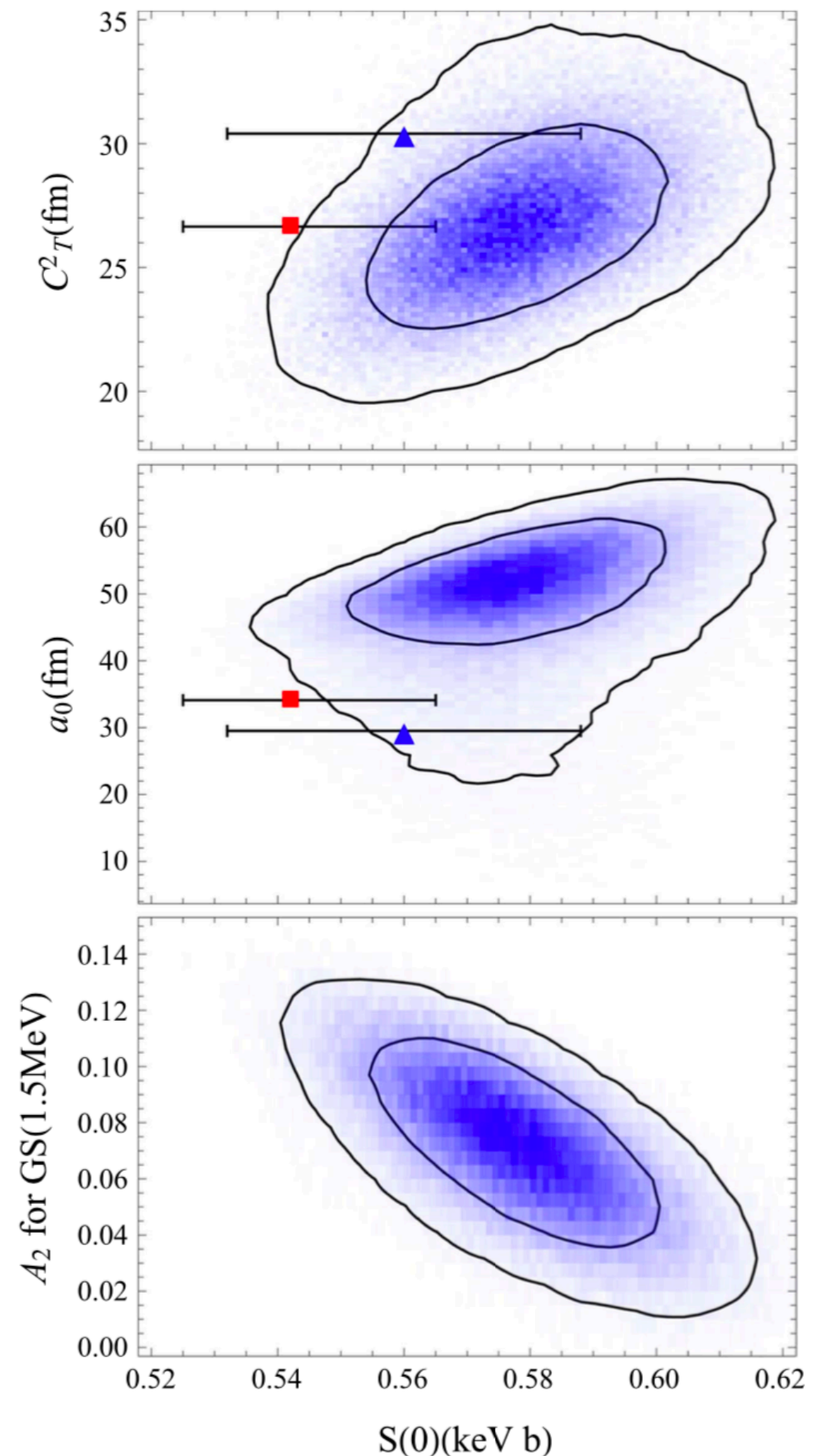
$S(0)$ and its correlants

$$S(0) = 0.578^{+0.015}_{-0.016} \text{ keV b}$$

cf. SFII: $S(0) = 0.56 \pm 0.03 \text{ keV b}$

$$Br(0) = 0.406^{+0.013}_{-0.011}$$

Mostly consistent with other analyses, but 1.5σ higher than that of deBoer et al.



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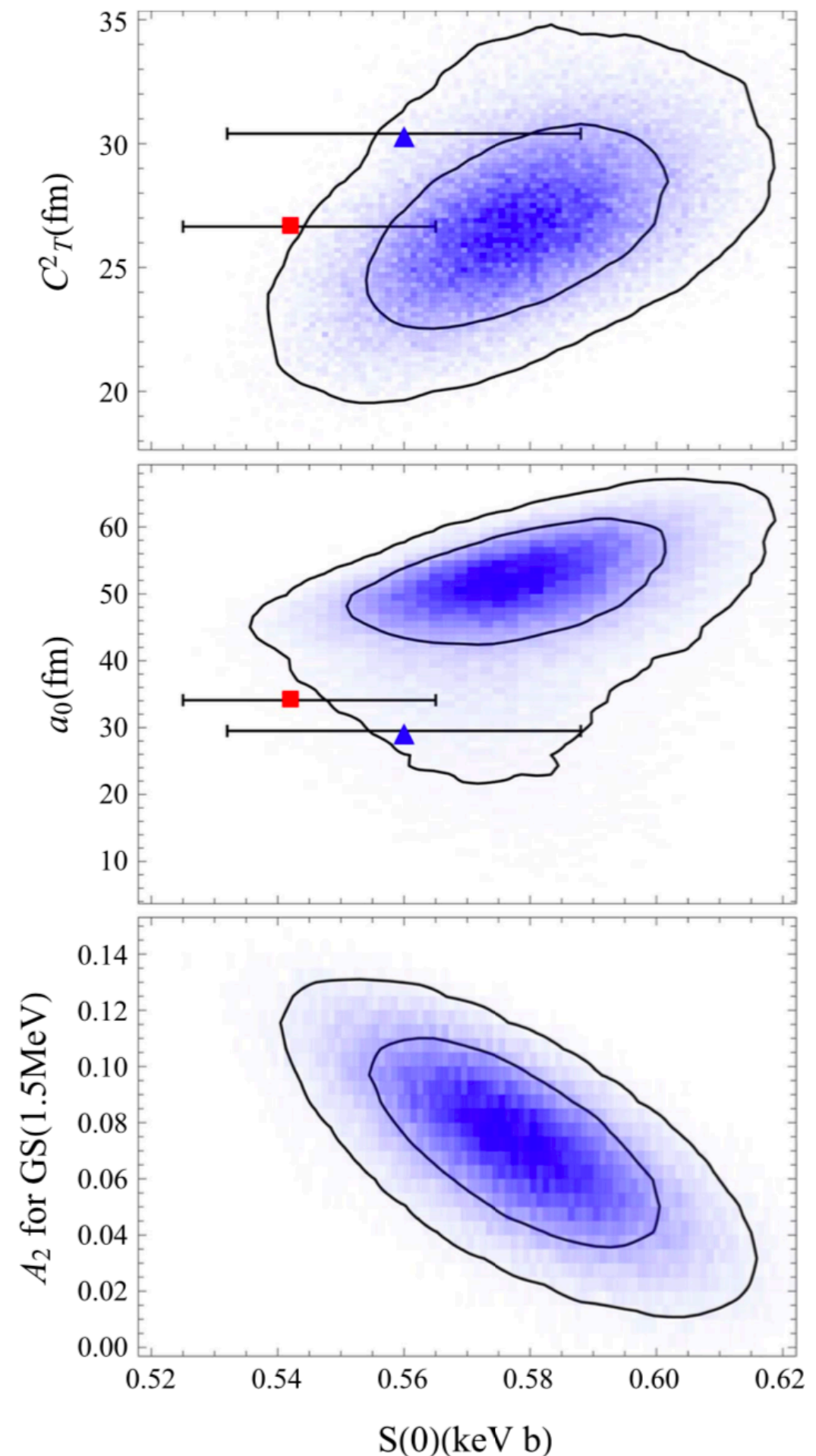
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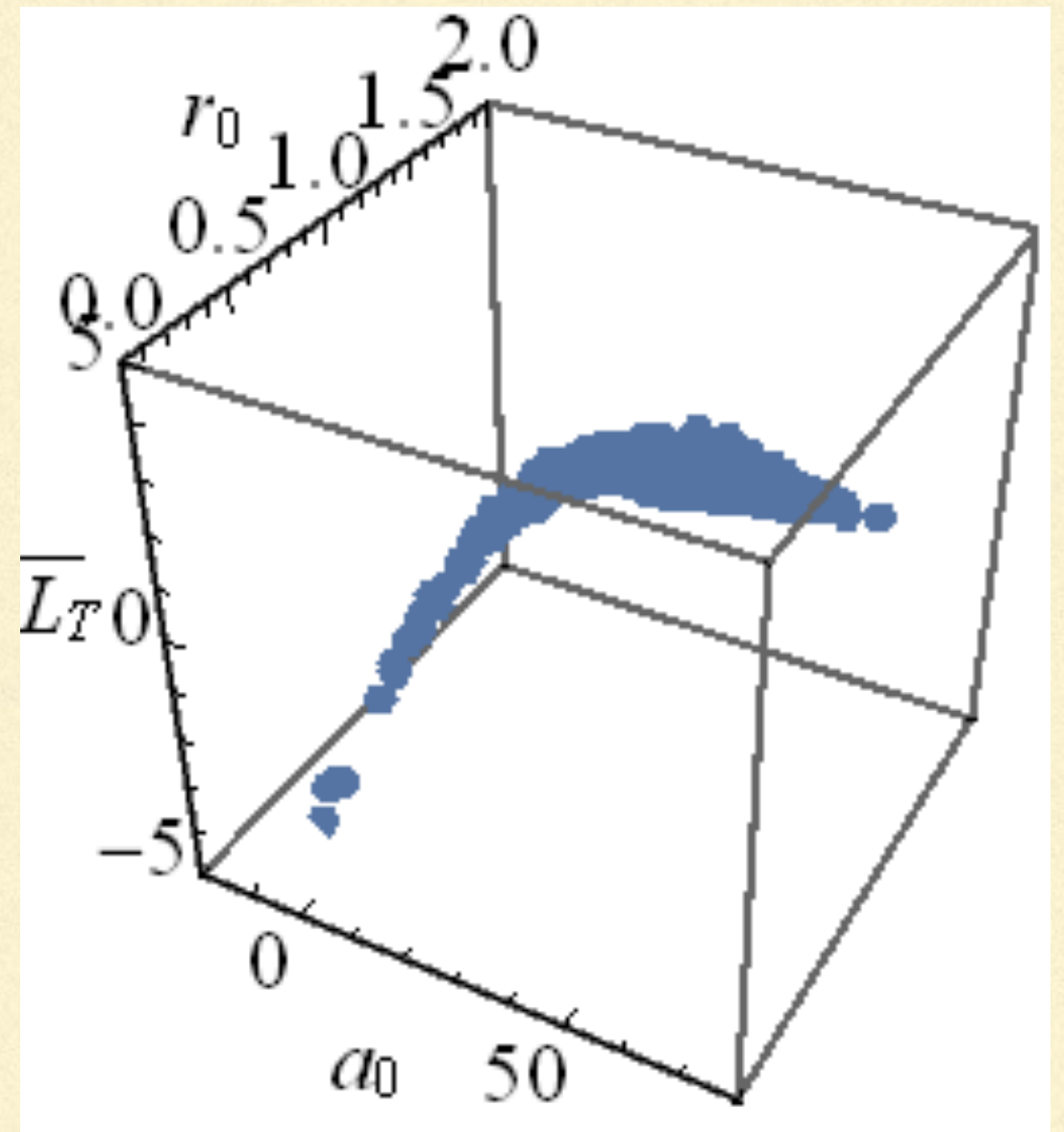
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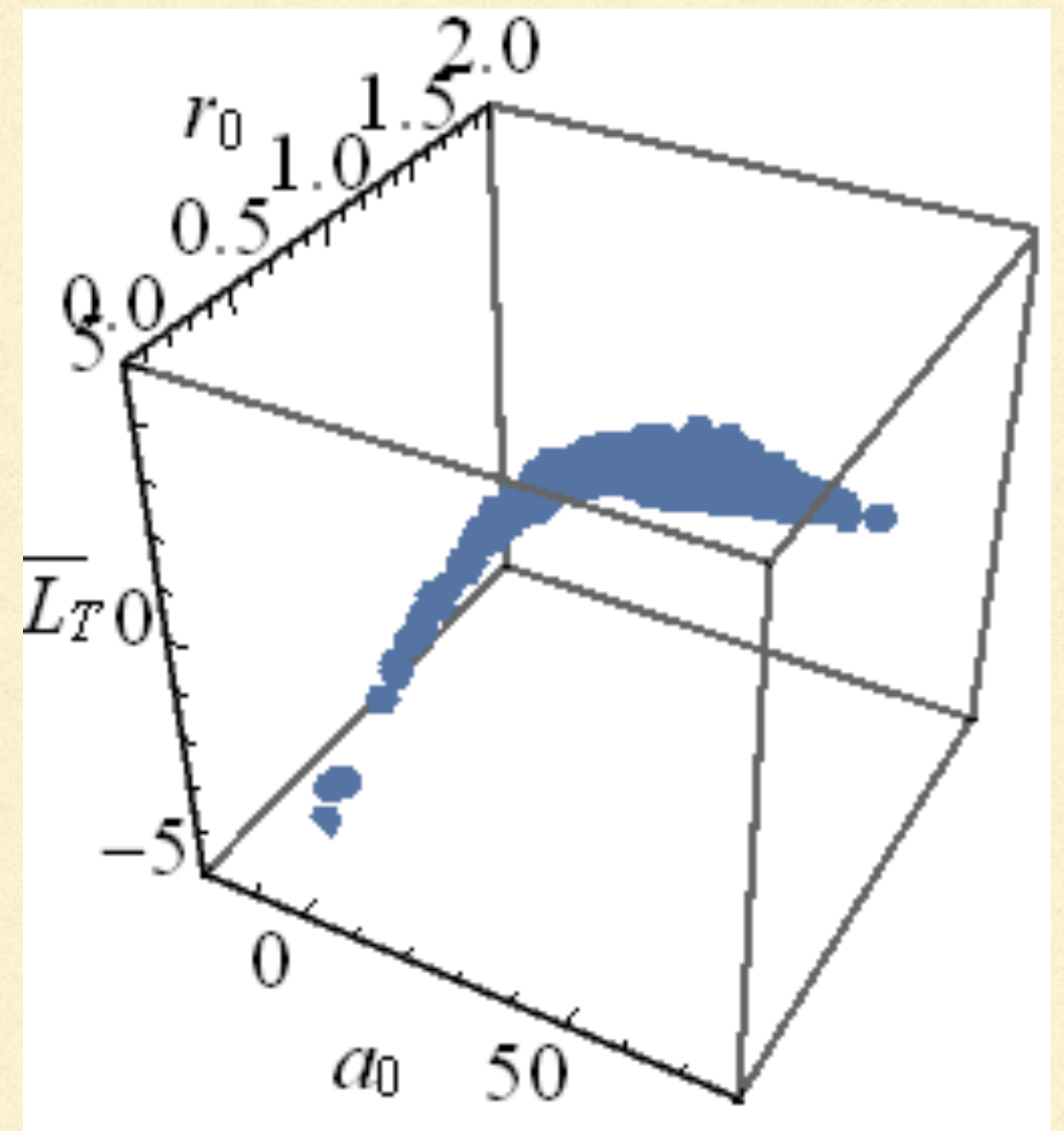
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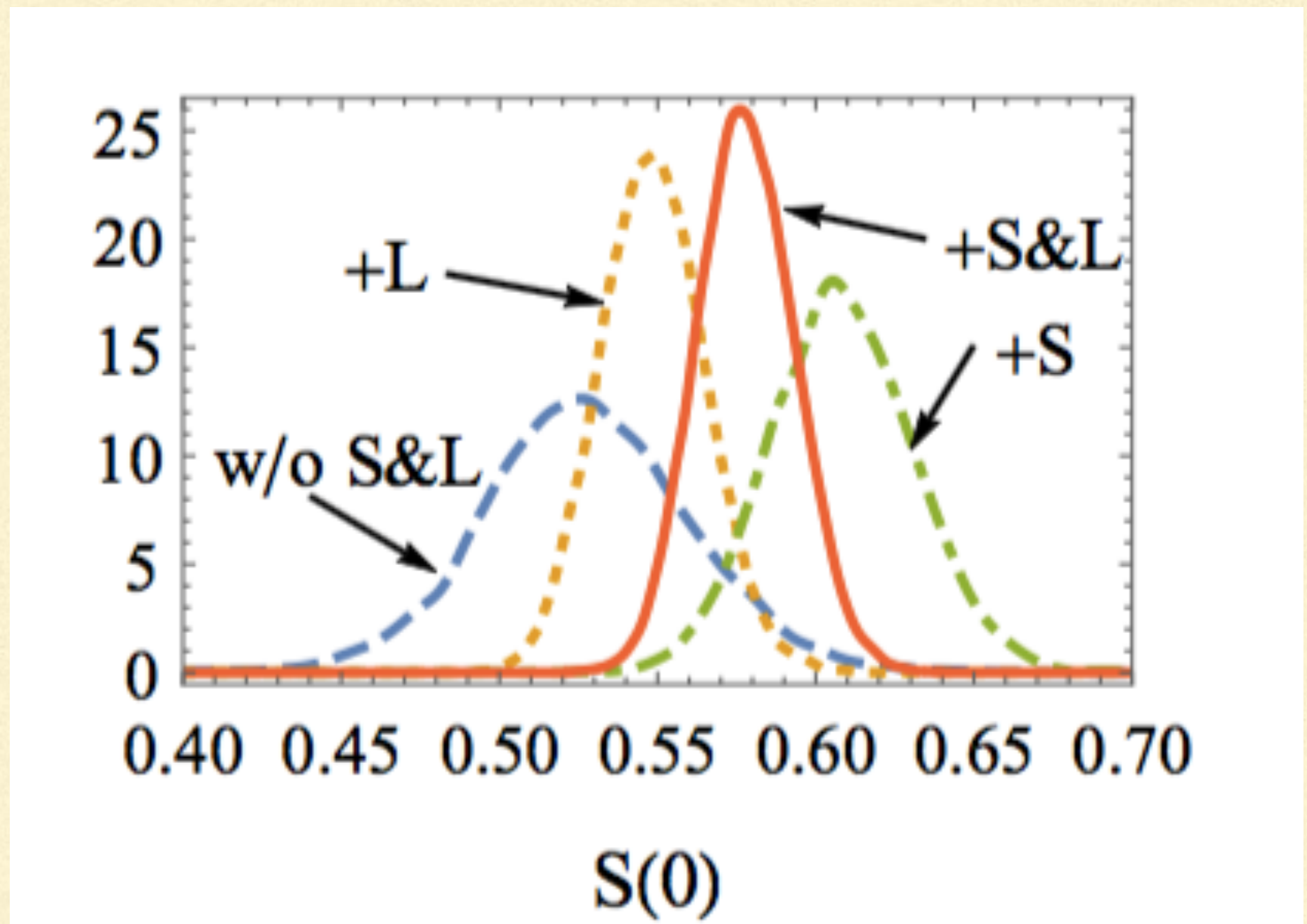
1. Measure $P_2(\cos \Theta)$ dependence
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$$a_0 = 50^{+7}_{-6} \text{ fm}$$

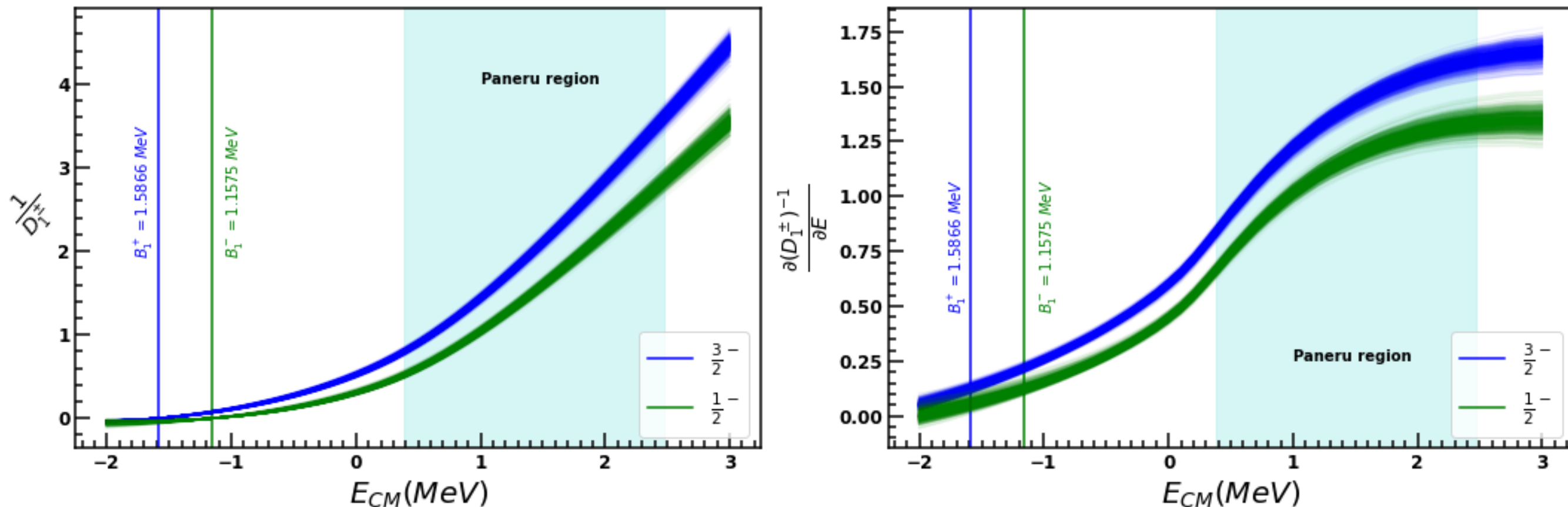
Impact of different data sets

- Floating data within quoted CME crucial for achieving data consistency
- Pdf gets narrower when either of the precise, low-energy data sets are included
- Seattle data push $S(0)$ to higher values, but still possible to find concordance between Seattle, Luna, and older data



Extrapolating to the bound-state pole

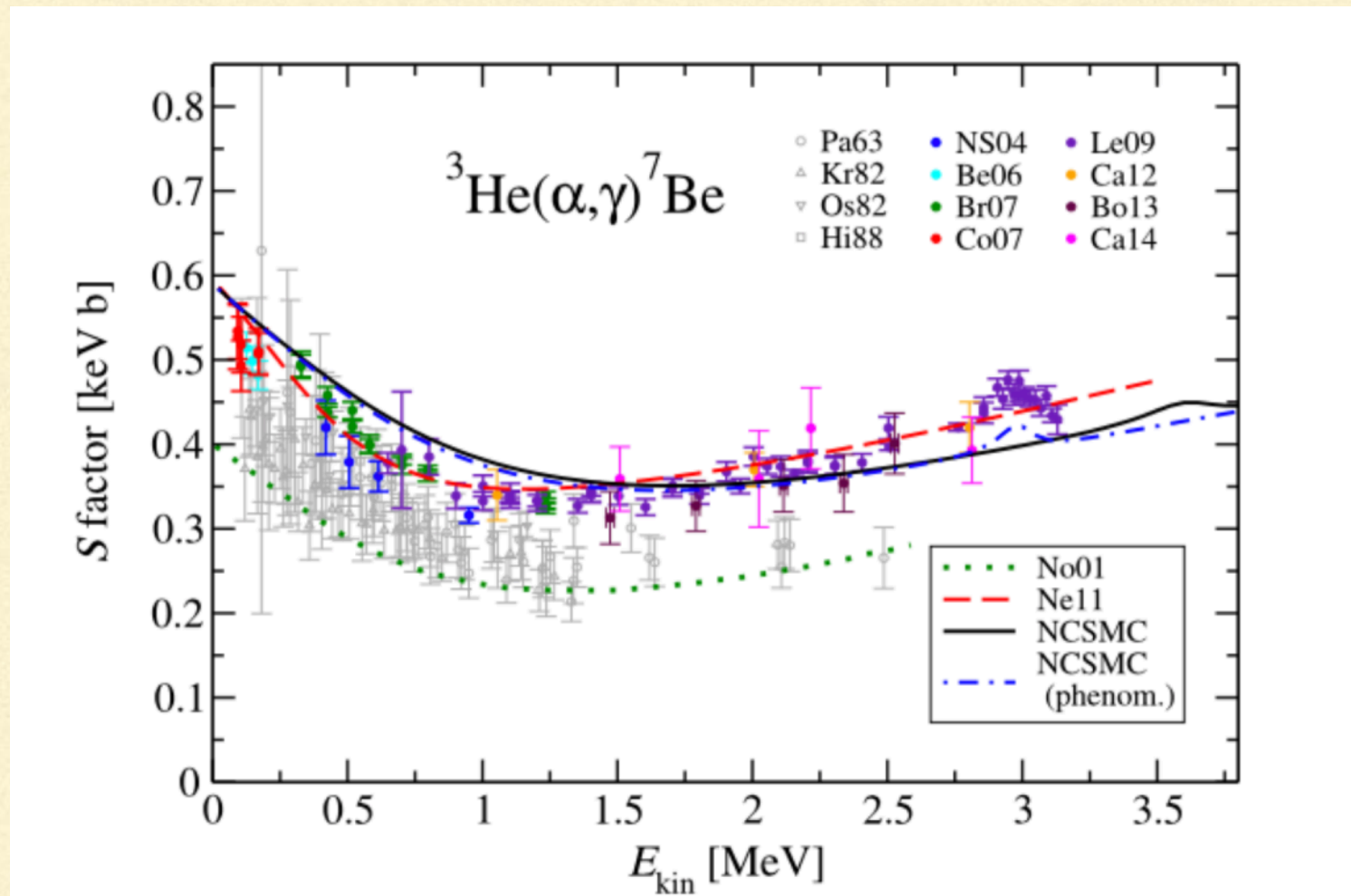
Analysis is stabilized by information from capture



$$C_1^{+2} = 16.0 \pm 1.4 \text{ fm}; \quad C_1^{-2} = 14.2 \pm 1.6 \text{ fm}$$

Extrapolation to pole has large relative error. Model error?

Connecting to *ab initio* calculations

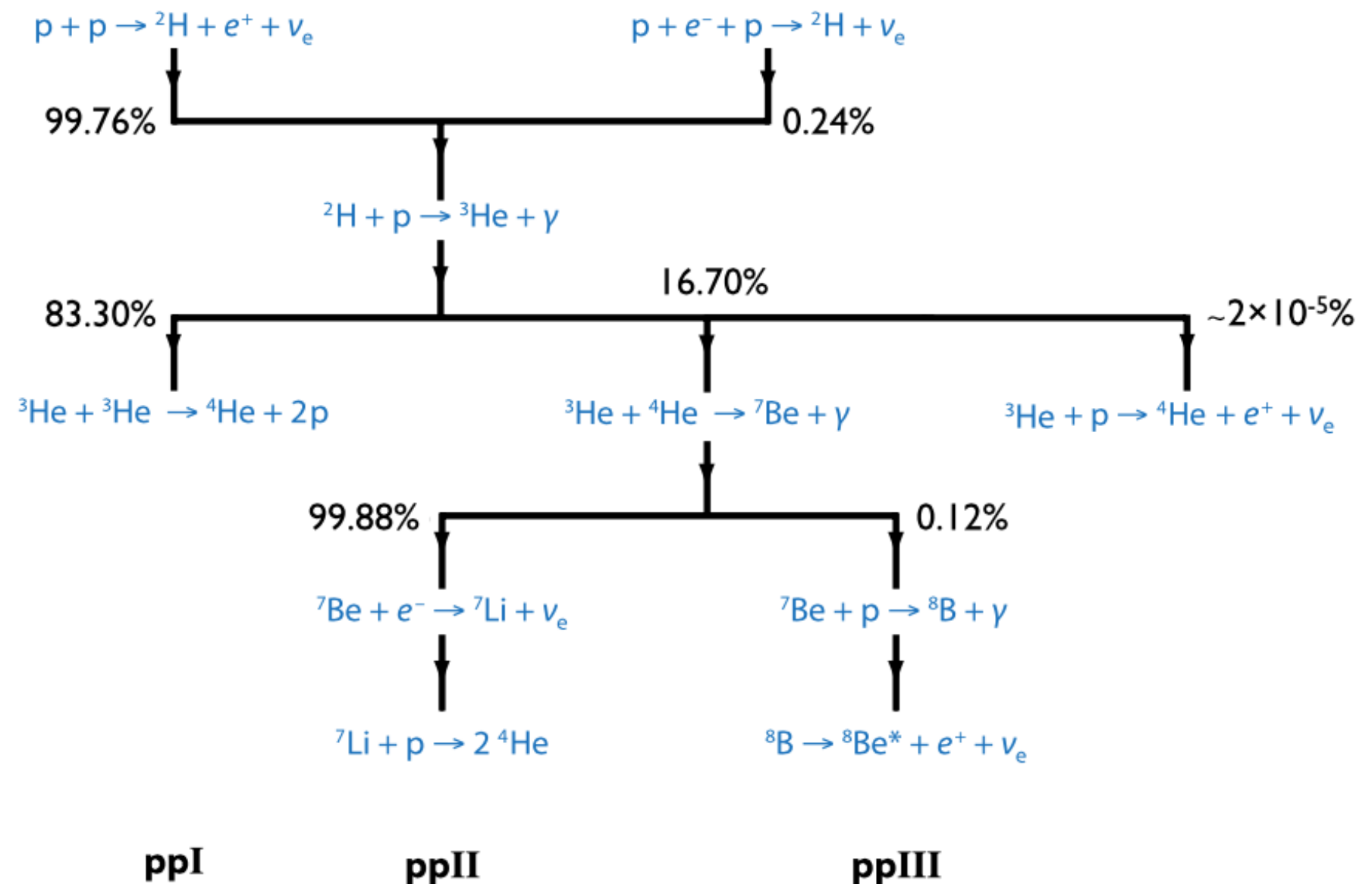


Dohet-Eraly et al., PLB (2016)

- ANC extracted from capture data: $C_{P1/2}^2 + C_{P3/2}^2 = 27 \pm 3 \text{ fm}^{-1}$
- Significant constraints on s-wave scattering parameters already from capture
- Short-distance parameter L_{EI} is smaller for data and for Nollett's *ab-initio* based calculation than for cluster models. Pauli principle?

Why is ${}^7\text{Be} + \text{p} \rightarrow {}^8\text{B} + \gamma$ important?

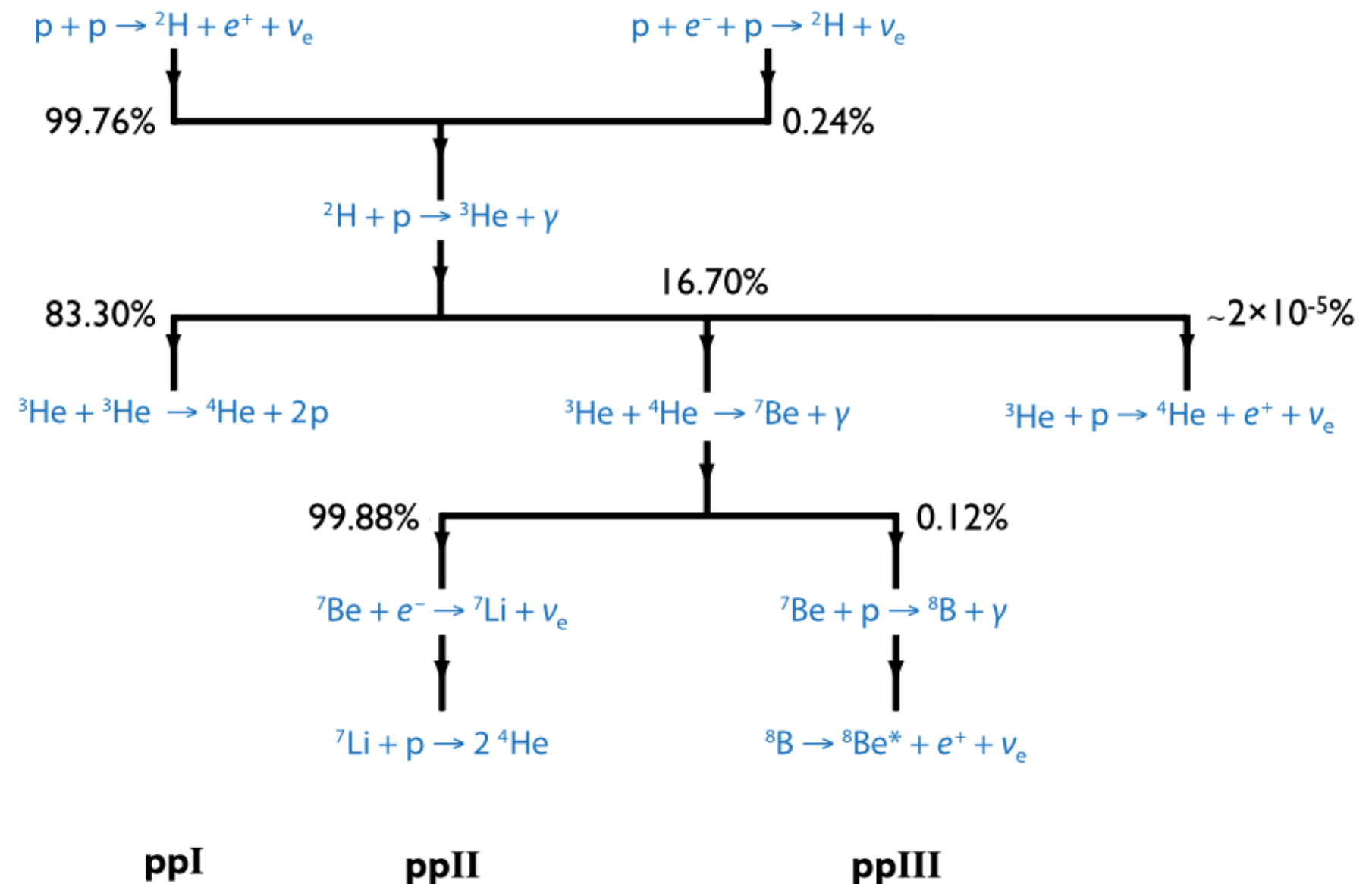
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- Part of pp chain (ppIII)

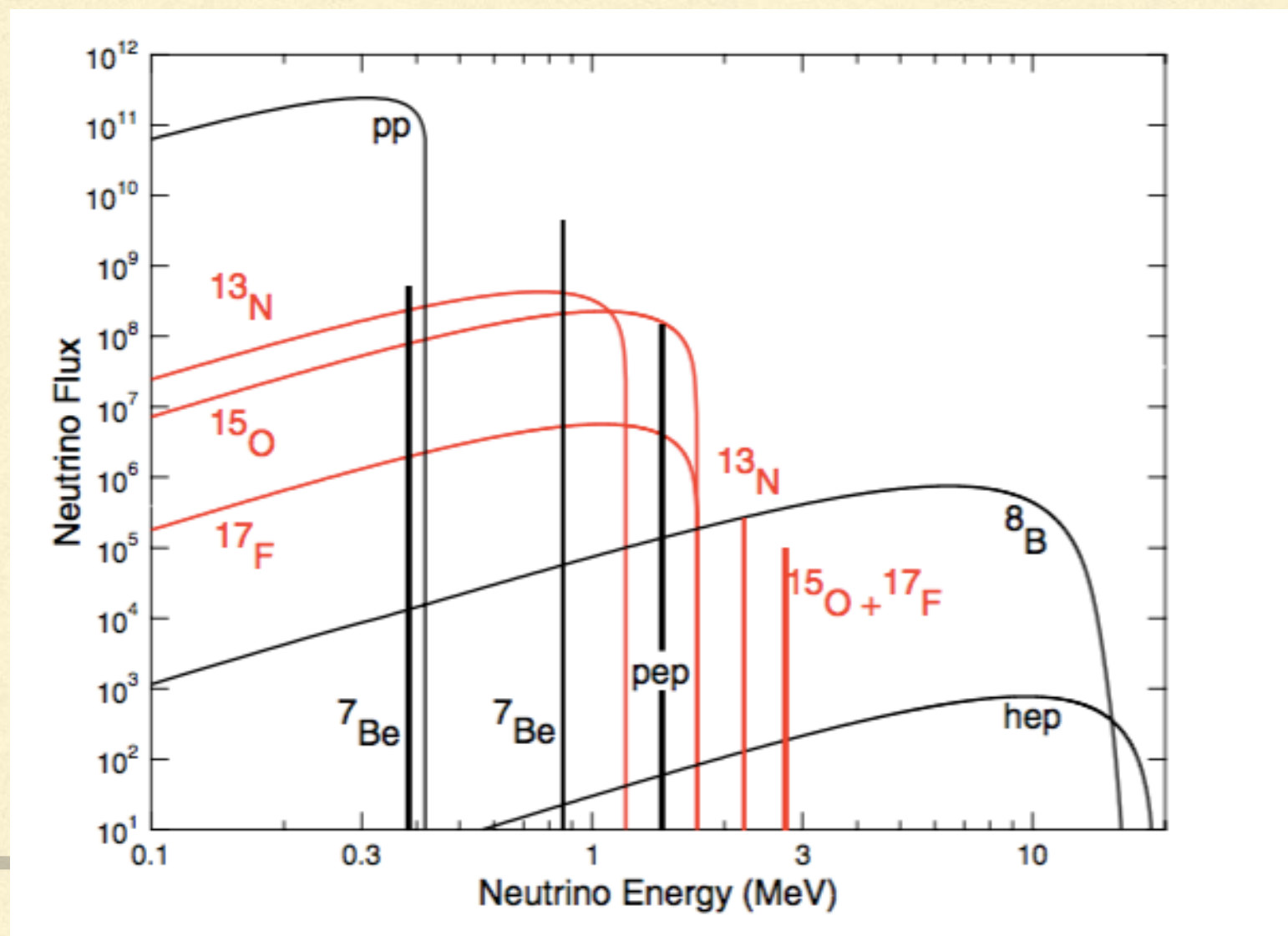
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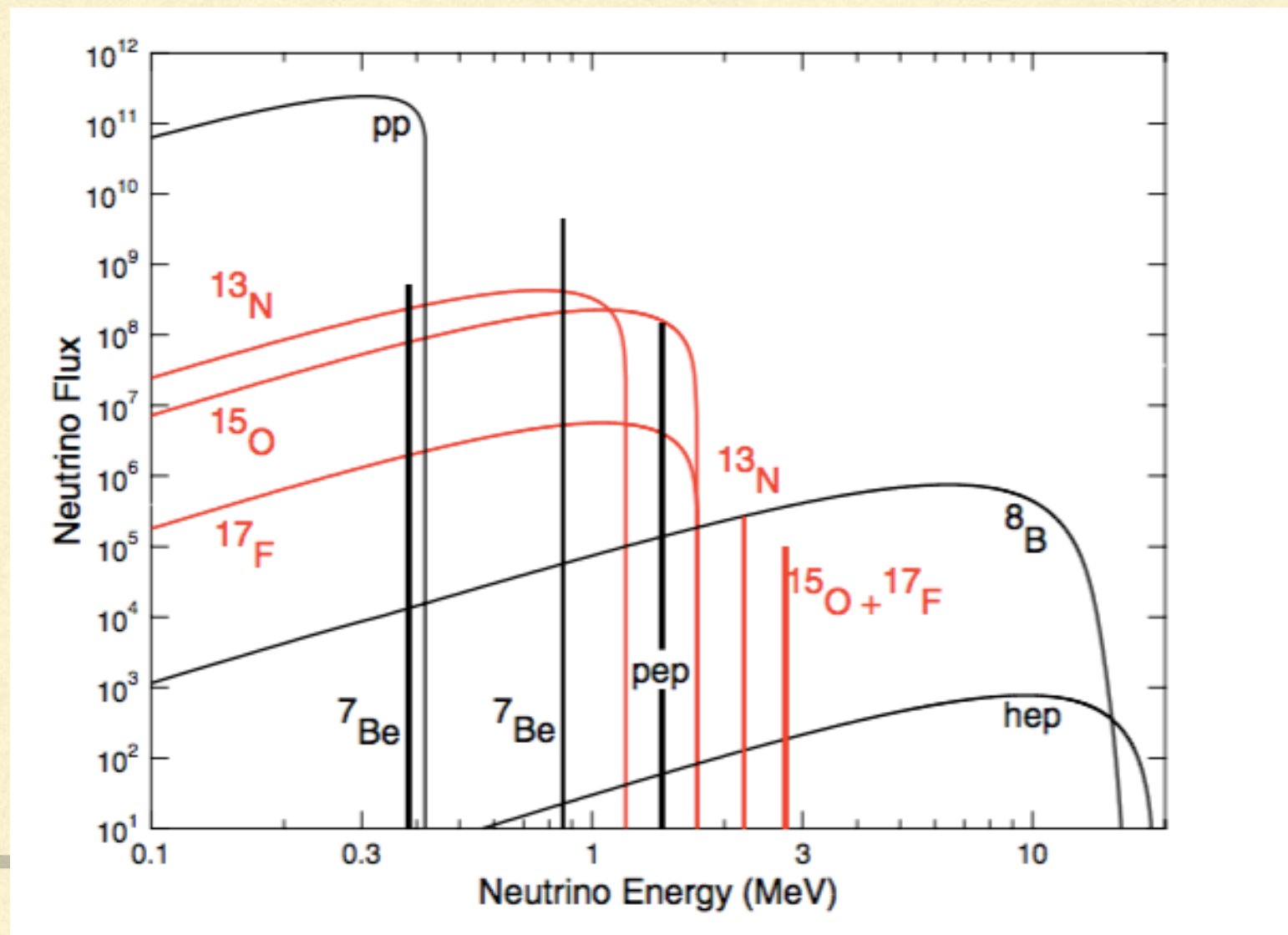
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- Part of pp chain (ppIII)
- Key for predicting flux of solar neutrinos, especially high-energy (${}^8\text{B}$) neutrinos
- Accurate knowledge of ${}^7\text{Be}(p,\gamma)$ needed for inferences from solar-neutrino flux regarding solar composition → solar-system formation history



NLO for ${}^7\text{Be}(p,\gamma)$

Zhang, Nollett, Phillips, PRC (2014)

cf. Ryberg, Forssen, Hammer, Platter, EPJA (2014)

Zhang, Nollett, Phillips, PLB (2015); PRC (2018)

- LO calculation: ISI in $S=2$ & $S=1$ into p-wave bound state. Scattering wave functions are linear combinations of Coulomb wave functions F_0 and G_0 . Bound state wave function=the appropriate Whittaker function
- We also incorporate a low-lying excited state ($1/2^-$) in ${}^7\text{Be}$
- NLO: piece of the amplitude representing capture at short distances, represented by a contact operator $\Rightarrow$ there is an LEC that must be fit

$$S(E) = f(E) \sum_s C_s^2 \left[|\mathcal{S}_{\text{EC}}(E; \delta_s(E)) + \bar{L}_s \mathcal{S}_{\text{SD}}(E; \delta_s(E)) + \epsilon_s \mathcal{S}_{\text{CX}}(E; \delta_s(E))|^2 + |\mathcal{D}(E)|^2 \right]$$

Four parameters at LO;
five more at NLO

- ANCs in ${}^5\text{P}_2$ and ${}^3\text{P}_2$: $A_{5\text{P}_2}$ and $A_{3\text{P}_2}$
- Scattering lengths and effective ranges in both ${}^5\text{S}_2$ and ${}^3\text{S}_1$: a_2, r_2 and a_1, r_1
- Core excitation: determined by ratio of ${}^8\text{B}$ couplings of ${}^7\text{Be}^*p$ and ${}^7\text{Be}-p$ states: ϵ_1
- LECs associated with contact interaction, one each for $S=1$ and $S=2$: L_1 and L_2

Data for ${}^7\text{Be} + p \rightarrow {}^8\text{B} + \gamma_{\text{EI}}$

- 42 data points for $100 \text{ keV} < E_{\text{c.m.}} < 500 \text{ keV}$
 - Junghans (BEI and BE3)
 - Fillipone
 - Baby
 - Hammache (1998 and 2001)

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 - CMEs
 - 2.7% and 2.3%
 - 11.25%
 - 5%
 - 2.2% (1998)
 - Subtract M1 resonance: negligible impact at 500 keV and below
 - Deal with CMEs by introducing five additional parameters, ξ_i
-

Building the pdf

- Bayes:

$$\text{pr}(\vec{g}, \{\xi_i\} | D; T; I) \propto \text{pr}(D | \vec{g}, \{\xi_i\}; T; I) \text{pr}(\vec{g}, \{\xi_i\} | I),$$

- First factor: likelihood

$$\ln \text{pr}(D | \vec{g}, \{\xi_i\}; T; I) = c - \sum_{j=1}^N \frac{[(1 - \xi_j)S(\vec{g}; E_j) - D_j]^2}{2\sigma_j^2},$$

- Second factor: priors

- Independent gaussian priors for ξ_i , centered at zero and with width=CME
- Gaussian priors for $a_{s=1}$ and $a_{s=2}$, based on Angulo et al. measurement
- Other EFT parameters, $r_{s=1}, r_{s=2}, L_1, L_2, \text{ANCs}, \epsilon_1$, assigned flat priors, corresponding to natural ranges
- No s-wave resonance below 600 keV

Extrinsic information

Questions we can answer

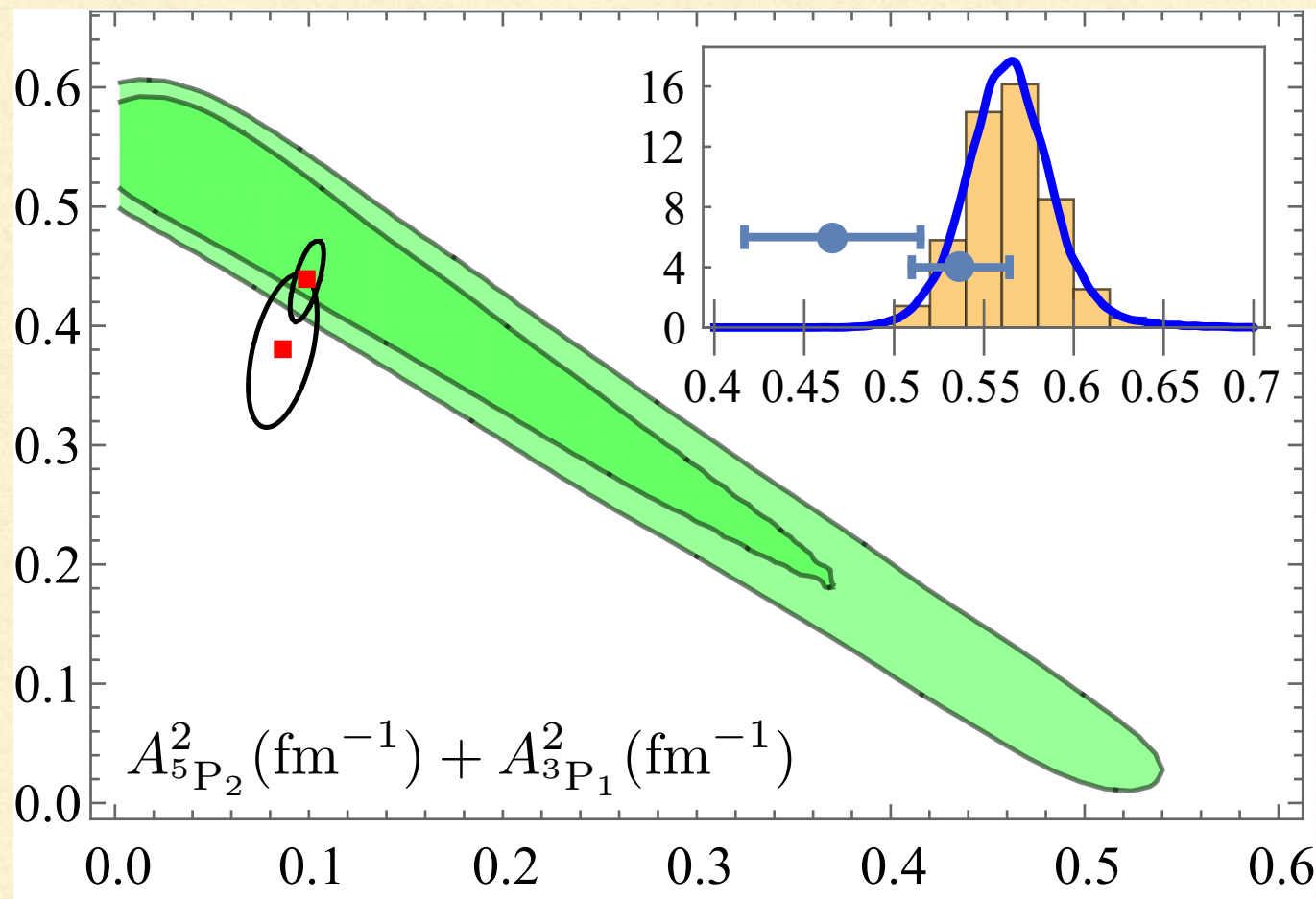
- Posteriors on parameters tell us about physics: which combinations are actually constrained?
 - Is it a “good fit”?
 - How do we see when parameters are not well constrained?
 - Do systematic errors seem to be well estimated?
 - Extrapolation
 - Is a higher-order calculation useful?
 - What quantities should future experiments go after?
-

Posterior plots $\Rightarrow$ Physics

$$\text{pr} (g_1, g_2 | D; T; I) = \int \text{pr} (\vec{g}, \{\xi_i\} | D; T; I) d\xi_1 \dots d\xi_5 dg_3 \dots dg_9$$

Posterior plots $\Rightarrow$ Physics

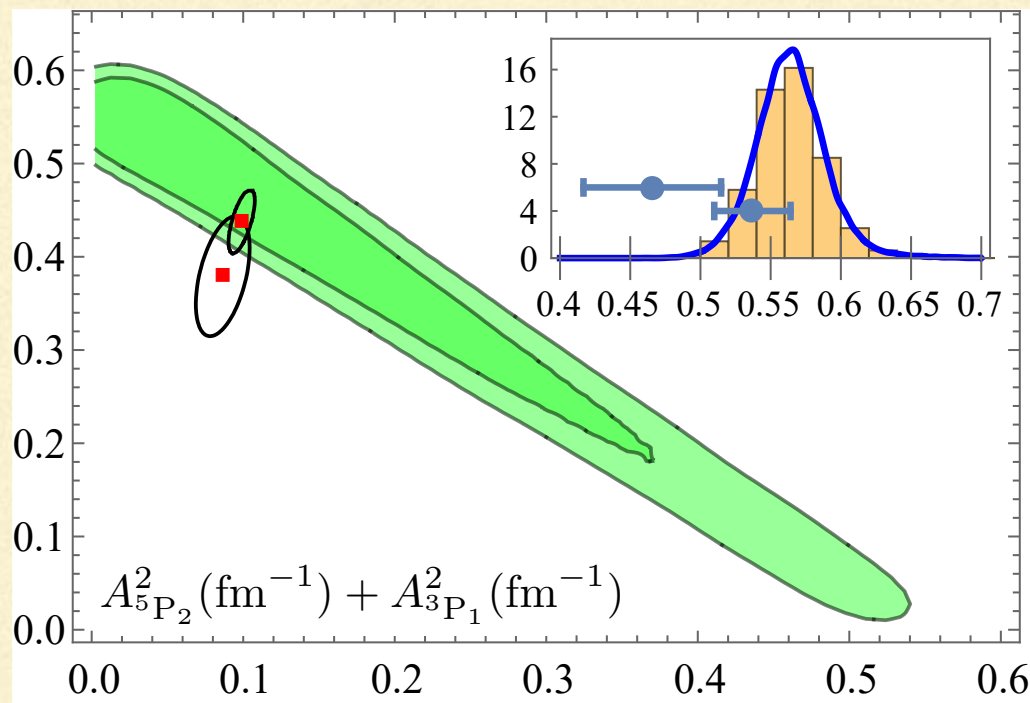
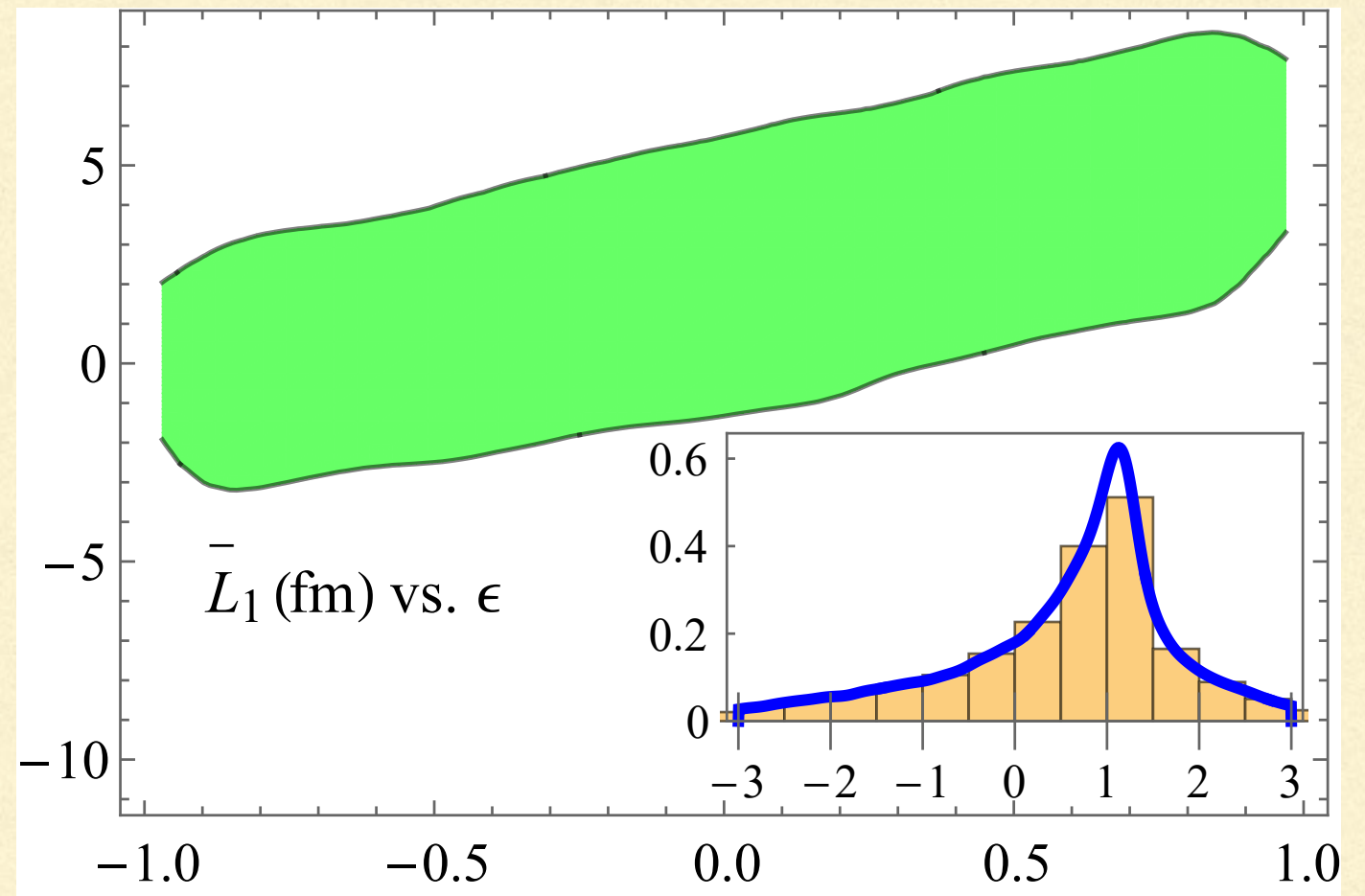
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- ANCs are highly correlated but sum of squares strongly constrained

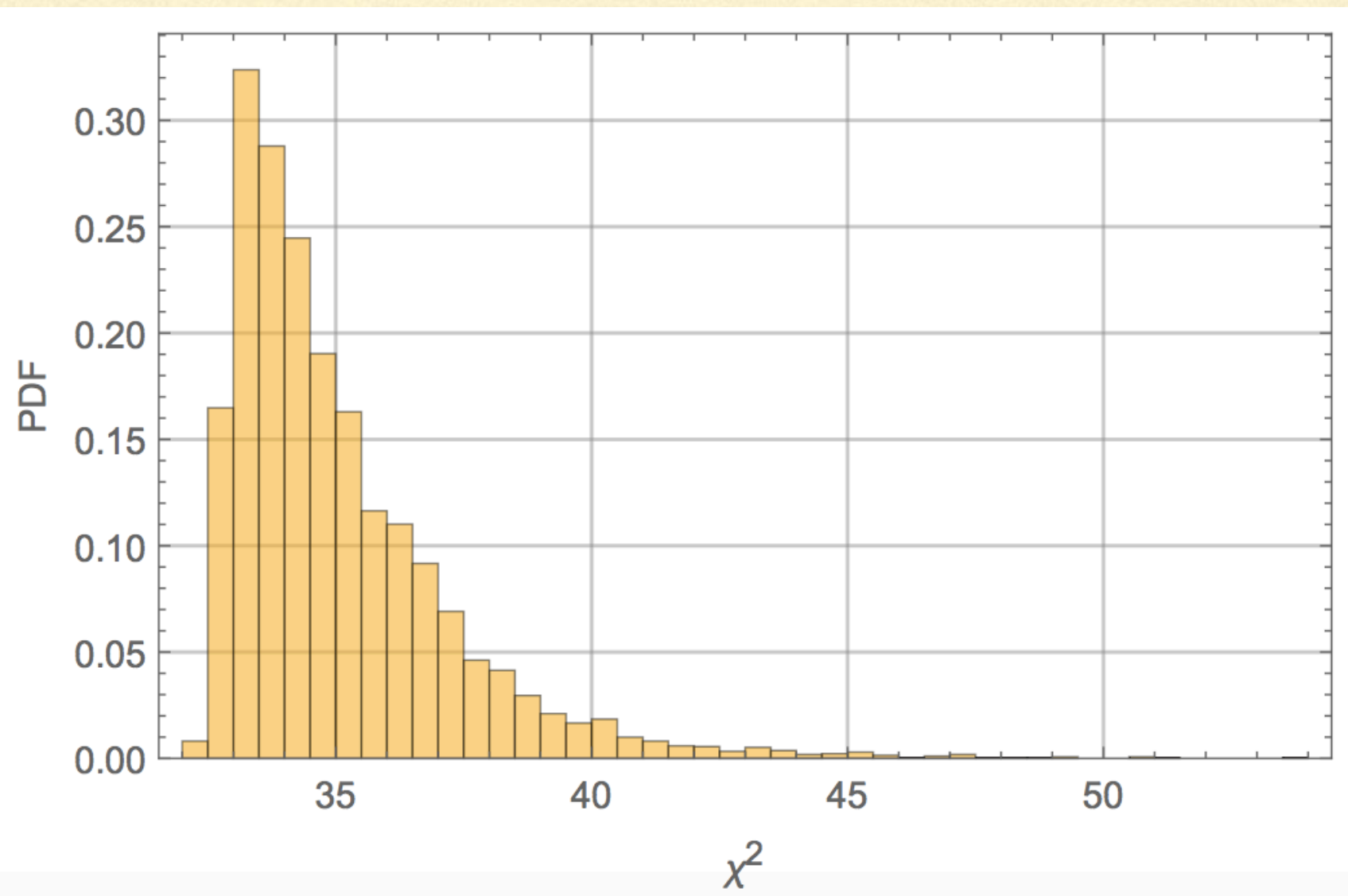
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- ANCs are highly correlated but sum of squares strongly constrained
- One spin-I short-distance parameter: $0.33 \bar{L}_1 / (\text{fm}^{-1}) - \epsilon_1$

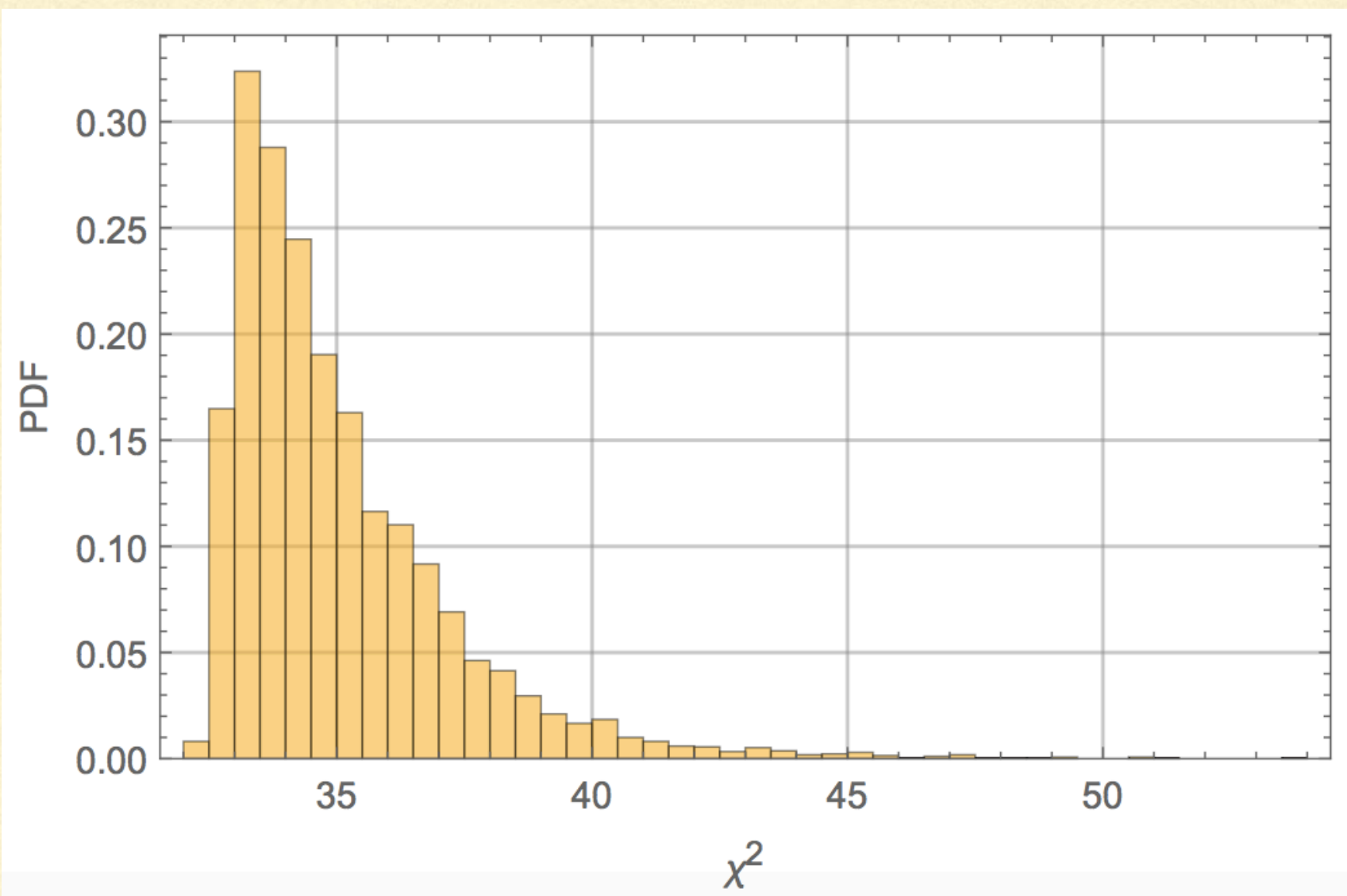
More questions we can answer



42 data points,
7 parameters “fit”
to these data,
5 ξ_i ’s fixed to their
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More questions we can answer

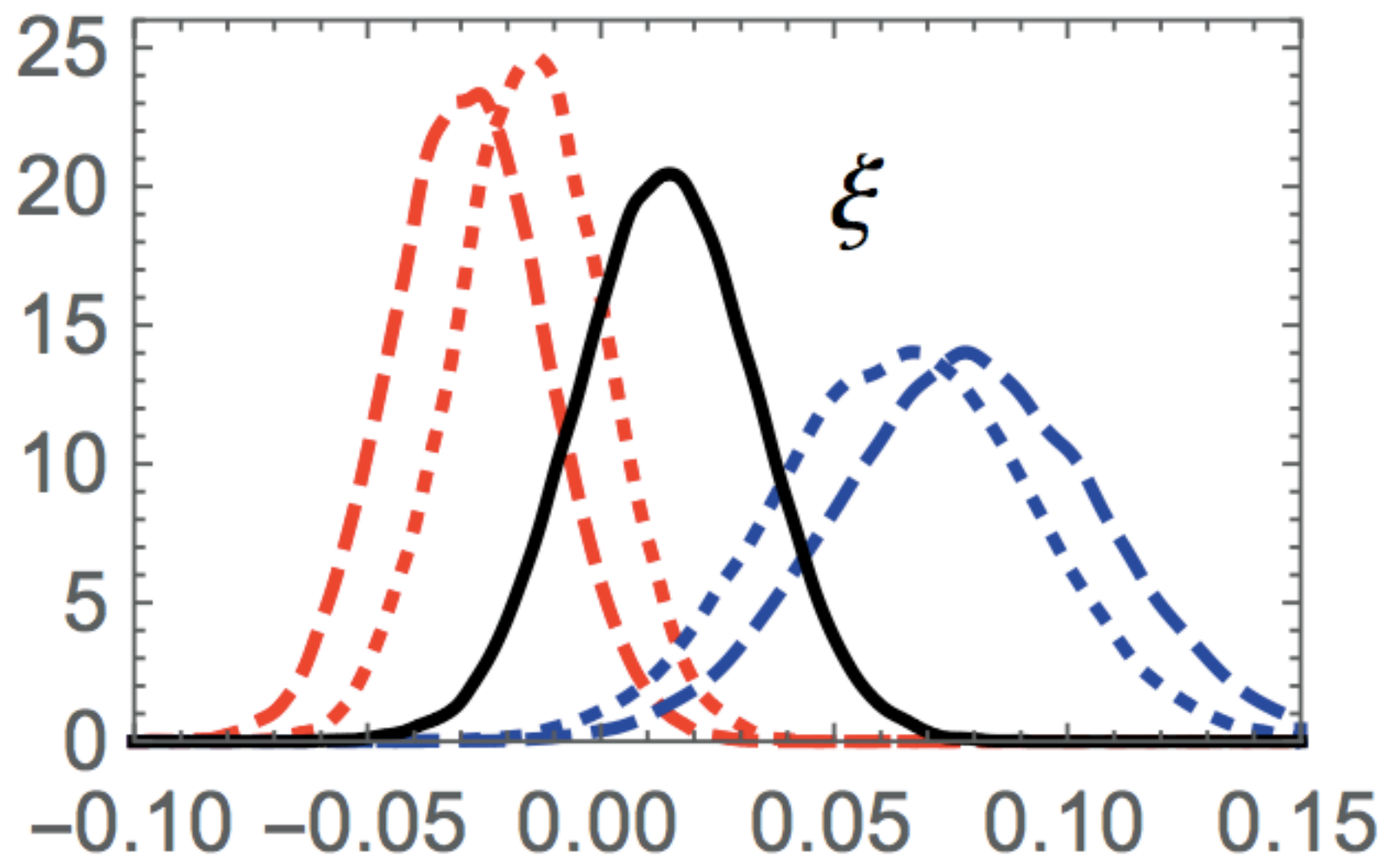
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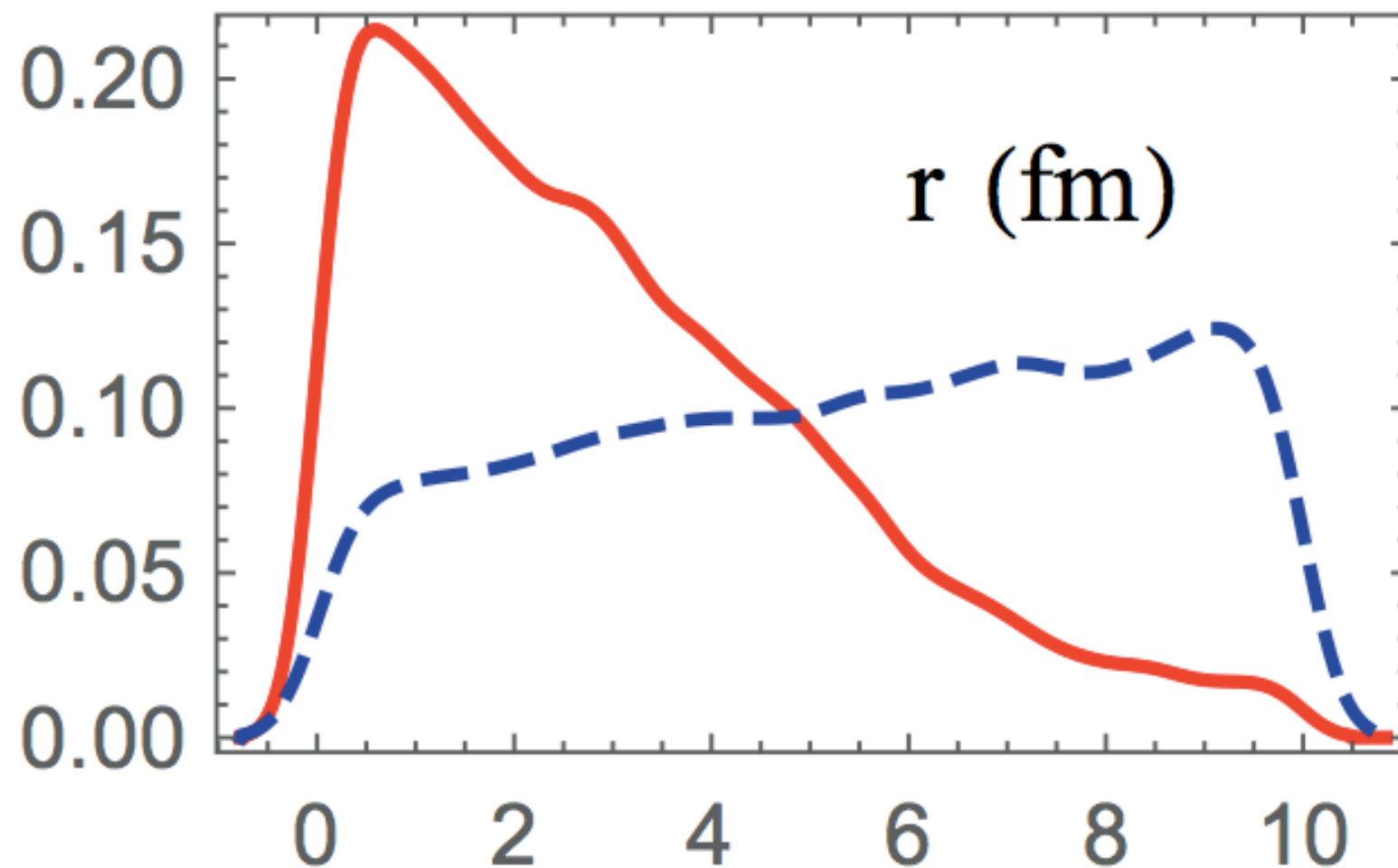
More questions we can answer

- Is it a “good fit”?
- Did the experimentalists understand their systematic errors?



More questions we can answer

- Is it a “good fit”?
- Did the experimentalists understand their systematic errors?
- Are there parameters that are not well constrained by these data?



Final result

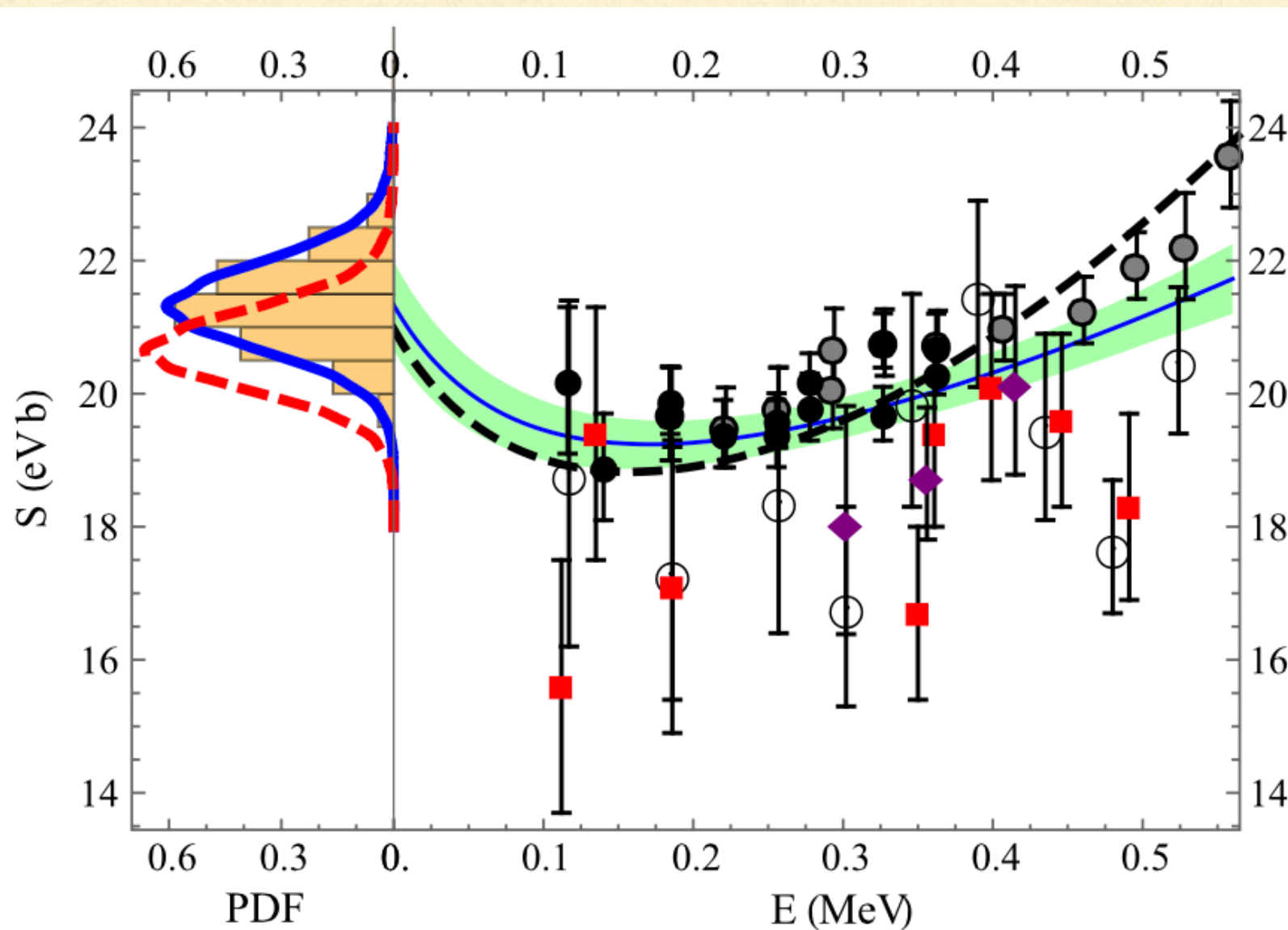
Zhang, Nollett, DP, PLB, 2015

$$\text{pr}(\bar{F}|D;T;I) = \int \text{pr}(\vec{g}, \{\xi_i\}|D;T;I) \delta(\bar{F} - F(\vec{g})) d\xi_1 \dots d\xi_5 d\vec{g}$$

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Zhang, Nollett, DP, PLB, 2015

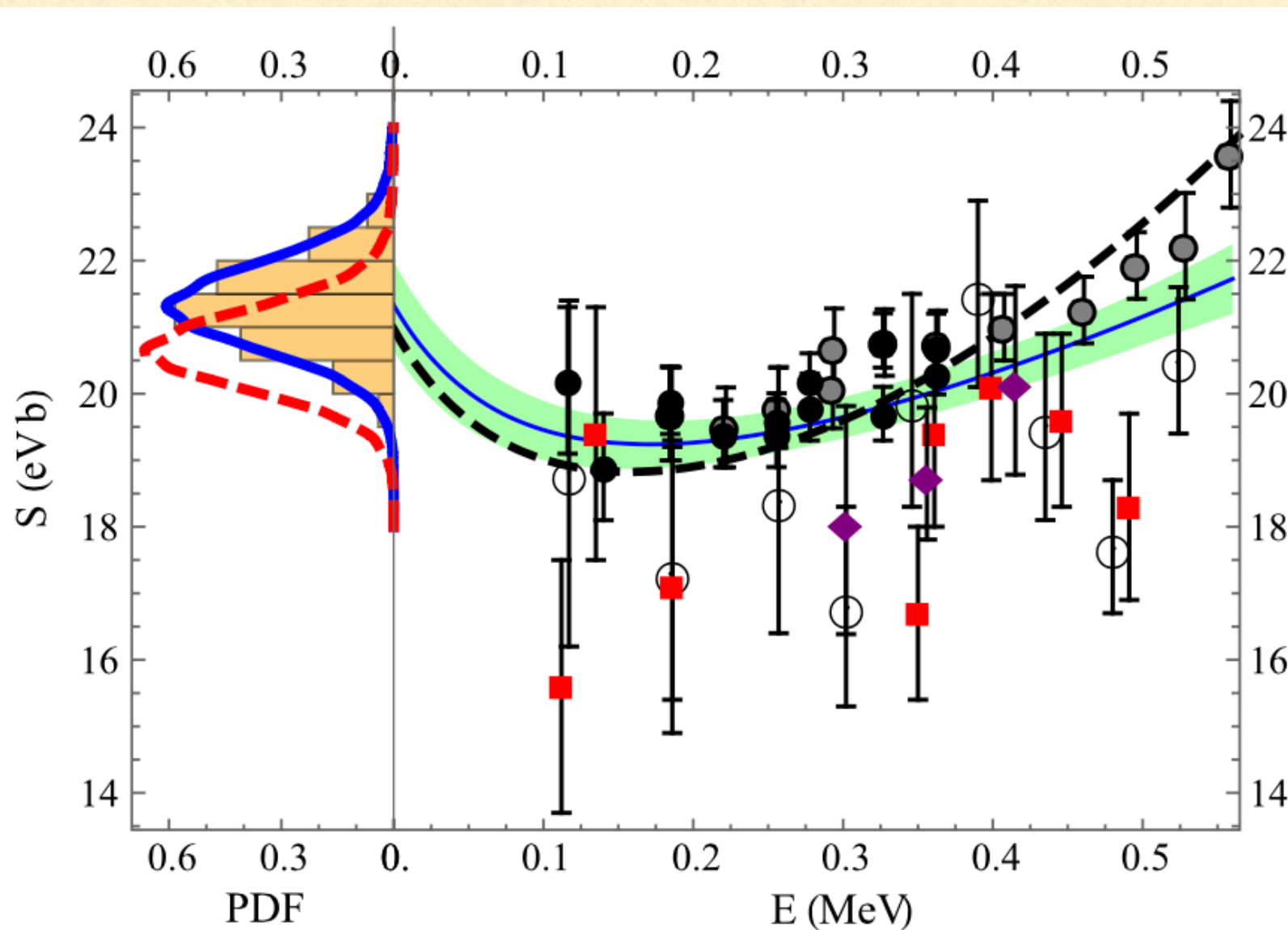
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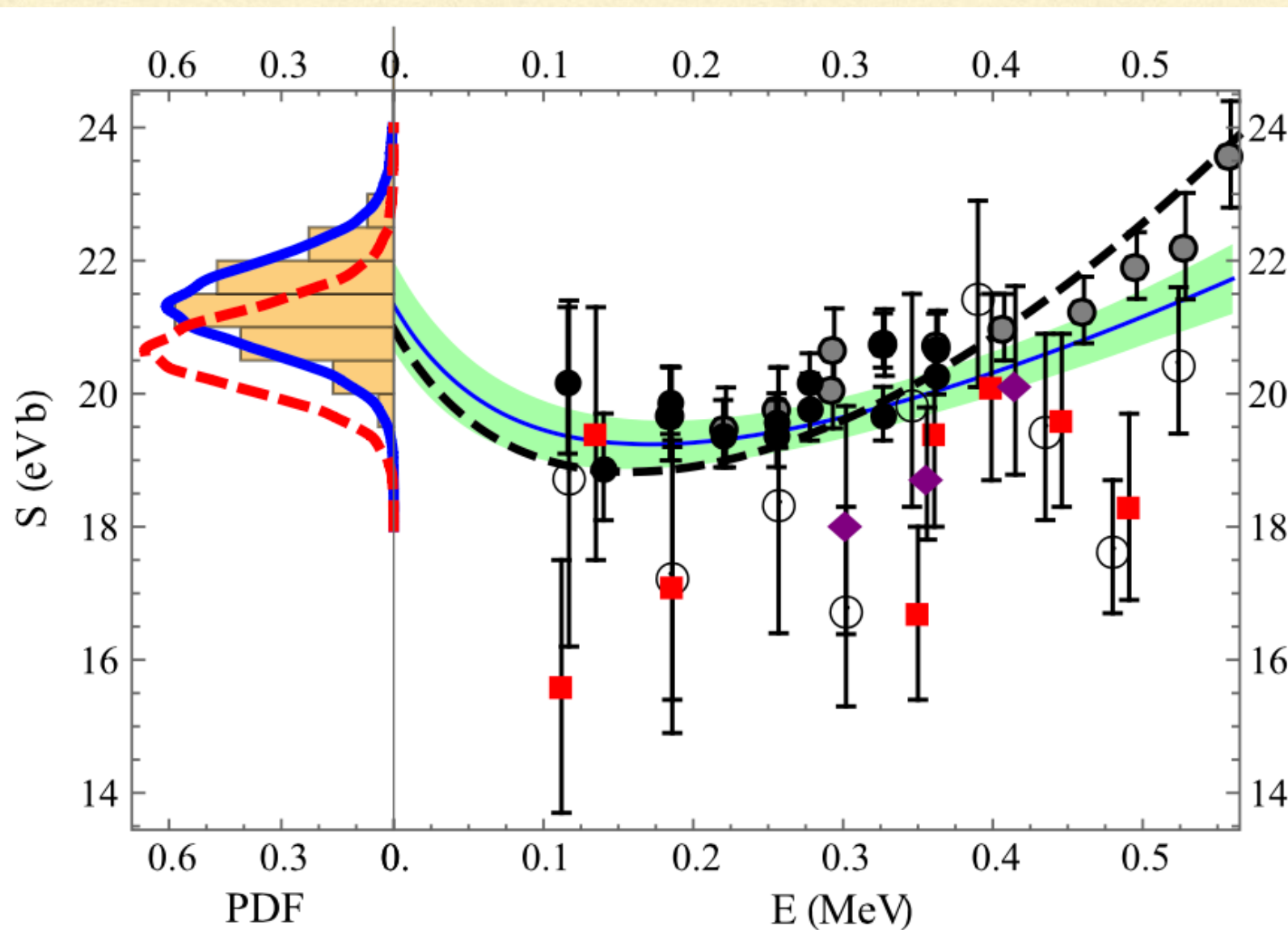


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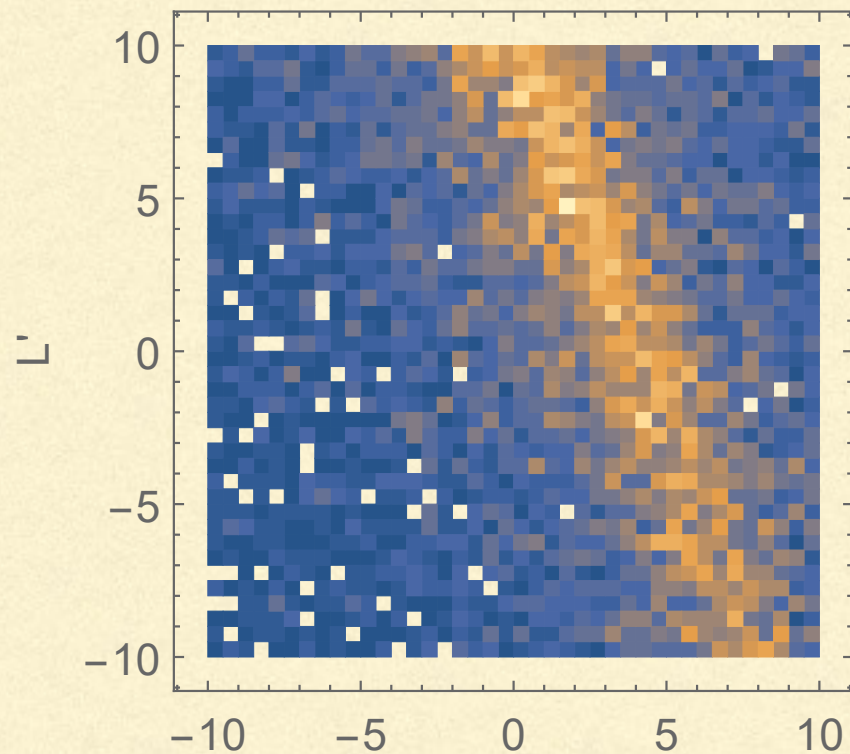


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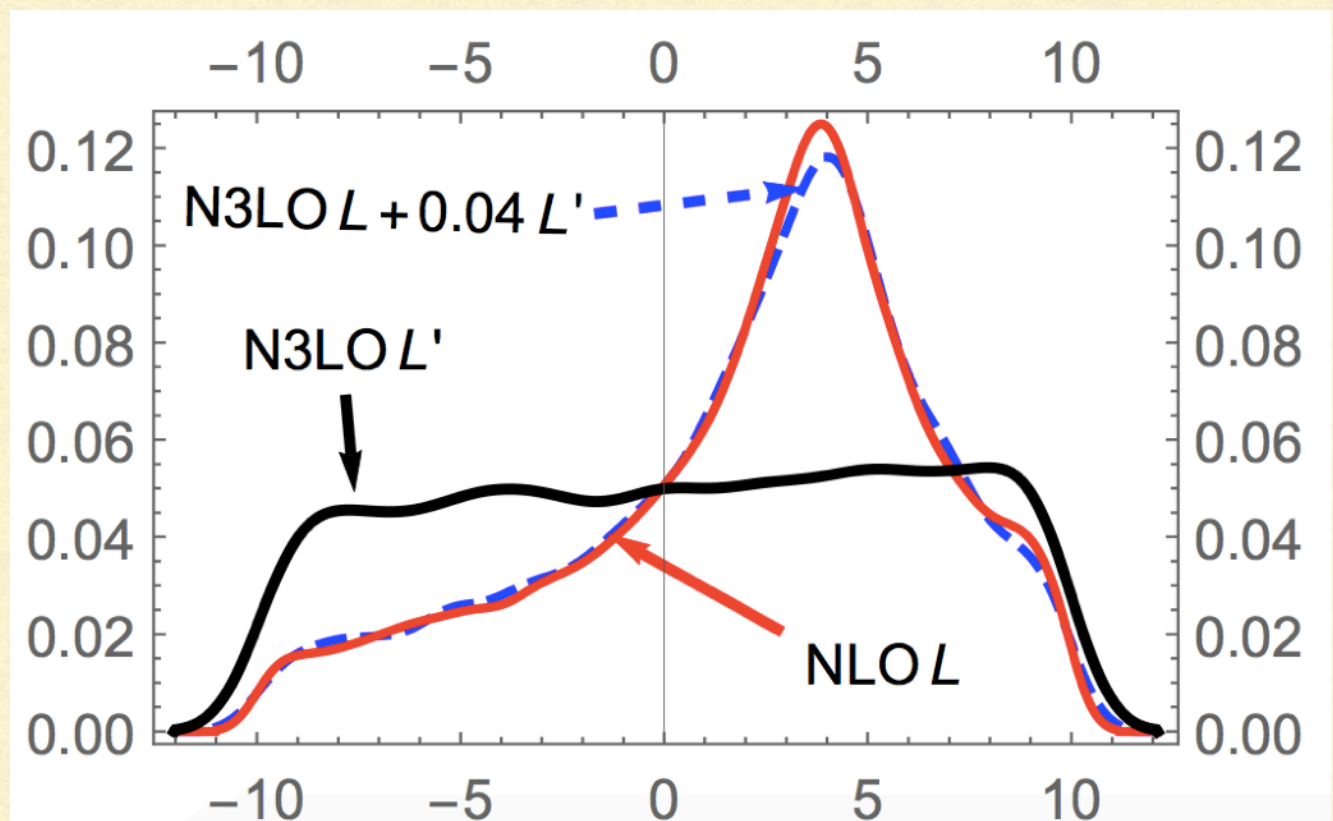
Uncertainty reduced by
factor of two: model
selection

Truncation error

- N2LO correction=0 (technically only in absence of excited state)
- EFT s-wave scattering corrections (shape parameter)~0.8%
- E2, M1 contributions < 0.01%, Radiative corrections: ~0.1%
- So first correction is at N3LO, i.e., $\bar{L}_i \rightarrow \bar{L}_i + k^2 \bar{L}'_i$

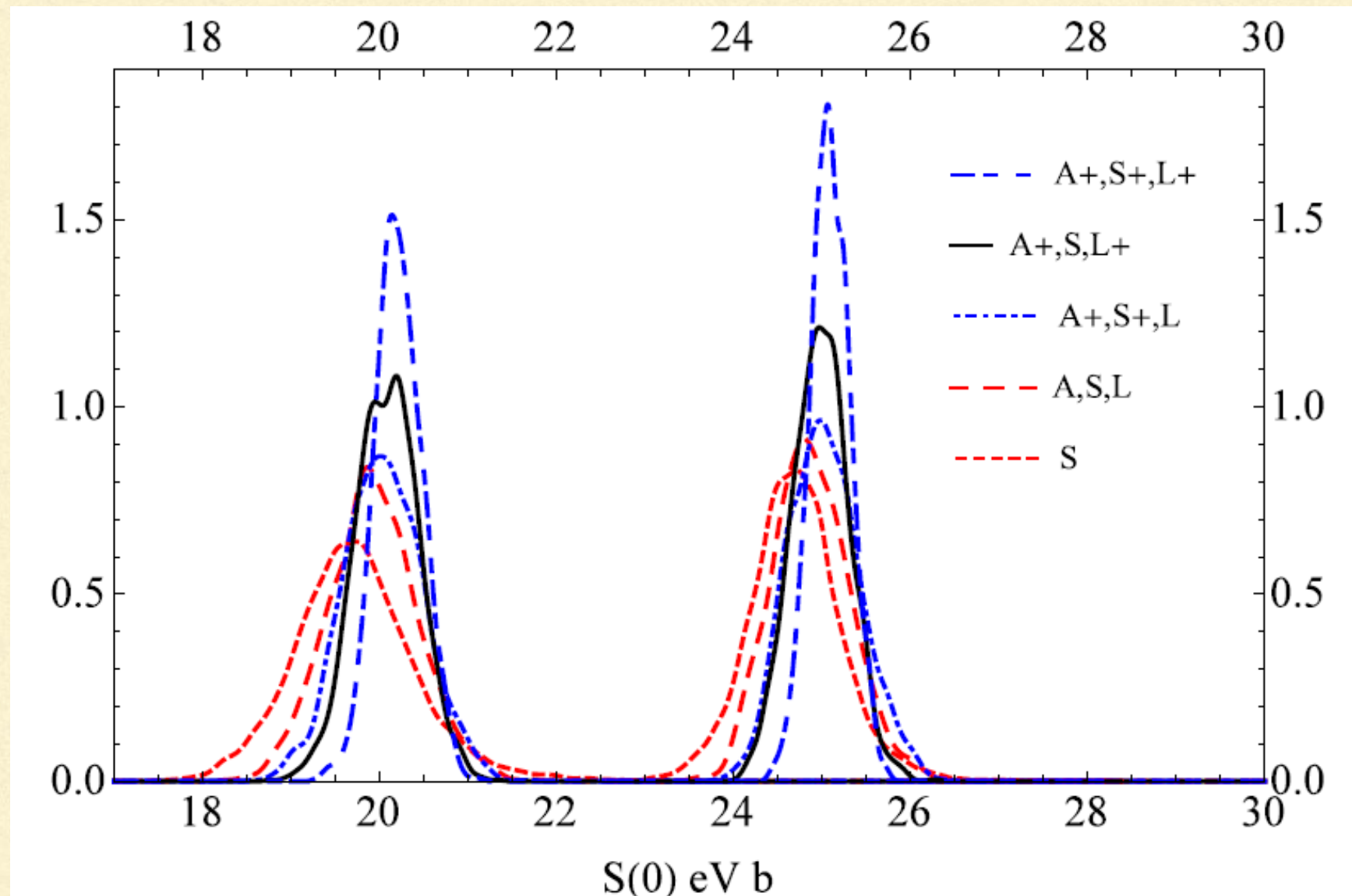


L



Planning improvements

Use extrapolant to simulate impact of hypothetical future data that could inform posterior pdf for $S(0)$



Left-to-right:
42 data points all of
similar quality
to Junghans et al.

A: ANC
S: $a_{S=1}$ and $a_{S=2}$
L: short-distance

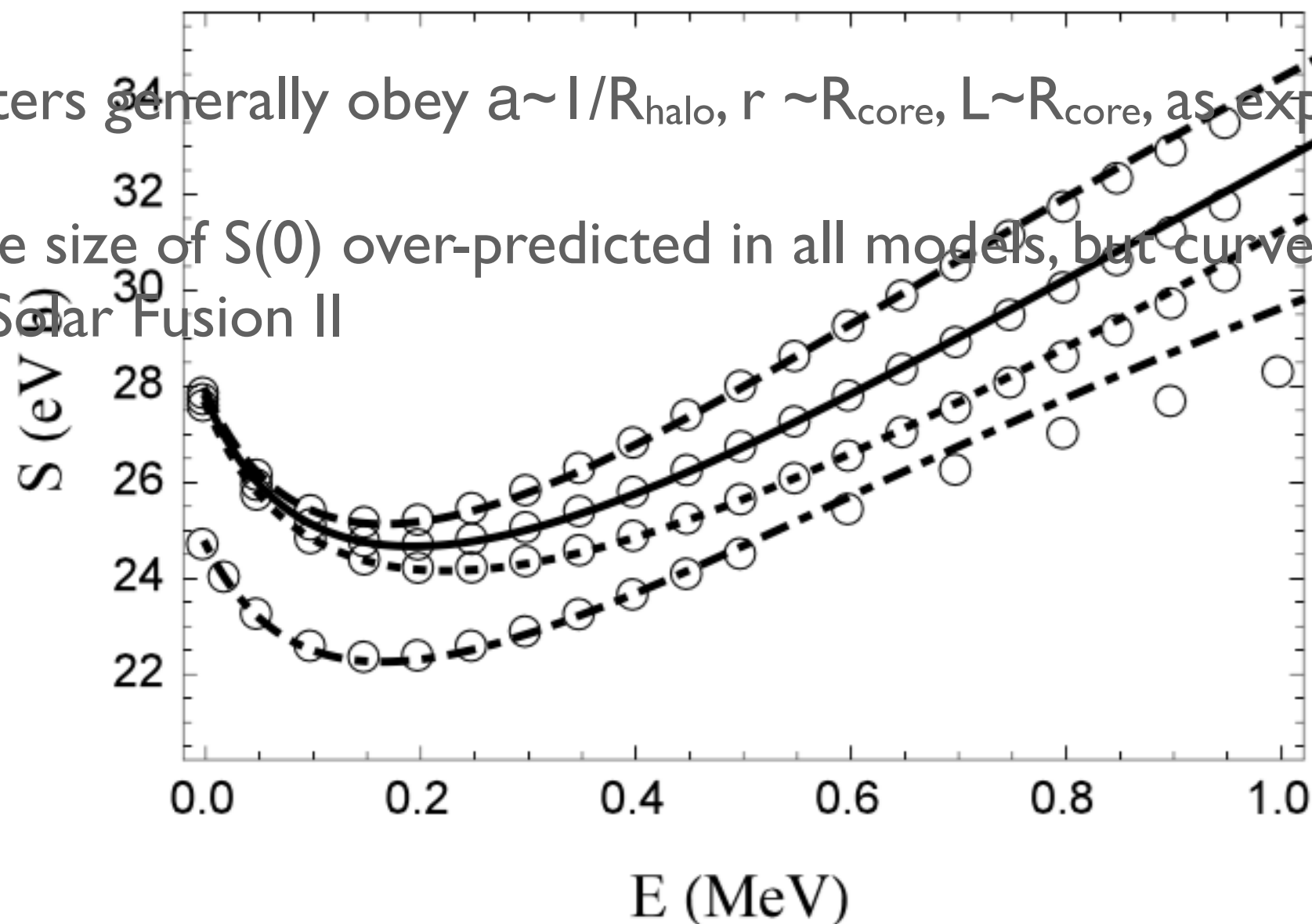
Note that 1 keV uncertainty in S_{Ip} of ^8B may not be negligible effect

Halo EFT as a “super model”

- Halo EFT is also the EFT of all the models used to extrapolate the cross section in “Solar Fusion II”
- Differences are sub-% level between 0 and 0.5 MeV

- Parameters generally obey $a \sim 1/R_{\text{halo}}$, $r \sim R_{\text{core}}$, $L \sim R_{\text{core}}$, as expected

- Absolute size of $S(0)$ over-predicted in all models, but curves rescaled in fits for Solar Fusion II



C_2
($^3P_2$)
0.20068
0.20066
0.20065
0.10900

TABLE
for scat
assumed

L_2
2.68987
3.10464
4.18777
3.73317
is fm^{-1} ,
explicitly