

Quantum dynamics in carbon burning

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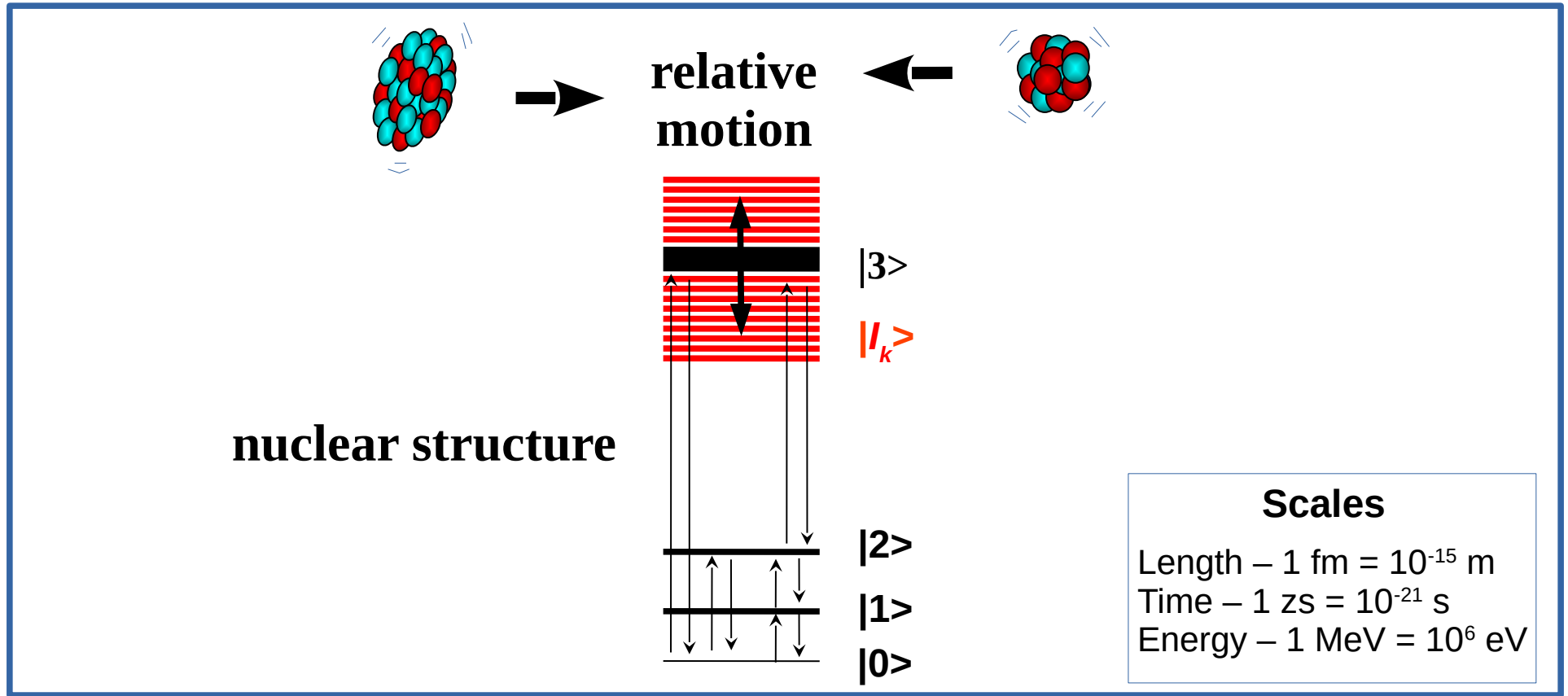
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Michael Wiescher (University of Notre Dame, USA)

Outline

- **TDWP**
- **CCDM**

The Physics of Low-Energy Nuclear Reactions



Interplay between **relative motion** and **nuclear structure** determines reaction dynamics and reaction outcomes (cross sections).

Quantum Wave-Packet Dynamics

♦ **Preparation:** the initial state $\Psi(t = 0)$



♦ **Time propagation:** $\Psi(0) \rightarrow \Psi(t)$
guided by the operator, $\exp(-i \hat{H} t / \hbar)$
 $\hat{H}$ is the model Hamiltonian

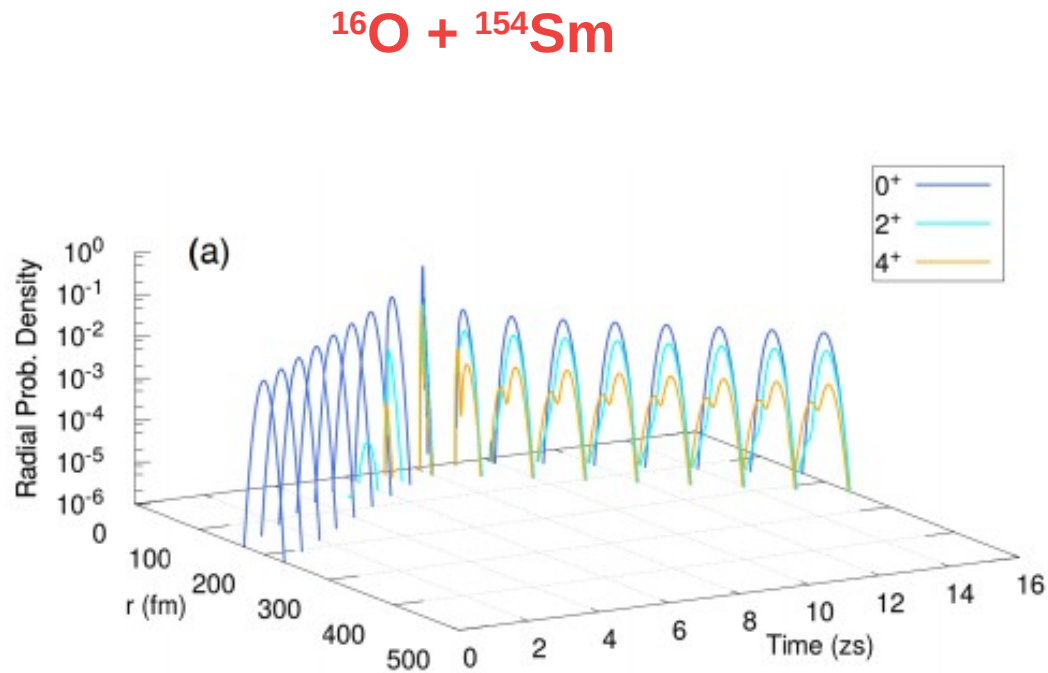
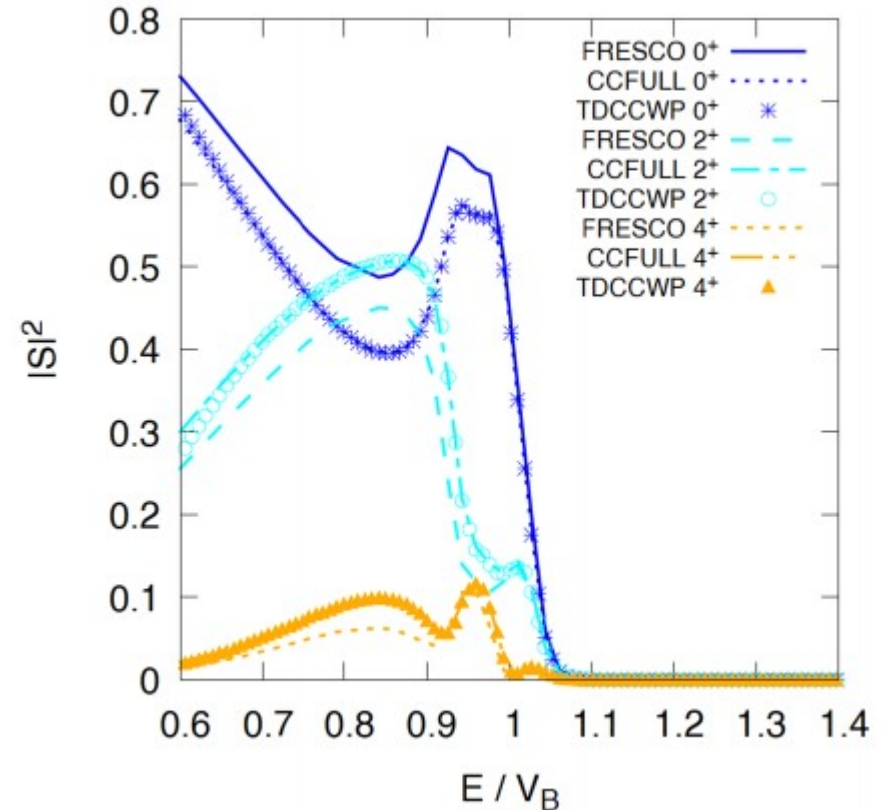


♦ **Analysis:** extraction of probabilities from
the time-dependent wave function



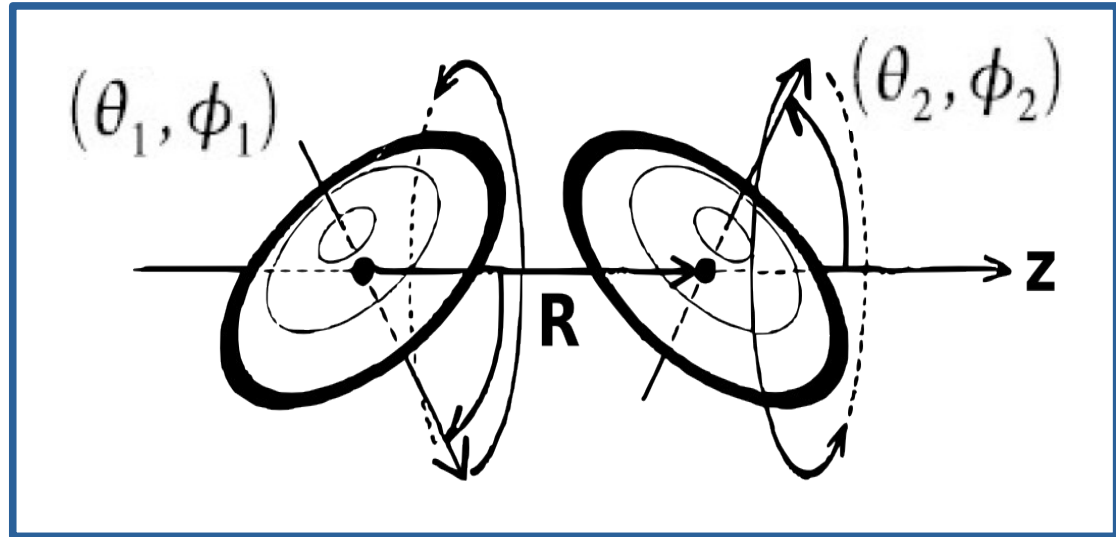
Quantum Coupled-Channels Wave-Packet Dynamics

Vockerodt & AD-T, PRC **104**, 064601 (2021); PRC **100**, 034606 (2019)



The $^{12}\text{C} + ^{12}\text{C}$ Molecular Structure & Carbon Fusion

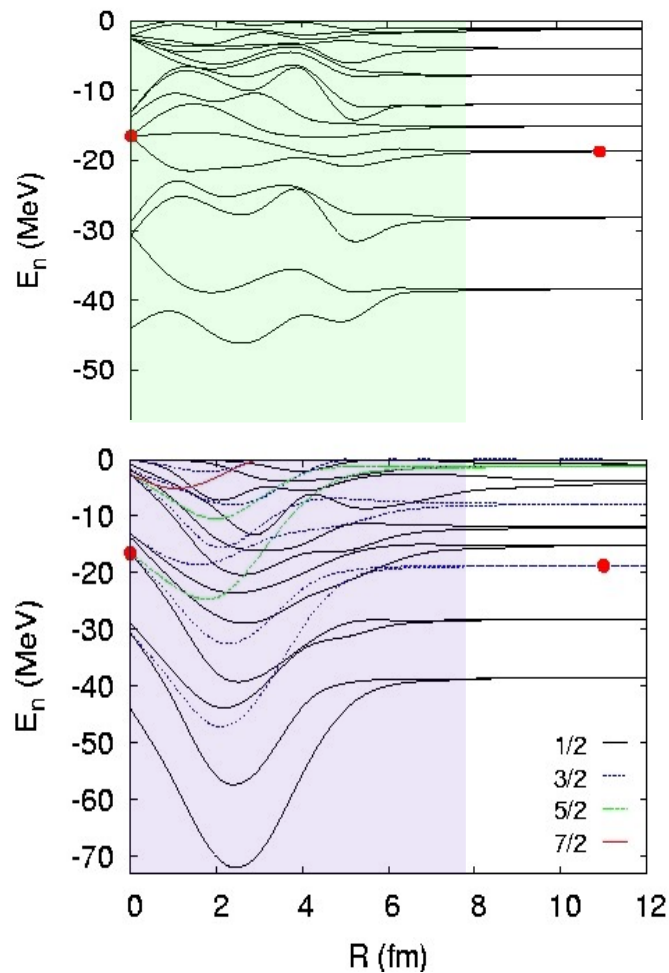
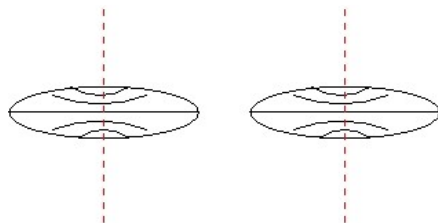
Greiner, Park & Scheid, in Nuclear Molecules, World Scientific, 1994



Quadrupole deformation of ^{12}C : ~ -0.5

How does this molecular structure affect low-energy fusion?

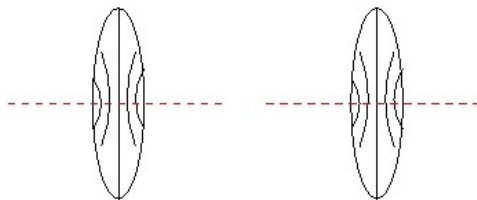
Sensitivity of Molecular Shell Structure to the ^{12}C Alignment



$$V = \sum_{s=1}^2 e^{-i\mathbf{R}_s \hat{k}} \hat{U}(\Omega_s) V_s \hat{U}^{-1}(\Omega_s) e^{i\mathbf{R}_s \hat{k}}$$

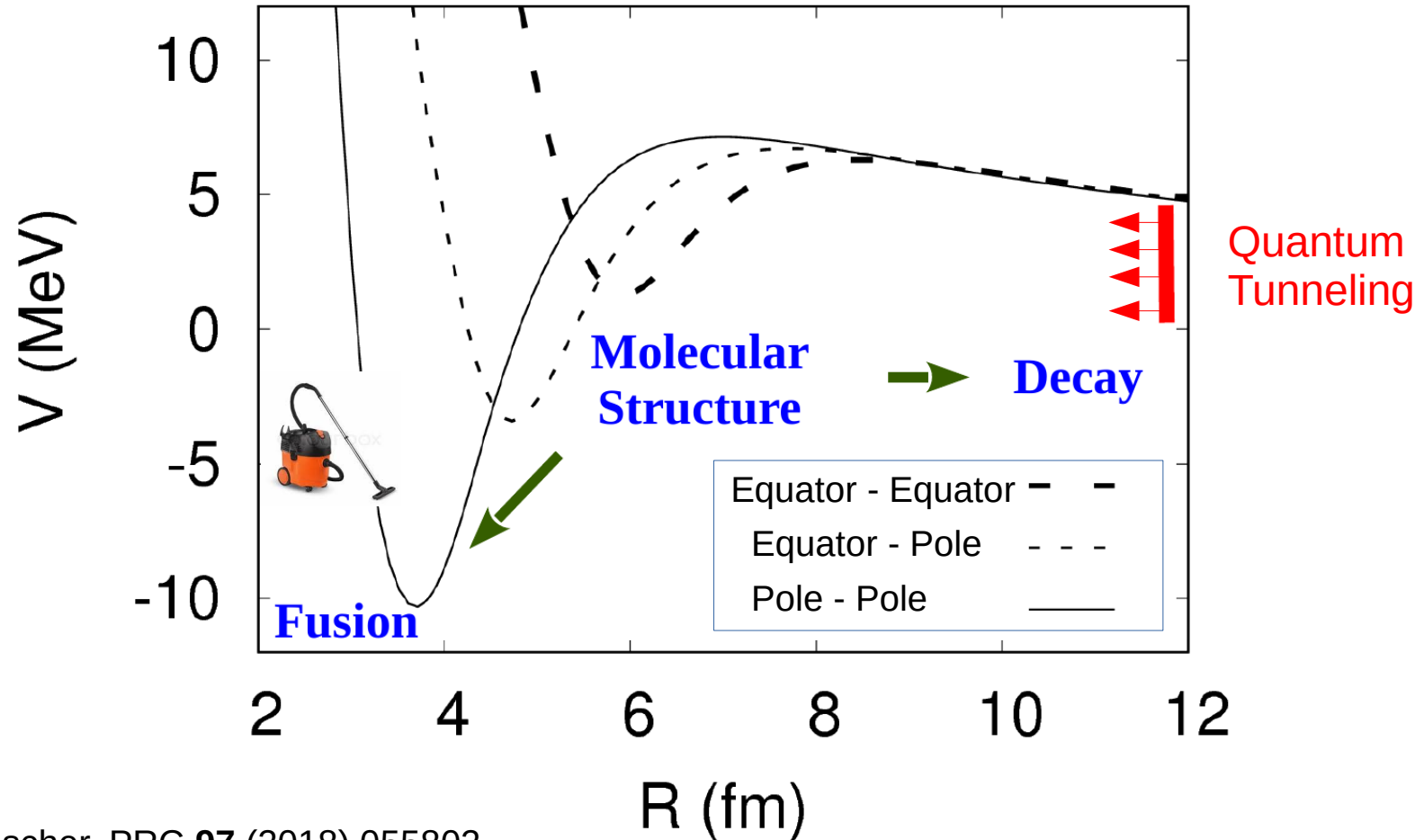
$$V_s \approx \sum_{\nu\mu}^N |s\nu\rangle V_{\nu\mu}^s \langle s\mu|$$

Potential Separable Expansion Method



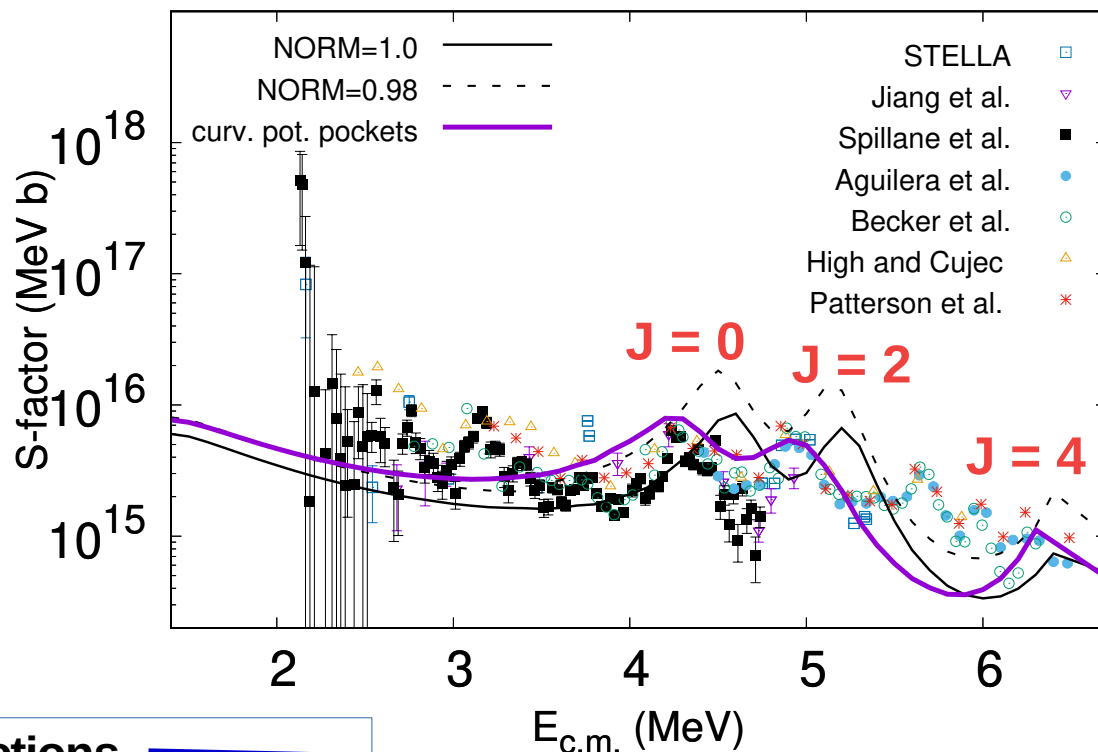
AD-T, PRL **101** (2008) 122501

Potential Energy Landscape: Molecular structure influences fusion and decay rates



Astrophysical S-Factor for $^{12}\text{C} + ^{12}\text{C}$ Fusion

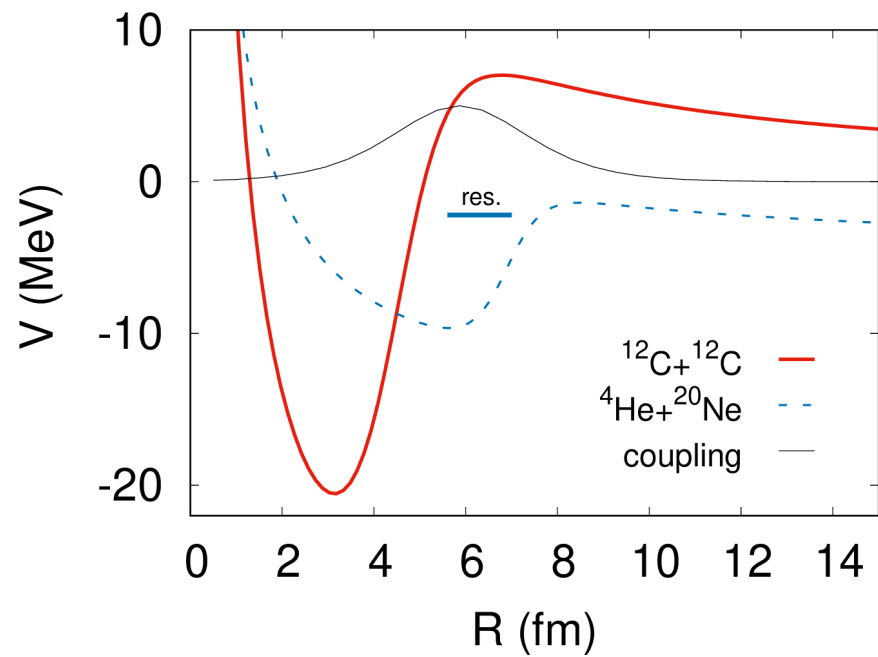
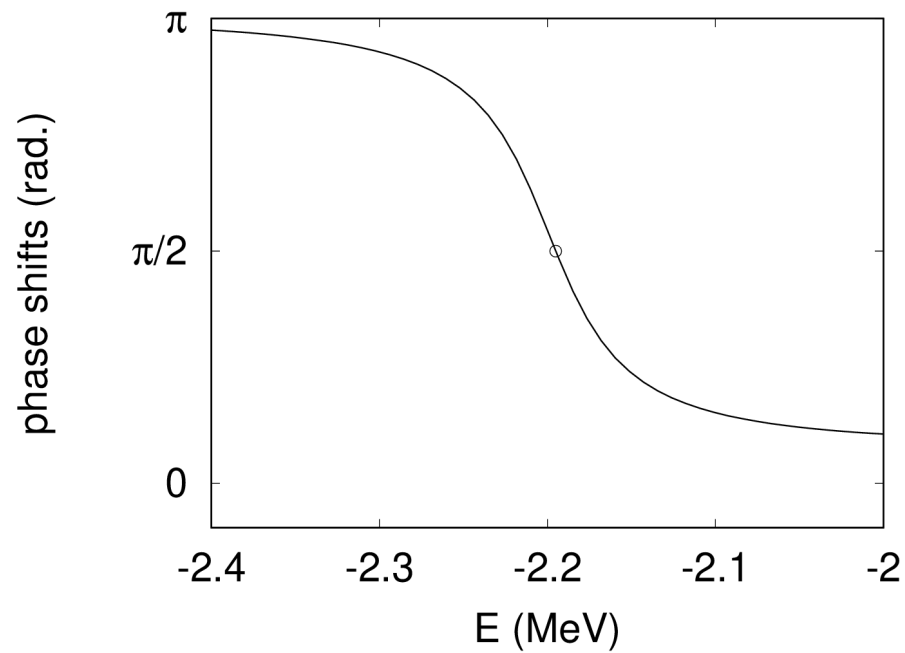
AD-T & Wiescher, PRC **97** (2018) 055802



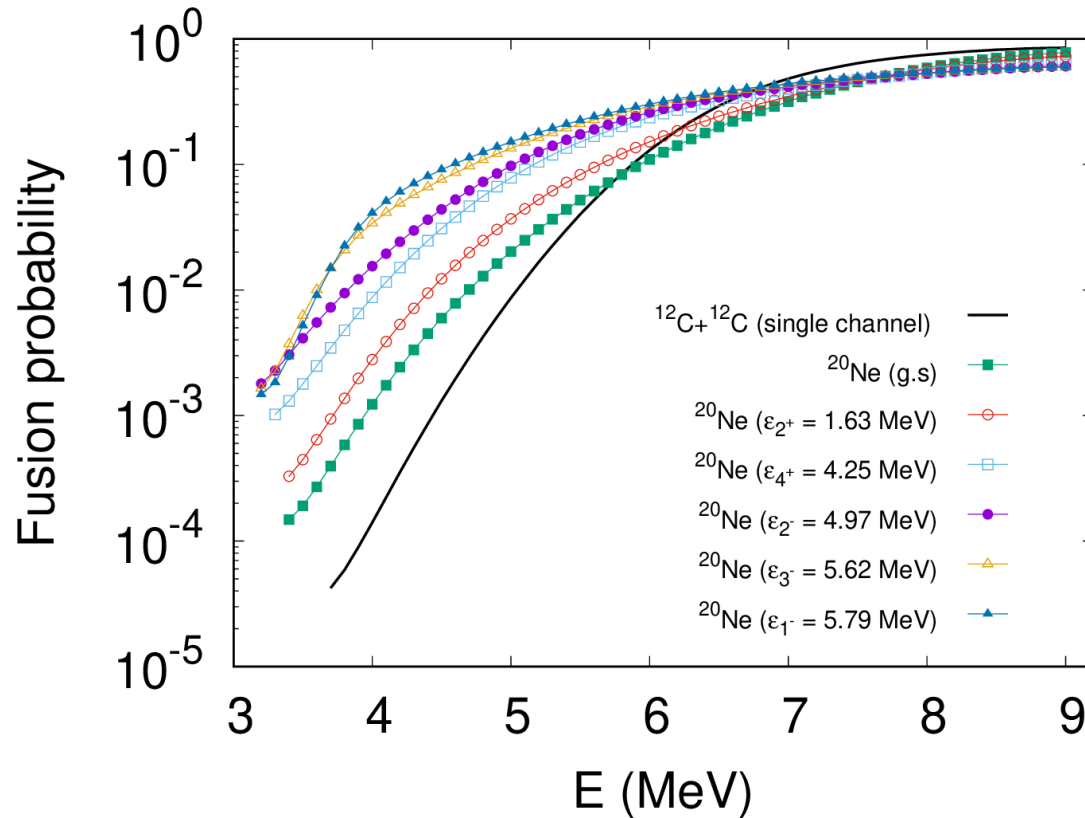
range of cross sections
at this energy window

1pb 1nb

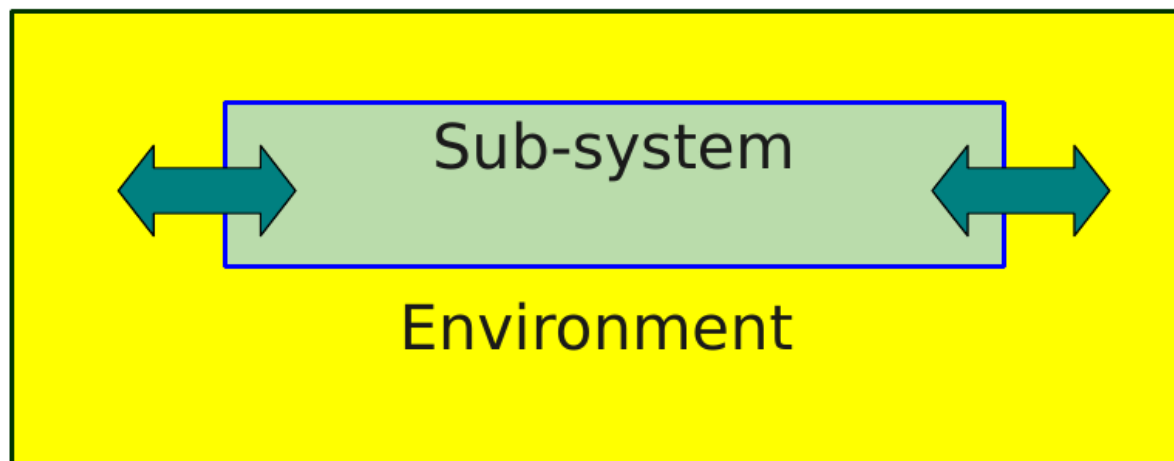
Cluster Effects on Carbon Burning



Cluster Effects on Carbon Burning



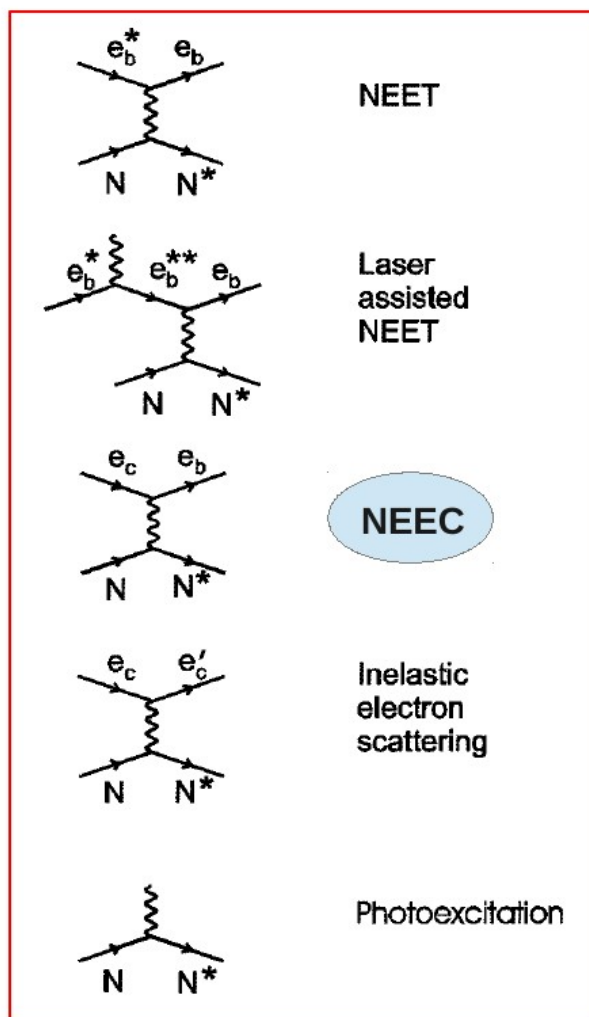
Open Quantum System Dynamics



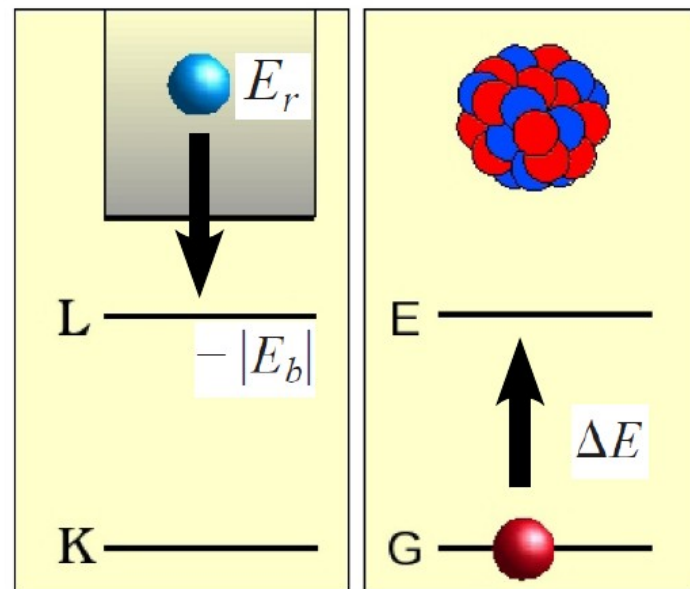
Sub-system: Low-Energy Collision in the Vacuum

Environment: Plasma

Mechanisms of Nuclear Excitation in Plasmas



NEEC



$$E_r \approx \Delta E - |E_b|.$$

Gosselin & Morel, PRC **70** (2004) 064603

Harston & Chemin, PRC **59** (1999) 2462

Coupled-Channels Density-Matrix Framework

AD-T, PRC **82** (2010) 054617; Lee & AD-T, PLB **827** (2022) 136970

$$\begin{aligned}
 i\hbar \frac{\partial \rho_{\alpha\alpha'}^{rs}}{\partial t} = & \sum_t (T^{rt} \rho_{\alpha\alpha'}^{ts} - \rho_{\alpha\alpha'}^{rt} T^{ts}) \\
 & + [U_{\alpha}(r) - U_{\alpha'}(s)] \rho_{\alpha\alpha'}^{rs} \\
 \text{Plasma screening effects} \rightarrow & + \sum_{\beta} [V_{\alpha\beta}(r) \rho_{\beta\alpha'}^{rs} - \rho_{\alpha\beta}^{rs} V_{\beta\alpha'}(s)] \\
 & + (\varepsilon_{\alpha} - \varepsilon_{\alpha'}) \rho_{\alpha\alpha'}^{rs} \\
 & + i\hbar \left\{ \delta_{\alpha\alpha'} \sum_{\mu} \sqrt{\Gamma_{\alpha\mu}^{rr}} \rho_{\mu\mu}^{rs} \sqrt{\Gamma_{\alpha\mu}^{ss}} \right. \\
 & \left. - \frac{1}{2} \sum_{\mu} (\Gamma_{\mu\alpha}^{rr} + \Gamma_{\mu\alpha'}^{ss}) \rho_{\alpha\alpha'}^{rs} \right\}
 \end{aligned}$$

Excitations caused by coupling to atomic shells

Initial Density Matrix

$$\hat{\rho}_{\alpha\alpha'}^{rs}(0) = \underbrace{\langle \alpha | \hat{\sigma}(0) | \alpha' \rangle}_{\text{Internal states}} \otimes \underbrace{g^*(z_r) g(z_s)}_{\text{Radial motion}}$$

$$\hat{\sigma}(0) = \sum_i w_i |i\rangle \langle i|$$

$$w_i = \delta_{ij}$$

Pure state

$$w_i = \frac{e^{-\frac{E_i}{k_B T}}}{\text{Tr}[e^{-\frac{\hat{H}_0}{k_B T}}]}$$

Mixed state

Key Messages

LEVERHULME
TRUST



- Some resonant structures in the $^{12}\text{C}+^{12}\text{C}$ astrophysical S-factor are due to the **quantum partner-dance** of ^{12}C in nuclear molecule.
- Resonance structures in the $^{12}\text{C}+^{12}\text{C}$ fusion probability **echo** molecular resonances of different cluster configurations of ^{24}Mg .
- Dynamical models (CCDM & TDCCWP) enable new exciting areas of research (e.g., **plasma-assisted** & **laser-assisted** nuclear reactions).

EXTRA SLIDES

Nuclear Excitation Rate by NEEC & Photon Absorption

Gosselin & Morel, PRC **70** (2004) 064603

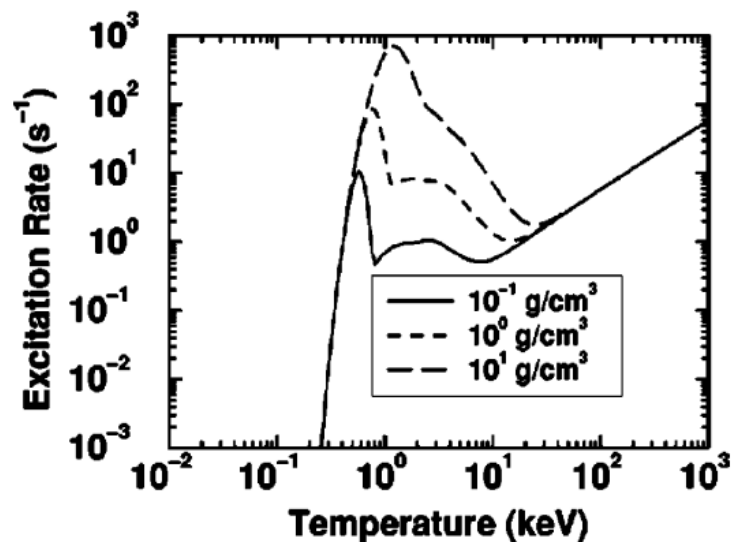
- ✓ Relativistic atom model
- ✓ Fermi golden rule

NEEC rate: Γ_e^{NEEC}

$$\frac{2\pi}{\hbar} n(E_r) |\langle \psi_f \varphi_b | H | \psi_i \varphi_r \rangle|^2 (2J_f + 1) f_{\text{FD}}(E_r) \\ \times [1 - f_{\text{FD}}(E_b)] \frac{1}{2} \left[\text{erf}\left(\frac{E_r}{\varepsilon\sqrt{2}}\right) - \text{erf}\left(\frac{E_b}{\varepsilon\sqrt{2}}\right) \right]$$

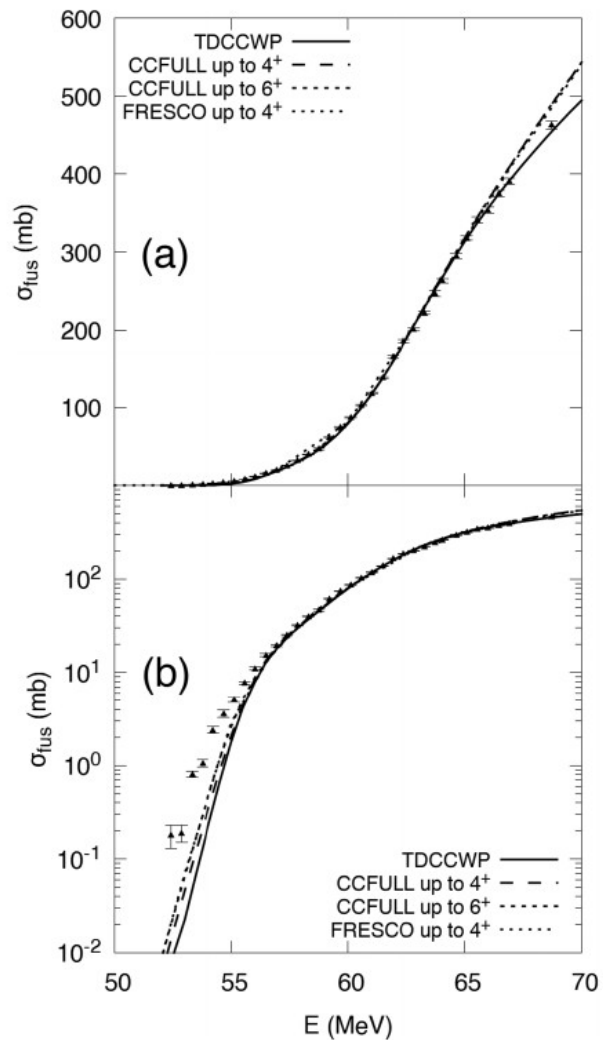
Photon-Excitation rate: Γ_e^{PE}

$$\frac{2J_f + 1}{2J_i + 1} \frac{\ln 2}{T_{J_f \rightarrow J_i}^\gamma} \frac{1}{e^{\Delta E/k_B T} - 1}$$



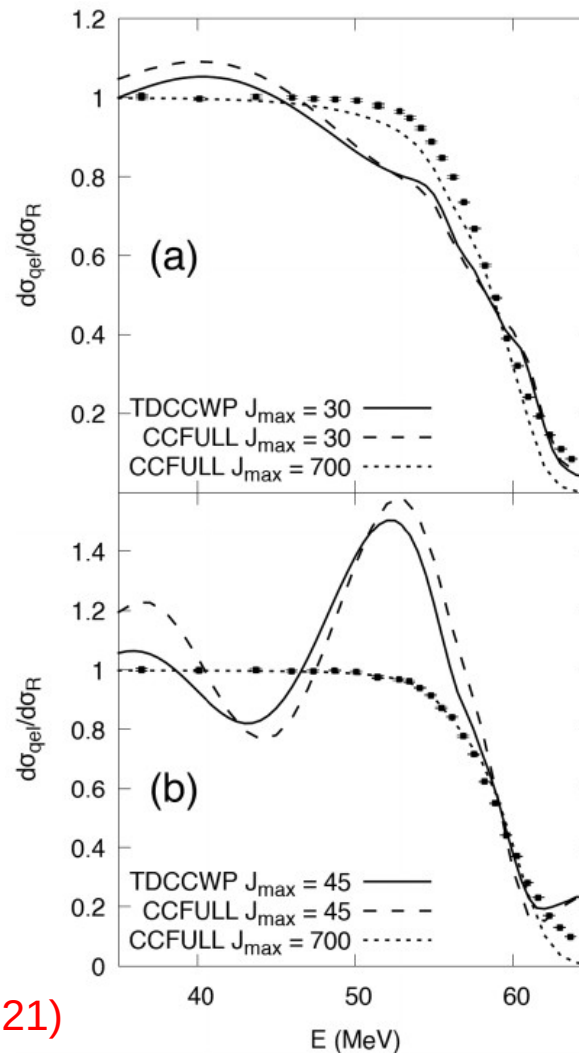
$\Gamma_e^{\text{NEEC}} + \Gamma_e^{\text{PE}}$ for the 4.8 keV
E2 transition in ^{93}Mo .

FUSION $^{16}\text{O} + ^{154}\text{Sm}$



Vockerodt & AD-T
 PRC **104**, 064601 (2021)

QUASI-ELASTIC SCATTERING



$^{16}\text{O} + ^{154}\text{Sm}$

$\theta = 175^\circ$

$^{16}\text{O} + ^{152}\text{Sm}$

CCDM vs. CCFULL: $^{16}\text{O} + ^{144}\text{Sm}$

Lee & AD-T, PLB **827** (2022) 136970

