Quantum many-body physics near decay thresholds

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Supported by the US Department of
Energy Award number: DE-SC0009883

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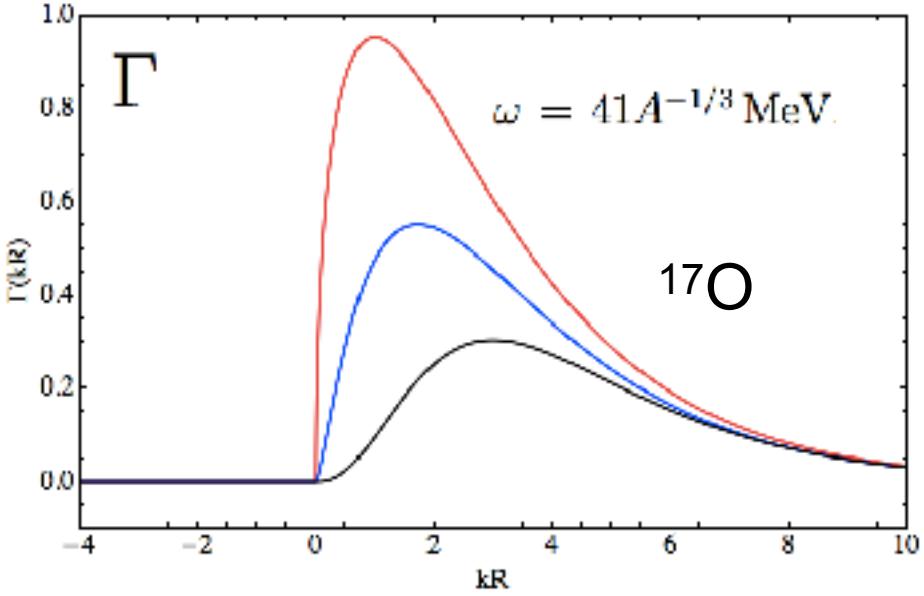
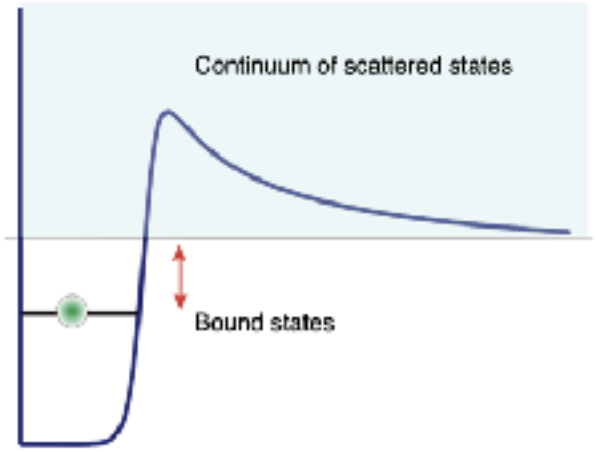
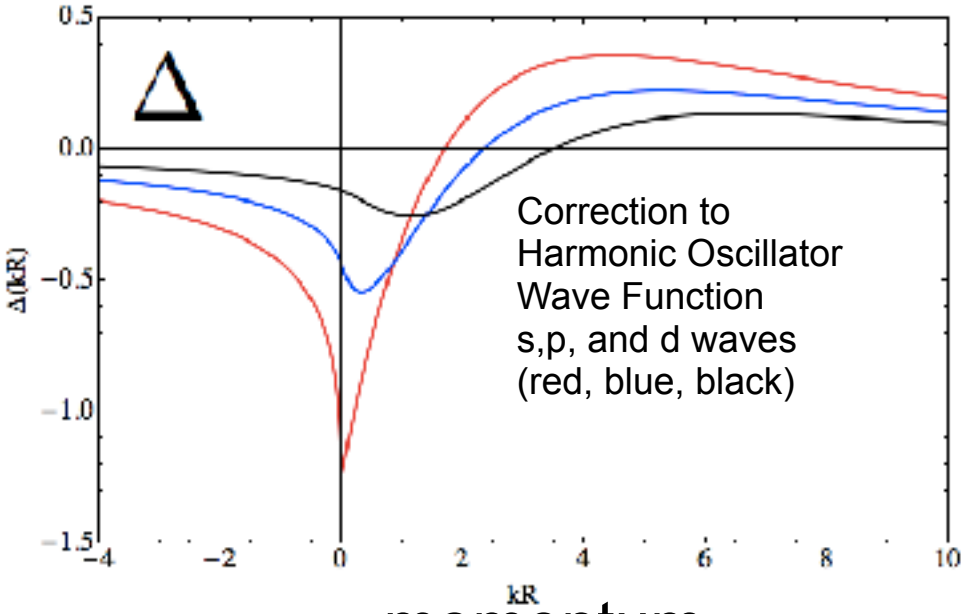
Threshold discontinuity

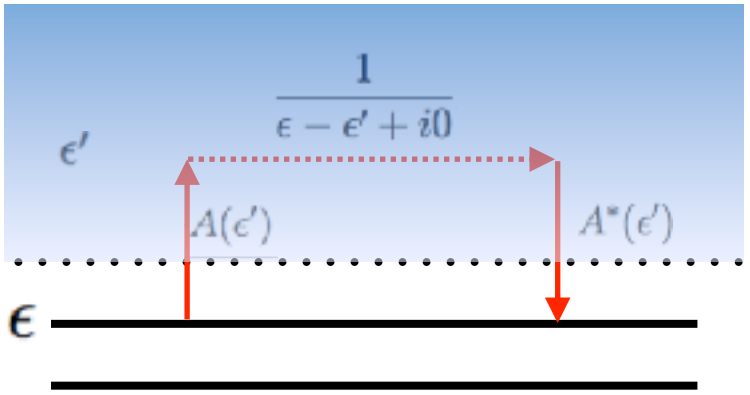






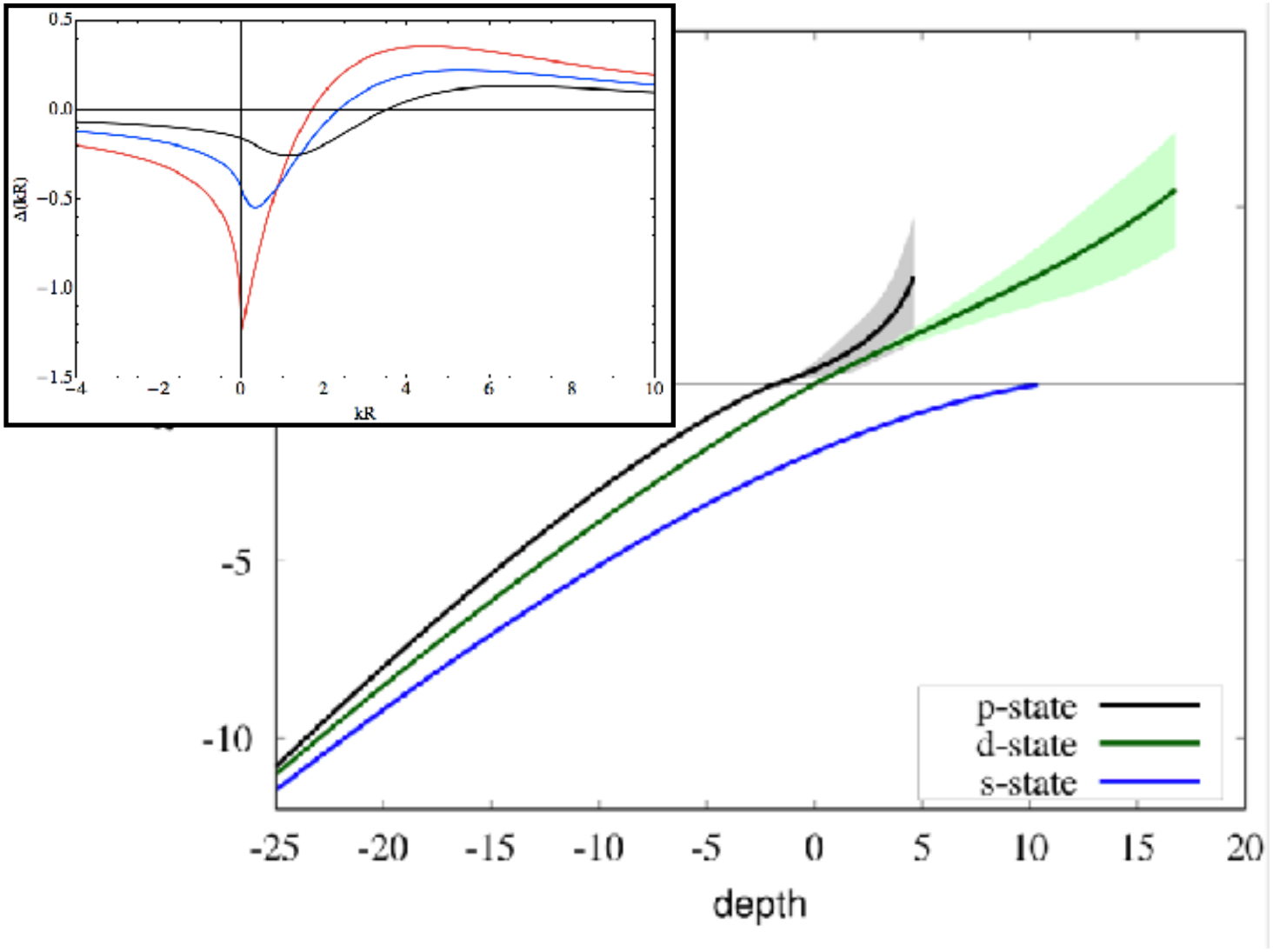
Correction to
Harmonic Oscillator
Wave Function
s,p, and d waves
(red, blue, black)

momentum



$$H'(\epsilon) = \int_0^\infty d\epsilon'\, A^*(\epsilon') \frac{1}{\epsilon - \epsilon' + i0} A(\epsilon')$$

Effect of weak binding



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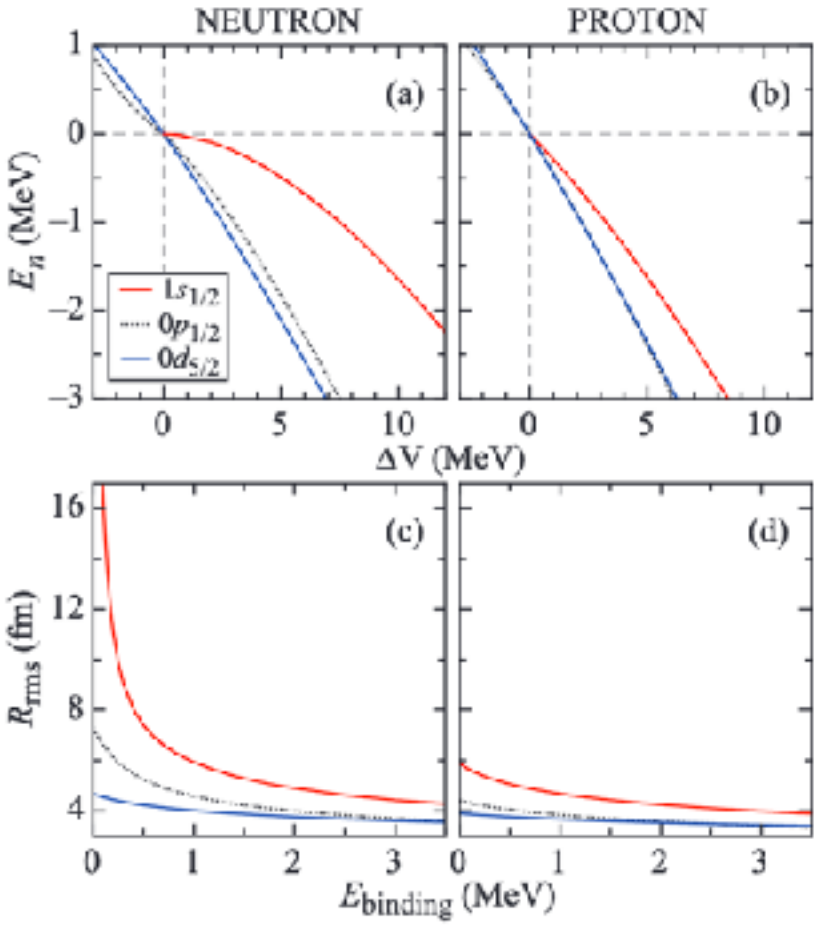
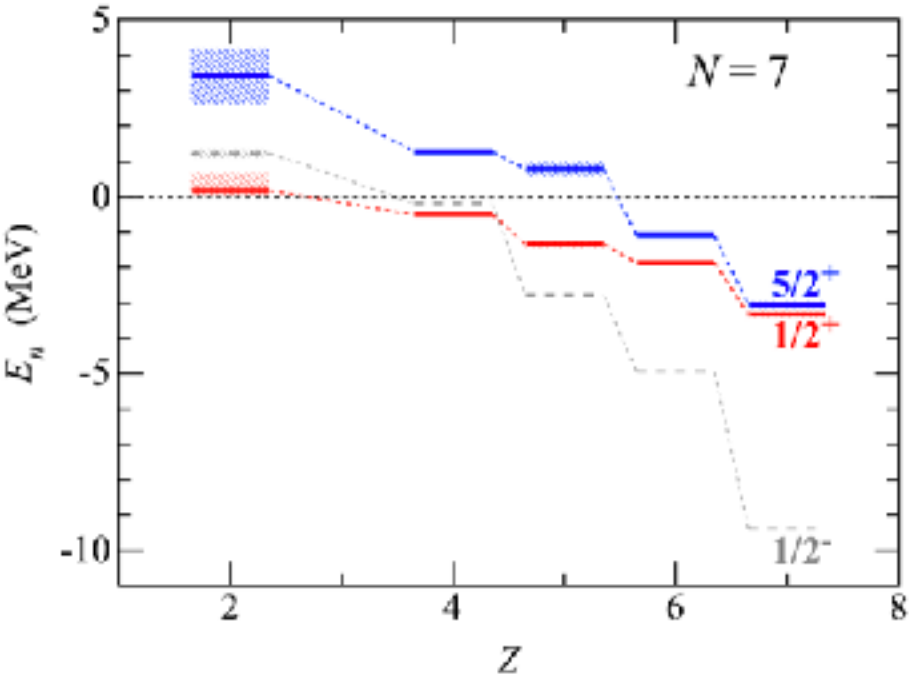


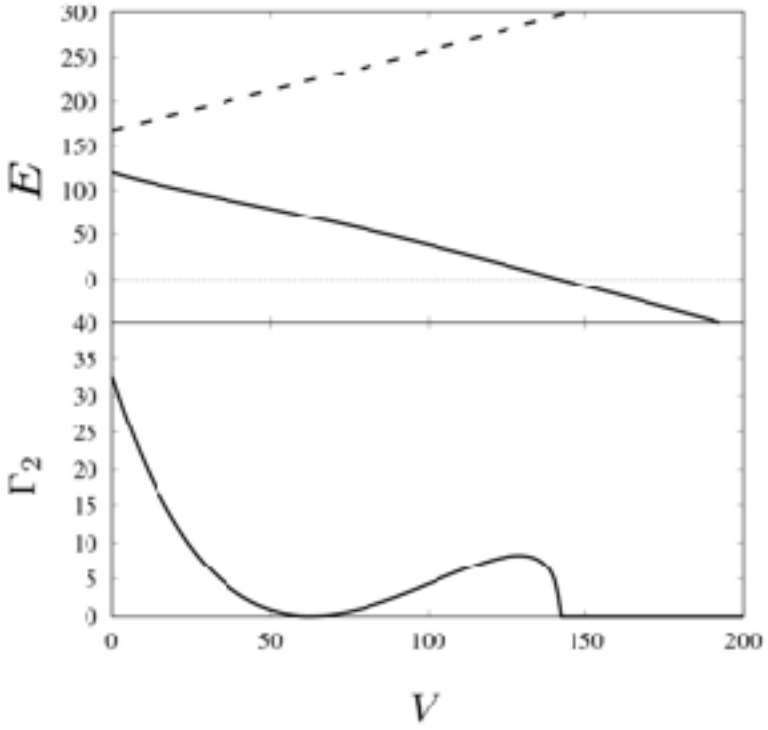


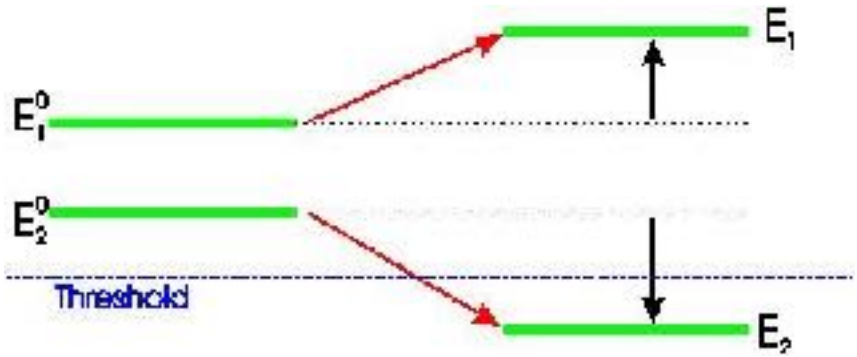
C. R. Hoffman, B. P. Kay, and J. P. Schiffer Phys. Rev. C 89, 061305(R)
B. P. Kay, C. R. Hoffman, and A. O. Macchiavelli Phys. Rev. Lett. 119, 182502

Wave function realignment
Superradiance

- Model $\mathcal{H}$

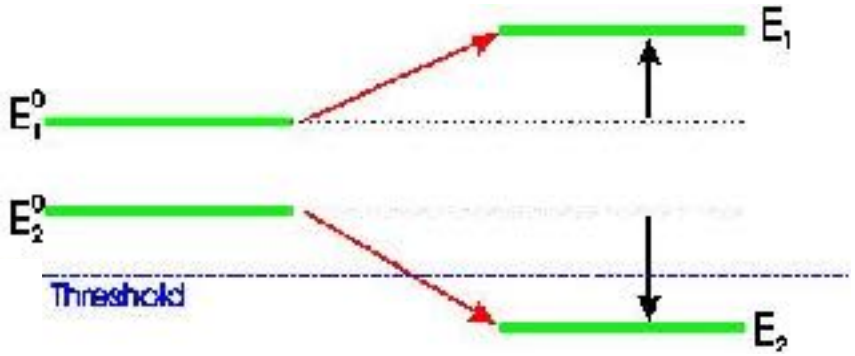
$$\mathcal{H}(E) = \begin{pmatrix} \epsilon_1 - \frac{i}{2}\gamma_1 & v - \frac{i}{2}A_1A_2 \\ v - \frac{i}{2}A_1A_2 & \epsilon_2 - \frac{i}{2}\gamma_2 \end{pmatrix}$$



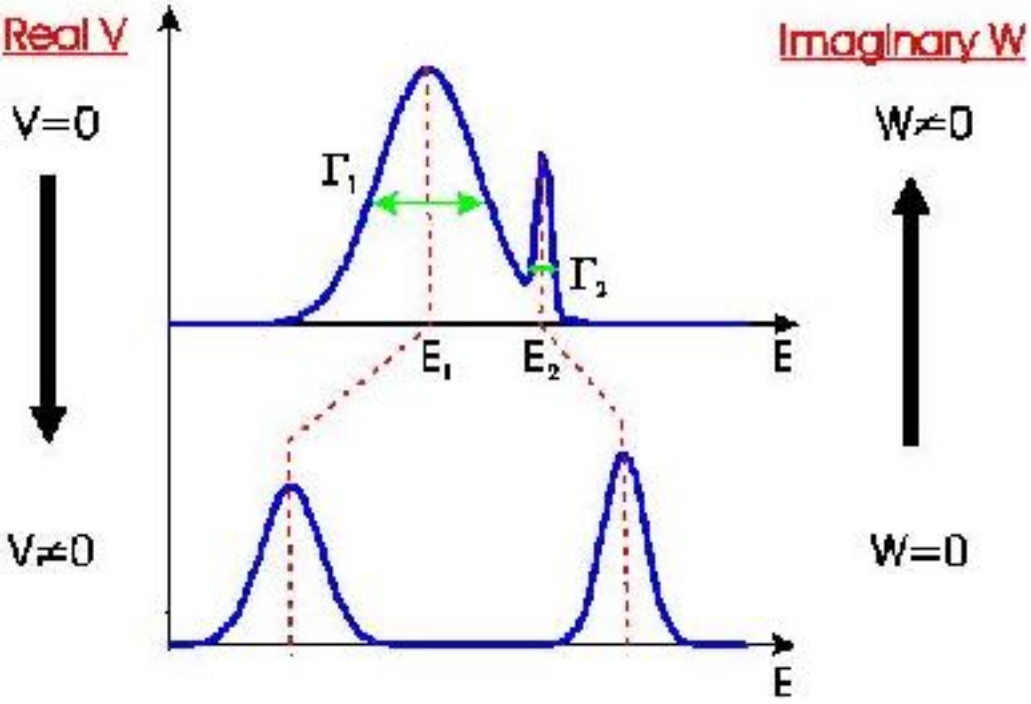


Example of interacting resonances

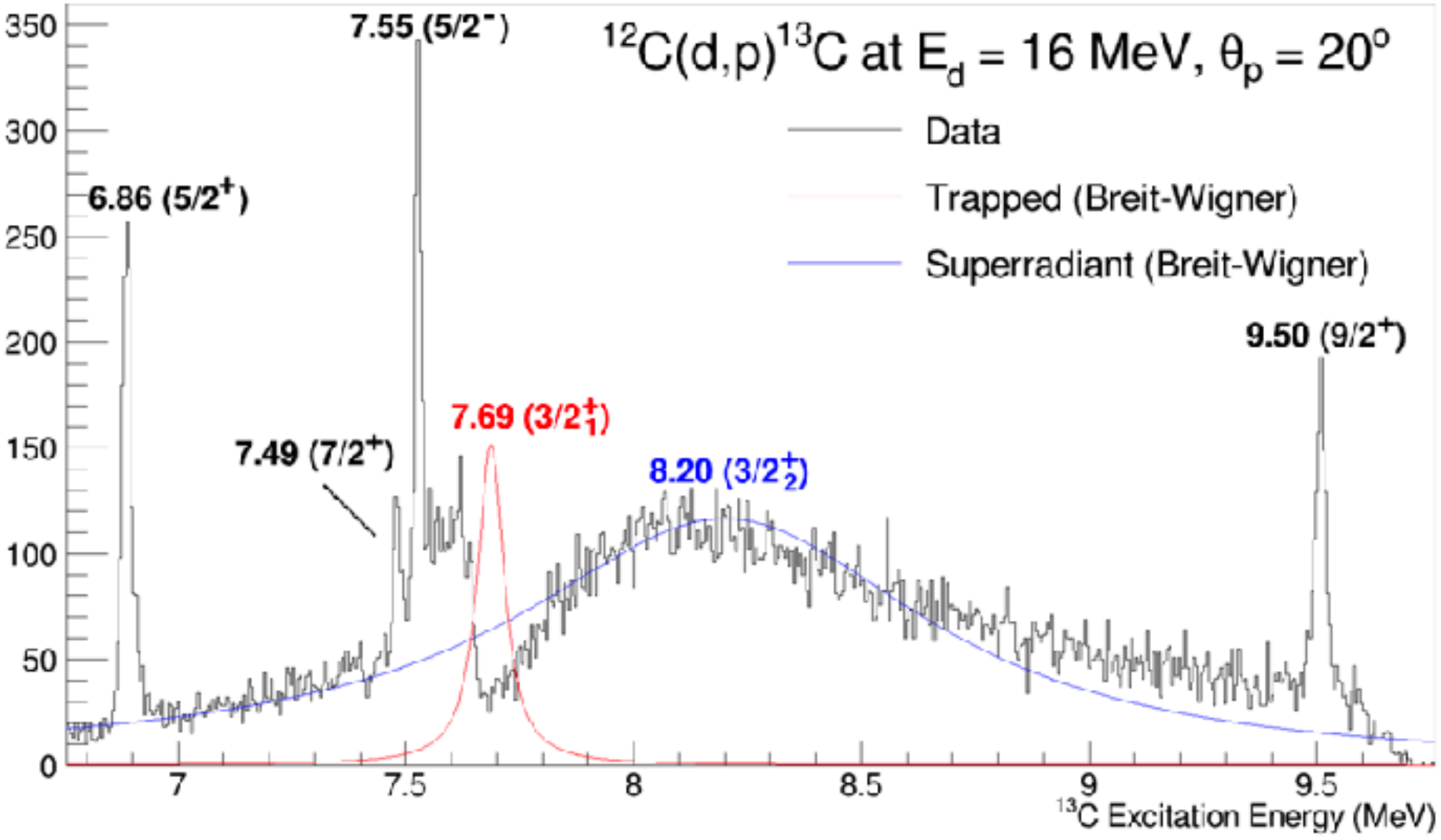
^{13}C, ^{13}N, ^{11}Li



$\mathcal{H}=H^0+V-iW/2$



^{13}C interference of levels



K. Hanselman *Direct Reactions And Decay Correlations Studying The Proton-Unbound Superradiant State In 13N,* Thesis, Florida State University 2022

^{13}C interference of levels

	Exp. Energy	Exp. Width	SM Width	CSM Width
$3/2^+(1)$	7.686(6)	0.070(5)	0.809	0.094
$3/2^+(2)$	8.2(1)	1.1(3)	0.369	0.984

K. Kravvaris and A. Volya, “Quest for superradiance in atomic nuclei,” AIP Conf. Proc., vol. 1912, no. 1, p. 020010, 2017, doi: 10.1063/1.5016135.

K. Hanselman *Direct Reactions And Decay Correlations Studying The Proton-Unbound Superradiant State In 13N,* Thesis, Florida State University 2022

Wave function realignment
Superradiance

$$H = \begin{pmatrix} \epsilon - \frac{i}{2}\Gamma & v \\ v & 0 \end{pmatrix} = H_0 - \frac{i\Gamma}{2} A^\dagger A \qquad A = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Stationary system $\Gamma = 0$

Energies $$E_{1,2} = \frac{1}{2}\left(\epsilon \pm \sqrt{\epsilon^2 + 4v^2}\right)$$

Spectroscopic Factors $$\mathrm{SF}_{1,2} = \frac{1}{2}\left(1 \pm \frac{\epsilon}{\sqrt{\epsilon^2 + 4v^2}}\right)$$

Observing superradiance

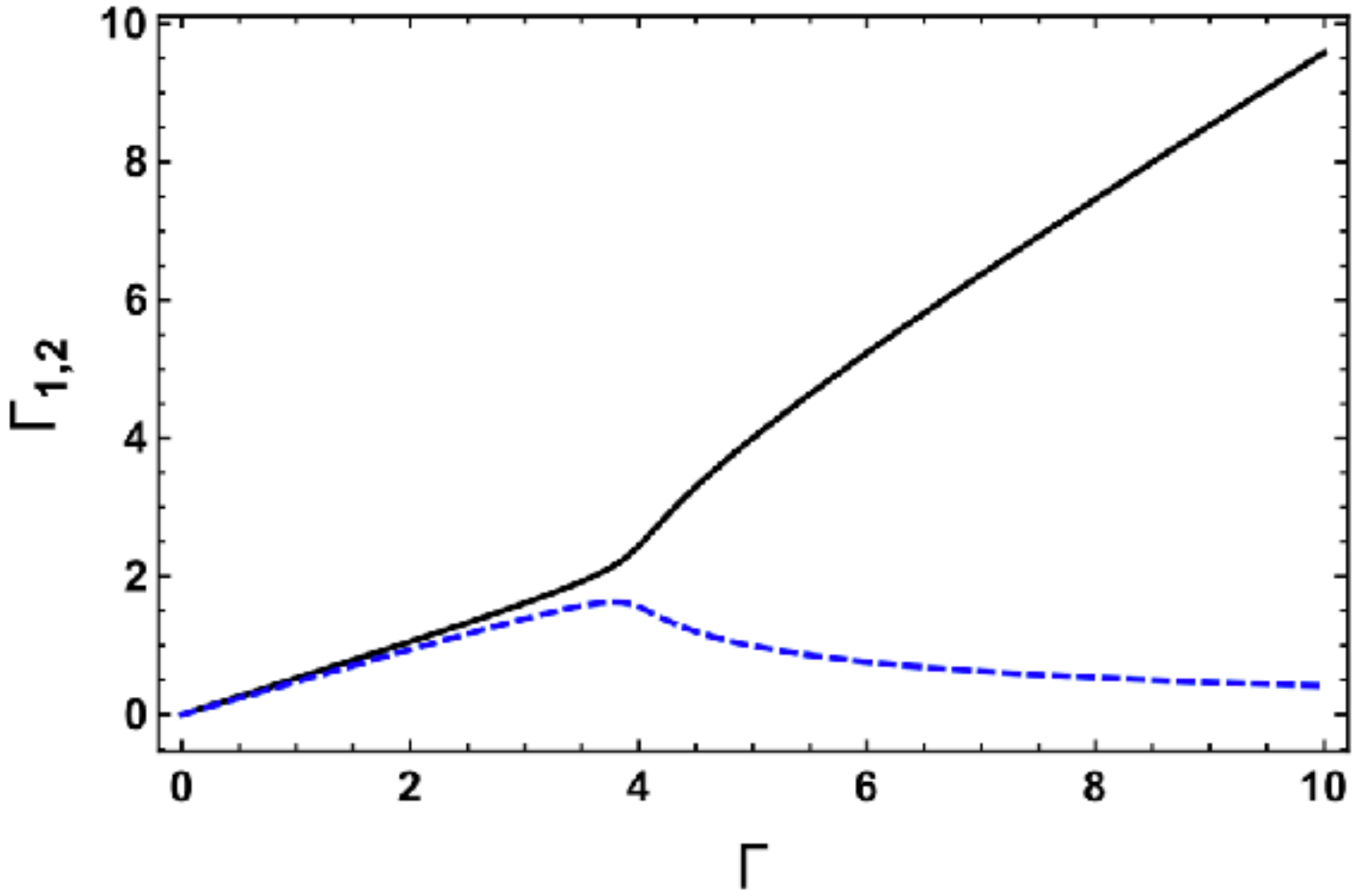
$$H = \begin{pmatrix} \epsilon - \frac{i}{2}\Gamma & v \\ v & 0 \end{pmatrix} = H_0 - \frac{i\Gamma}{2} A^\dagger A \qquad A = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Energies $$\mathcal{E}_{1,2} = \frac{1}{2}\left(\epsilon - \frac{i}{2}\Gamma \pm \sqrt{\left(\epsilon - \frac{i}{2}\Gamma\right)^2 + 4v^2}\right)$$

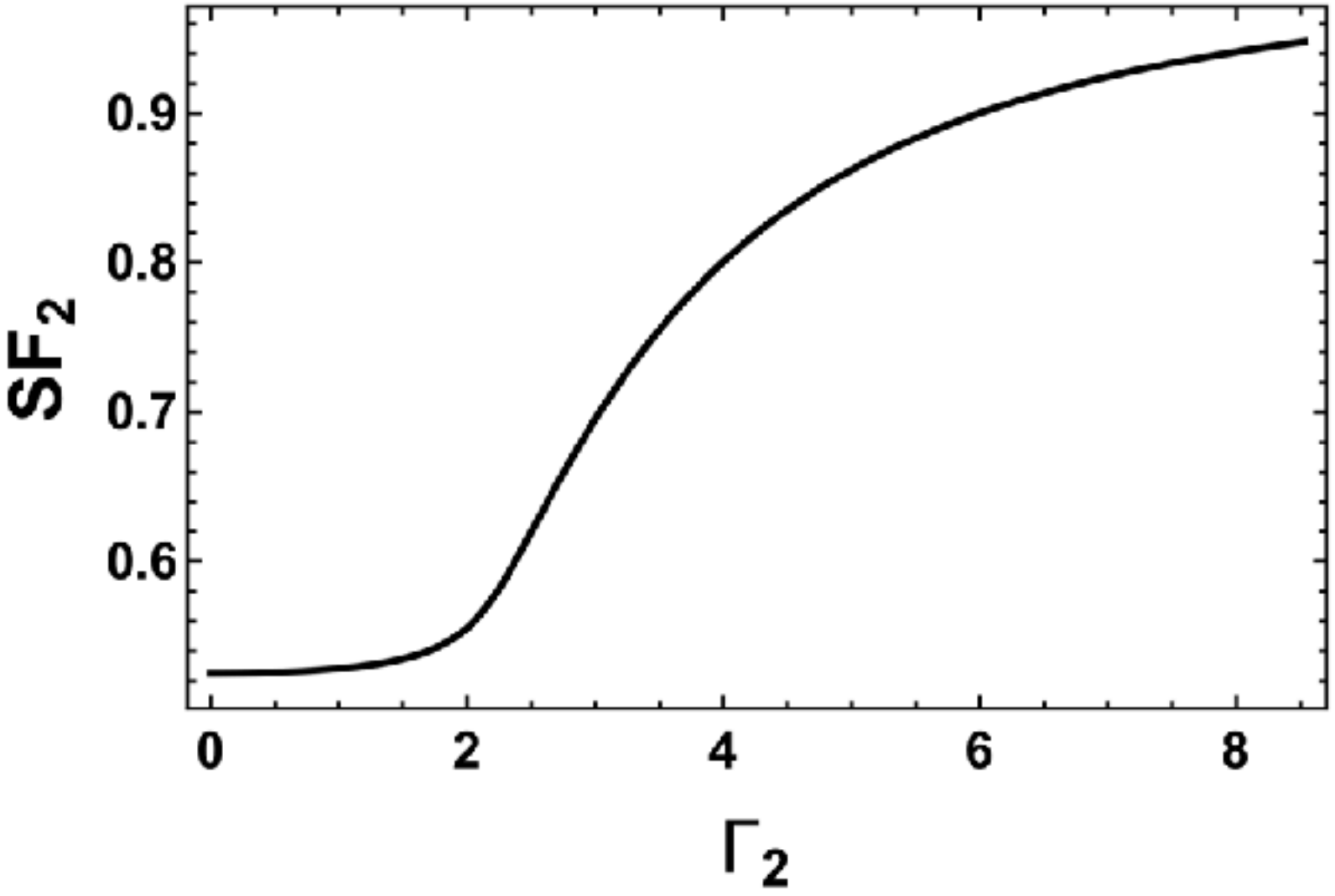
Width $$\Gamma_{1,2} = -2\,\mathrm{Im}\left(\mathcal{E}_{1,2}\right)$$

Spectroscopic Factors $$\mathrm{SF}_{1,2} = \Gamma_{1,2}/\Gamma$$

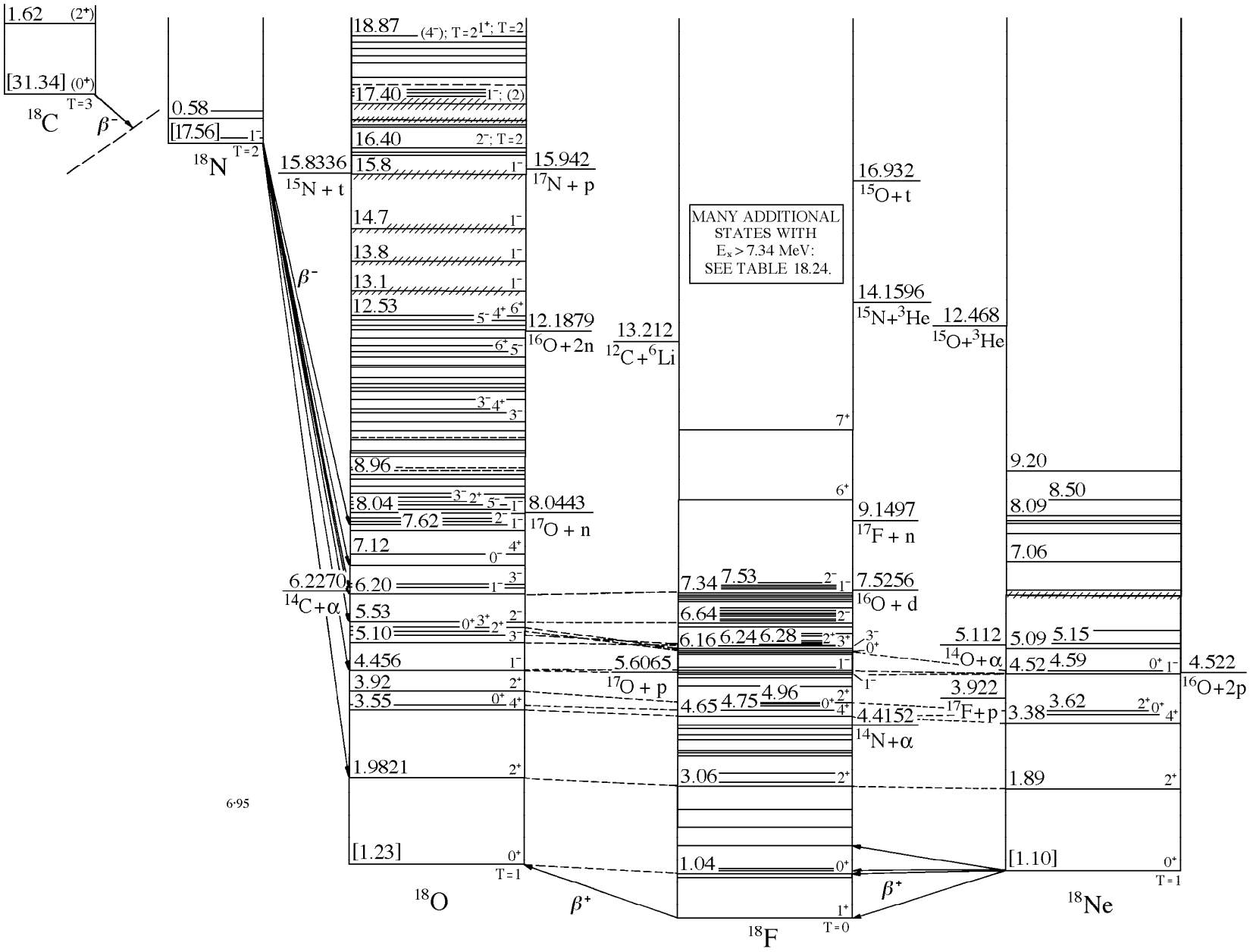
Observing superradiance



Spectroscopic factor for superradiant state



Clustering in A=18 mirror systems



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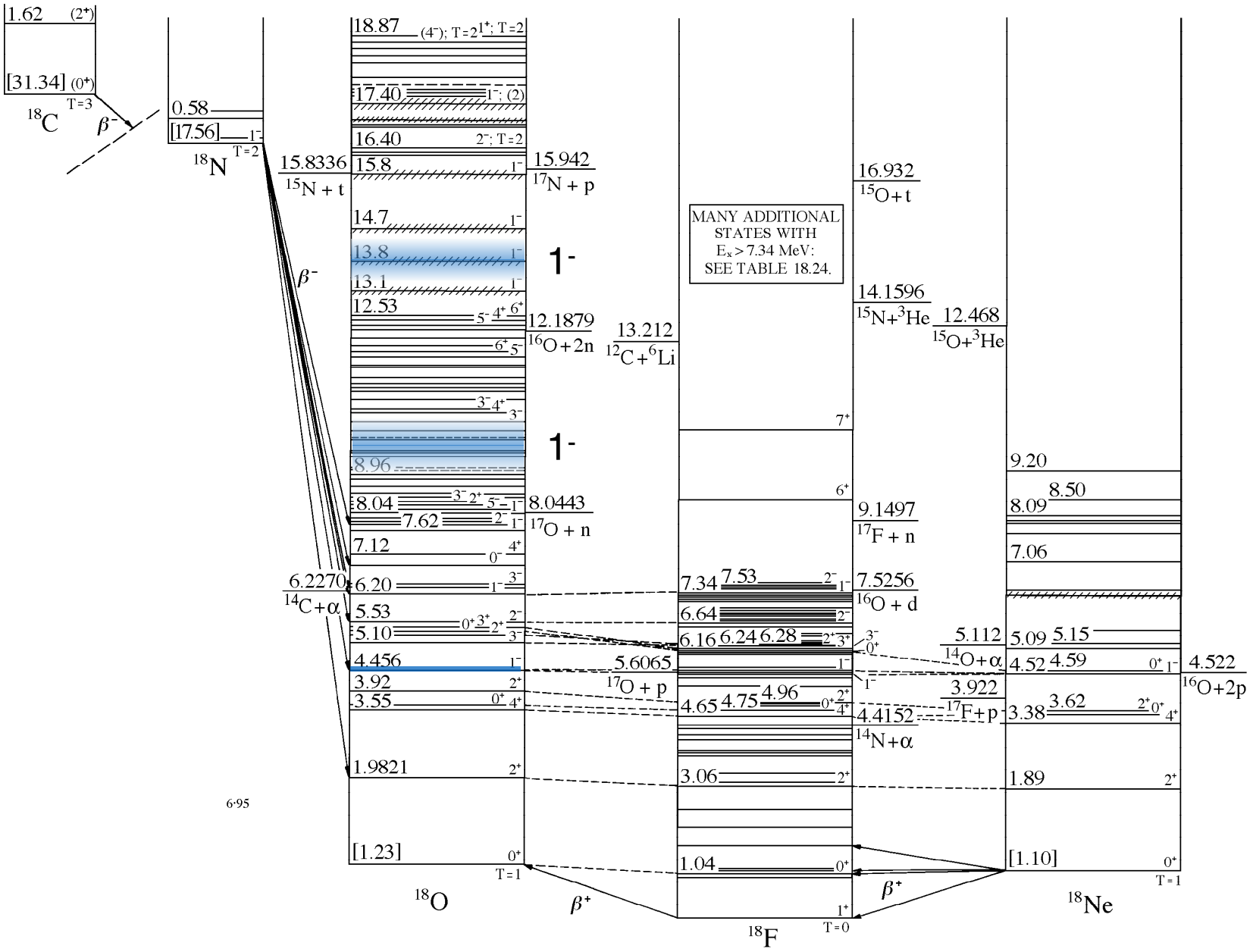
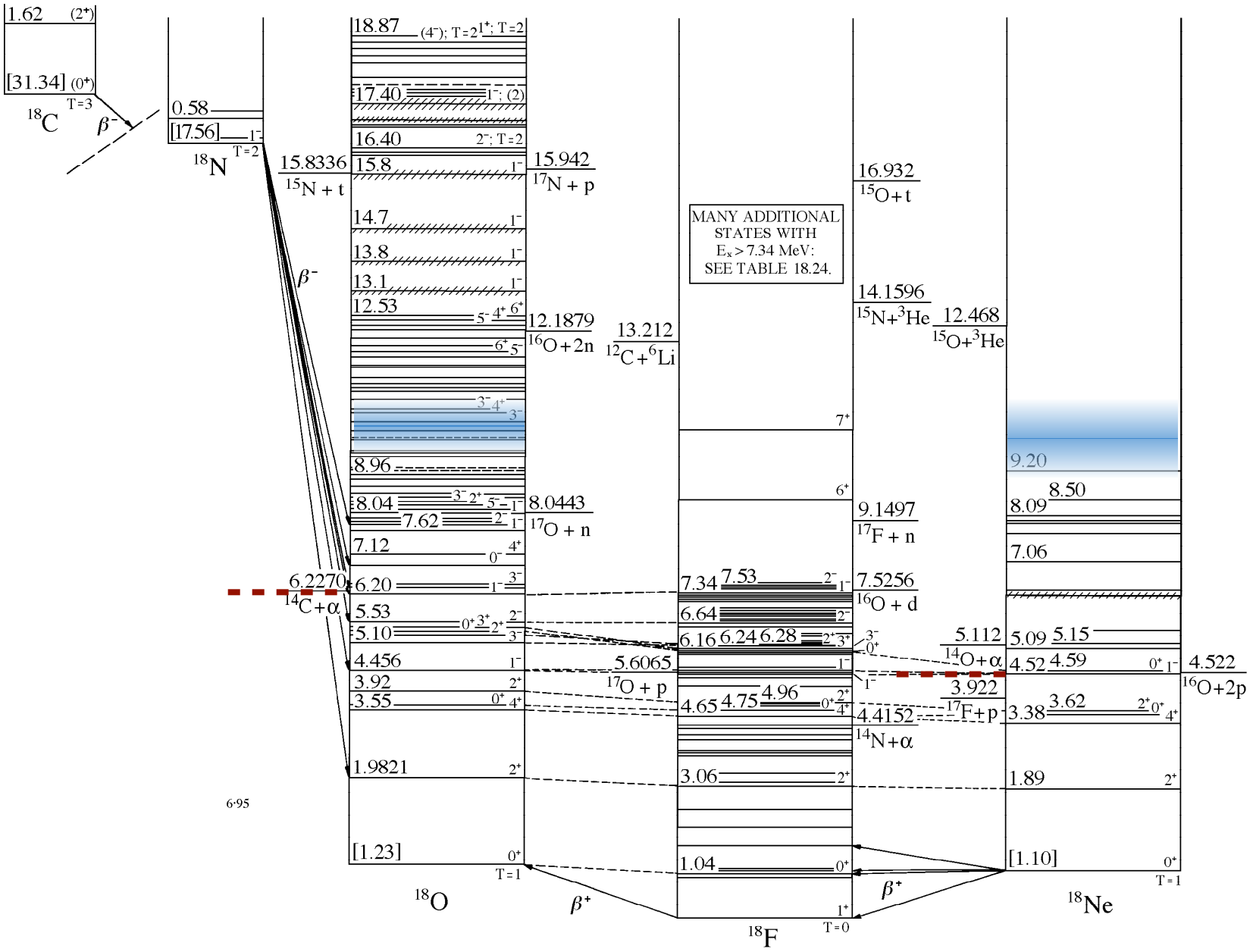


Isospin symmetry



	^{18}Ne			^{18}O		
J	E (MeV)	Γ (keV)	SF	E (MeV)	Γ (keV)	SF
1^-	9.08(1)	357	**0.21(1)**	9.19(2)	200	**0.20(1)**
1^-	9.57(1)	1062	**0.51(5)**	9.76(2)	630	**0.46(4)**
1^-	10.58(4)	416	**0.15(5)**	10.8(3)	630	**0.29(4)**
1^-	13.730(2)	780	**0.2(1)**	14.3(3)	400	**0.10(4)**
2^+	9.19(3)	265	**0.21(2)**	9.79(6)	90	**0.10(3)**
2^+	10.94(6)	1302	**0.52(3)**	12.21(8)	1000	**0.37(9)**
2^+	13.4 (2)	1755	**0.45(8)**	12.8(3)	4800	**1.56(13)**
2^+	16.9(2)	1515	**0.3(2)**			
3^-				8.29(6)	2.9	**0.18(1)**
3^-	8.77(8)	419	**1.0(4)**	9.35(2)	110	**0.48(13)**
3^-	11.0(1)	497	**0.28(7)**	11.95(1)	300	**0.17(2)**
3^-	12.7(2)	2025	**0.7(2)**	12.98(4)	770	**0.32(5)**
3^-	14.8(2)	3967	**1.0(2)**	14.0(2)	2100	**0.7(1)**
4^+	8.16	31	**0.8(3)**	7.11*		
4^+	13.3(3)	845	**0.37(4)**	13.46(2)	210	**0.12(1)**
4^+	14.15(21)	375	**0.14(10)**	14.77(5)	680	**0.28(2)**
5^-	11.31(4)	15	0.03(2)	11.63(1)	30	**0.13(1)**
5^-	12.9(2)	532	**0.48(12)**	13.08(1)	120	**0.17(1)**
5^-	13.79(8)	219	**0.14(10)**	14.1(1)	260	**0.23(2)**
5^-	14.6(7)	521	**0.27(20)**	14.7(1)	230	**0.16(6)**
6^+	11.8(2)	54	**0.30(7)**	11.69(5)	12	**0.23(1)**
6^+	12.4(2)	167	**0.56(26)**	12.57(1)	50	**0.38(8)**

M. Barbui *et al.*, "α-cluster structure of ^{18}Ne," *Phys. Rev. C*, vol. 106, no. 5, p. 054310, Nov. 2022, doi: 10.1103/PhysRevC.106.054310.

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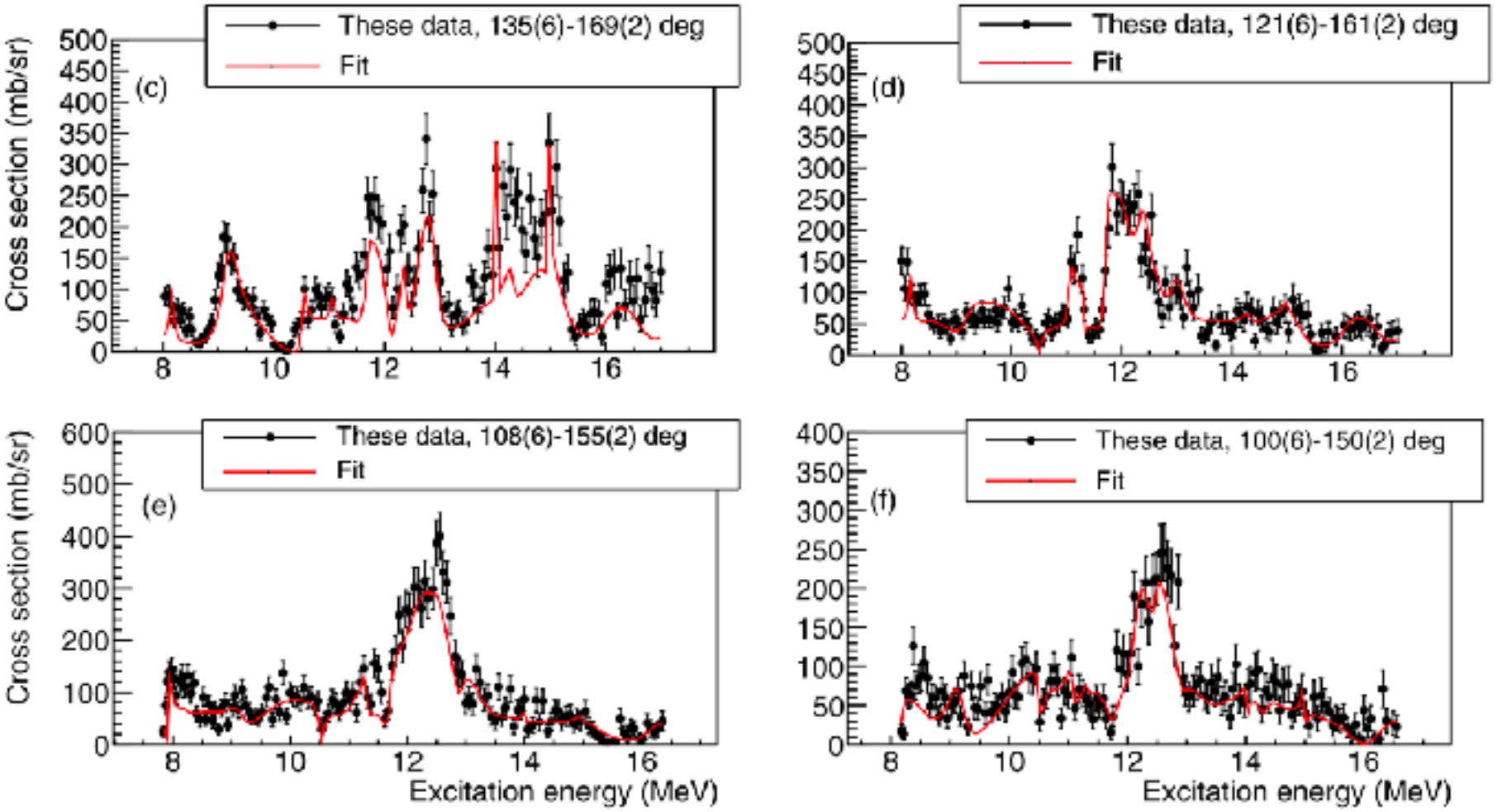

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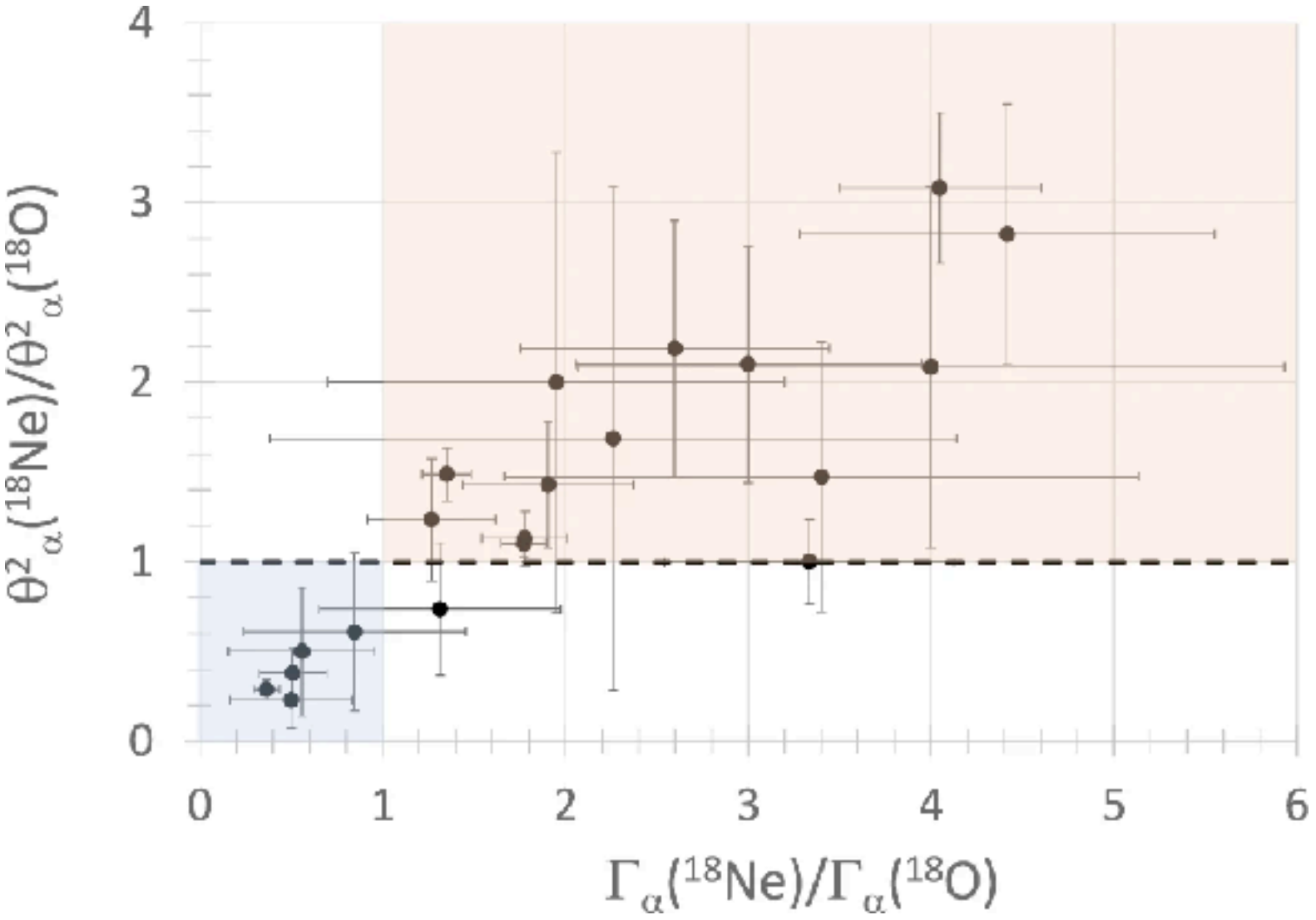
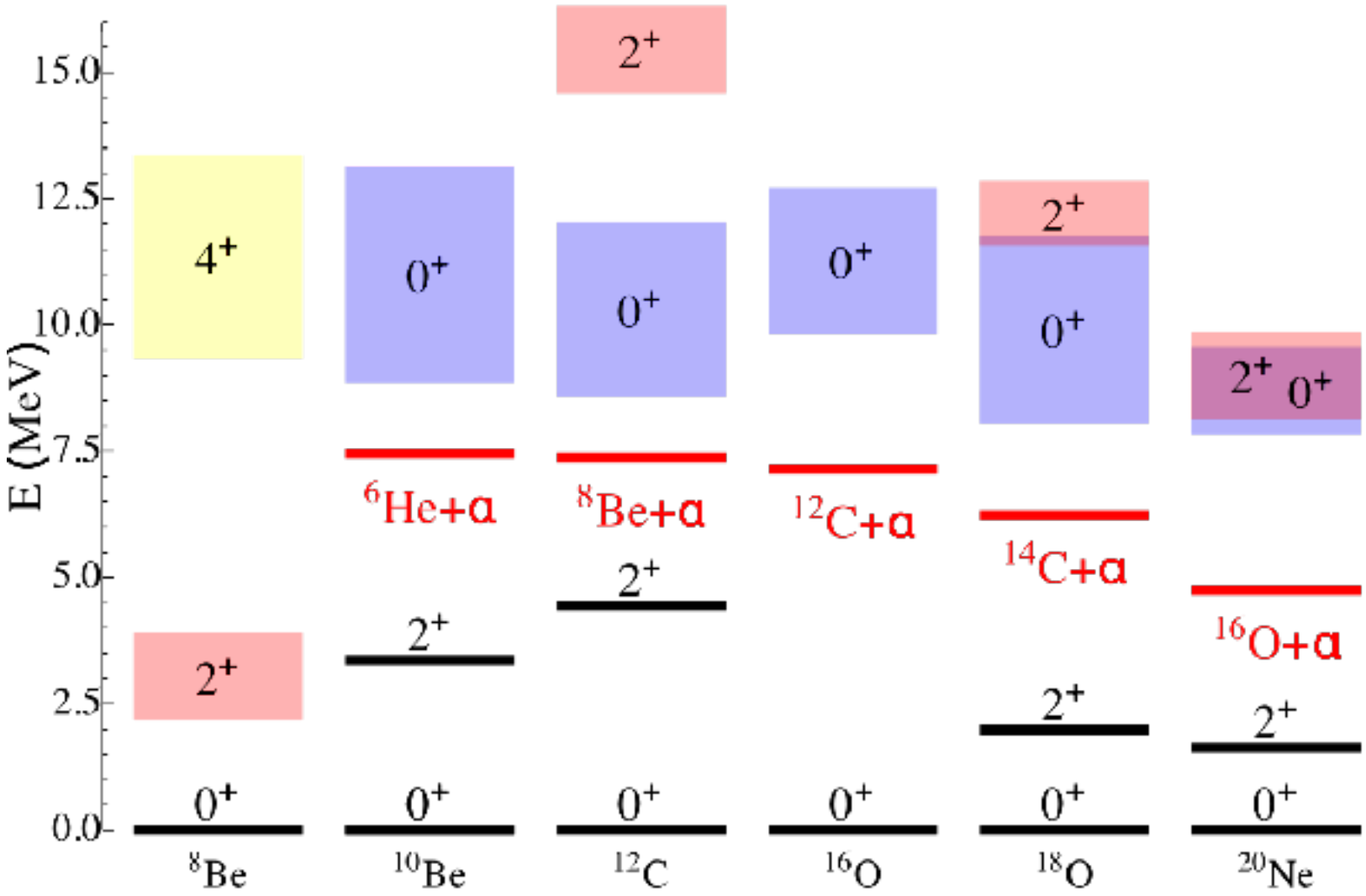
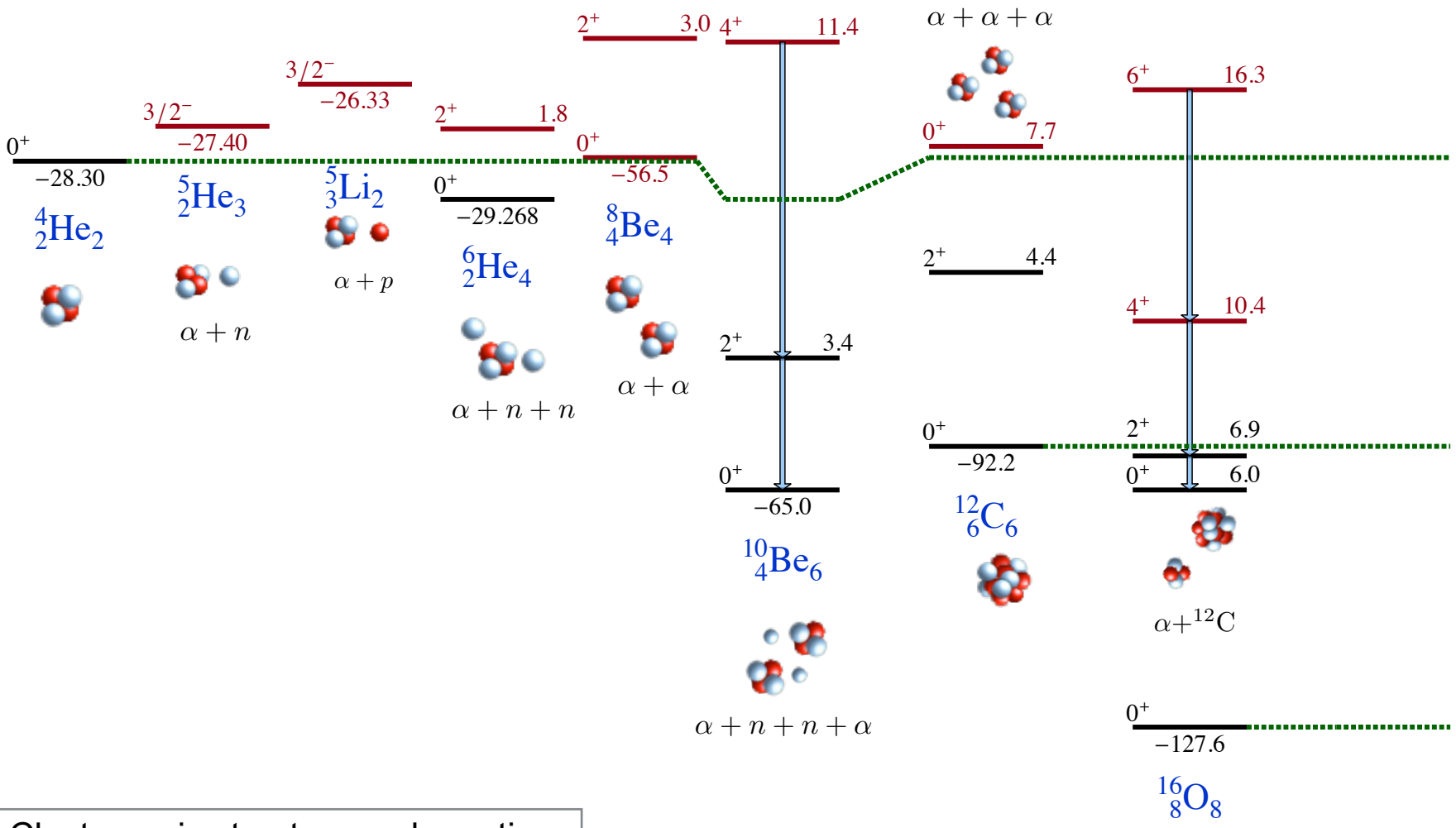


A. Volya, M. Barbui, V. Z. Goldberg, and G. V. Rogachev, "Superradiance in alpha clustered mirror nuclei," *Commun Phys*, vol. 5, no. 1, Art no. 1, Dec. 2022, doi: 10.1038/s42005-022-01105-9.

Clustering and continuum



Clustering in light nuclei



- Clusters mix structure and reactions
- Strength distribution

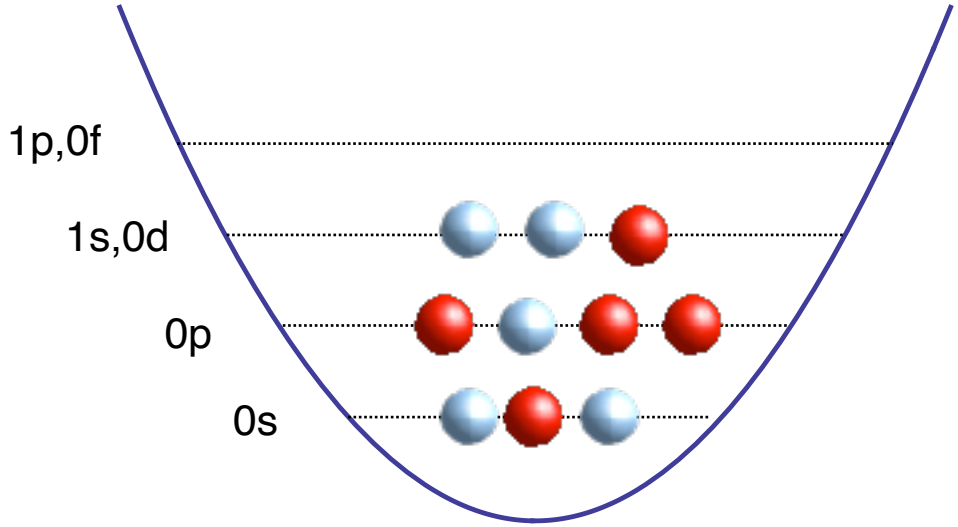
The Nuclear Shell Model

$$H = \frac{\mathbf{P}^2}{2MA} + H_{\rm int} \qquad H(\omega_0) = H + \frac{AM\omega_0^2}{2}\mathbf{R}^2 = H_{\rm cm}(\omega_0) + H_{\rm int}$$

$$H_{\rm cm}(\omega_0) = \frac{\mathbf{P}^2}{2MA} + \frac{AM\omega_0^2}{2}\mathbf{R}^2 \qquad \Psi_{n\ell m} = \phi_{n\ell m}(\mathbf{R})\,\Psi'$$

$$E(\omega_0) = \hbar\omega_0(N_{\rm cm} + 3/2) + E'$$

$$N_{\rm cm} = 2n + \ell$$



Translational invariance and Center of Mass (CM)

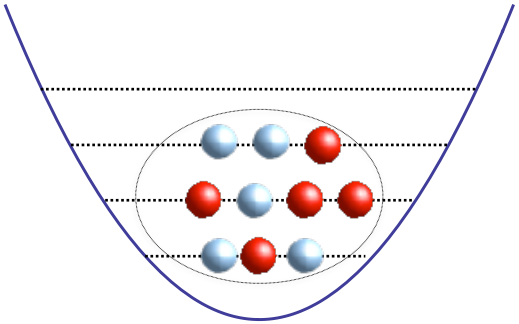
Shell model, Glockner-Lawson procedure

$$\Psi_D = \phi_{000}(\mathbf{R}_D)\,\Psi'_D$$

SM state

Center-of-mass vibration

Intrinsic state



Controlling CM with operator $\mathbf{R}$

Control only CM quanta

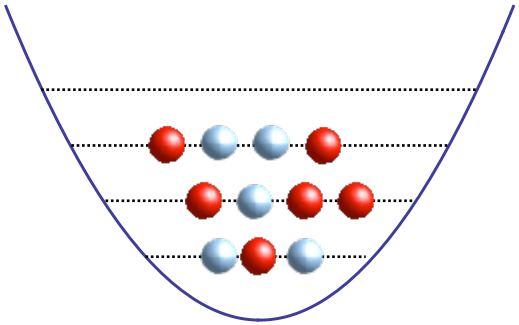
$$D_\mu = \sqrt{\frac{4\pi}{3}} R_\mu \qquad R_\mu = \sqrt{\frac{\hbar}{2Am\omega}}(\mathcal{B}^\dagger_\mu + \mathcal{B}_\mu)$$

K. Kravvaris and A. Volya, "Study of nuclear clustering from an ab initio perspective," *Phys. Rev. Lett.*, vol. 119, no. 6, p. 062501, 2017.
K. Kravvaris and A. Volya, "Clustering in structure and reactions using configuration interaction techniques," *Phys. Rev. C*, vol. 100, no. 3, p. 034321, Sep. 2019, doi: 10.1103/PhysRevC.100.034321.

Configuration interaction approach and clustering

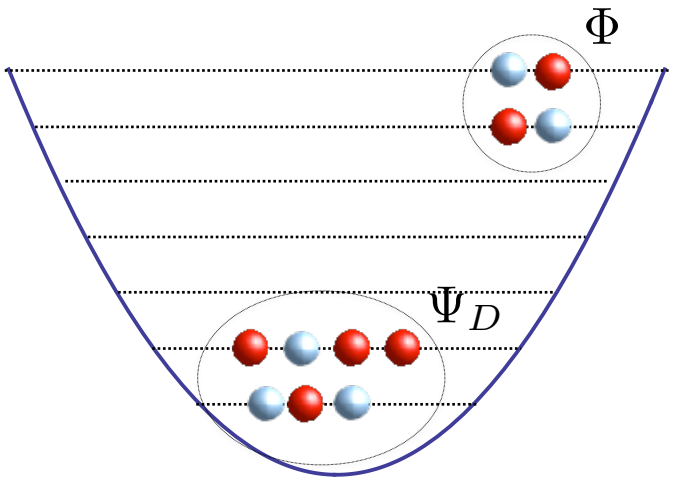
Traditional shell model configuration
m-scheme

$$|\Psi\rangle = \Psi^\dagger|0\rangle \sim a_1^\dagger a_2^\dagger \ldots a_A^\dagger|0\rangle$$



Cluster configuration

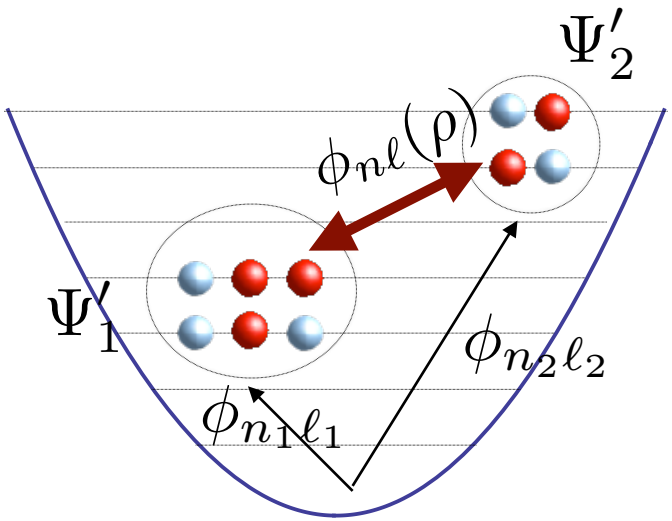
$$|\text{channel}\rangle \sim |\Phi\Psi_D\rangle \equiv \Phi^\dagger \Psi_D^\dagger|0\rangle$$



$$|\Psi\rangle \quad + \quad \Phi^\dagger|\Psi_D\rangle$$

Clustering reaction basis channel

(basis states for clustering)



$$\Psi = \phi_{000}(\mathbf{R})\Psi'$$

Boost

$$\Psi_{n\ell m} = \phi_{n\ell m}(\mathbf{R})\,\Psi'$$

CM-Recouple

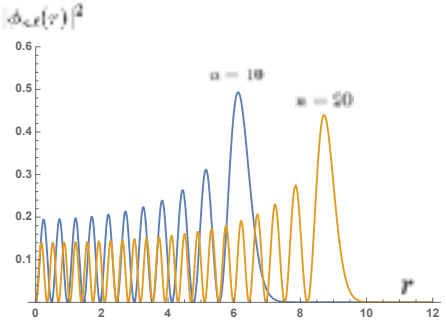
$$\Phi_{n\ell m} = \mathcal{A}\left\{\phi_{000}(\mathbf{R})\phi_{n\ell m}(\boldsymbol{\rho})\Psi'^{(1)}\Psi'^{(2)}\right\}$$

$$\Phi^{\dagger}_{n\ell} = \sum_{\substack{n_1\ell_1\\ n_2\ell_2}} \mathcal{M}^{n\ell 00;\ell}_{n_1\ell_1 n_2\ell_2}\left[\Psi^{\dagger}_{n_1\ell_1 m_1} \times \Psi^{\dagger}_{n_2\ell_2 m_2}\right]_{\ell}$$

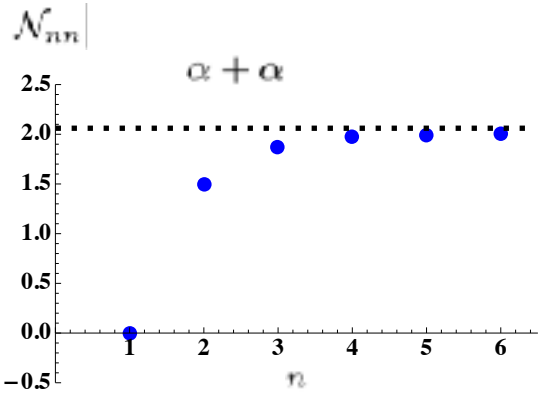
Resonating group method and reactions

$$\sum_n \mathcal{H}^{(\ell)}_{nn'}\chi_{n'} = E\sum_n \mathcal{N}^{(\ell)}_{nn'}\chi_{n'}$$

$$\begin{pmatrix} \mathcal{H}_{00} & \dots & \mathcal{H}_{0n} & 0 & 0 & 0 \\ \vdots & \ddots & \vdots & 0 & \vdots & \vdots \\ \mathcal{H}_{n0} & \dots & \mathcal{H}_{nn} & T_{nn+1} & 0 & \vdots \\ 0 & 0 & T_{n+1n} & T_{n+1n+1} & T_{n+1n+2} & 0 \\ 0 & \dots & 0 & T_{n+2n+1} & T_{n+2n+2} & \ddots \\ 0 & \dots & \dots & 0 & \ddots & \ddots \end{pmatrix} \begin{pmatrix} \chi_0 \\ \vdots \\ \chi_n \\ x_{n+1} \\ x_{n+2} \\ \vdots \end{pmatrix} = E \begin{pmatrix} \mathcal{N}_{00} & \dots & \mathcal{N}_{0n} & 0 & 0 & 0 \\ \vdots & \ddots & \vdots & 0 & \vdots & \vdots \\ \mathcal{N}_{n0} & \dots & \mathcal{N}_{nn} & 0 & 0 & \vdots \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & \dots & 0 & 0 & 1 & 0 \\ 0 & \dots & \dots & 0 & 0 & \ddots \end{pmatrix} \begin{pmatrix} \chi_0 \\ \vdots \\ \chi_n \\ x_{n+1} \\ x_{n+2} \\ \vdots \end{pmatrix}$$

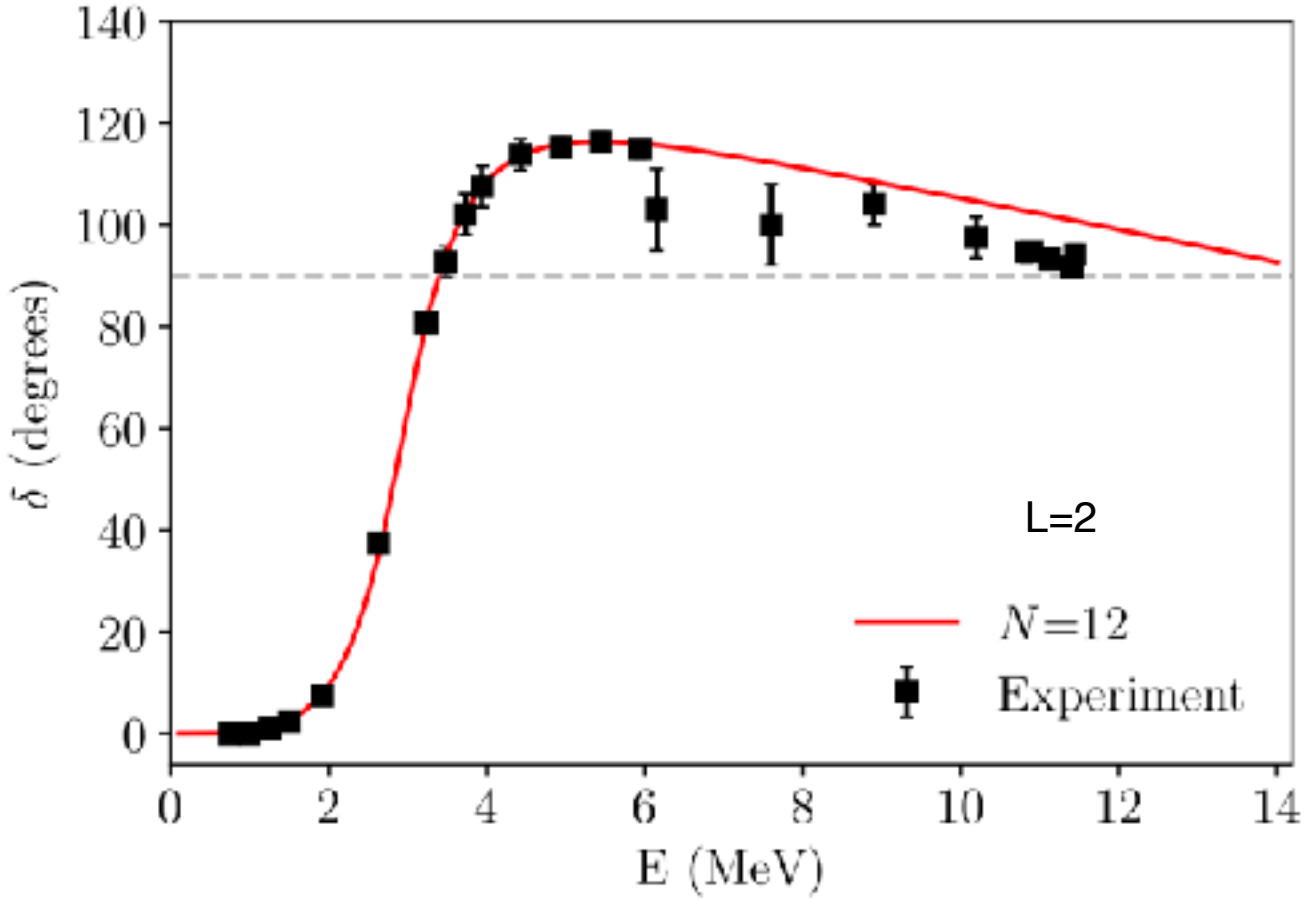


Asymptotic solution with phase shift



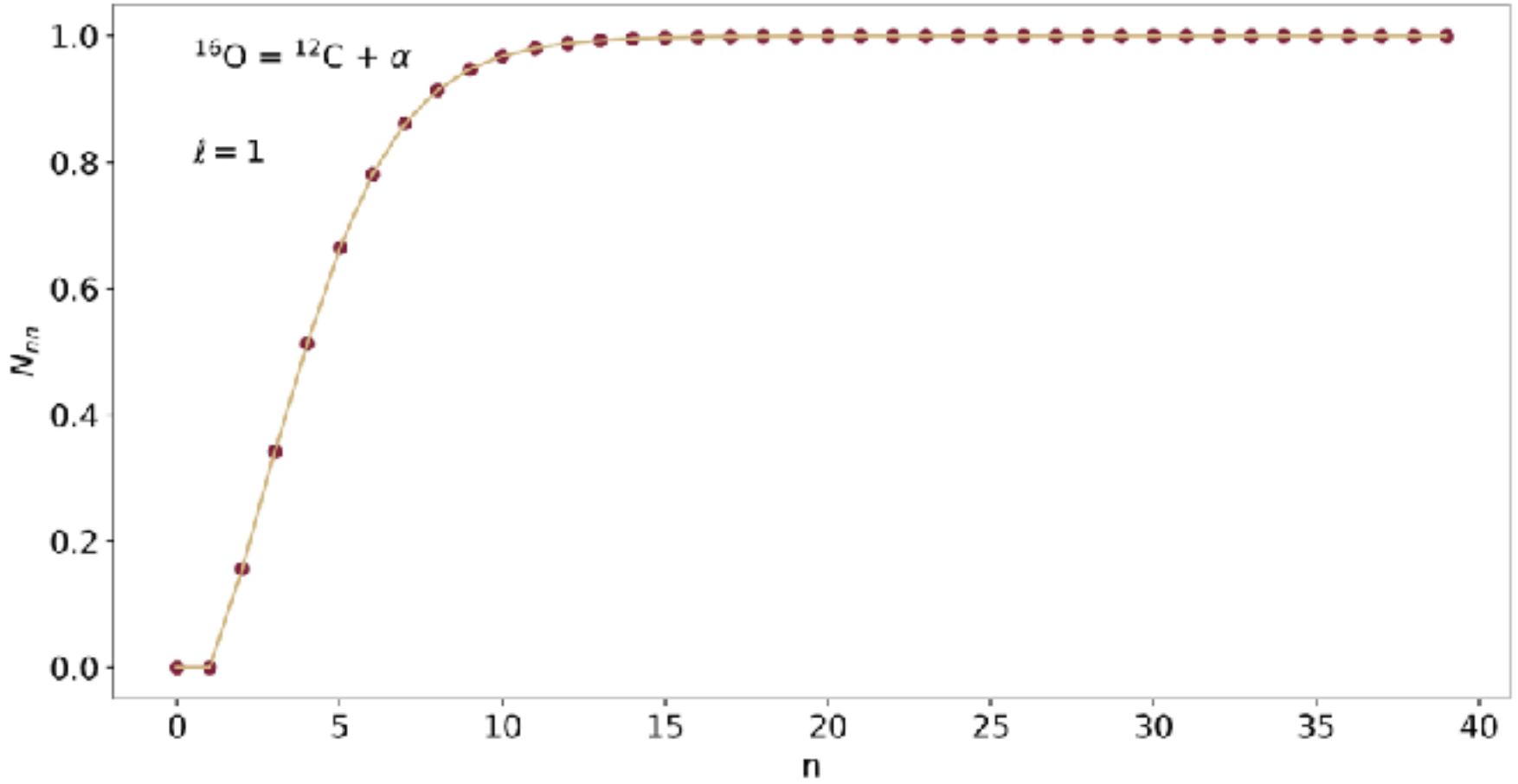
J-matrix (or HORSE) method: J. M. Bang, Annals of Physics **280**, 299 (2000)
Experimental data: Phys. Rev. 168, 1114 (1968); Nucl. Phys. **A287**, 317 (1977)

alpha+alpha scattering phase shifts

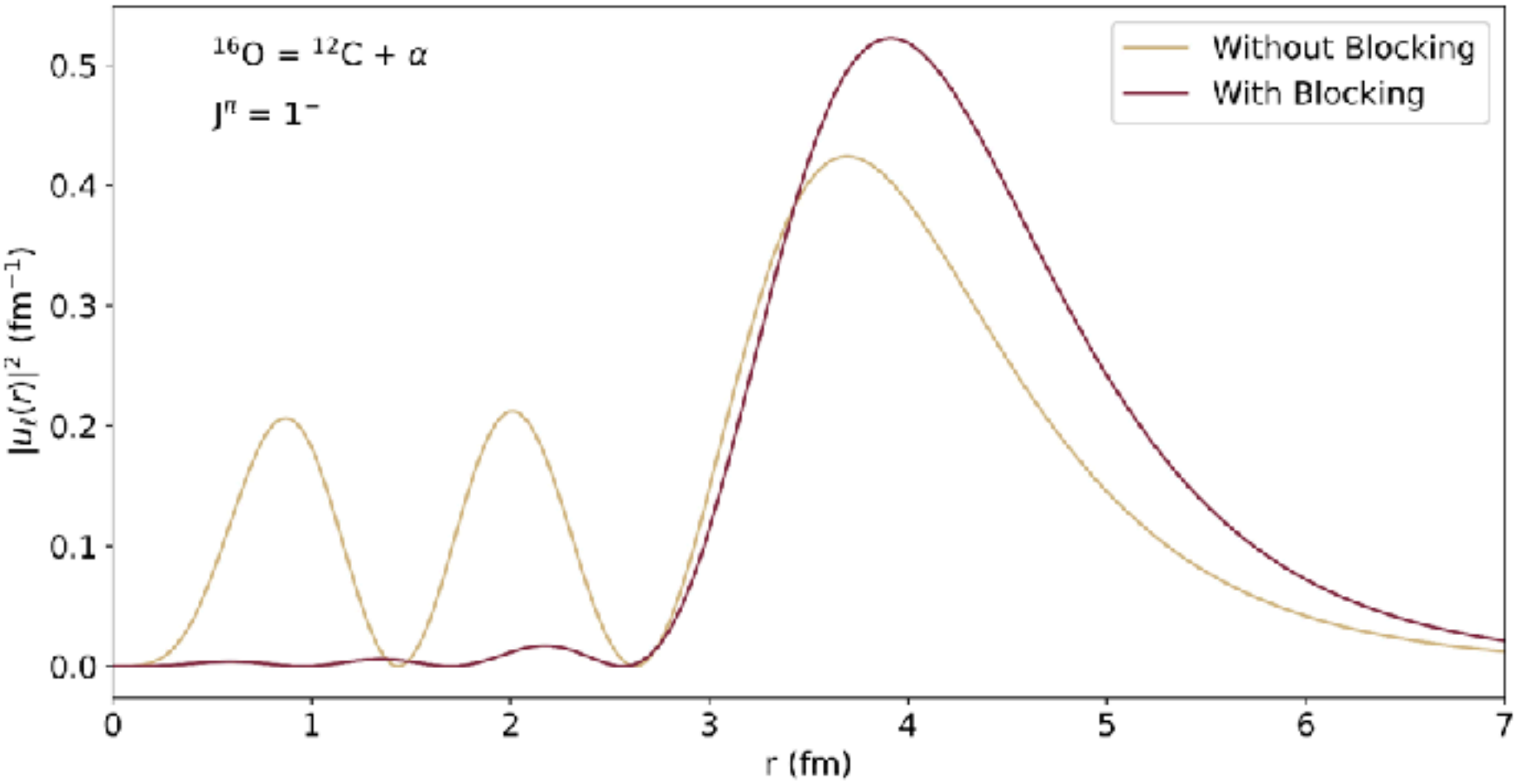


Experimental data from S. A. Afzal, A. A. Z. Ahmad, and S. Ali, Rev. Mod. Phys. 41, 247 (1969).

Asymptotic Normalization and blocking

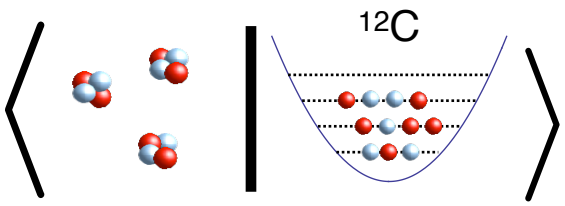


ANC and role of blocking



Kevin Liguori, *Asymptotic Normalization Coefficients for Alpha Clusters* Honors Thesis, Florida State University 2023.

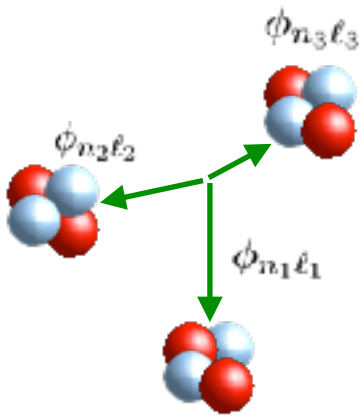
Spectroscopic amplitudes

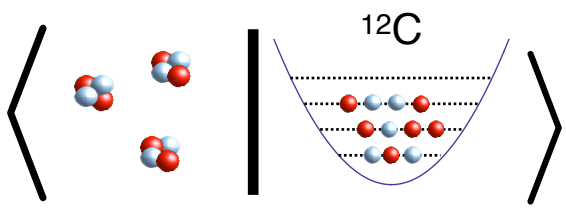


parent	J^π	channel	$\lvert\langle\Psi\lvert\mathcal{F}_\ell\rangle\rvert$
^{8}Be[4]	0^+	$\alpha[0]+\alpha[0]$	0.905
^{8}Be[4]	2^+	$\alpha[0]+\alpha[0]$	0.898
^{8}Be[4]	4^+	$\alpha[0]+\alpha[0]$	0.874
^{8}Be[4]	0^+	$\alpha[2]+\alpha[2]$	0.961
^{8}Be[4]	2^+	$\alpha[2]+\alpha[2]$	0.957
^{8}Be[4]	4^+	$\alpha[2]+\alpha[2]$	0.943
^{10}Be[4]	0^+	^{6}He[0] + α[0]	0.844
^{10}Be[4]	0^+	^{6}He[4] + α[0]	0.820
^{10}Be[4]	2^+	^{6}He[0] + α[0]	0.834
^{10}Be[4]	2^+	^{6}He[4] + α[0]	0.796
^{12}C[4]	0^+_1	$\alpha[0]+\alpha[0]+\alpha[0]$	0.841
^{12}C[4]	0^+_2	$\alpha[0]+\alpha[0]+\alpha[0]$	0.229

Spectroscopic amplitudes.

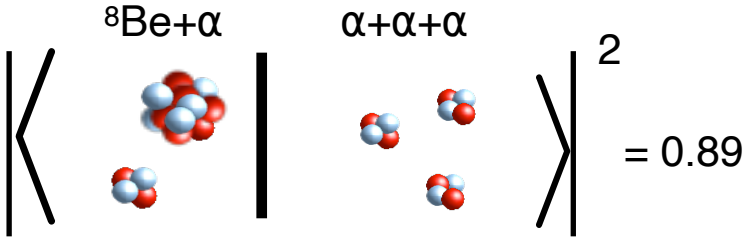
Ttriple-alpha RGM



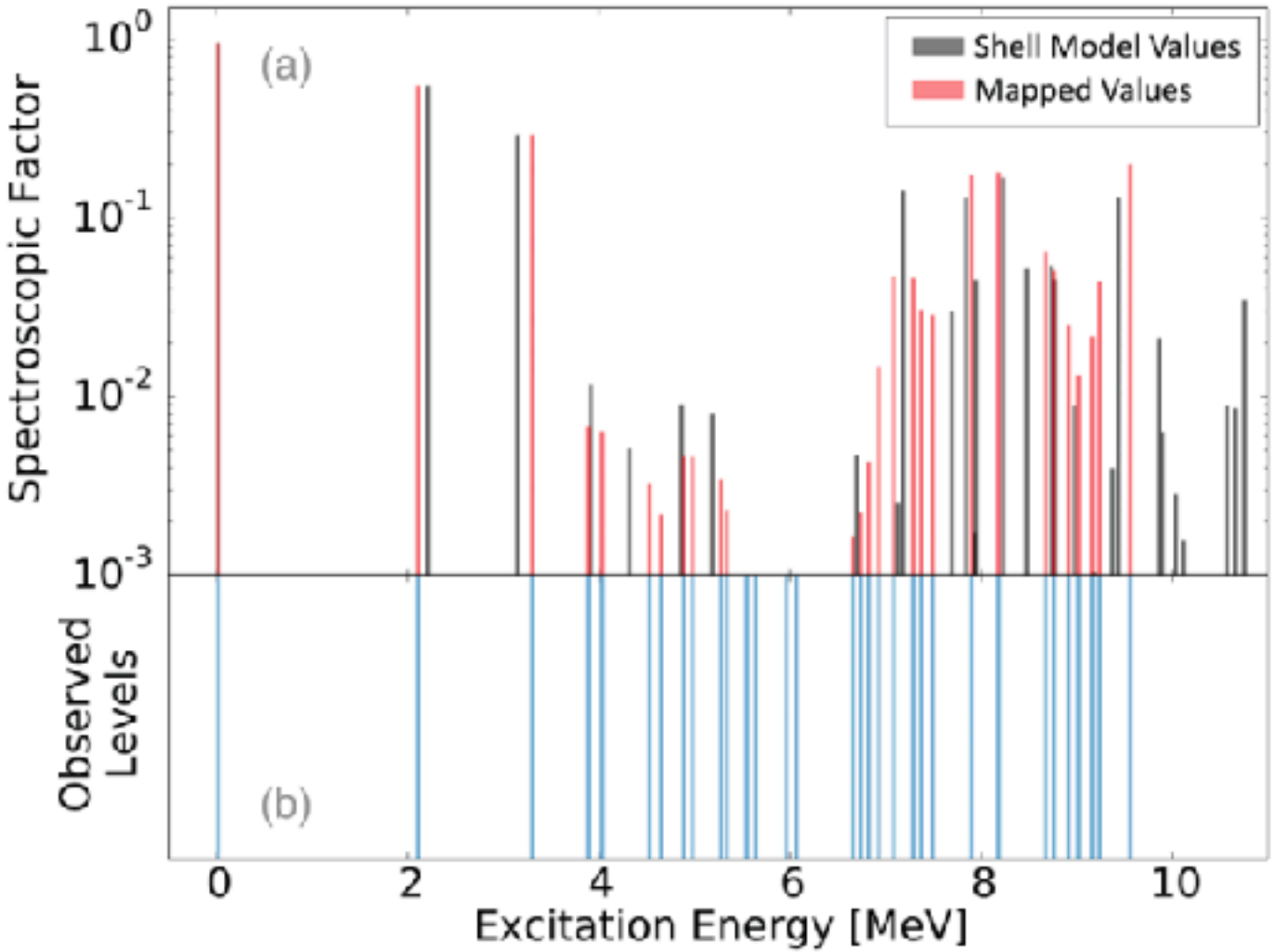


parent	channel	overlap
$^{12}\mathrm{C}[4](0_1^+)$	$\alpha[0]+\alpha[0]+\alpha[0]$	0.841
$^{12}\mathrm{C}[4](0_2^+)$	$\alpha[0]+\alpha[0]+\alpha[0]$	0.229

N_{max}(rel)=12



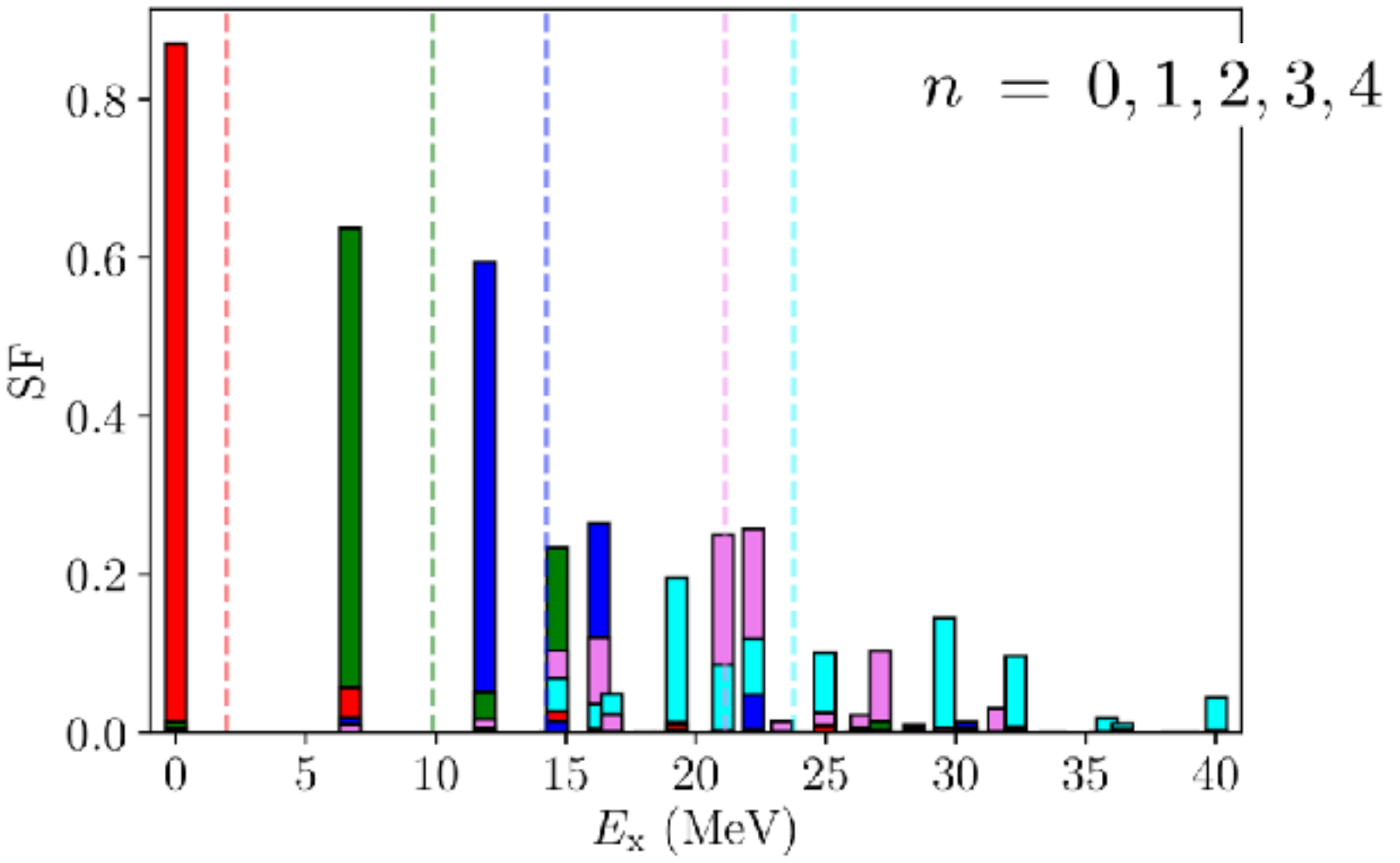
Searching for clustering strength



α-spectroscopic factors for states in 34Ar calculated using the shell model

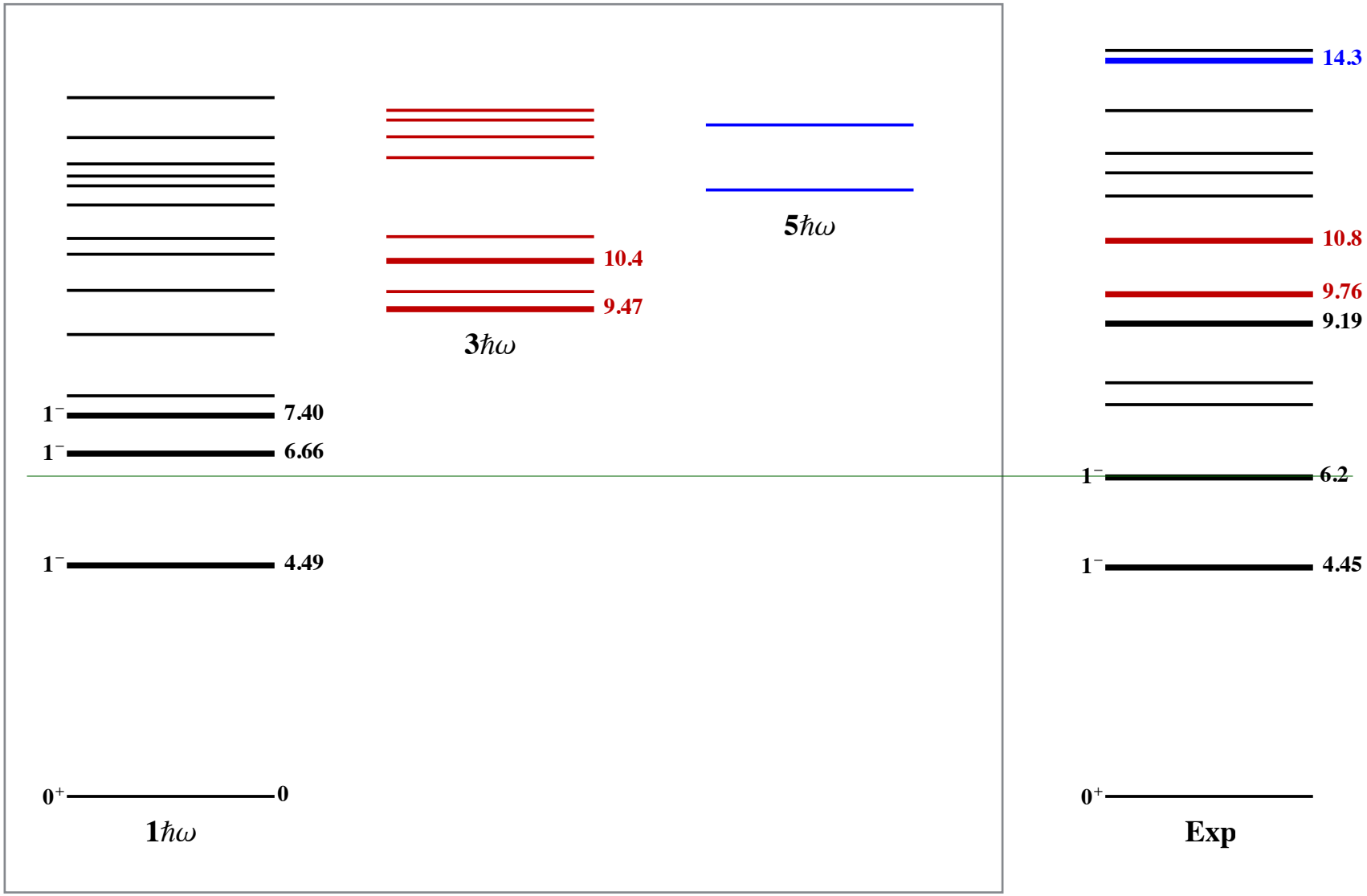
A. M. Long *et al.*, "alpha-unbound levels in Ar-34 from Ar-36(p,t)Ar-34 reaction measurements and implications for the astrophysical S-30(alpha, p)Cl-33 reaction rate," *Phys. Rev. C*, vol. 97, no. 5, p. 054613, May 2018, doi: 10.1103/PhysRevC.97.054613.

Searching for clustering strength

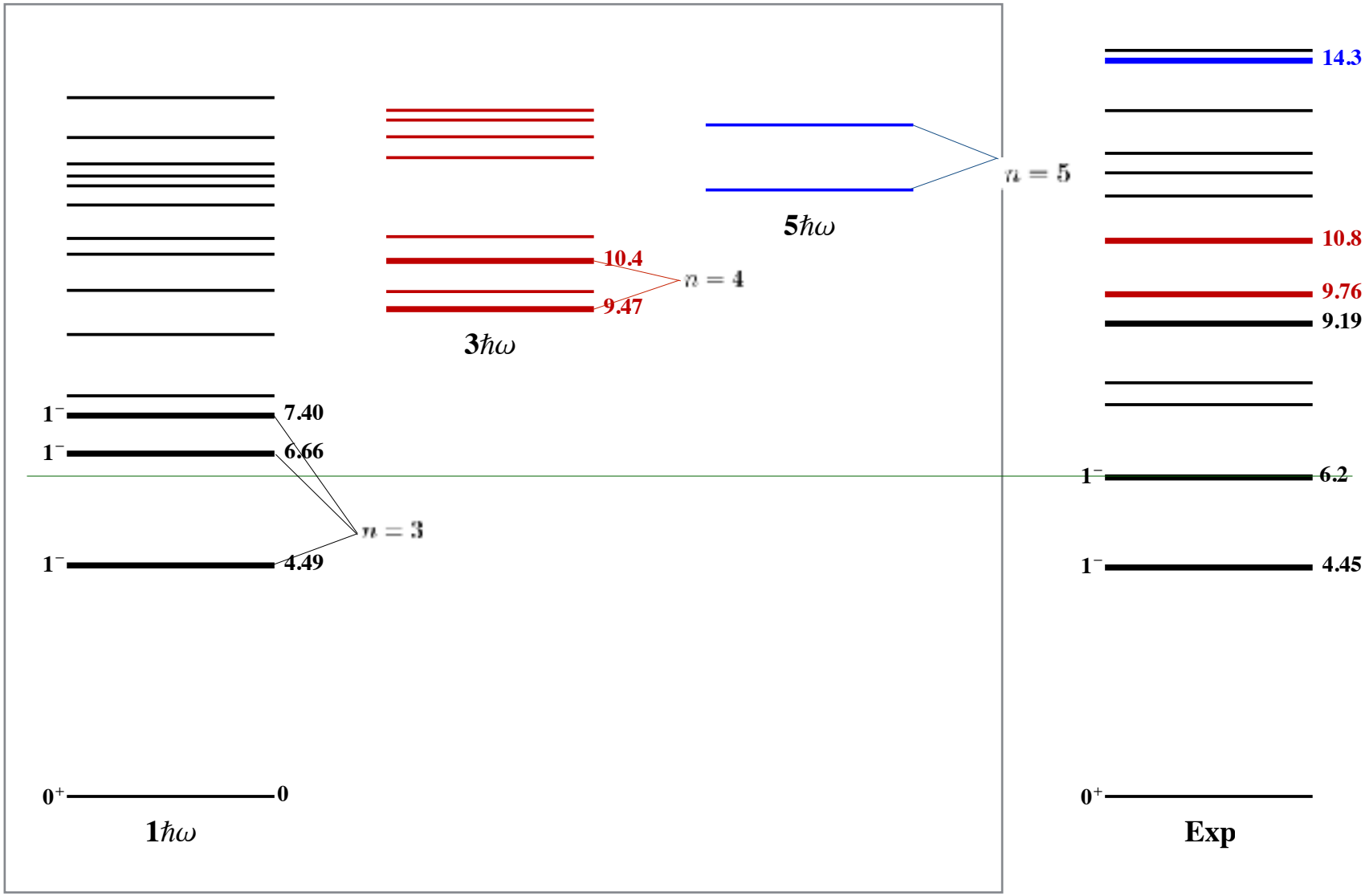


Distribution of dynamic spectroscopic factors for ^{20}Ne → ^{16}O(g.s.) + α. The dashed lines correspond to the RGM energies for each decay channel.

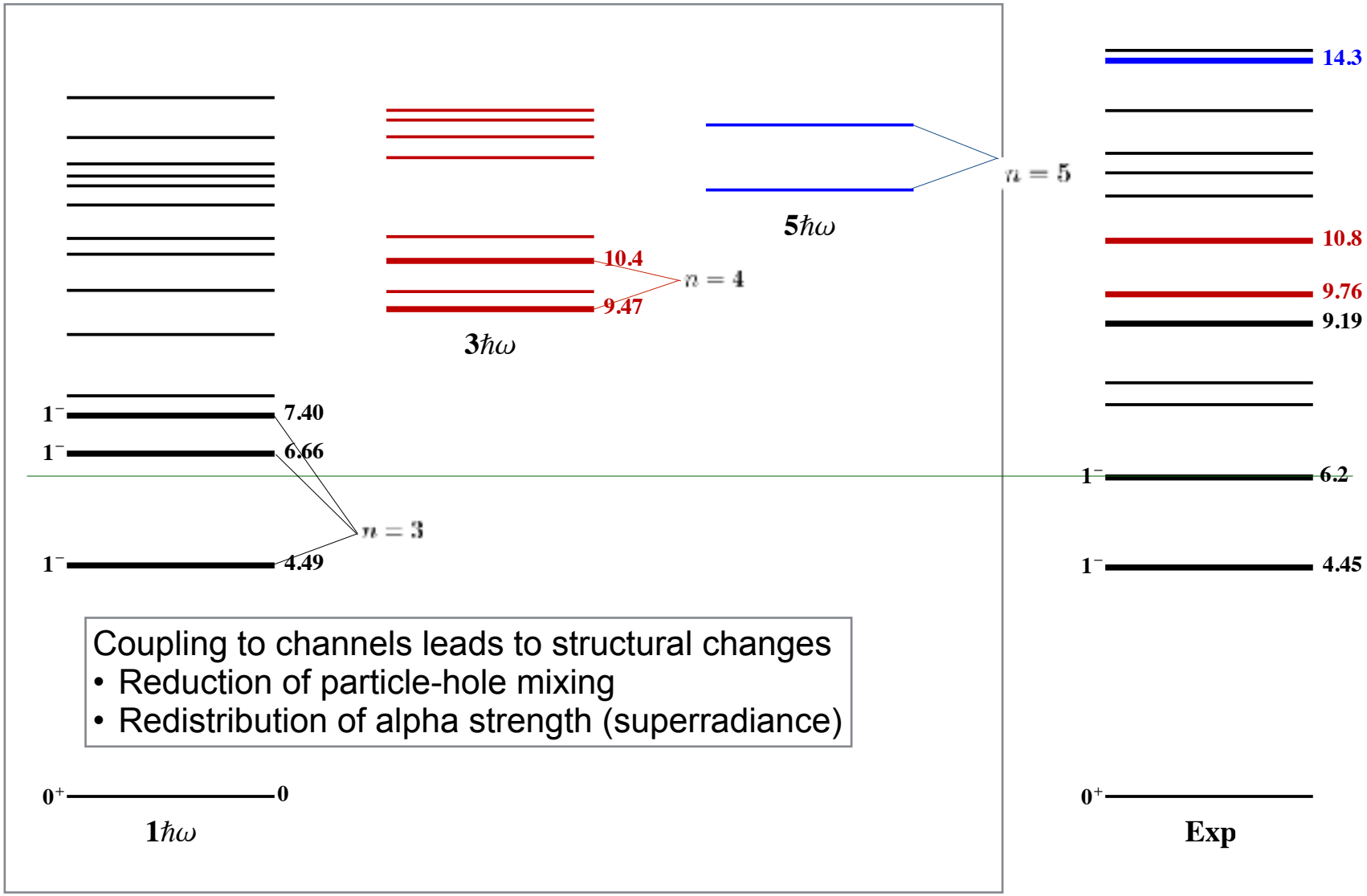
Channel coupling in ^{18}O
l=1 channel



Channel coupling in ^{18}O
l=1 channel

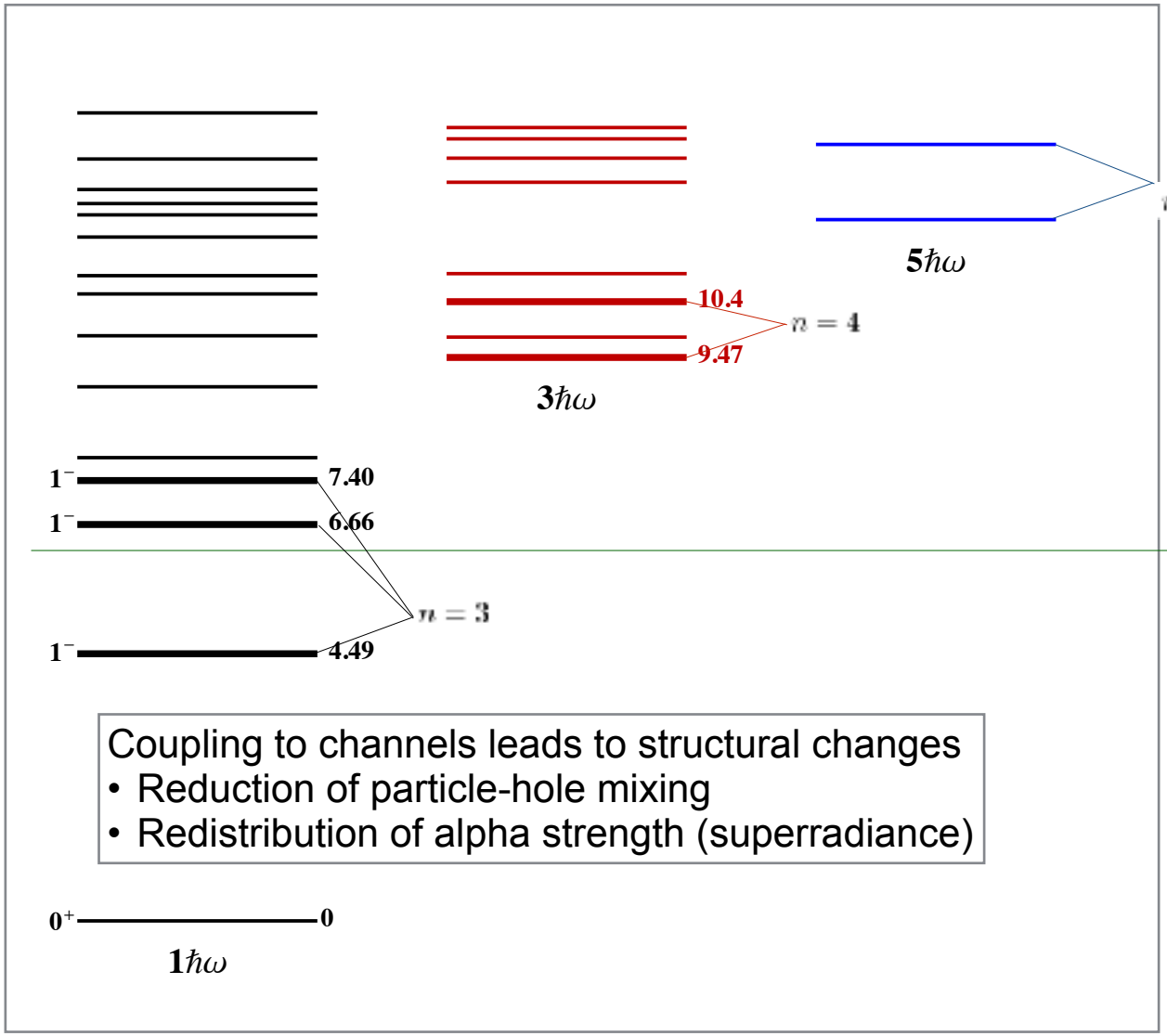


Channel coupling in ^{18}O
l=1 channel



Coupling to channels leads to structural changes
- Reduction of particle-hole mixing
- Redistribution of alpha strength (superradiance)

Channel coupling in ^{18}O
I=1 channel



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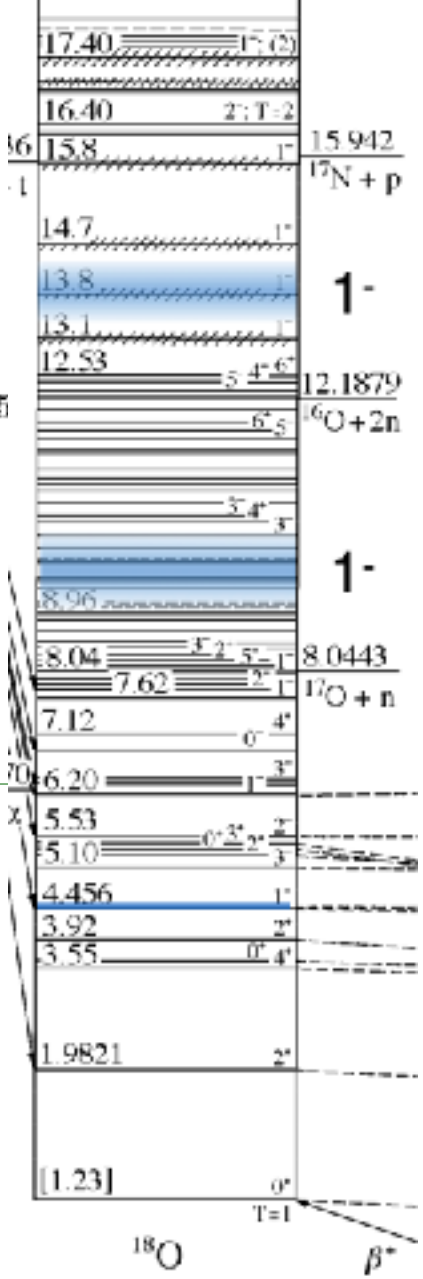
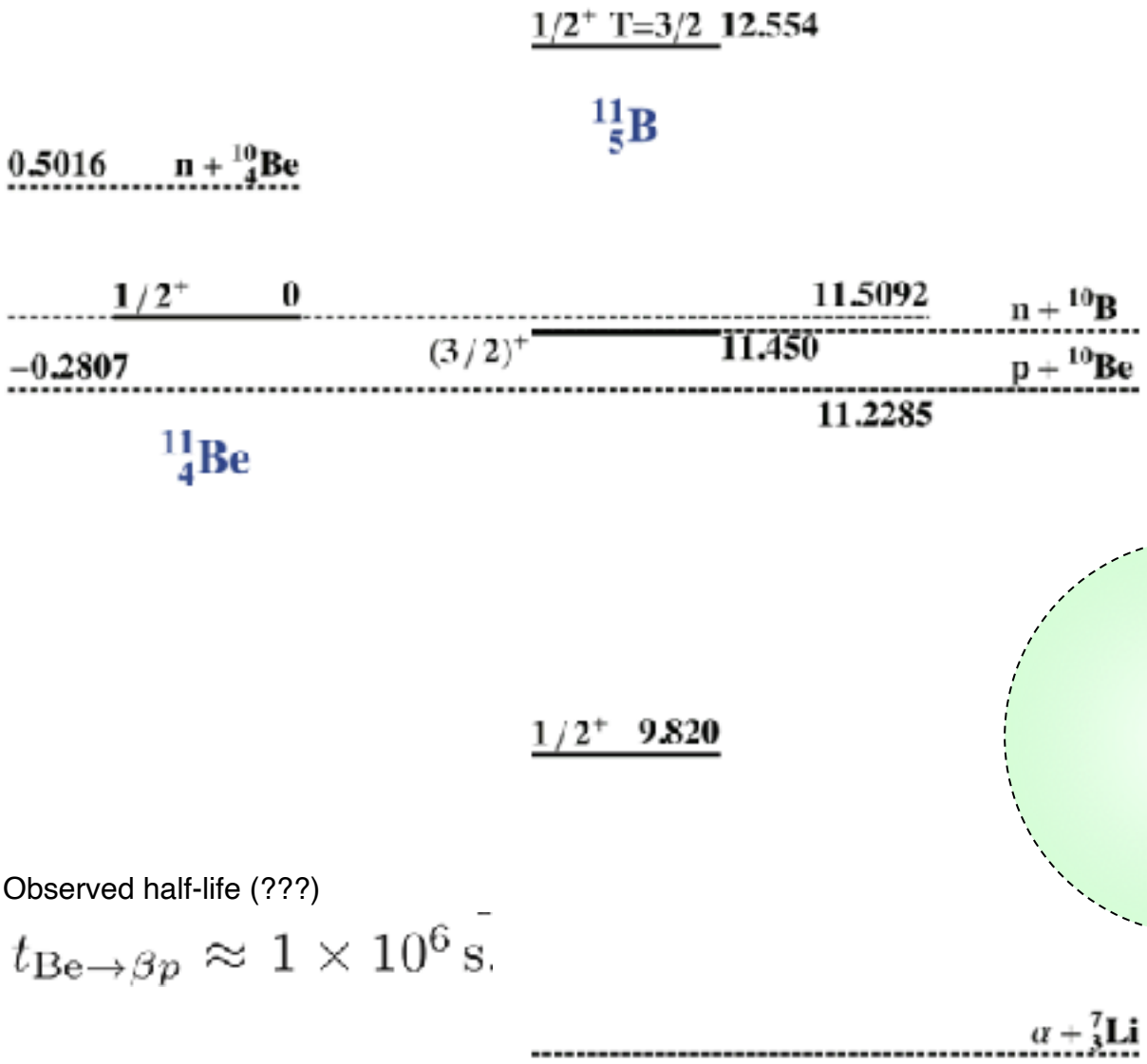
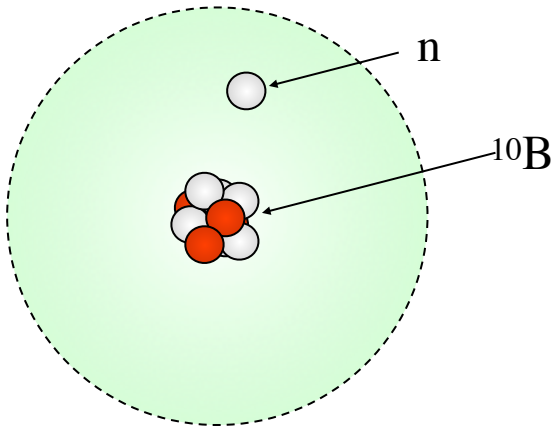


11Be beta-delayed proton decay

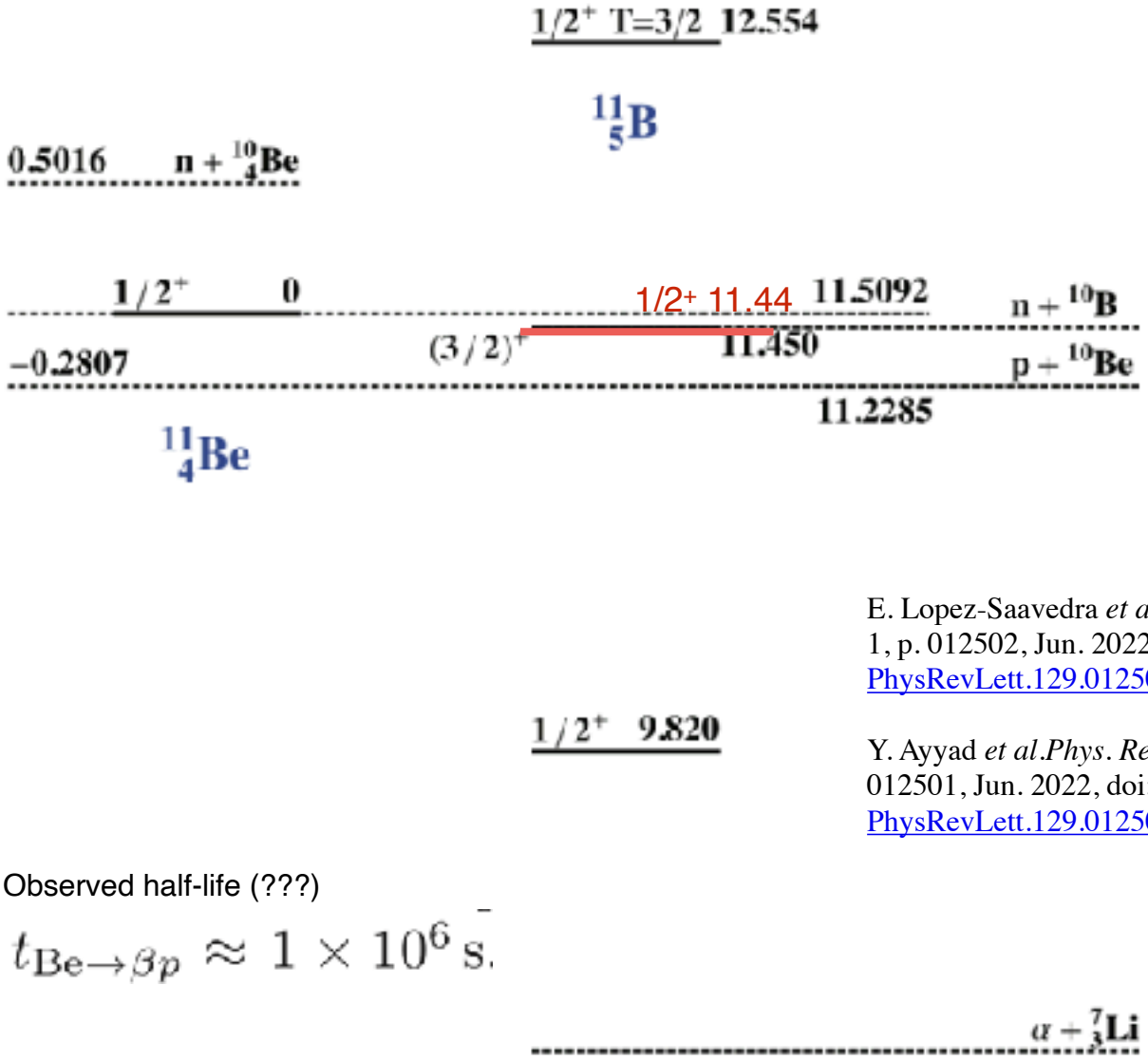




Observed half-life (???)

$$t_{^{11}\mathrm{Be}\to\beta p} \approx 1 \times 10^{6}\ \mathrm{s}.$$

11Be beta-delayed proton decay



E. Lopez-Saavedra *et al.*, *Phys. Rev. Lett.*, vol. 129, no. 1, p. 012502, Jun. 2022, doi: 10.1103/PhysRevLett.129.012502.

Y. Ayyad *et al.Phys. Rev. Lett.*, vol. 129, no. 1, p. 012501, Jun. 2022, doi: 10.1103/PhysRevLett.129.012501.

Observed half-life (???)

$$t_{^{11}\mathrm{Be}\to\beta p} \approx 1 \times 10^{6}\ \mathrm{s}.$$

Notes on beta-decay rates

$$ft = \frac{\mathcal{T}}{B(F) + \lambda_A^2 B(GT)} \qquad \mathcal{T} = 6145\,\mathrm{s},$$

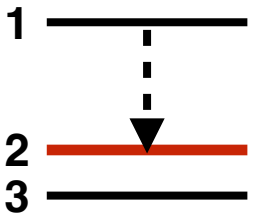
$$\lambda_A = 1.27$$

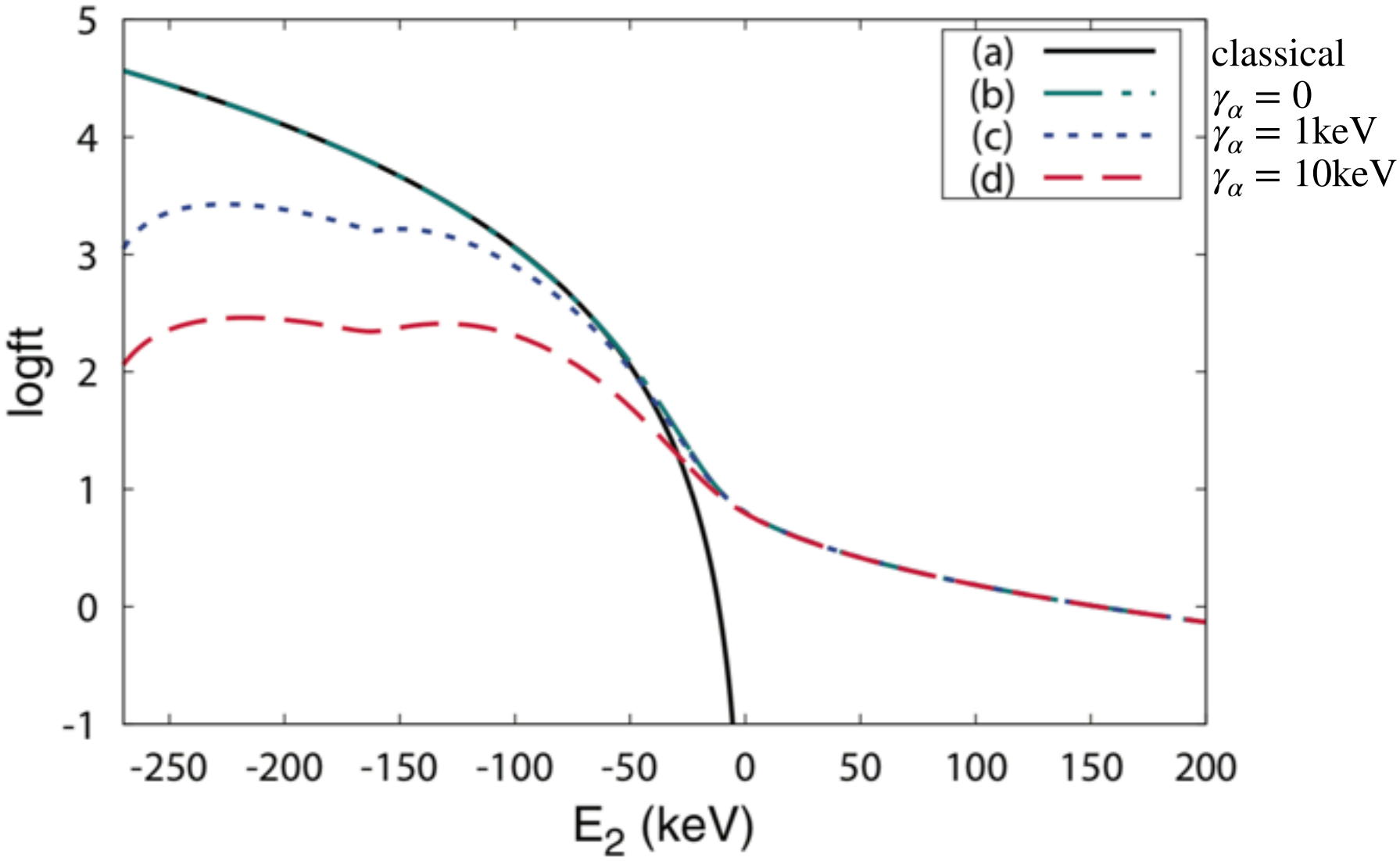
Neutron decay $n \rightarrow p + e^- + \bar{\nu}_e \quad B(F) = 1 \quad B(GT) = 3$

$$f(Q) = 1.715$$

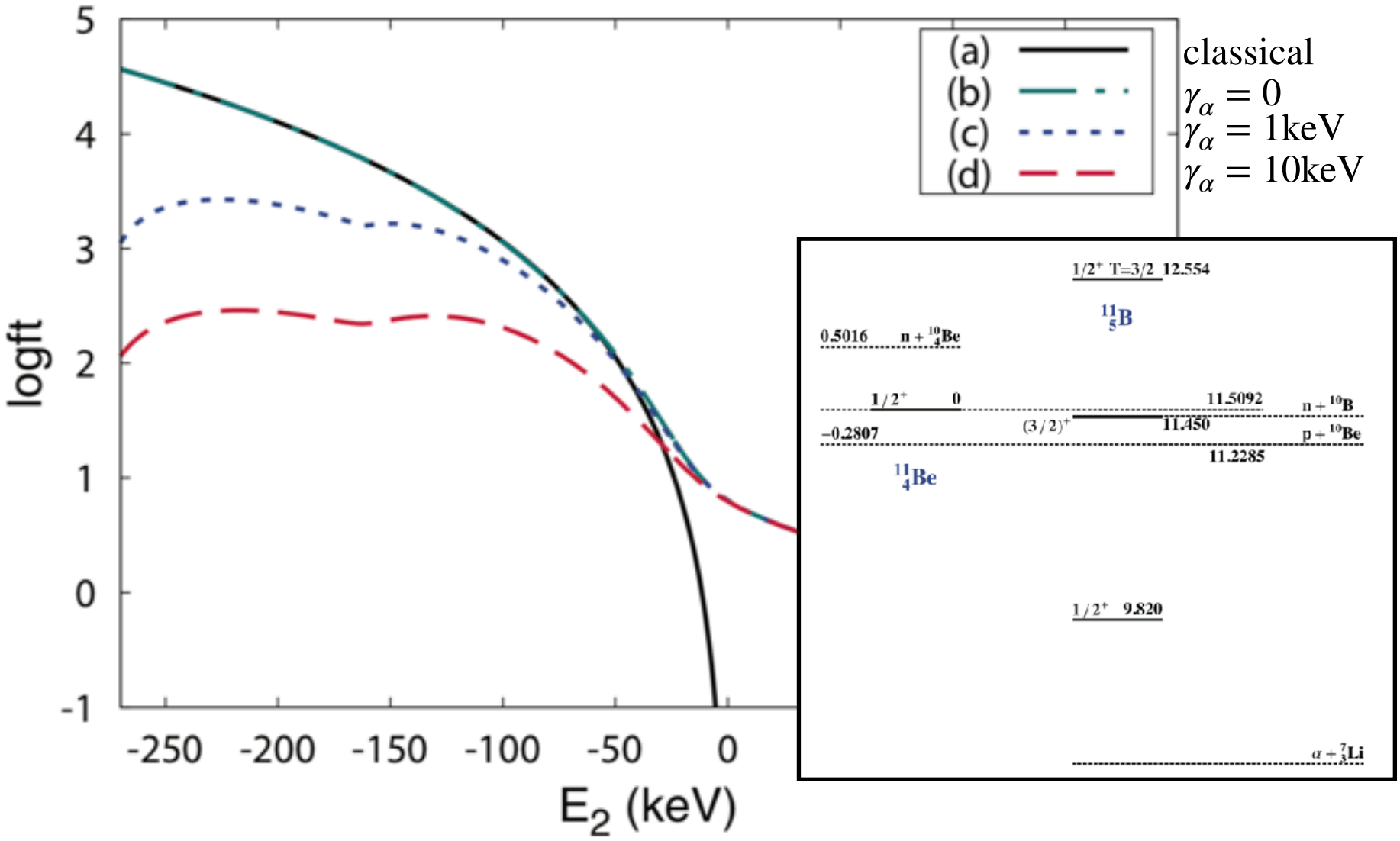
Sequential beta decay

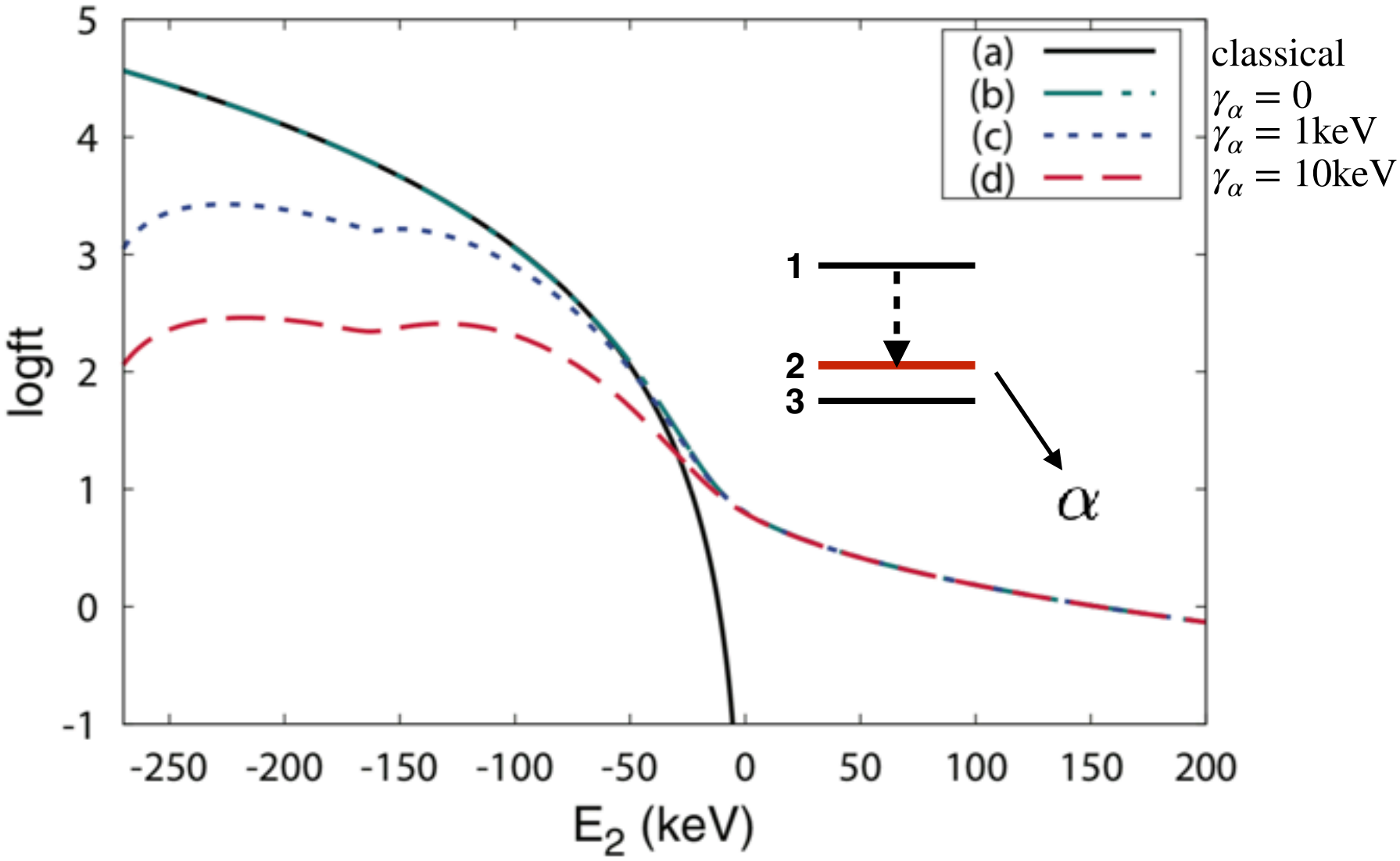
For sequential decay $$\mathcal{F}(Q) = \int_0^Q \frac{\mathrm{d}\epsilon}{2\pi} \frac{f(\epsilon)\gamma_2(Q-\epsilon)}{(\epsilon + E_2)^2 + \Gamma_2^2(Q-\epsilon)/2}$$





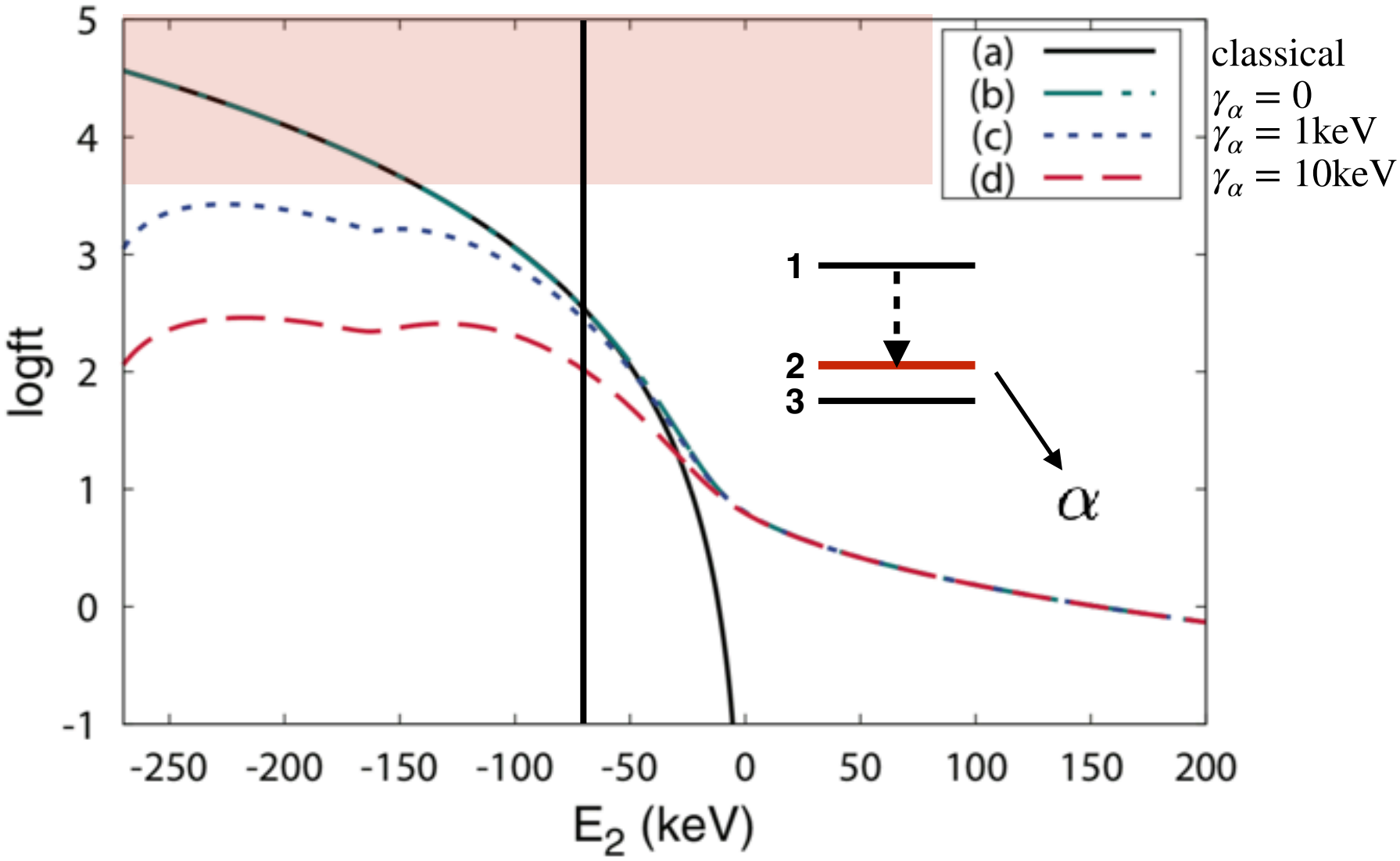
$t_{^8\mathrm{Be}\to\beta p} \approx 1 \times 10^6\,\mathrm{s}.$


5
4
3
2
1
0
-1
logft
-250
-200
-150
-100
-50
0
E2 (keV)
(a)
(b)
(c)
(d)
classical
$\gamma_\alpha = 0$
$\gamma_\alpha = 1$keV
$\gamma_\alpha = 10$keV
1/2+ T=3/2 12.554
11 5 B
0.5016 n + 10 4 Be
1/2+ 0
11.5092
n + 10B
−0.2807
(3/2)+
11.450
p + 10Be
11.2285
11 4 Be
1/2+ 9.820
α + 7 3 Li


5
4
3
2
1
0
-1
logft
-250
-200
-150
-100
-50
0
50
100
150
200
E2 (keV)
(a)
(b)
(c)
(d)
classical
$\gamma_\alpha = 0$
$\gamma_\alpha = 1$keV
$\gamma_\alpha = 10$keV
1
2
3
α

1/2+ states in 11B

	Theory (FSU)			Experiment	
J	**E(MeV)**	**log(ft)**	**SF(p)**	**E(MeV)**	**SF(p)**
1/2+(1)	5.709	5.5	0.262	6.792	
1/2+(2)	10.545	3.4	0.117	9.820	
1/2+(3)	11.952	3.5	0.134	**11.44**	**0.27(6)**
1/2+(4) T=3/2	12.181		0.274	12.554	
1/2+(5)	12.827	4.0	0.028		
1/2+(6)	14.105	5.4	0.001		



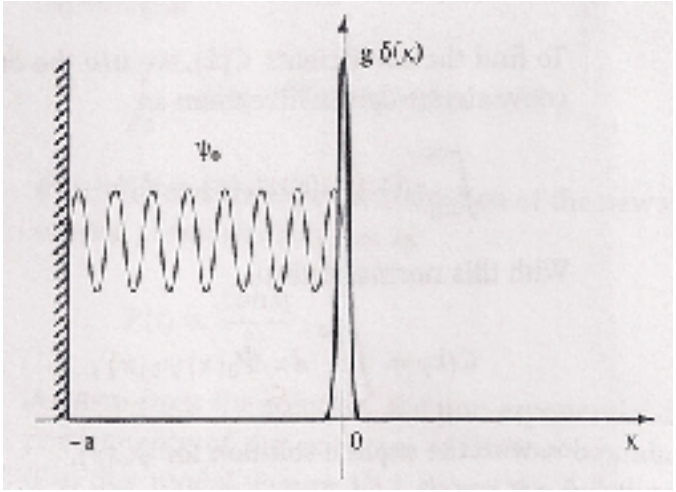
A. Volya, "Assessment of the beta-delayed proton decay rate of 11Be," *EPL*, vol. 130, no. 1, p. 12001, 2020, doi: 10.1209/0295-5075/130/12001.

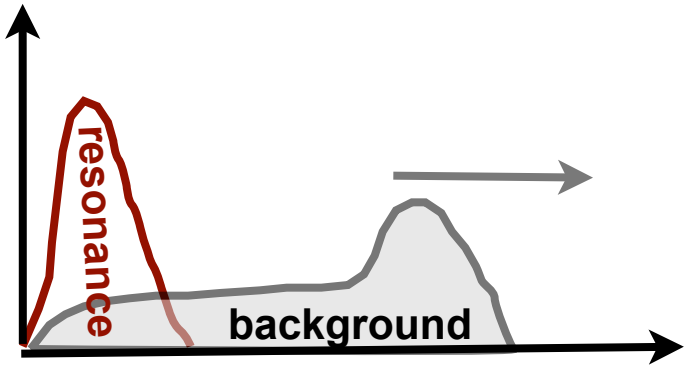
Observations

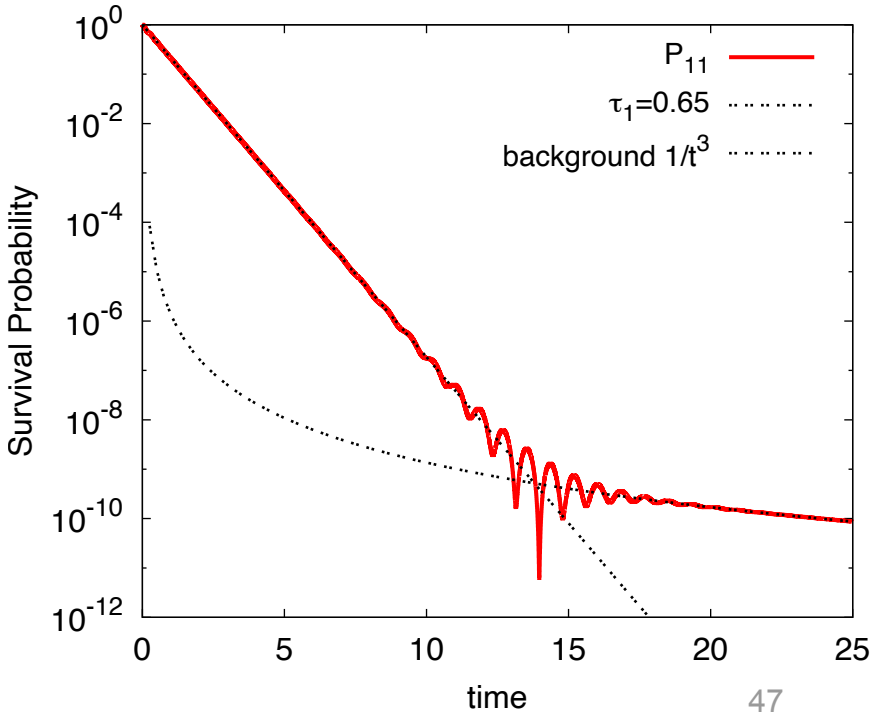
- The proton resonance is likely 1/2+(3), and it is lowered (predicted 12.2 MeV, observed at 11.44 MeV
- The proton SF is large, while there is no alpha decay?
- The strong GT transition is difficult to explain.

Time dependence of decay

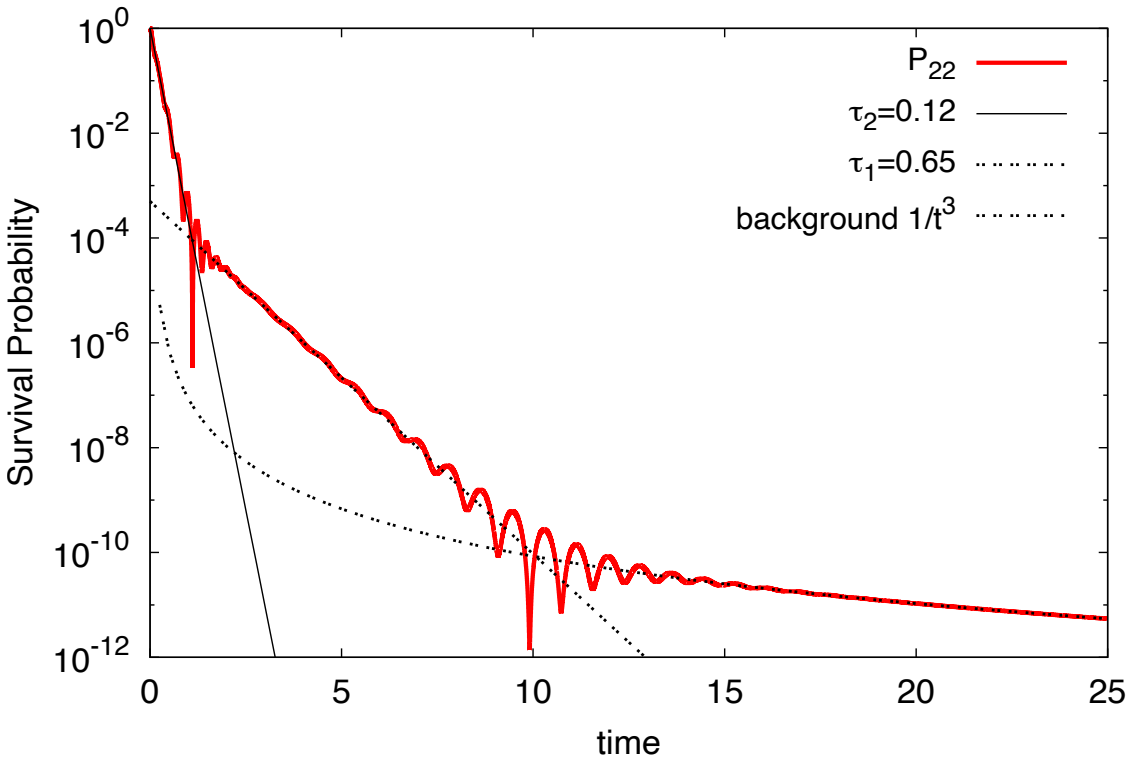
*Winter, Phys. Rev., **123**,1503 1961.*

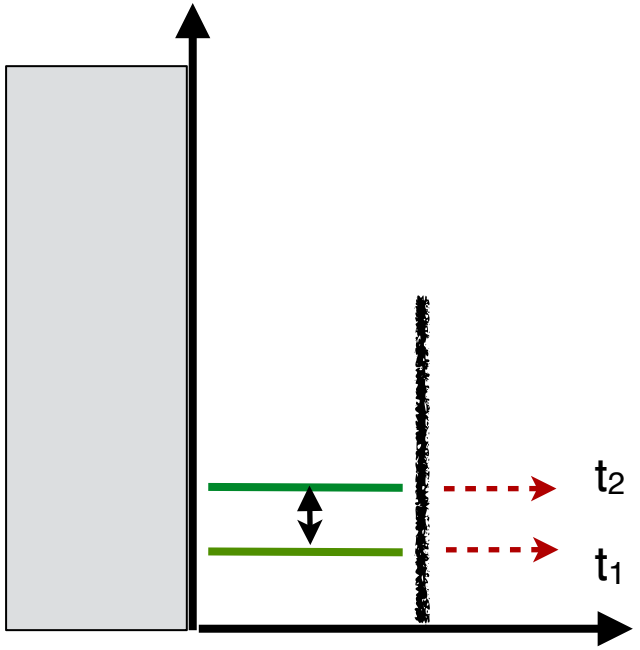






Internal dynamics in decaying system
Winter's model





M. Peshkin, A. Volya, and V. Zelevinsky, "Non-exponential and oscillatory decays in quantum mechanics," *EPL*, vol. 107, no. 4, p. 40001, 2014.

Time-dependent Continuum Shell Model Approach

•Expand Using evolution operator in Chebyshev polynomials

$$\exp(-iHt) = \sum_{n=0}^{\infty} (-i)^n (2 - \delta_{n0}) J_n(t) T_n(H)$$

•Chebyshev polynomial $T_n[\cos(\theta)] = \cos(n\theta)$

•Use iterative relation and matrix-vector multiplication to generate

$$|\lambda_n\rangle = T_n(H)|\lambda\rangle$$

$$|\lambda_0\rangle = |\lambda\rangle, \quad |\lambda_1\rangle = H|\lambda\rangle \quad |\lambda_{n+1}\rangle = 2H|\lambda_n\rangle - |\lambda_{n-1}\rangle$$

$$\langle\lambda'|T_{n+m}(H)|\lambda\rangle = 2\langle\lambda'_m|\lambda_n\rangle - \langle\lambda'|\lambda_{n-m}\rangle, \quad n \geqslant m$$

•Use FFT to find return to energy representation

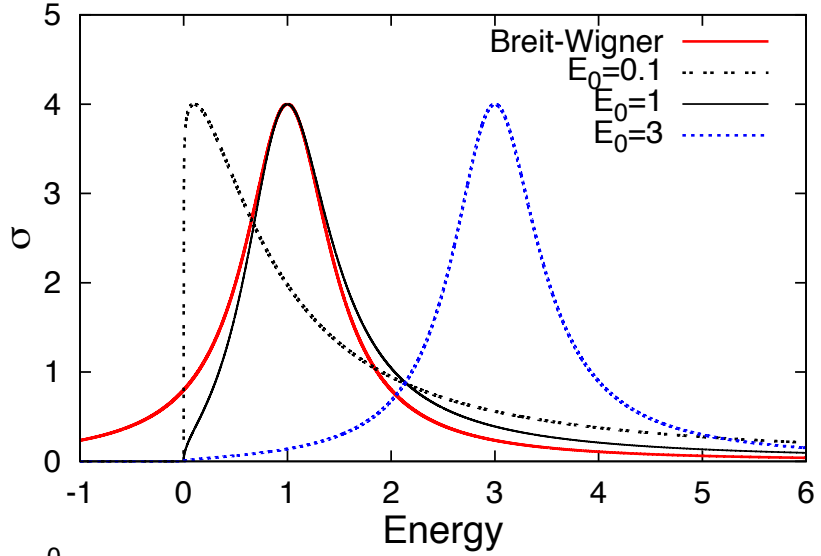
A. Volya, Phys. Rev. C 79, 044308 (2009).

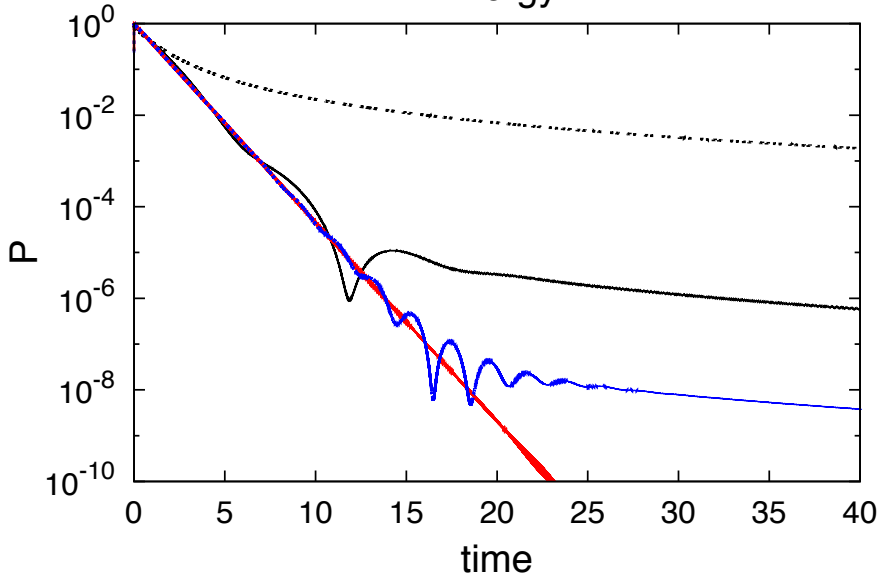
Probing the Non-exponential Decay Regime in Open Quantum Systems

- Broad threshold resonance (^{9}N,^{9}He)
  - Pronounced non-exponentiality
  - Very short half-life
- Three-body decay (^{6}Be, ^{13}Li, ^{16}Be)
  - Nucleon-nucleon correlations
  - Energy dependence
- Overlapping resonances (^{13}C, ^{13}N)
  - Interference, pronounced non-exponentiallity
  - Superradiance

S. M. Wang, W. Nazarewicz, A. Volya, and Y. G. Ma, "Probing the Non-exponential Decay Regime in Open Quantum Systems." arXiv, Nov. 21, 2022. doi: 10.48550/arXiv.2211.11619.

Time-dependent picture





$$\mathcal{G} = \frac{1}{E - E_o + i/2\,\Gamma(E)}$$

$$\Gamma(E) \propto \sqrt{E}$$

Power-law remote decay rate!

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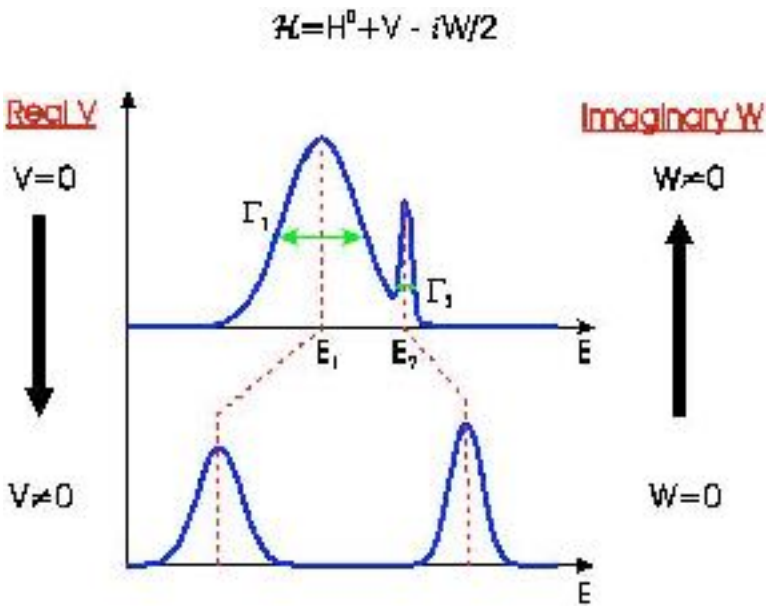
S. M. Wang, W. Nazarewicz, A. Volya, and Y. G. Ma, "Probing the Non-exponential Decay Regime in Open Quantum Systems." arXiv, Nov. 21, 2022. doi: 10.48550/arXiv.2211.11619.

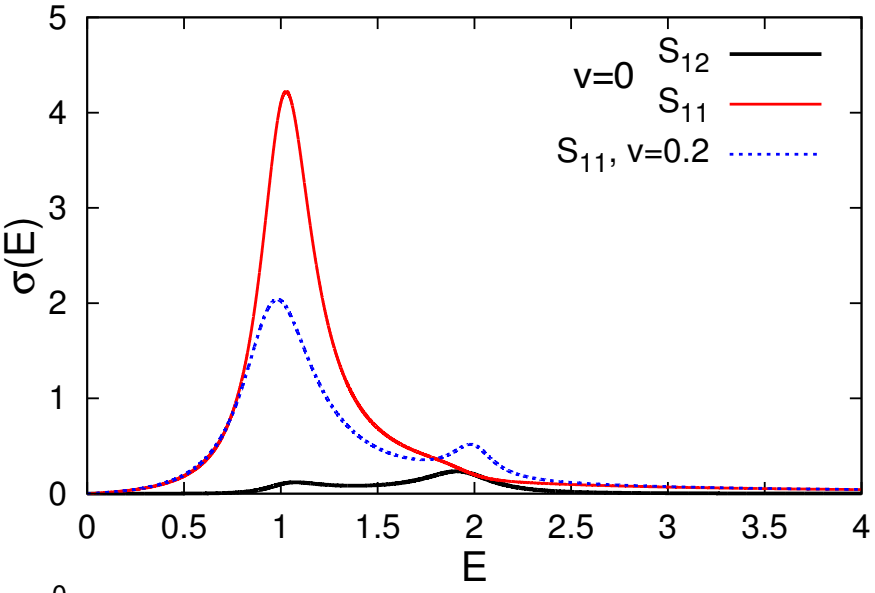
Two-level system

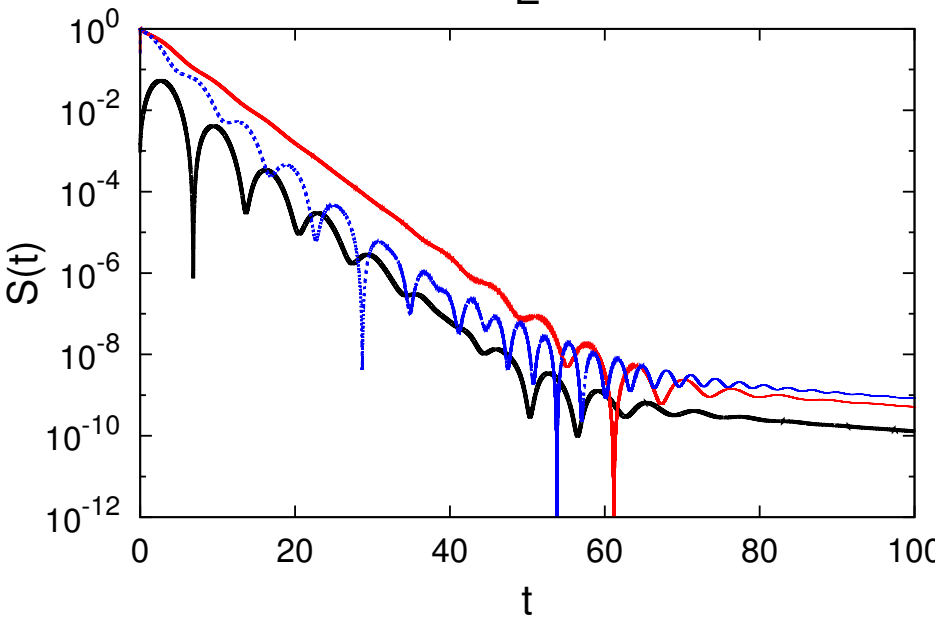
$$\mathcal{H} = \begin{pmatrix} \epsilon_1 - (i/2)\Gamma_1 & v - (i/2)A_1A_2 \\ v - (i/2)A_1A_2 & \epsilon_2 - (i/2)\Gamma_2 \end{pmatrix}$$

$$\Gamma_1 = A_1^2, \quad \Gamma_2 = A_2^2,$$

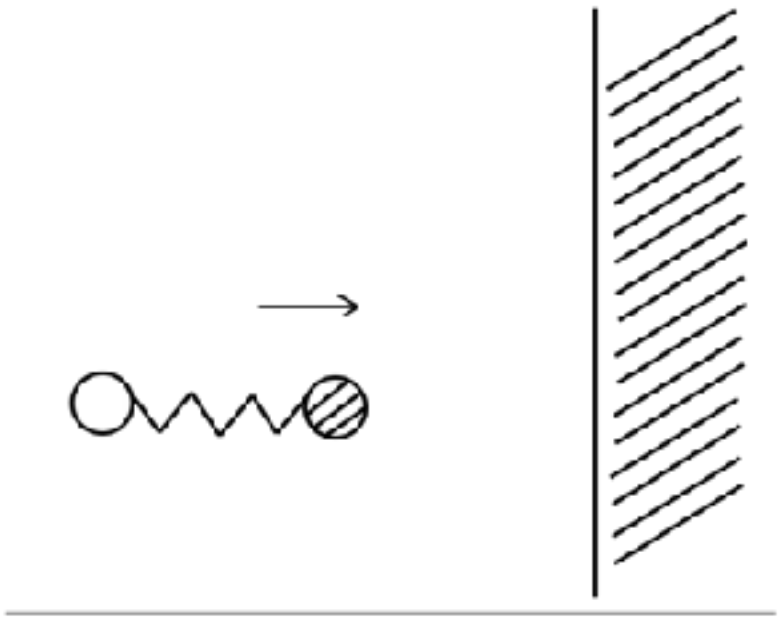
$$S(t) = |\langle \Psi(0) | \Psi(t) \rangle|^2$$

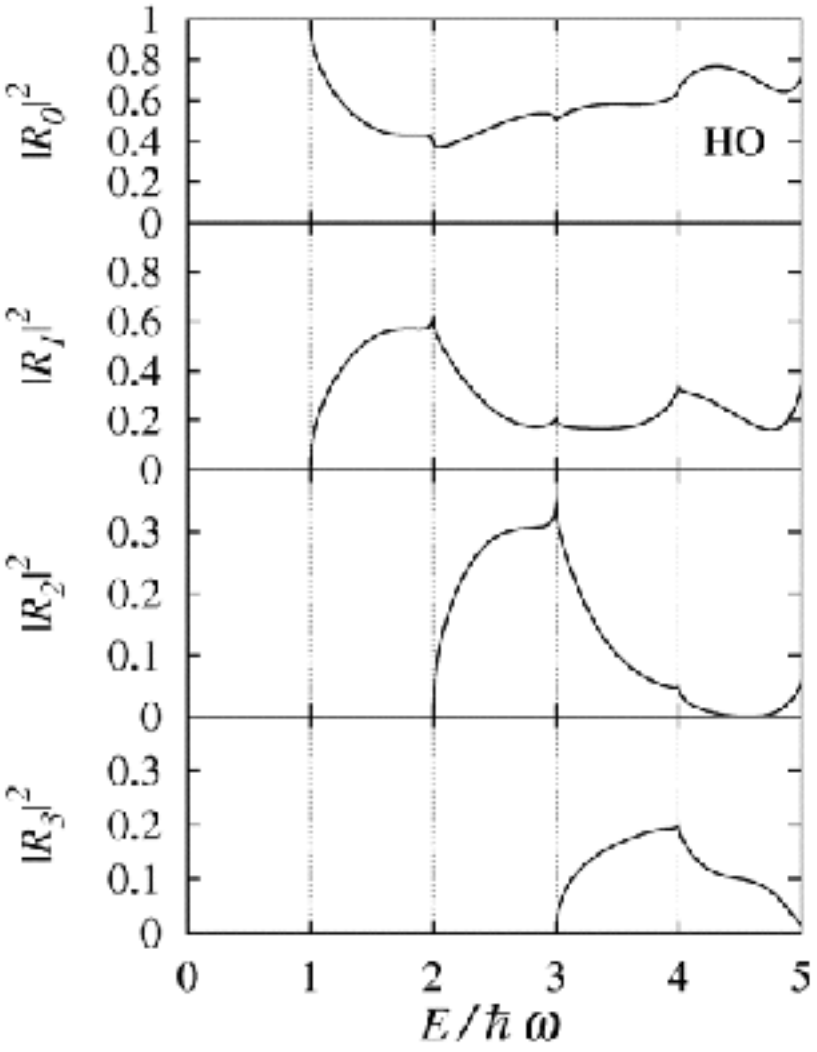






Wigner cusps, unitarity …



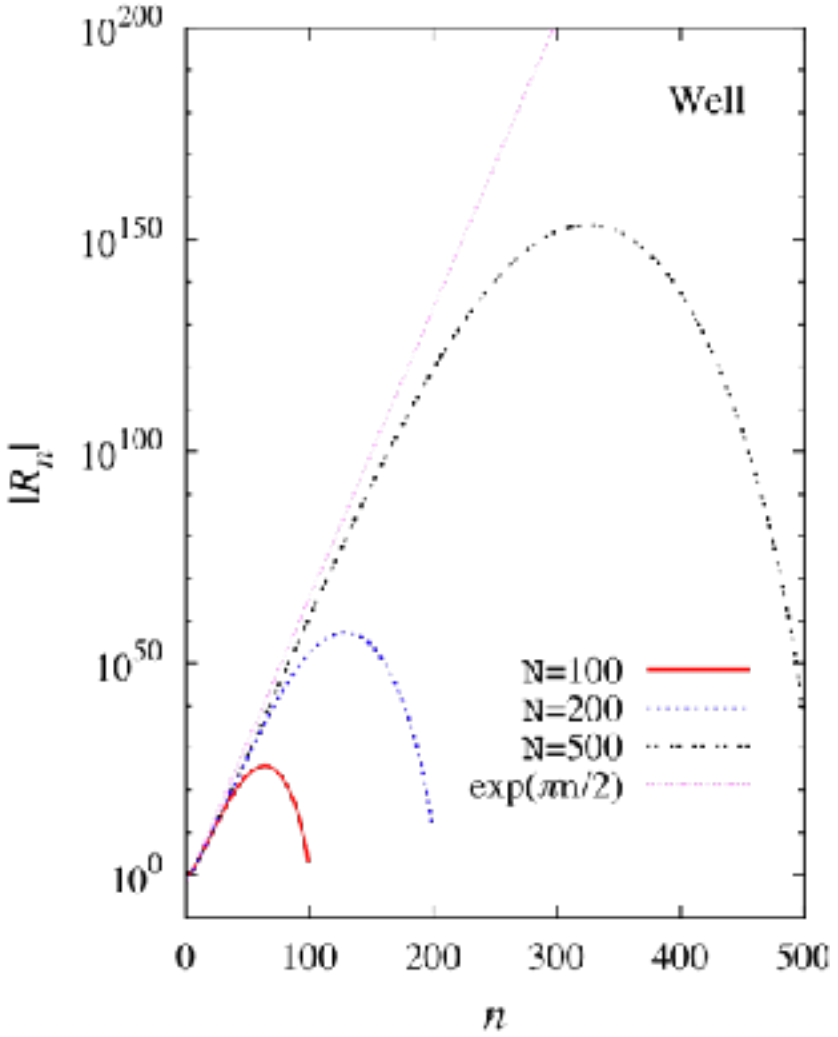


$$\Phi(X,x) = \frac{e^{iK_n X}}{\sqrt{|K_n|}}\psi_n(x) + \sum_{m=0}^{\infty} \frac{R_{mn}}{\sqrt{|K_m|}} e^{-iK_m X}\psi_m(x),$$

$$\Phi(X,x) = 0 \text{ at } x_2 = 0.$$

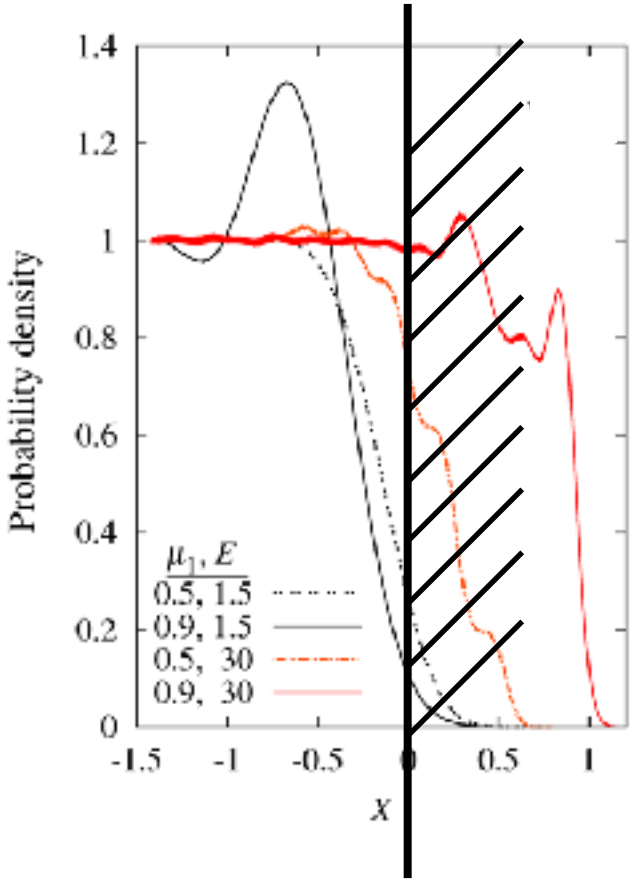
Two-nucleon system with continuum
finite square well binding

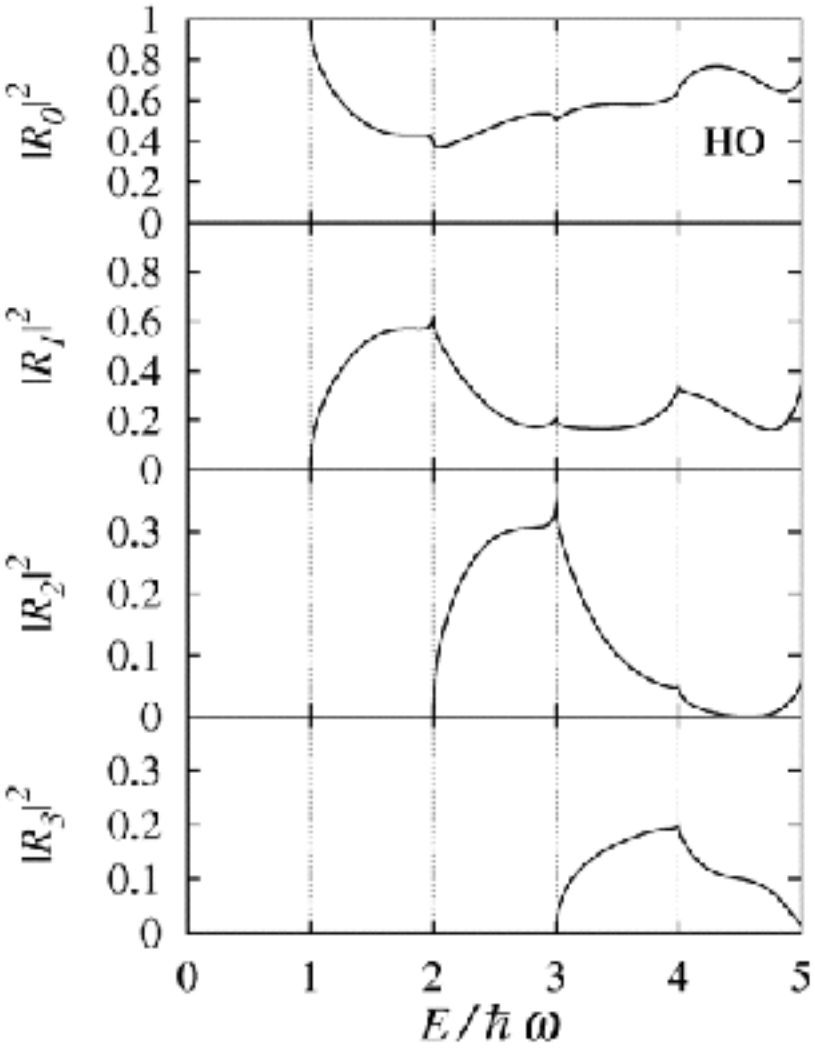
In the exact solution virtual amplitudes are exponentially large.

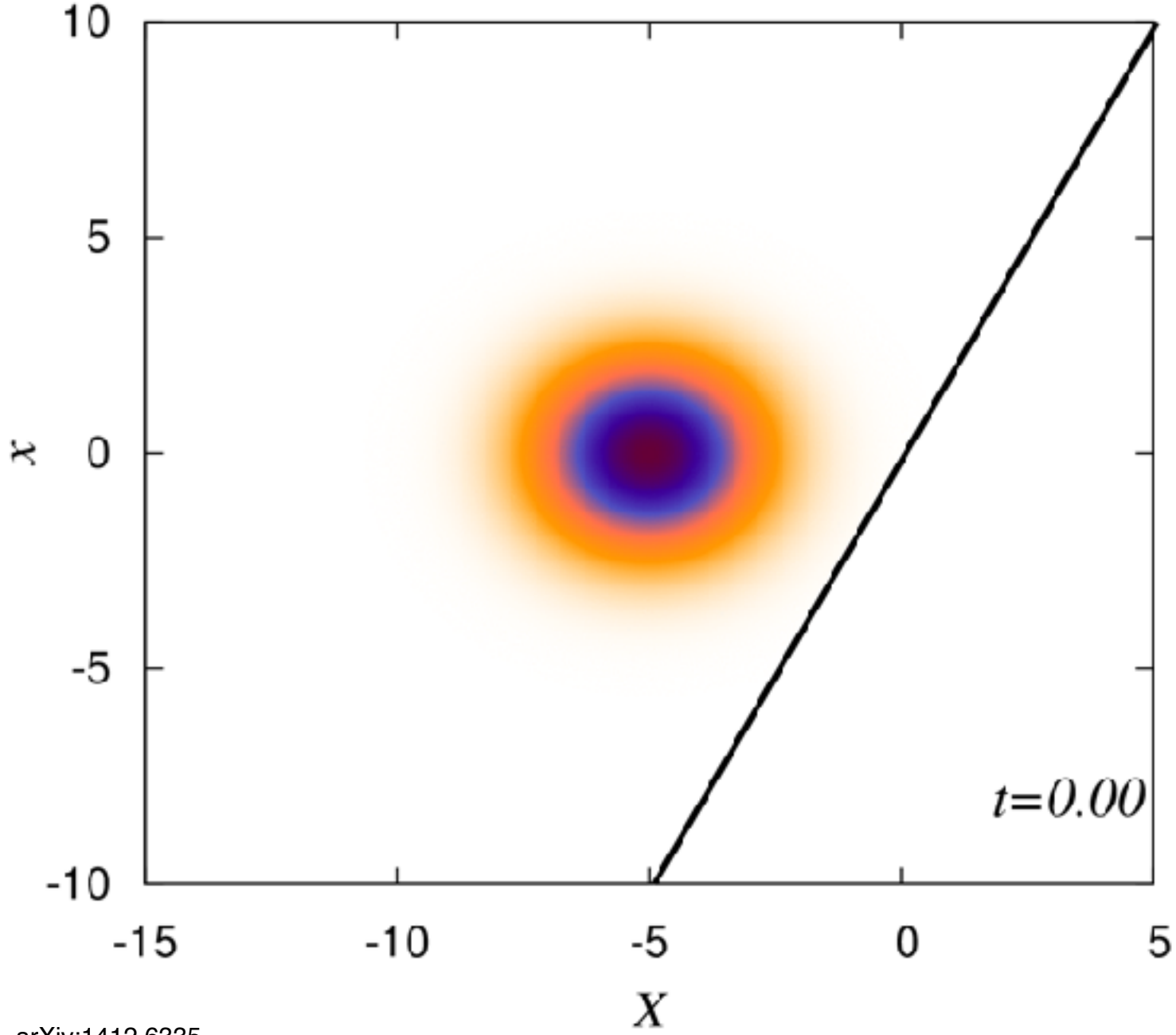


N. Ahsan and A. Volya, "Quantum tunneling and scattering of a composite object reexamined," *Phys. Rev. C*, vol. 82, no. 6, p. 064607, 2010.

Results: scattering off an infinite wall

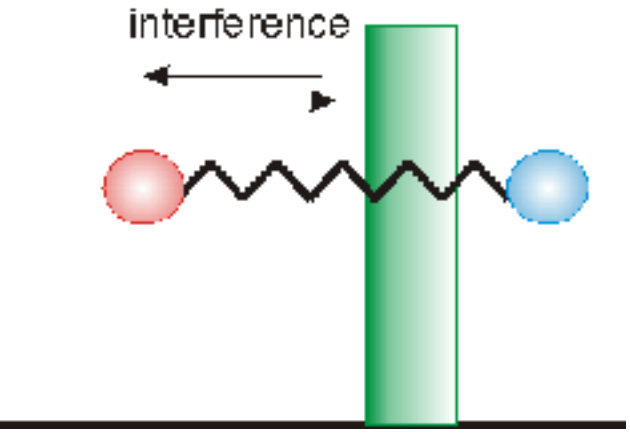


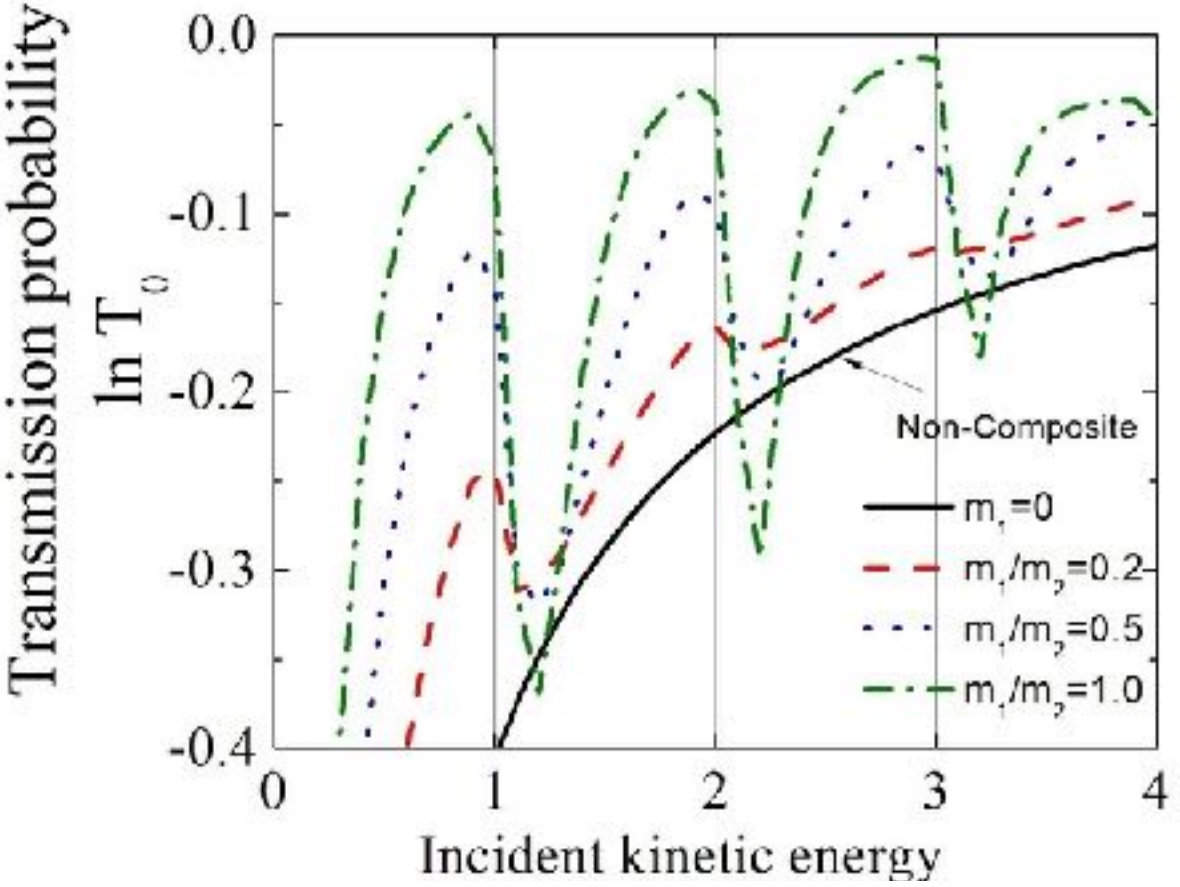


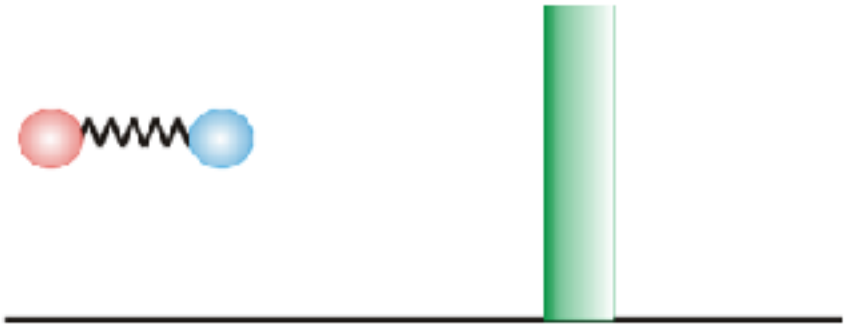


A.Volya, arXiv:1412.6335

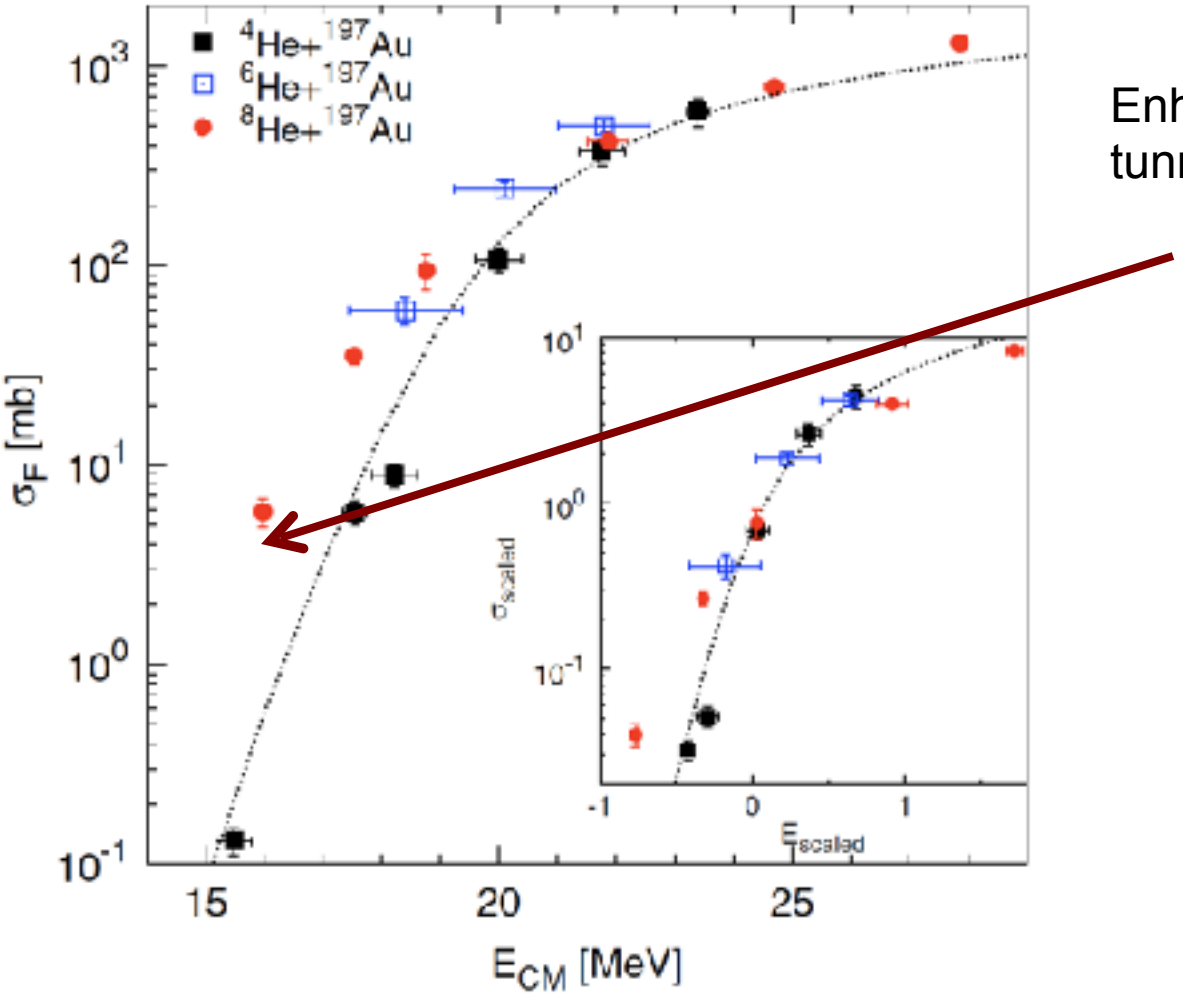
Resonant tunneling of composite objects



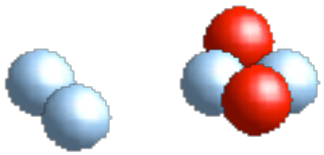




Enhanced tunneling probability for composite objects



Enhancement of tunneling probability.



^{6}He

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Funding Acknowledgements:

Funding: U.S. DOE contract DE-SC0009883.