

Near-threshold resonances and in atomic nuclei

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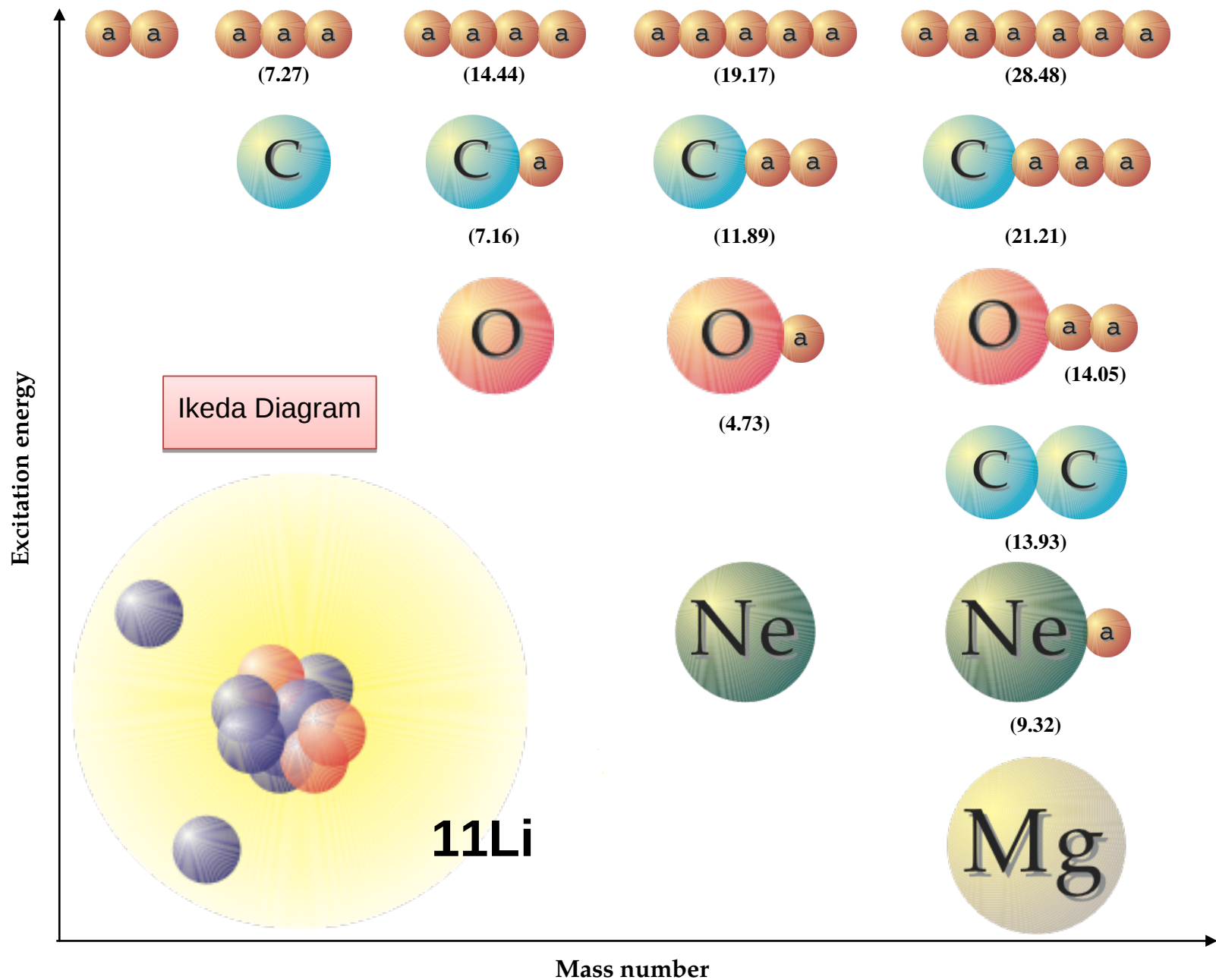
Workshop on Atomic and Nuclear Quantum Effects Near Threshold

Edinburgh, Scotland, May 31-June 2, 2023

With Marek Płoszajczak

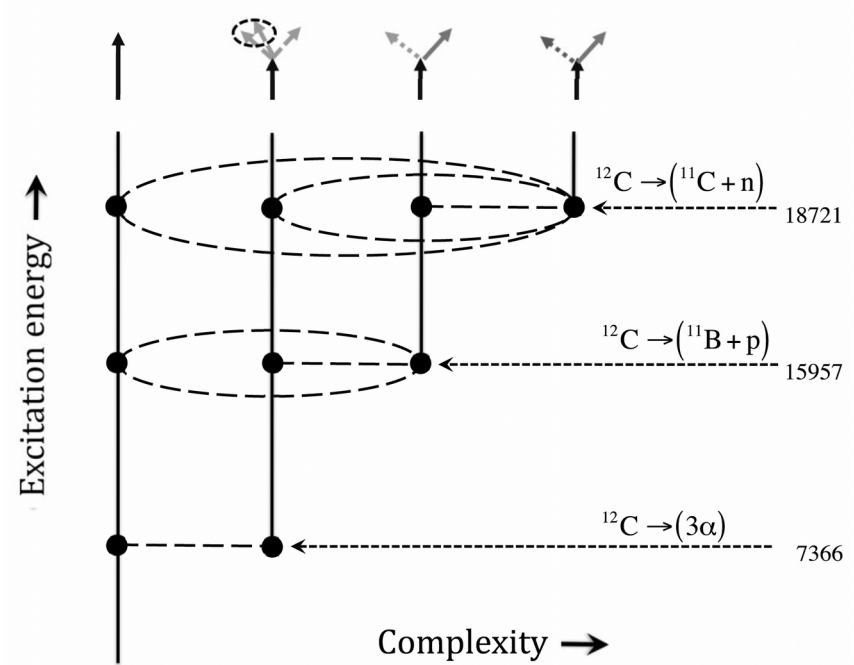
Menu

- Threshold resonances
- Resonant state properties
- Threshold effects and continuum coupling
- Examples of threshold-aligned states
- Perspective



K. Ikeda, N. Takigawa, and H. Horiuchi, Prog. Theor. Phys. Suppl. E68, 464 (1968)

- The threshold is a branching point (hence, nonanalytic behavior).
- The threshold effects are rooted in the *unitarity of scattering matrix* and the resulting *flux conservation*.
- If a new channel opens, a redistribution of the flux in other open channels appears, i.e., a modification of their reaction cross-sections.
- The shape of the near-threshold cusp depends strongly on the orbital angular momentum.



The multichannel representation of 12C

With the increasing excitation energy, subsequent decay channels open at threshold energies Q_n , leading to a complex *multichannel network of couplings*. When a new channel opens up at the threshold Q_i , the unitarity imposes the appearance of new channel couplings; hence, a *modification of all eigenfunctions*.

Real-energy picture: change of asymptotics

E.P. Wigner, Phys. Rev. 73, 1002 (1948), the Wigner cusp

G. Breit, Phys. Rev. 107, 923 (1957)

A.I. Baz', JETP 33, 923 (1957)

D.R. Inglis, Nucl. Phys. 30, 1 (1962)

F.C. Barker, Proc. Phys. Soc. 84, 681 (1964)

A.M. Lane, Phys. Lett. 33B, 274 (1970)

S.N. Abramovich et al., Part. and Nucl. 23, 305 (1992).

R-matrix approach:

anomaly is a result of different asymptotic conditions below
and above the threshold

A characteristic behavior (**a cusp**) of scattering and reaction cross sections of *neutral* particles in the vicinity of a reaction threshold (Wigner threshold law)

$$\sigma(i \rightarrow j) \sim k_j^{2\ell_j+1} \sim E_j^{\ell_j+\frac{1}{2}}$$

Endoergic reaction with the
production of slow neutral particles

$$\sigma(i \rightarrow j) \sim k_i^{2\ell_i-1} \sim E_i^{\ell_i-\frac{1}{2}}$$

Exoergic reaction (e.g., the absorption of slow neutrons by nuclei)

For charged particles, the angular momentum dependence is smooth.

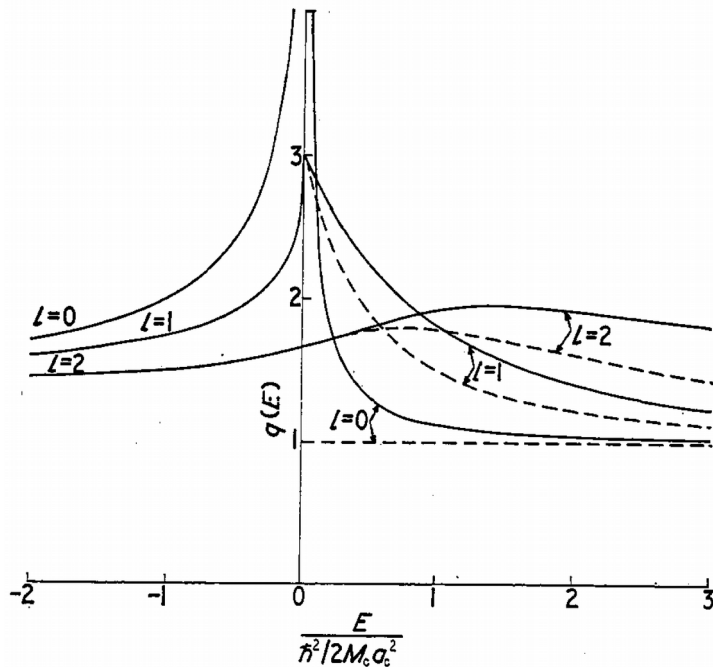
Such a behavior is seen in all related quantities (scattering matrix, spectroscopic factors,...)

R-matrix in the Lane-Thomas formulation. One level approximation

$$g_\ell(E) = \frac{1}{\pi} \frac{d\delta_\ell(E)}{dE}$$

This relation connects the Wigner cusp phenomenon with the appearance of threshold resonances and anti-bound (or virtual) states.

neutral particles



charged particles

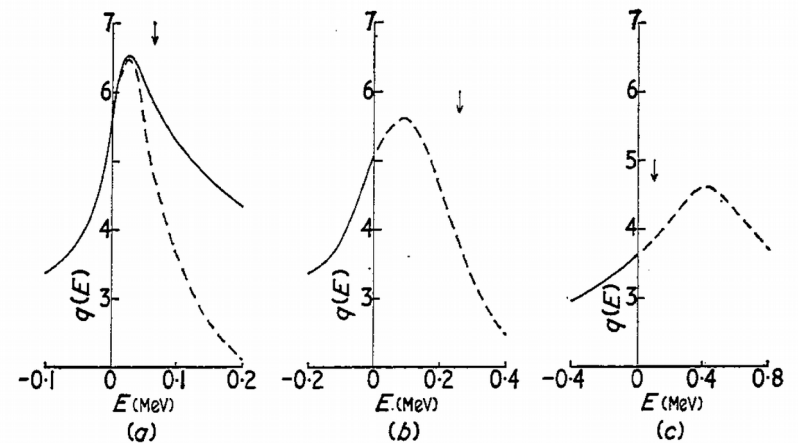
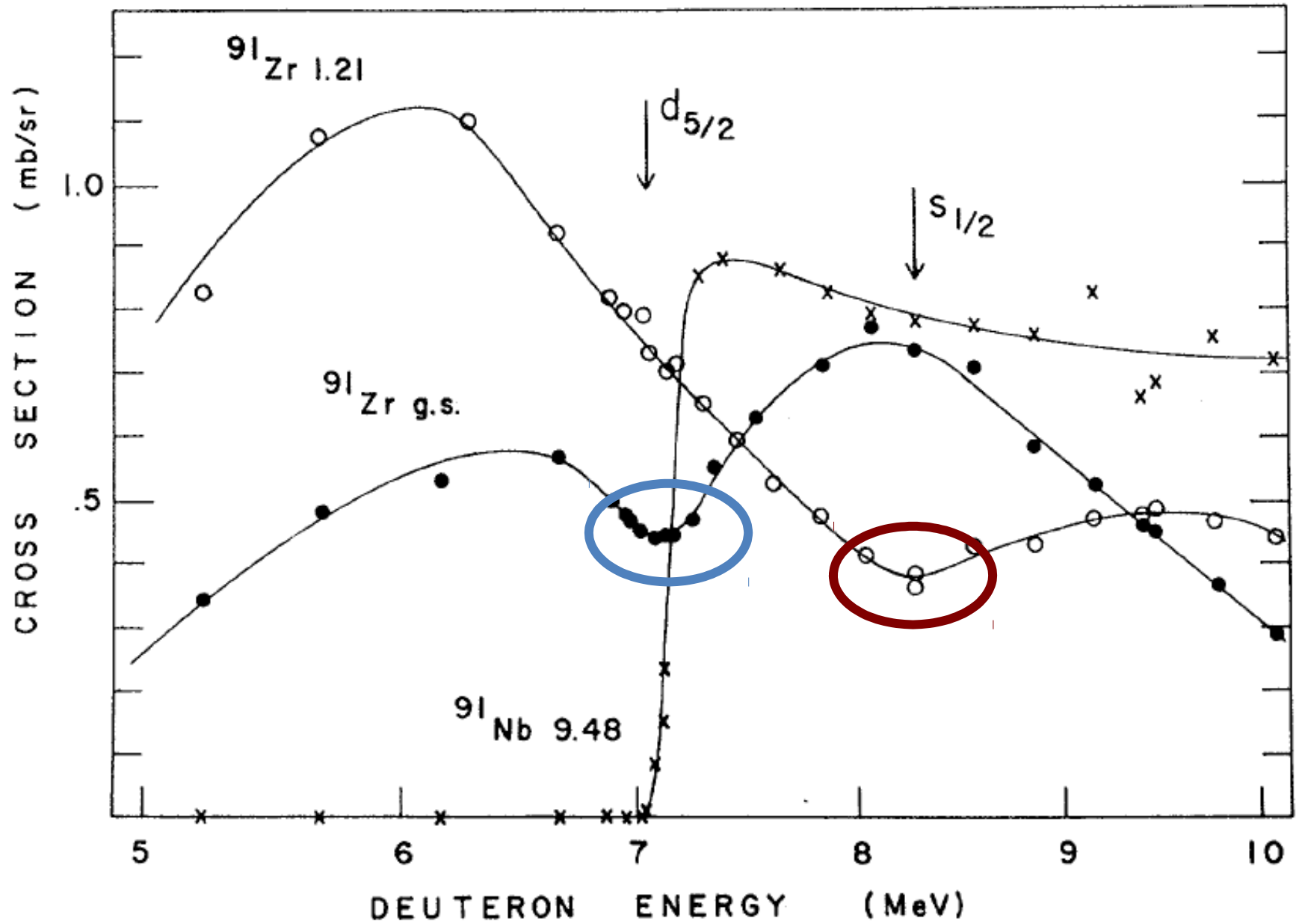
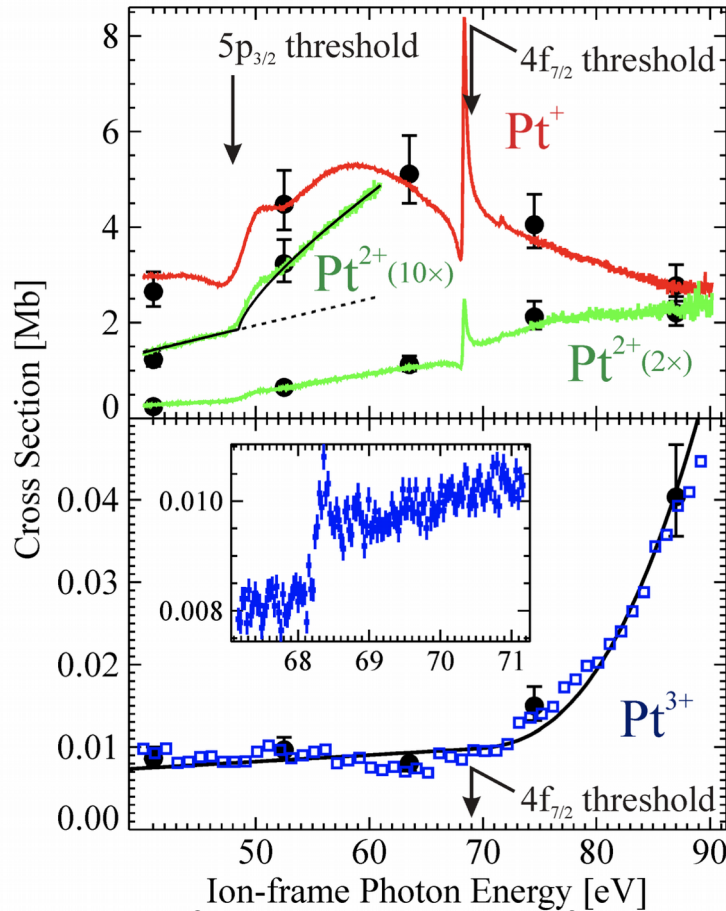


Figure 2. Enhancement factors for channels (a) ${}^3\text{H} + d$, (b) ${}^3\text{He} + d$, (c) ${}^4\text{He} + {}^4\text{He}$, all with $l = 0$ and with values of a_c and $\gamma_{\lambda c}^2$ given in the text. Full curves give values of $q(E)$, broken curves values of $q_1(E)$. Arrows indicate energies of observed levels of ${}^5\text{He}$, ${}^5\text{Li}$ and ${}^8\text{Be}$.

Large enhancement factor for the probability of finding the eigenenergy around the threshold



Threshold effects in atoms and molecules



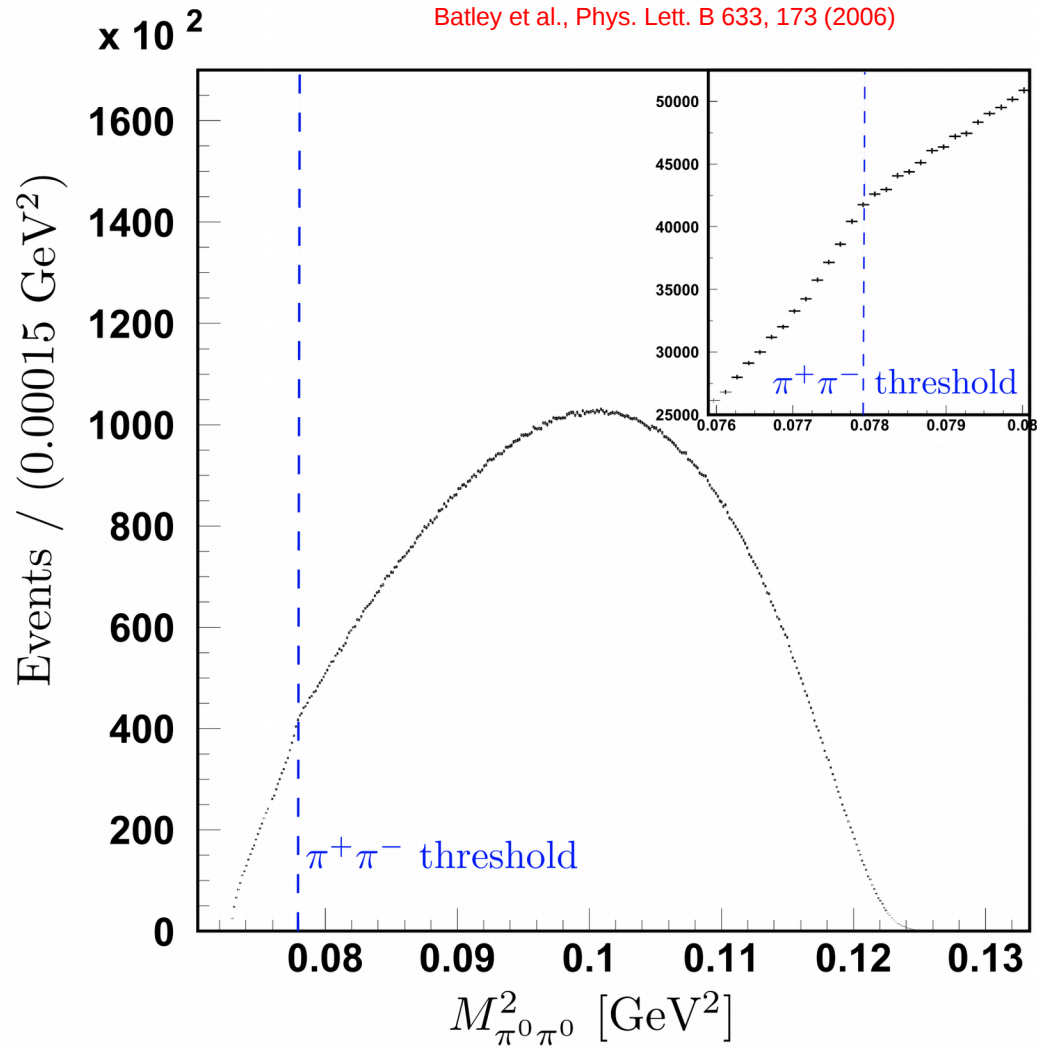
Inner-shell photodetachment of transition metal negative ions

Bilodeau et al.

J. Phys. Conf. Ser. 194 (2009) 022095

Fig. 1. Pt⁺, Pt²⁺ (upper panel), and Pt³⁺ (lower panel) production following photoabsorption in Pt, scaled to measured absolute cross sections (circles). The solid curve in the lower panel is a d-wave law. Inset: Higher statistics scan of the region near the 4f threshold in Pt³⁺.

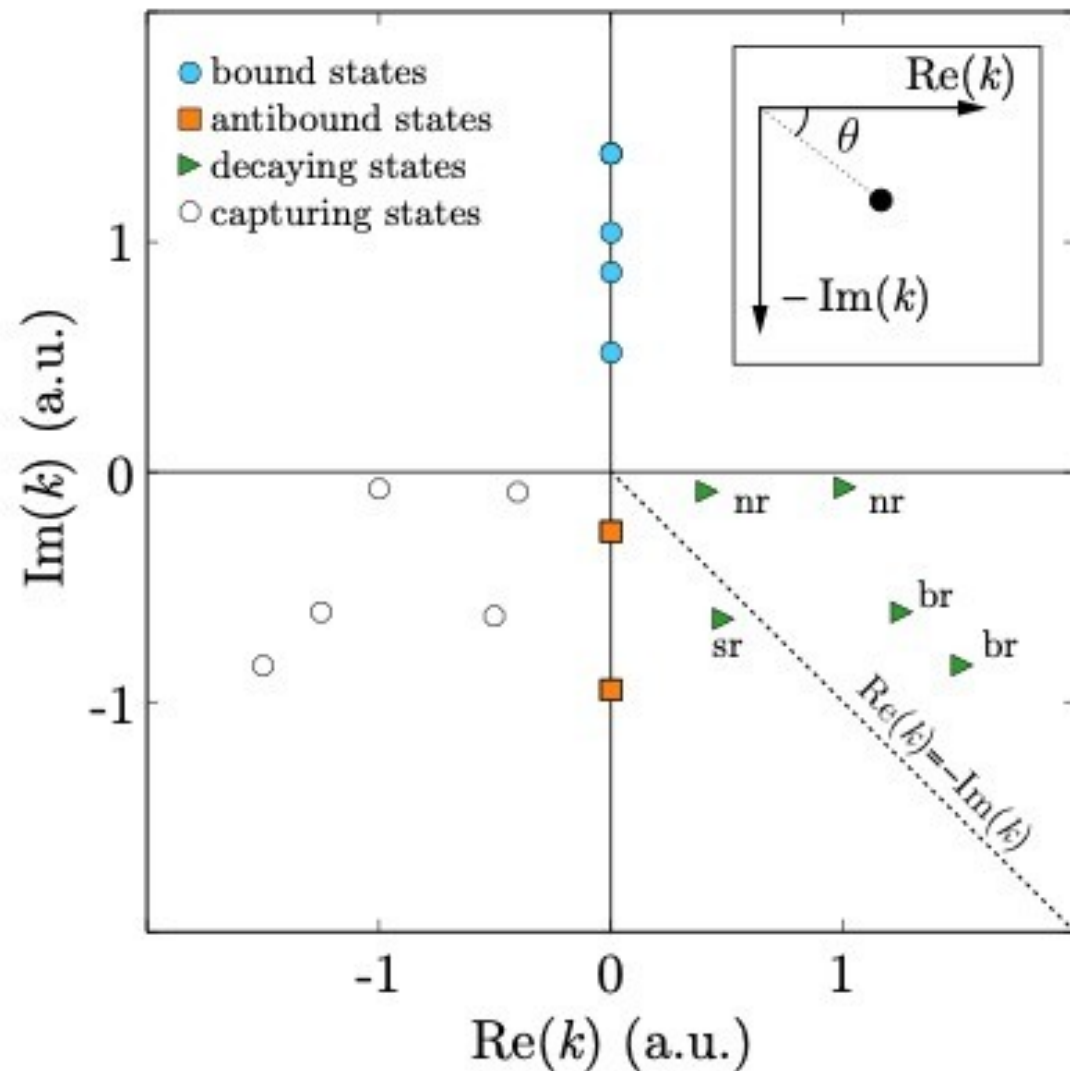
Threshold effects in hadrons and hadronic molecules



$$K^+ \rightarrow \pi^+ \pi^0 \pi^0$$

anomaly in the $\pi^0\pi^0$ invariant mass

Resonant states in the complex-k plane



- Resonant states with $\text{Re}(E) > 0$ and small Γ can be associated with narrow resonances (nr).
- For antibound (virtual) states $\text{Re}(E) < 0$ and $\Gamma = 0$.
- For subthreshold resonant states $\text{Re}(E) < 0$ and $\Gamma > 0$.
- The antibound and subthreshold resonant states lie on the second Riemann energy sheet.
- Low-momentum antibound and threshold resonant states result in the low-energy cross-section enhancement. These poles should be viewed as *scattering features* rather than physical states of the system.

Resonant states of the NN system

$T=0$

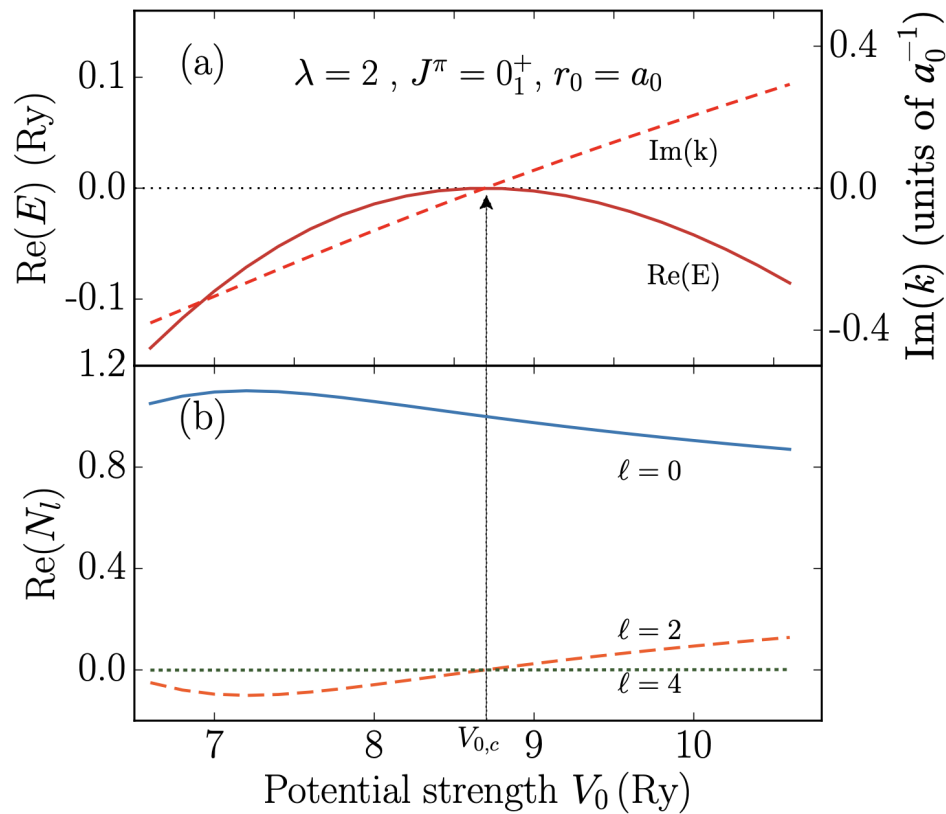
- np : bound state (deuteron), $k=+i0.2315 \text{ fm}^{-1}$

$T=1$

- np : virtual state, $k=-i0.044 \text{ fm}^{-1}$
- nn : virtual state, $k=-i0.0559(33) \text{ fm}^{-1}$ [Babanko and Petrov, Phys. At. Nucl. 76, 684 (2013)]
- pp : threshold resonant state, $k=(0.0647-i0.0870) \text{ fm}^{-1}$ [Kok, Phys. Rev. Lett. 45, 427 (1980)]

For the $4n$ system, see Deltuva and Lazauskas Phys. Rev. C 100, 044002 (2019)

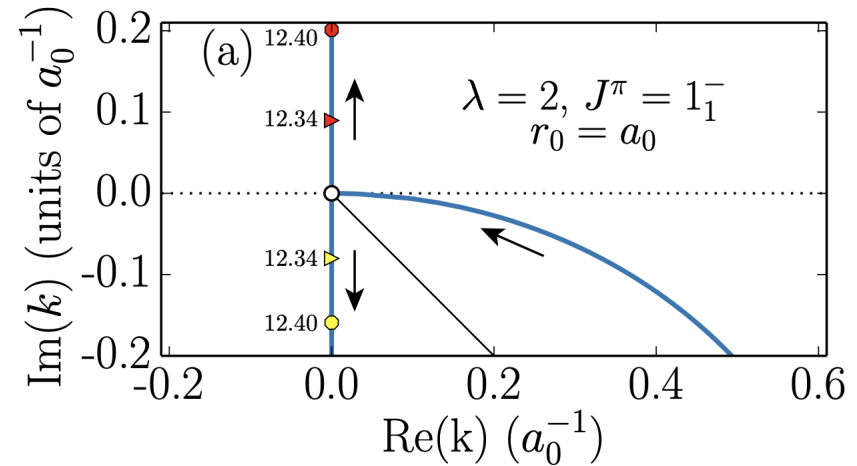
$\ell = 0$ component present



Bound state becomes virtual state
after crossing the threshold

$\ell > 0$

Generic behavior: W. Domcke, J. Phys. B 14, 4889 (1981)

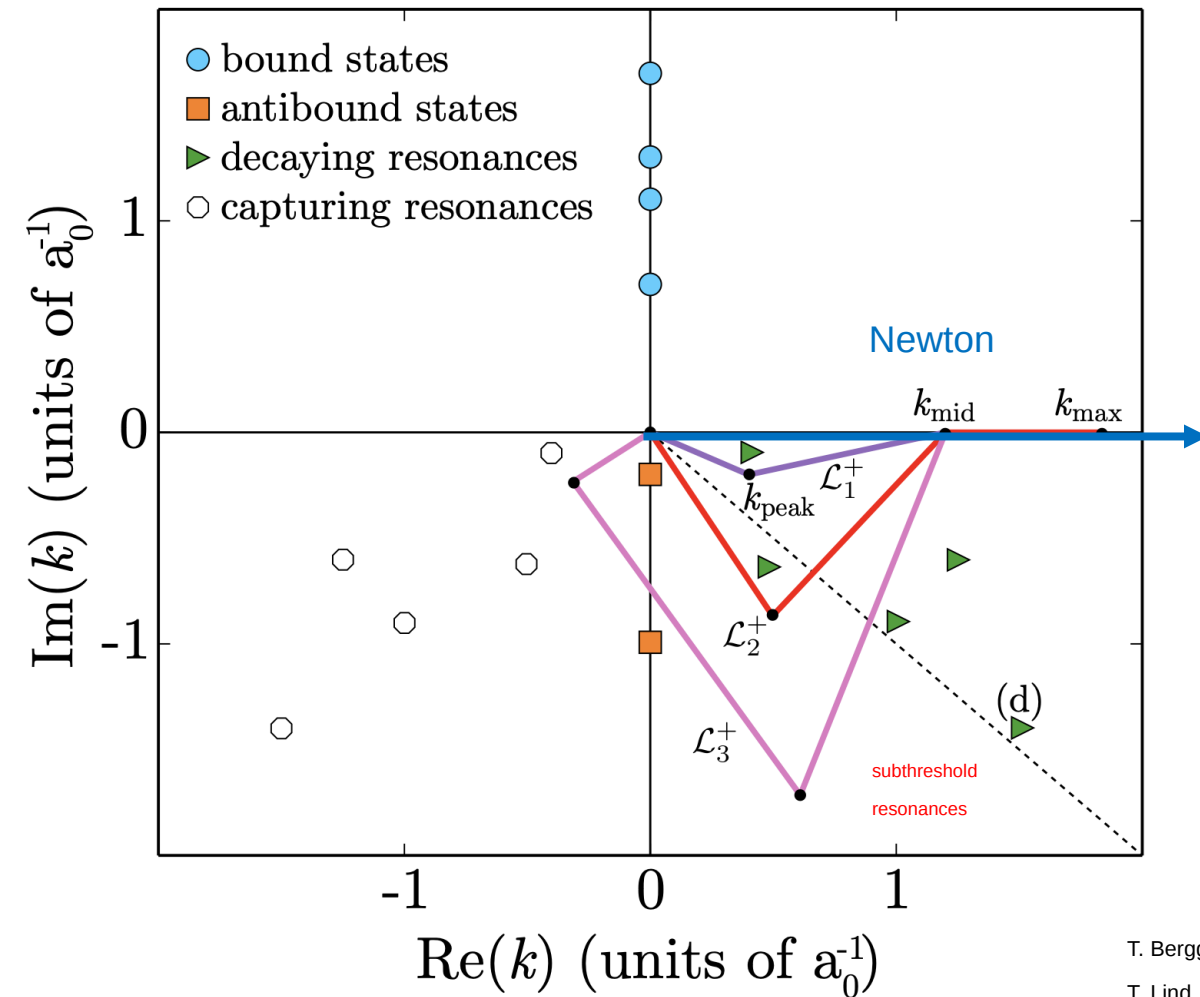


X. Mao et al., Phys. Rev. A 98, 062515 (2018)

Bound state and virtual state

coalesce at the threshold forming an exceptional point (a double pole).
As the potential strength decreases, two Gamow resonant states are
born: one decaying and one capturing.

Complex-energy picture: threshold Gamow poles



$$E = \frac{\hbar^2 k^2}{2m}$$

$$f(k) = f(-k^*)$$

(time reversal property)

For subthreshold resonances

$\text{Re}(E) < 0$ and $\Gamma > 0$

T. Berggren, Nucl. Phys. A109, 265 (1968)

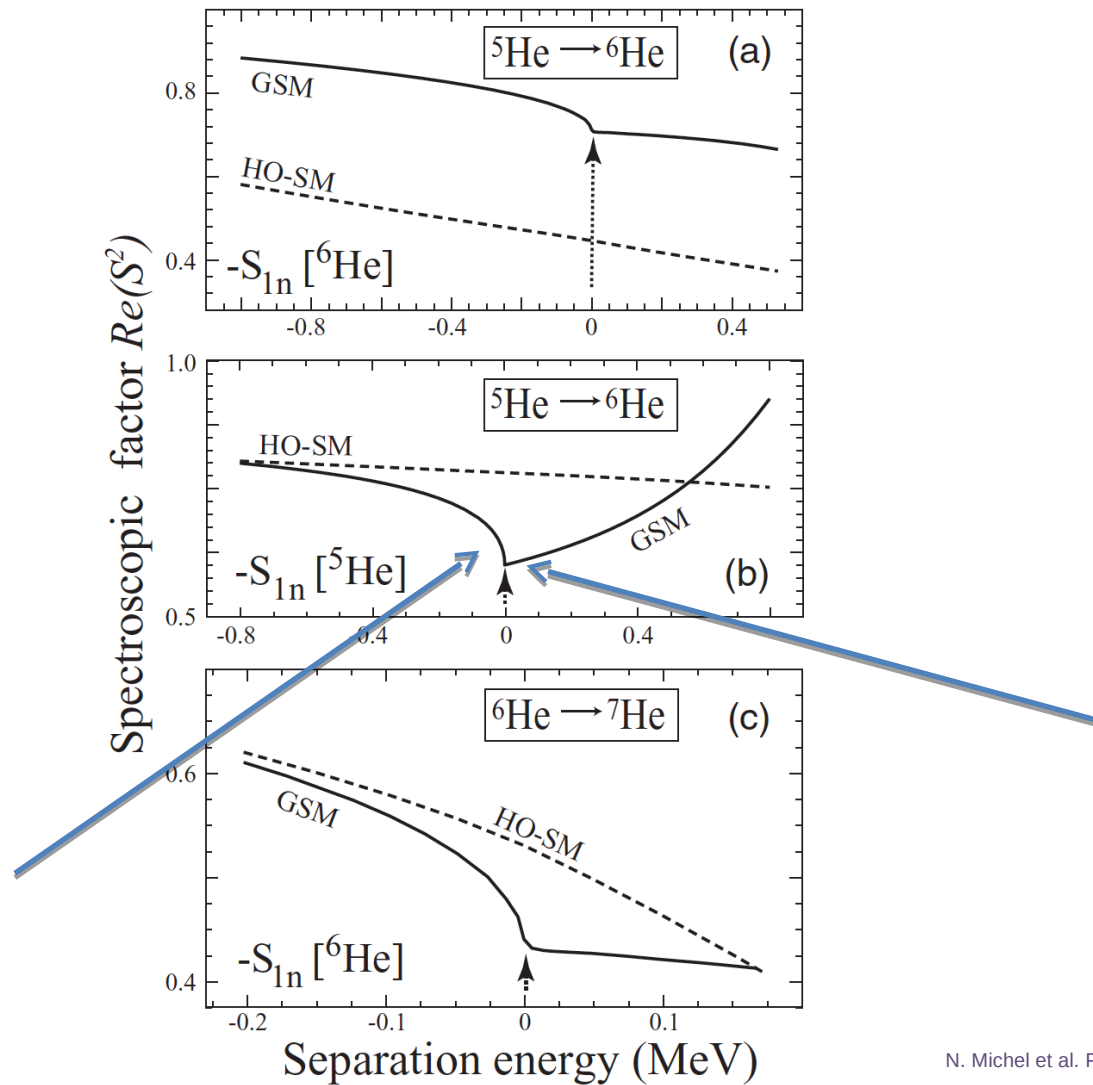
T. Lind, Phys. Rev. C47, 1903 (1993)

$$\sum_{n \in (b,d)} |u_n\rangle \langle u_n| + \int_{L^+} |u(k)\rangle \langle u(k)| dk = 1.$$

One-body basis

$$S_{\ell j}^2 = \sum A_{\ell j}^2(k_p) \quad A_{\ell j}(k_p) = \langle \Psi_A || a_{\ell j}^+(k_p) || \Psi_{A-1} \rangle / \sqrt{2J_A + 1}$$

independent on the choice of the single-particle basis



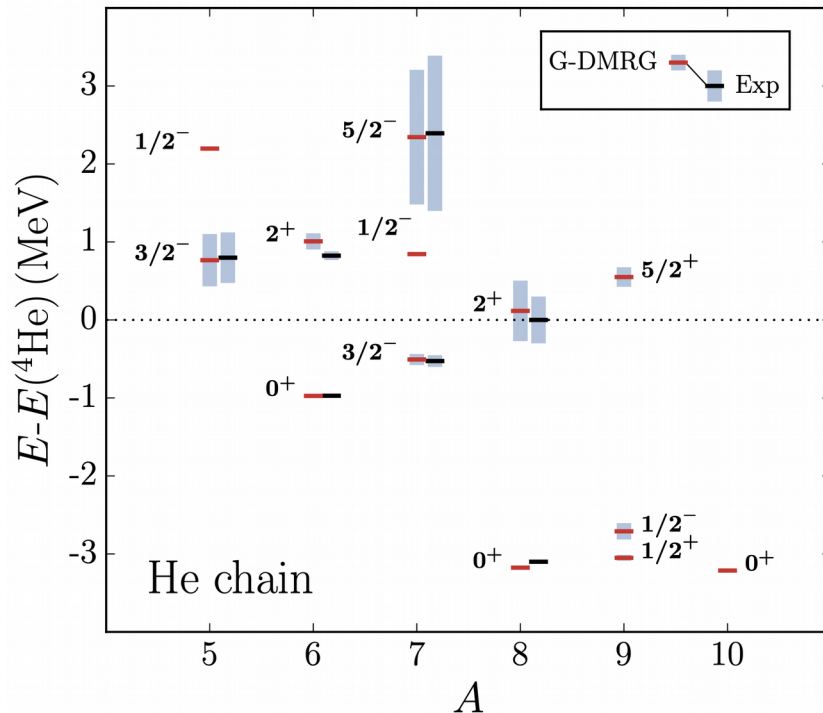
N. Michel et al. PRC 75, 031301(R) (2007)

Virtual (antibound), subthreshold resonant states in nuclei

(scattering features in the low-energy continuum)

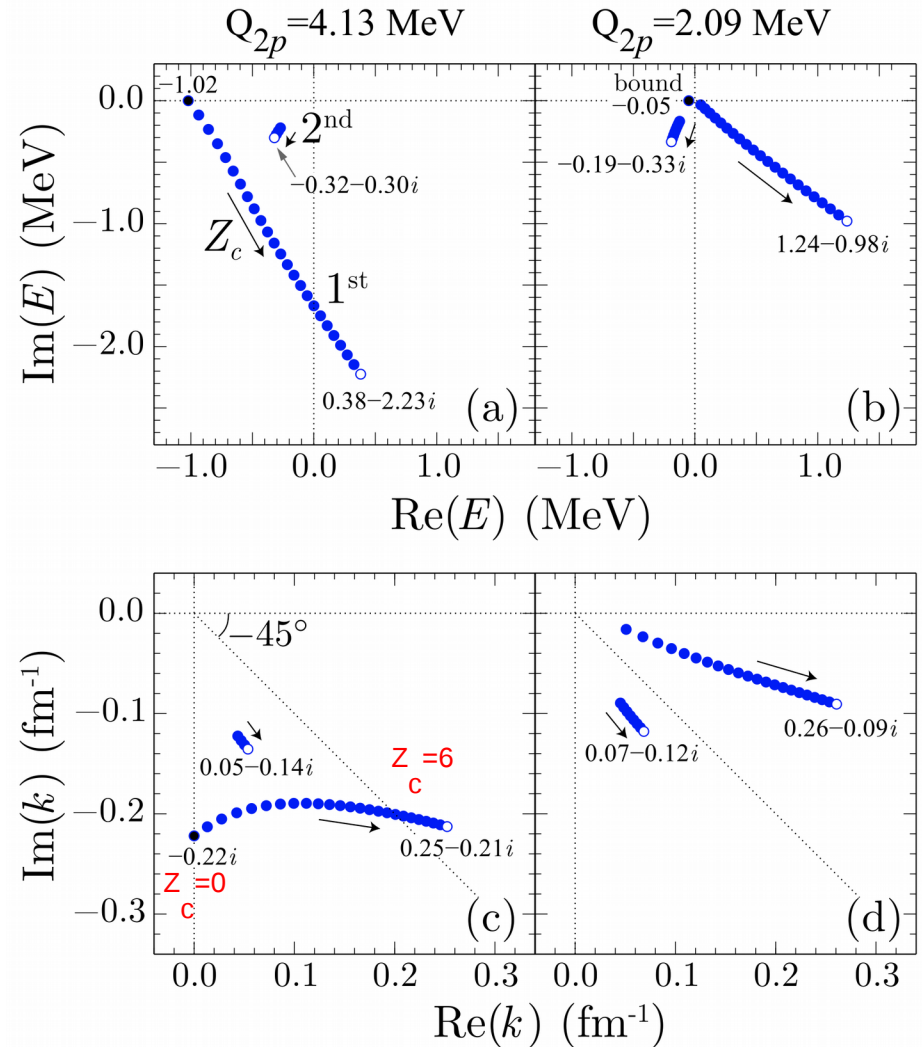
- The broad threshold resonant state in ^{10}N is an isobaric analog of the virtual state in ^{10}Li . These states impact the g.s. structures of ^{11}O and ^{11}Li .
- The $\frac{1}{2}^+$ g.s. of ^9He is a virtual state.

K. Fossez et al., Phys. Rev. C 98, 061302(R) (2018)



S.M. Wang et al., Phys. Rev. C 99, 054302 (2019)

$^9\text{Li}+n \rightarrow ^9\text{C}+p$



Time-dependent framework

$$\Psi(t) = \int \sum_i a_i(E) e^{-i \frac{E}{\hbar} t} |E, c_i\rangle dE$$

asymptotic

configuration

survival

amplitude

$$\mathcal{A}(t) = \langle \Psi(0) | \Psi(t) \rangle = \int_0^{+\infty} \rho(E) e^{-i \frac{E}{\hbar} t} dE$$

spectral function

$$\rho(E) = |\langle E | \Psi \rangle|^2$$

D. F. Ramírez Jiménez and N. G. Kelkar, Different manifestations of S-matrix poles, Ann. Phys. 396, 18 (2018).

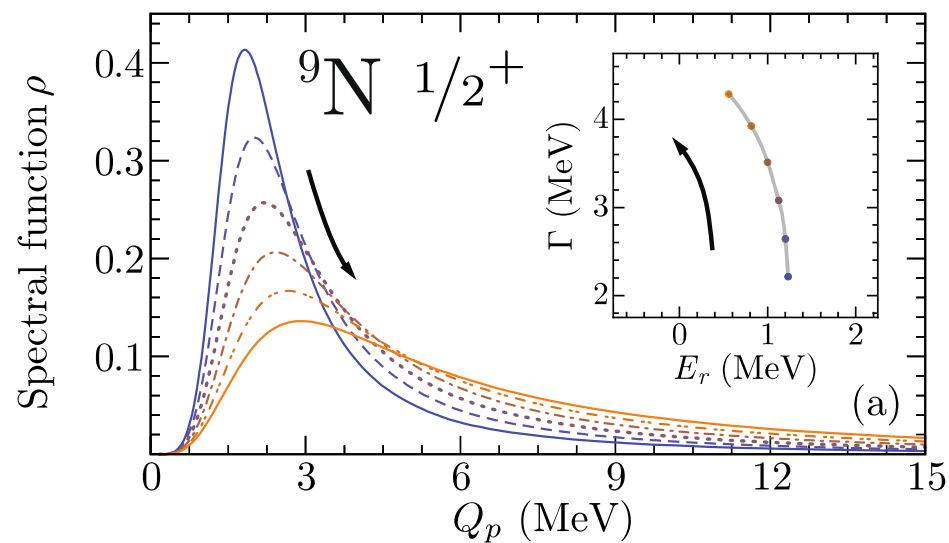
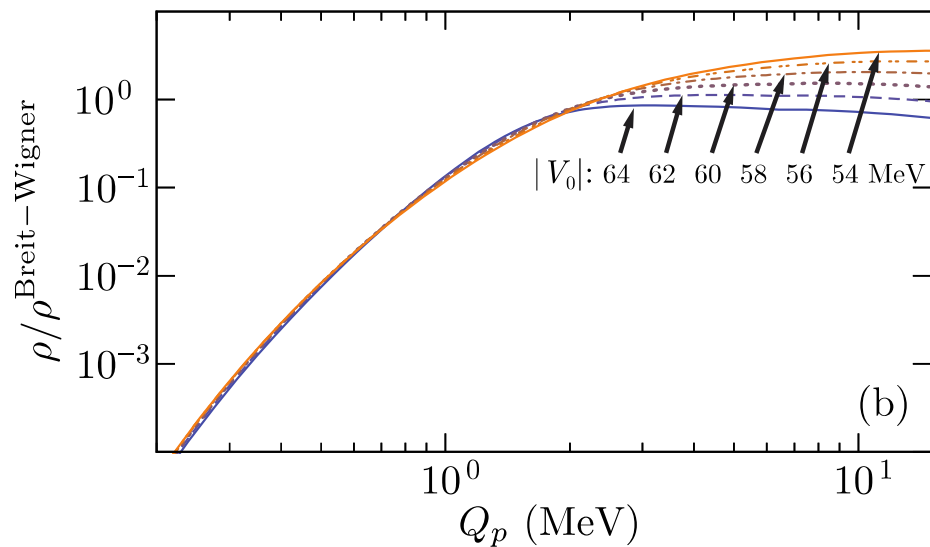
By making use of the Mittag-Leffler expansion, in the absence of Coulomb interaction, the near-threshold spectral function becomes

$$\rho(E) = \text{Im} \sum_n \frac{1}{\tilde{E}_n^{\ell+1/2}} \frac{E^{\ell+1/2}}{\tilde{E}_n - E}$$

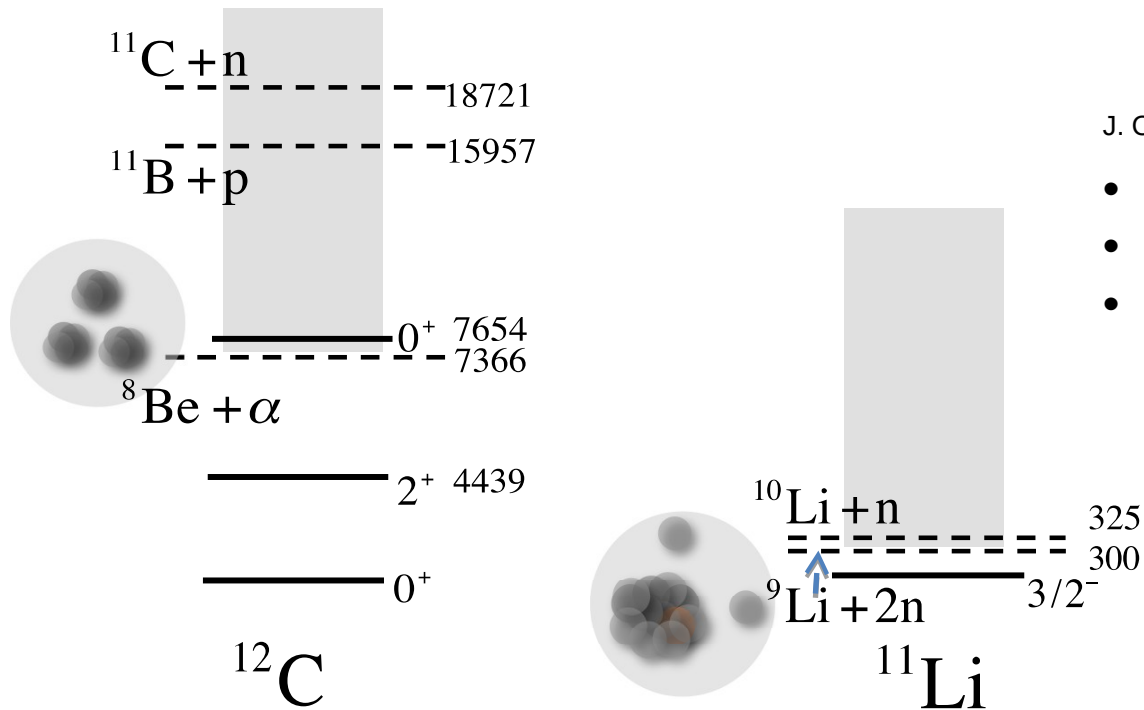
$$\tilde{E} = E_r - i\Gamma/2$$

For a narrow, isolated resonance, the spectral function assumes the Breit-Wigner shape.

$$E_r < \Gamma/2$$



The origin of nuclear clustering: collective effect due to the continuum coupling



J. Okołowicz, M. Płoszajczak, WN,

- Fortschr. Phys. 61, 66 (2013)
- Prog. Theor. Phys. Supplement **196**, 230 (2012)
- Acta. Phys. Pol. 45, 331 (2014)

The clustering is the generic near-threshold phenomenon in open quantum system which does not originate from any particular property of nuclear forces or any dynamical symmetry of the nuclear many-body problem. See Marek's talk.

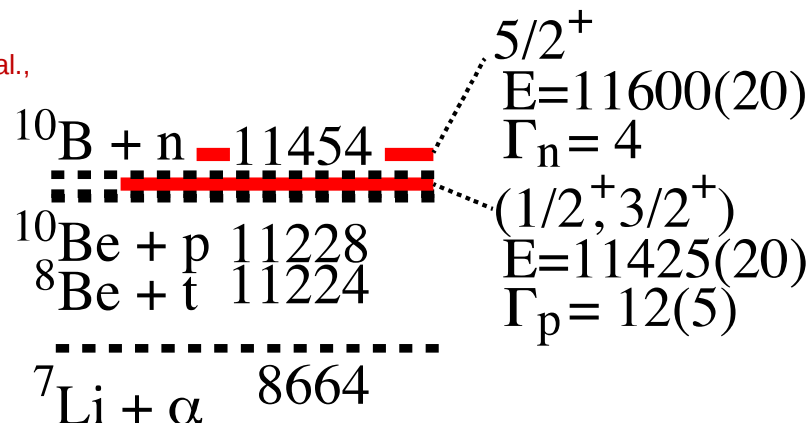
Many experimental examples!

Near-Threshold Proton-Emitting Resonance in ^{11}B

β -p and β - α decay of the ^{11}Be neutron halo ground state

- Evidence of a near-threshold resonance in ^{11}B , Ayyad et al., PRL 123, 082501 (2019); PRL 129, 012501 (2022)
- Observation of a near-threshold proton resonance in ^{11}B , Lopez-Saavedra et al., PRL 129, 012502 (2022)

- J. Okołowicz et al. Phys. Rev. Lett. 124, 042502 (2020)
- A. Volya, Europhys. Lett. 130, 12001 (2020)
- W. Elkamhawy et al., Phys. Lett. B 821, 136610 (2021)
- J. Okołowicz et al., J. Phys. G 49, 10LT01 (2022)



$\approx$

$3/2^-$
 ^{11}B

- Two close lying resonances: proton and neutron threshold- aligned states.
- Very important states!

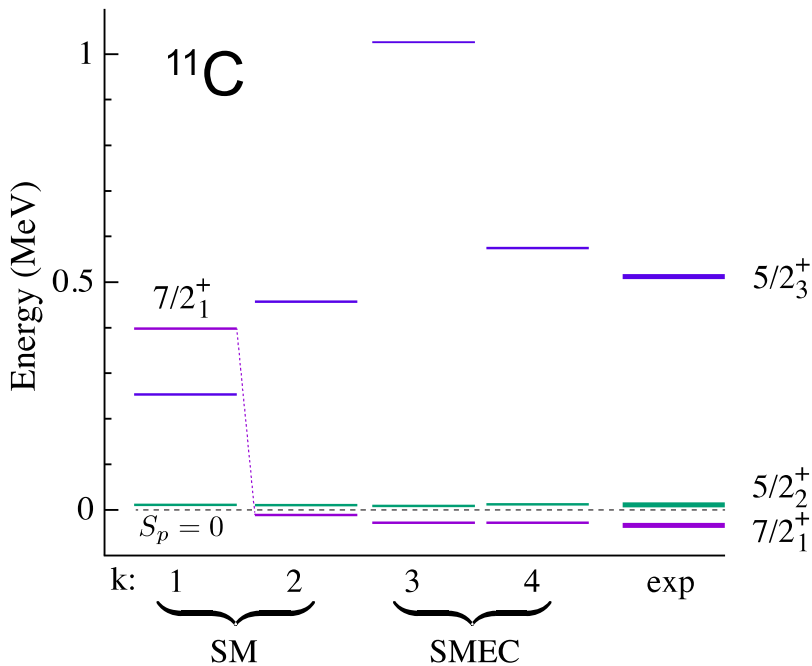


TABLE I. The proton $s_{1/2}$ and $d_{5/2}$ spectroscopic factors $\mathcal{S}_{\ell j}^{(k)}$ of the $7/2_1^+$, $5/2_2^+$, and $5/2_3^+$ resonances in the vicinity of the proton threshold in ^{11}C obtained in different calculation variants.

J^π	$\mathcal{S}_{s1/2}^{(1)}$	$\mathcal{S}_{s1/2}^{(2)}$	$\mathcal{S}_{s1/2}^{(3)}$	$\mathcal{S}_{s1/2}^{(4)}$	$\mathcal{S}_{d5/2}^{(1)}$	$\mathcal{S}_{d5/2}^{(2)}$	$\mathcal{S}_{d5/2}^{(3)}$	$\mathcal{S}_{d5/2}^{(4)}$
	SM		SMEC		SM		SMEC	
$7/2_1^+$	0.15	0.09	0.12	0.10	0.36	0.37	0.34	0.38
$5/2_2^+$	0.29	0.32	0.38	0.33	0.12	0.19	0.19	0.19
$5/2_3^+$	0.08	0.06	0.02	0.04	0.01	0.01	0.04	~ 0

- Boron-proton fusion reactor utilizes the aneutronic fusion reaction $^{11}\text{B}(\text{p},2\alpha)^4\text{He}$
- The natural boron fuel contains ^{10}B whose abundance amounts to 19%, and which produces the long-lived radioactive ^7Be through a reaction $^{10}\text{B}(\text{p},\alpha)^7\text{Be}$ that is poisoning the aneutronic energy source
- The low-energy cross section of $^{10}\text{B}(\text{p},\alpha)^7\text{Be}$ depends on the near-threshold states in ^{11}C

Editors' Suggestion

Featured in Physics

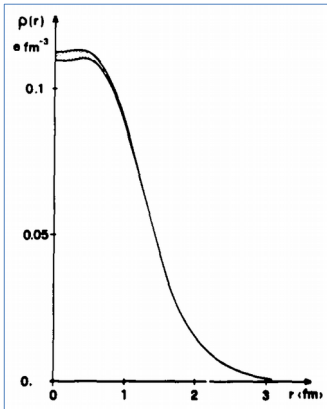
Measurement of the α -Particle Monopole Transition Form Factor |Challenges Theory:
A Low-Energy Puzzle for Nuclear Forces?

E (level) (keV)	XREF	J ^{π} (level)	T _{1/2} (level)
0.0		0+	STABLE
20210	E GHI K	0+	0.50 MeV % p = 100
21010	H	0-	0.84 MeV % n = 24 % p = 76

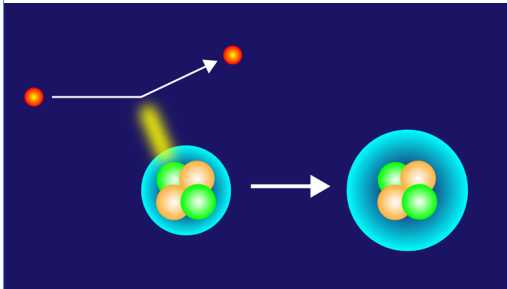
TABLE I. FWHM Γ_0 for the investigated resonance parametrizations σ_1 Eq. (1) and σ_2 Eq. (3) and the two background parametrizations BG1 and BG2.

	BG1 (keV)	BG2 (keV)
σ_1	268 ± 43	285 ± 33
σ_2	262 ± 47	288 ± 39

How reliable is this extraction?



S =19.8 MeV
p



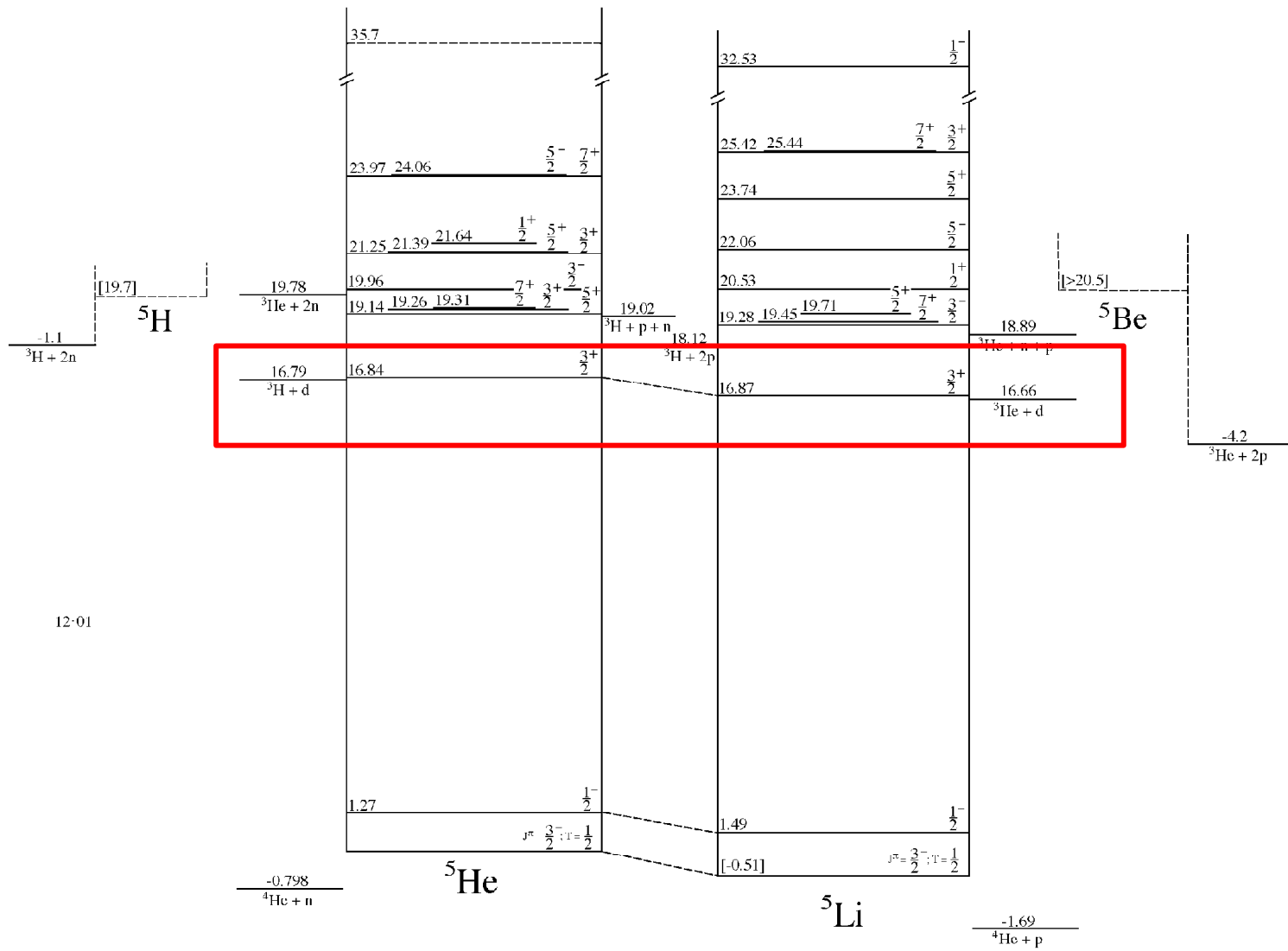
See Bacca et al. Phys. Rev. C 91, 024303 (2015) for a breathing mode interpretation

Kirscher and Gri  hammer, Eur. Phys. J. A 54, 137 (2018), with pionless EFT predicted a shallow, unstable state at an energy 0.38 ± 0.25 MeV above the triton-proton threshold.

Is the first excited state of 4He state NOT a breathing mode but a $t+p$ cluster state?

YES! Coupled-channel no-core Gamow shell model N. Michel, WN, M. P  oszajczak, in preparation

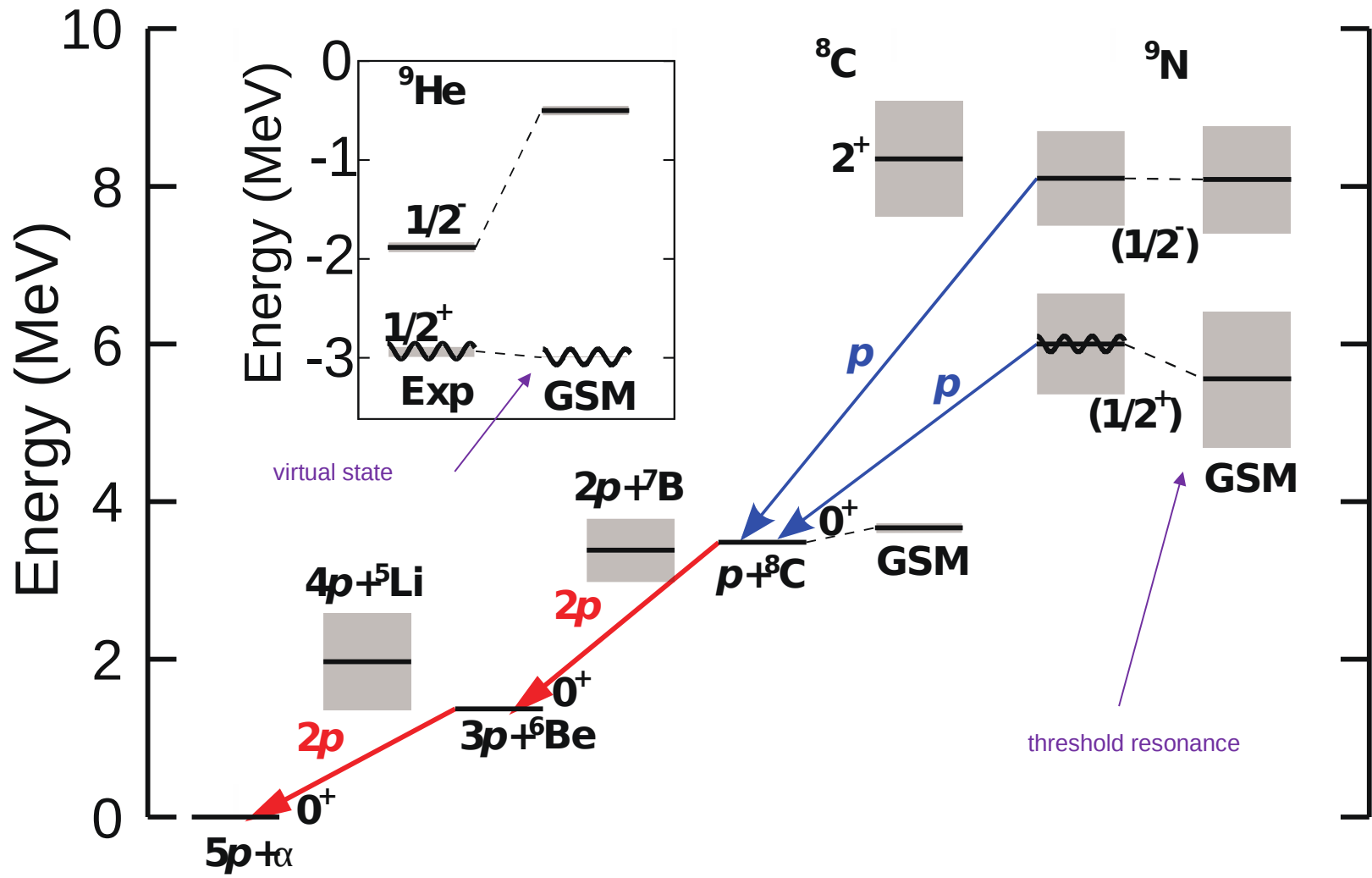
A=5 systems and fusion



D+T and D+3He fusion!

Exciting news: structure of 8C and 9N

R.J. Charity, J. Wylie, S.M. Wang et al., to be published



Astrophysical relevance

Near-threshold cluster resonances are important astrophysically

Many examples, see, e.g., Wiescher et al. Eur. Phys. J. A 57, 24 (2021)

Near-threshold α -cluster states in $^{10,11}\text{B}$, $^{14,15}\text{N}$, which enhance the reaction rates for α -capture reactions on lithium and boron isotopes

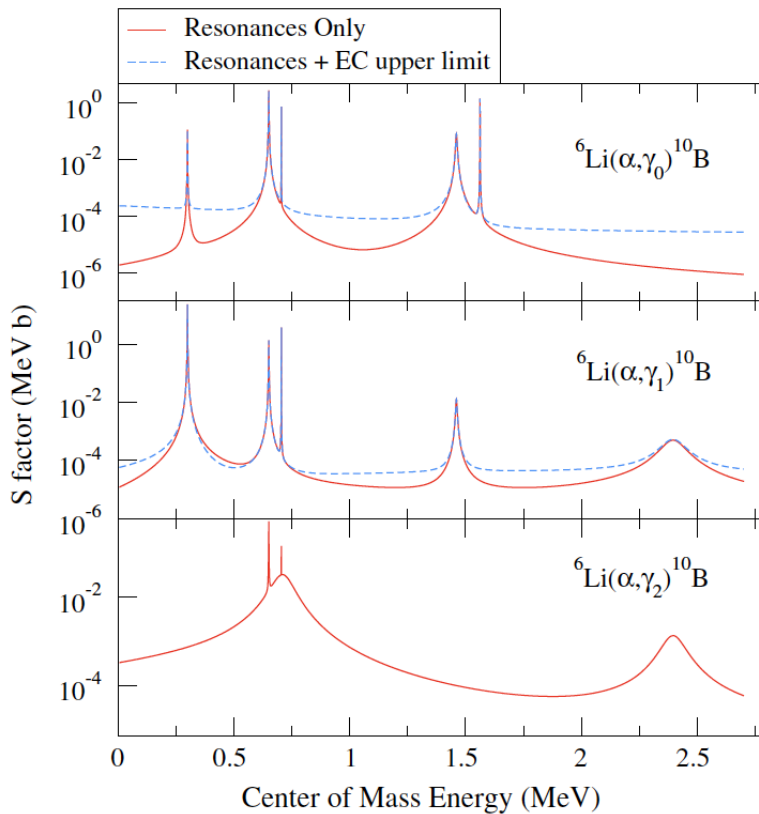


Fig. 1 Estimate of the $^6\text{Li}(\alpha, \gamma)^{10}\text{B}$ S-factor based on level parameters from the compilation [60] (red solid line). An estimate that includes an upper limit for the direct capture is also shown (blue dashed line)

deBoer et al. Phys. Rev. C 101, 045802, (2020)

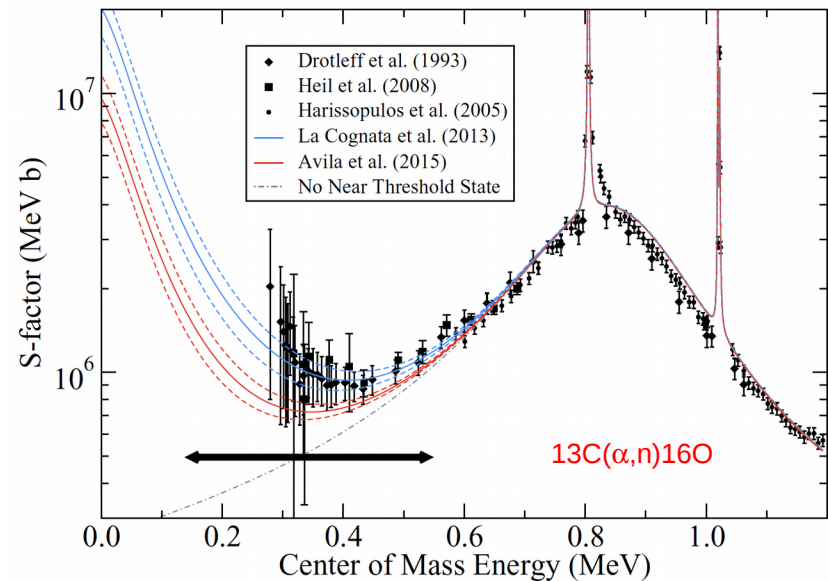


FIG. 8. S factors (solid lines) and their corresponding range of uncertainty (bounded by the dashed lines) for the $^{13}\text{C}(\alpha, n)^{16}\text{O}$ reaction using the $\tilde{\chi}^2$ of La Cognata *et al.* [31] (blue) and Avila *et al.* [33] (red). The S factor with no near-threshold state contribution is shown by the gray dashed-dotted line. The black arrow indicates the energy range of astrophysical interest.

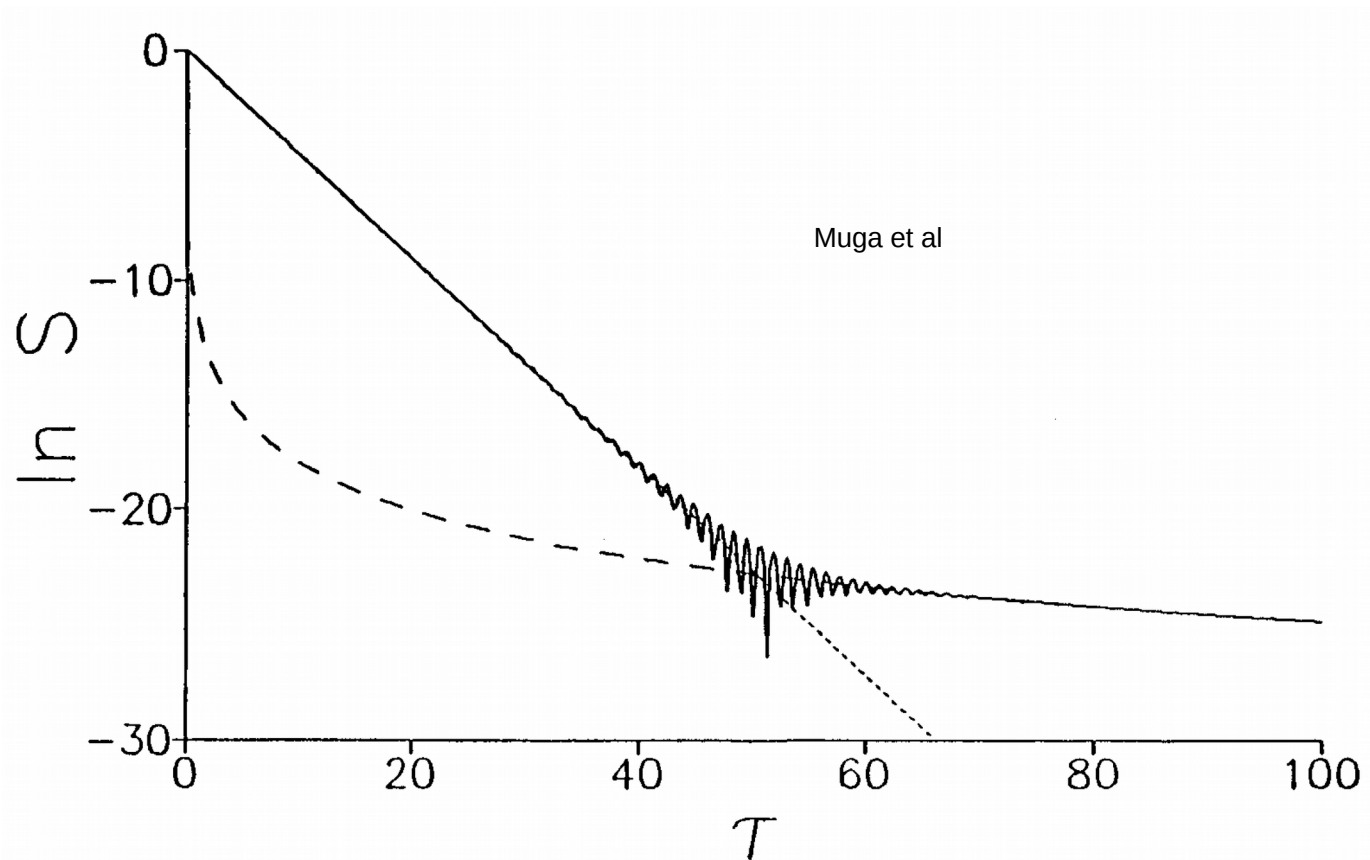
The Decay law at long times

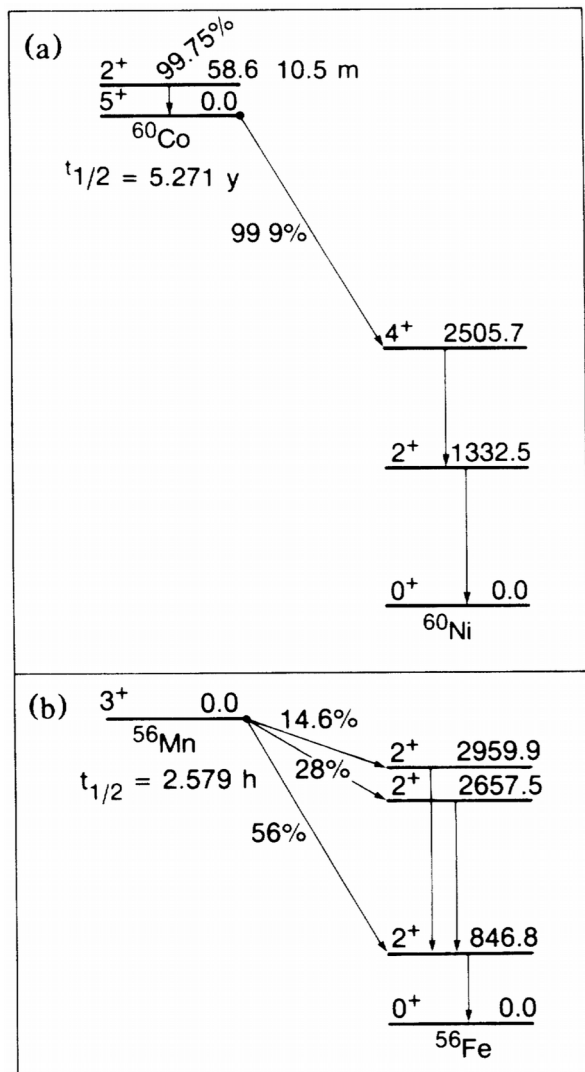
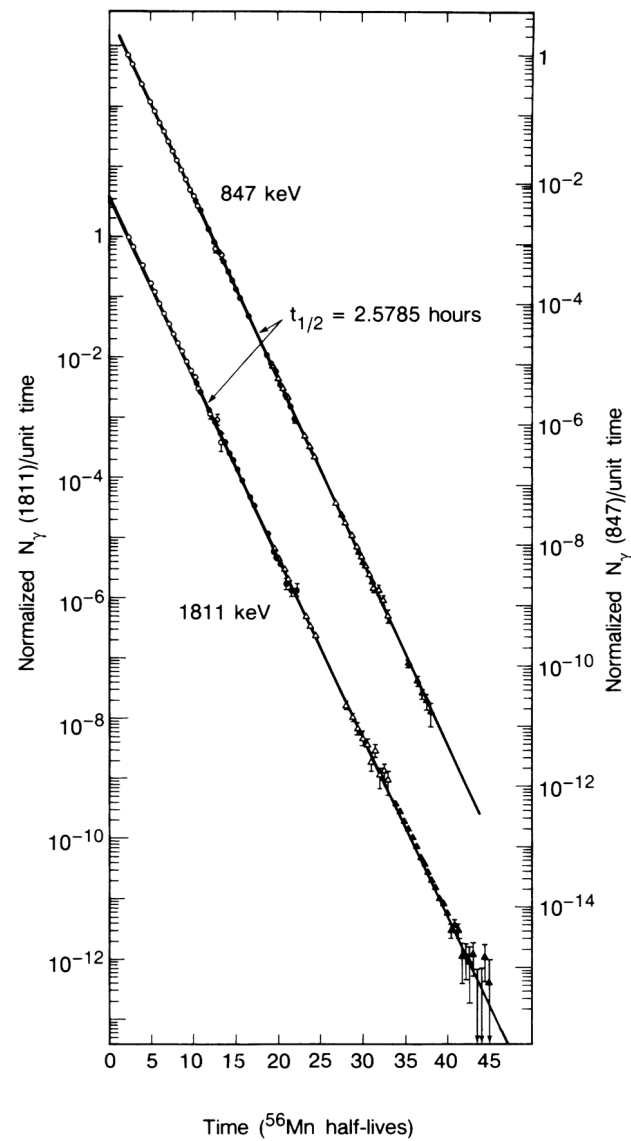
S.M. Wang, WN, A. Volya, Y.G. Ma, in preparation, arXiv:2211.11619

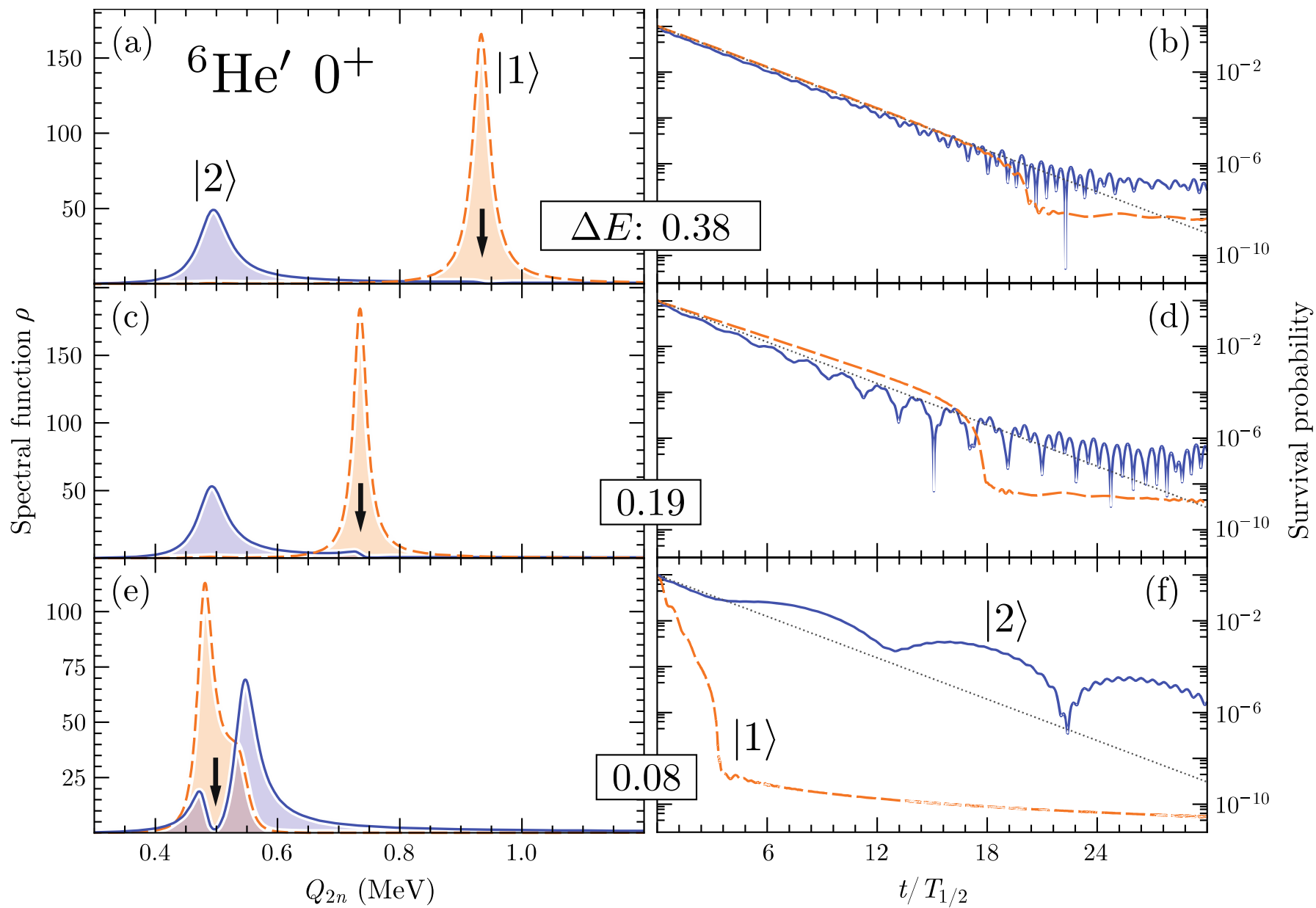
Fonda et al., Rep. Prog. Phys. 41 587 (1978)

Muga et al., Ann. Phys. 252, 336 356 (1996)

Peshkin et al., Europhys. Lett. 107, 40001 (2014)







Summary

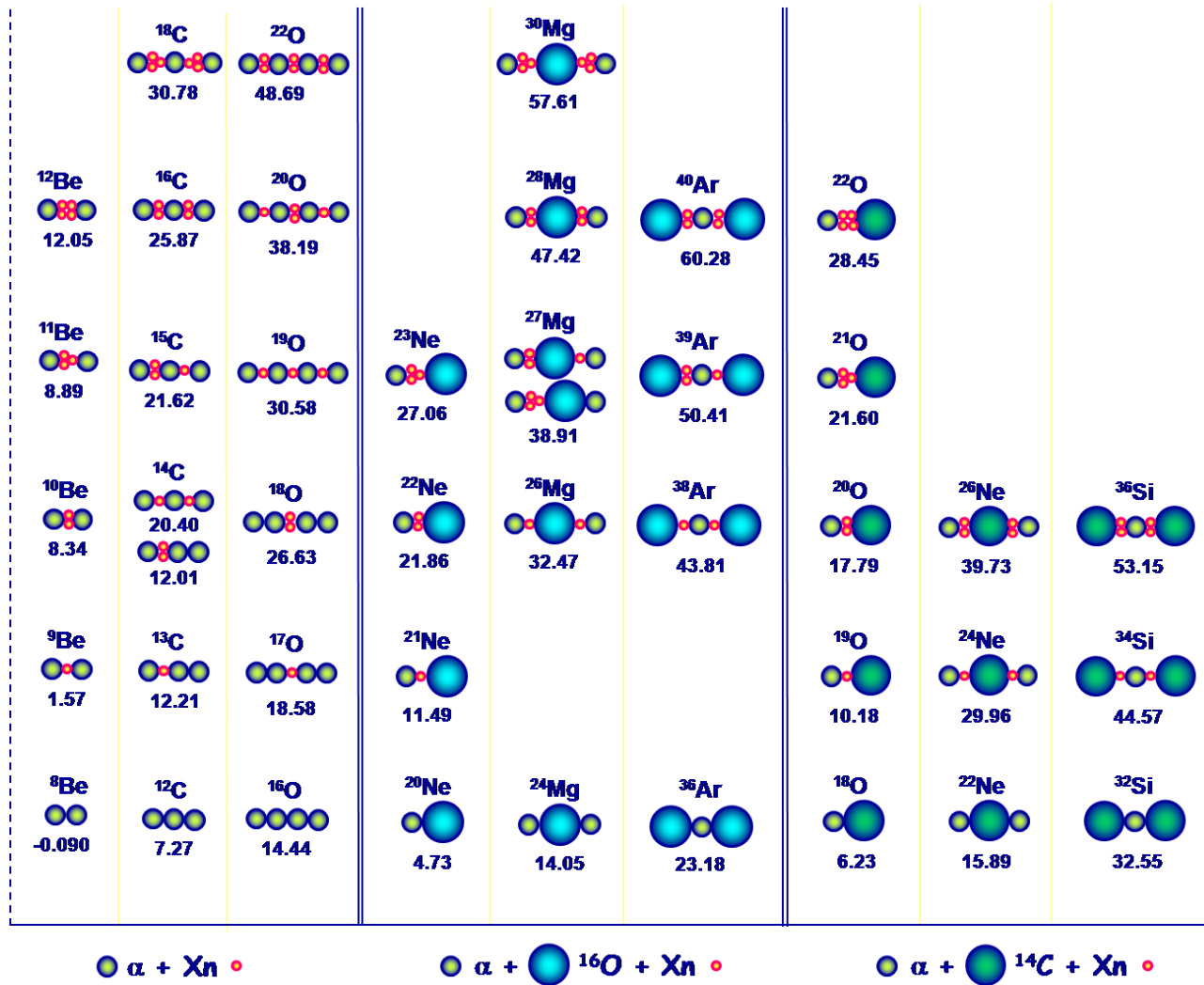
- Proximity of the threshold (branching point) induces the collective mixing of shell-model eigenstates resulting in a single aligned eigenstate of the OQS Hamiltonian (→ Chameleon resonance).
- The presence of charged-cluster states near the respective charged-cluster emission threshold (→ Ikeda Diagram) is a signature of a profound change of the near-threshold SM wave function and the direct manifestation of continuum-coupling correlations.
- A consistent description of the continuum coupling within the GSM and SMEC is currently being developed.
- Chameleon resonances are important for energy applications
- Chameleon resonances are important astrophysically.

The chameleon resonances in atomic nuclei are conveniently located but not unexpected!



BACKUP

Extended Ikeda diagram



W. von Oertzen et al., Eur. Phys. J. A 46, 345 (2010)

SMEC description

The scattering environment is provided by one-nucleon decay channels. The Hilbert space is divided into two orthogonal subspaces $\mathcal{Q}_0$ and $\mathcal{Q}_1$ containing 0 and 1 (n) particles in the scattering continuum, respectively.

Energy-dependent effective Hamiltonian (E is the scattering energy):

$$\mathcal{H}(E) = H_{\mathcal{Q}_0 \mathcal{Q}_0} + W_{\mathcal{Q}_0 \mathcal{Q}_0}(E)$$

standard SM (CQS) Hamiltonian

energy-dependent continuum coupling term

$$W_{\mathcal{Q}_0 \mathcal{Q}_0}(E) = H_{\mathcal{Q}_0 \mathcal{Q}_1} G_{\mathcal{Q}_1}^{(+)}(E) H_{\mathcal{Q}_1 \mathcal{Q}_0}$$

one-nucleon Green's function

coupling terms between subspaces $\mathcal{Q}_0$ and $\mathcal{Q}_1$

$$W_{\mathcal{Q}_0 \mathcal{Q}_0}(E) \approx V_0^2 h(E)$$

V_0 - the continuum coupling constant

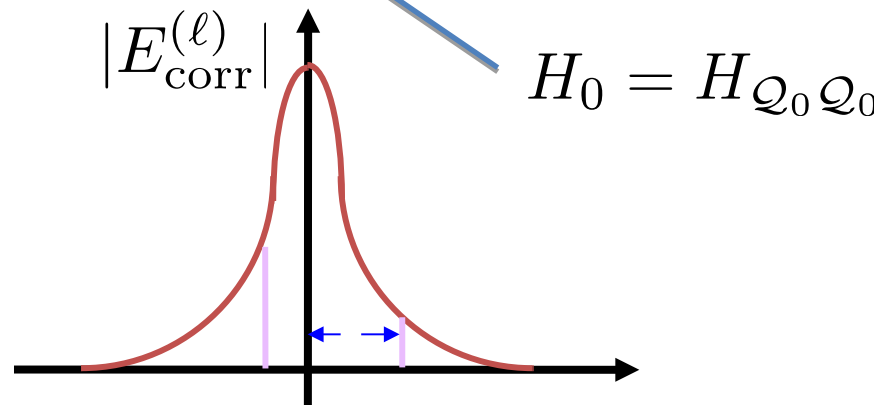
Appearance of the aligned state

$$\text{SM states } |\psi_i\rangle \rightarrow |\Phi_j\rangle = \sum_i b_{ji} |\psi_i\rangle$$

SMEC states

Continuum coupling correction to SM eigenstates

$$E_{\text{corr};i}^{(\ell)}(E) = \langle \Phi_{\bar{i}} | \mathcal{H} - H_0 | \Phi_i \rangle \simeq V_0^2 \langle \Phi_{\bar{i}} | h(E) | \Phi_i \rangle$$



Interaction through the continuum leads to the *collectivization* of SM eigenstates and formation of the *aligned* SMEC eigenstate which couples strongly to the decay channel and carries many of its characteristics.

Mixing of SM wave functions via the continuum

- The mixing of eigenfunctions (avoided crossing) is caused by a nearby exceptional point of the complex-extended Hamiltonian
- Exceptional points are generic features of open quantum systems.
- The configuration mixing of resonances is characterized by lines of coalescing eigenvalues (exceptional threads) of the complex-extended CSM Hamiltonian.



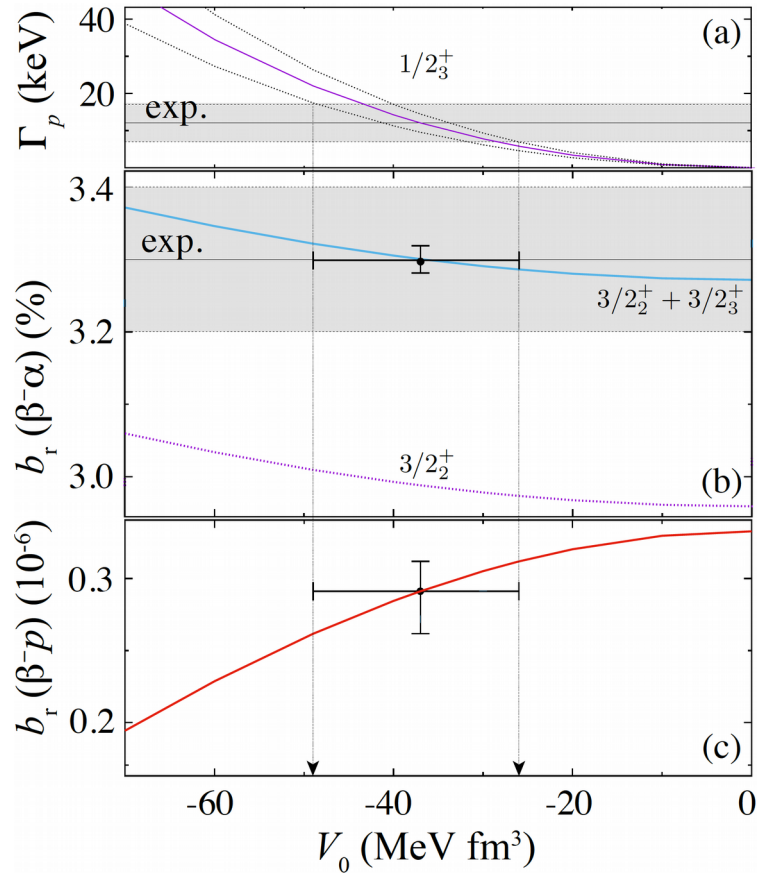
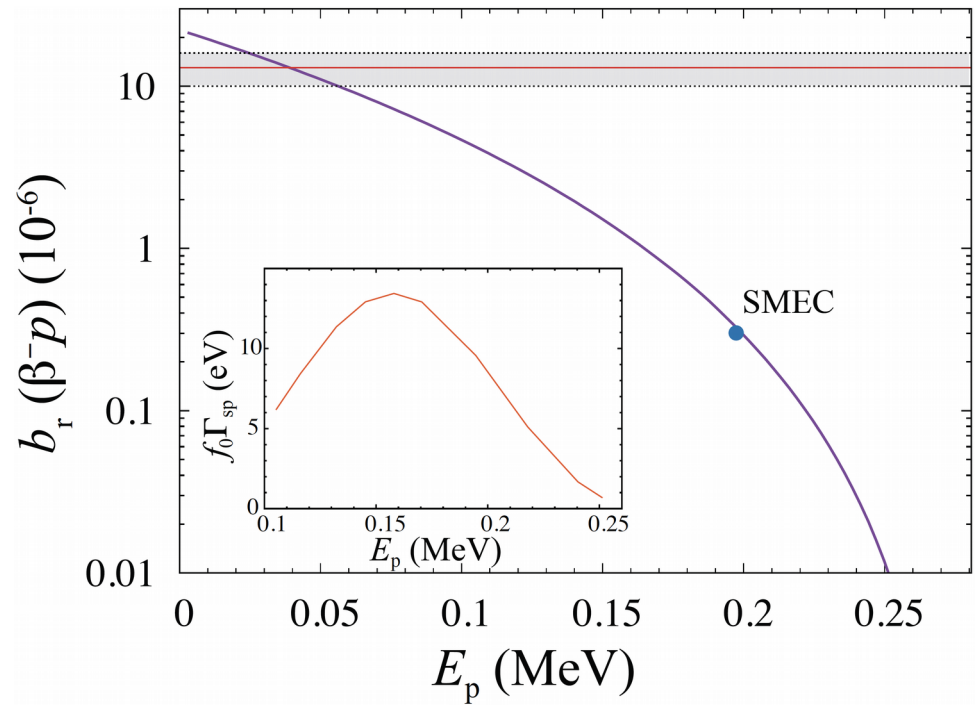
${}^8\text{Be}$

Cluster states may appear in the narrow energy window around the point of maximum continuum coupling. The continuum-coupling correlation energy and collectivity of the aligned state is reduced with increasing Coulomb barrier.

β -p and β - α decay of the ^{11}Be neutron halo ground state

J. Okołowicz et al., J. Phys. G 49, 10LT01 (2022): SMEC analysis

See also A. Volya, Europhys. Lett. 130, 12001 (2020); W. Elkamhawy et al., Phys. Lett. B 821,136610 (2021)



A tension between $b_r(\beta-\alpha)$ and $\Gamma_p(1/2_3^+)$ with $b_r(\beta-p)$.

$b_r(\beta-\alpha)$, $\Gamma_p(1/2_3^+)$, and EM transitions in ^{11}Be consistently described.