UnNuclear Physics: Conformal Symmetry in Nuclear Reactions

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References

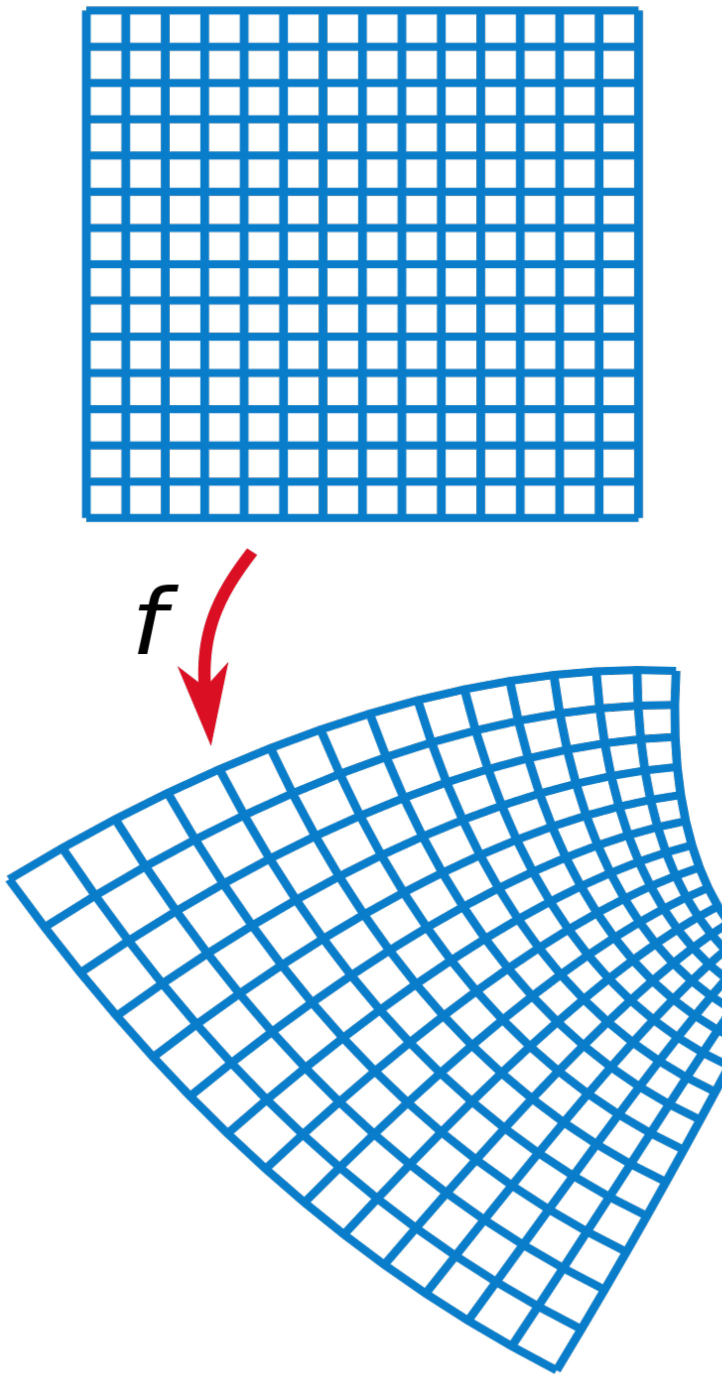
- H. Hammer and D.T. Son, arXiv:2103.12610 (PNAS 2021)
- T. Schäfer and G. Baym, arXiv:2109.06924 (PNAS 2021)

Role of symmetry in physics

- Symmetries play a very important role in physics
- Spacetime symmetry is key to understanding of elementary particles and matter
- In particle physics, Lorentz and Poincare symmetry
- Conformal symmetry are important for quantum field theory, theory of phase transitions

Conformal symmetry

- An extension of Poincaré group: conformal symmetry
- All transformations that preserve angle
- include: dilatation $x^\mu \to \lambda x^\mu$
- and 4 "proper conformal transformations"
- Field theory with this symmetry: conformal field theory
- applications in theoretical physics including phase transitions



Scale invariance in nonrelativistic physics

- Schrödinger equation of a free particle

$$i\frac{\partial\psi}{\partial t} = -\frac{1}{2m}\nabla^2\psi$$

- From a solution $\psi(t, \mathbf{x})$ one can construct a new one

$$\tilde{\psi}(t, \mathbf{x}) = \lambda^{3/2}\psi(\lambda^2 t, \lambda\mathbf{x})$$

- Valid also for N noninteracting particles interacting through a $1/r^2$ potential

- In fact, the symmetry of the free time-dependent Schrödinger equation is larger.
- If $\psi(t, \mathbf{x})$ is a solution, then

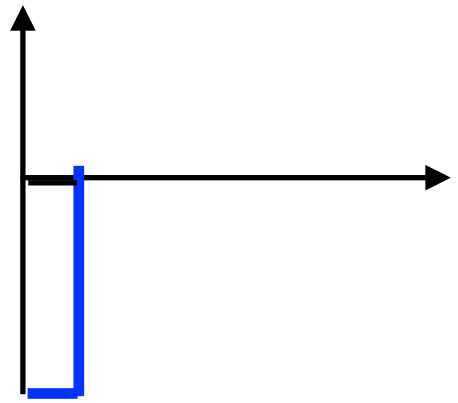
$$\tilde{\psi}(t, \mathbf{x}) = \frac{1}{(1 + \alpha t)^{3/2}} \exp\left(\frac{i}{2} \frac{m\alpha x^2}{1 + \alpha t} \right) \psi\left(\frac{t}{1 + \alpha t}, \frac{\mathbf{x}}{1 + \alpha t} \right)$$

  is also a solution

- The full symmetry group of the Schrödinger equation is called the Schrödinger symmetry
  - includes: time and spatial translations, Galilean boosts, rotation, scaling, and one "proper conformal transformation"

Beyond free theory

- Is the Schrödinger symmetry good only for non-interacting theory and $1/r^2$ interaction?
- Are there scale-invariant systems with short-ranged interaction?
- Answer: yes! the unitarity regime
  - $a \to \infty$
  - $r_0 \to 0$



Systems with large scattering length

- Helium-4 atoms $a \sim 100$ Å
- Neutrons $a \sim -20$ fm
- Ultracold trapped atoms: a can be tuned by a magnetic field
- In all these cases, interaction is short-ranged but particles “feel” each other at much larger distance

Nonrelativistic CFT

Y. Nishida, DTS, 2007

- One can build up the formalism of nonrelativistic conformal field theory in analogy with the relativistic theory
- Many notions can be extended
  - operator dimensions

$$\langle \mathcal{O}(t,\mathbf{x})\mathcal{O}^\dagger(0,\mathbf{0})\rangle \sim \frac{1}{t^{\Delta_\mathcal{O}}}\exp\left(\frac{ix^2}{2m_\mathcal{O}t}\right)$$

  - operator-state correspondence

Operator-state correspondence

Y. Nishida, DTS, 2007

- Dimension of a primary operator = energy of a state in a harmonic potential
- Example:

1 particle in h.p. $E = \frac{3}{2}\hbar\omega$ $\qquad [\psi] = \frac{3}{2}$

2 particles at unitarity in h.p.

$E = 2\hbar\omega$ $\qquad [\psi] = 2$

Operator-state correspondence

- Dimension of a primary operator = energy of a state in a harmonic potential

N	S	L	O	Δ
2	0	0	$\psi_\uparrow \psi_\downarrow$	2
3	1/2	1	$\psi_\downarrow \psi_\uparrow \boldsymbol{\nabla} \psi_\uparrow$	4.273
3	1/2	0	$\psi_\downarrow \boldsymbol{\nabla} \psi_\uparrow \cdot \boldsymbol{\nabla} \psi_\uparrow$	4.666
4	0	0	$\psi_\downarrow \psi_\uparrow \boldsymbol{\nabla} \psi_\downarrow \cdot \boldsymbol{\nabla} \psi_\uparrow$	5.0–5.1

“UnNuclear physics”

A nonrelativistic version of unparticle physics

field in NRCFT: “unnucleus”

H.-W. Hammer and DTS, 2103.12610

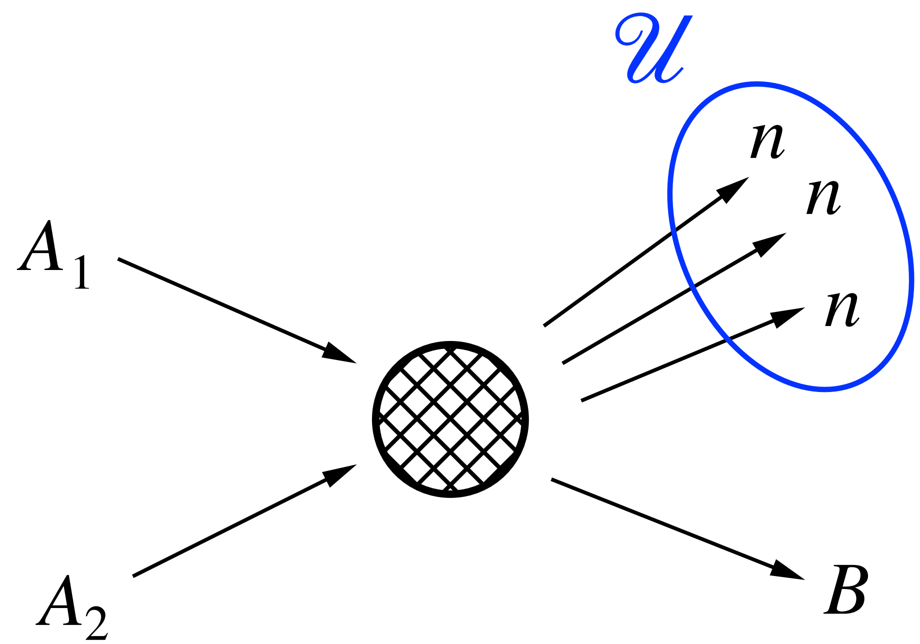
Few-neutron systems as unnuclei

- Neutrons have anomalously large scattering length: $a_{nn} \approx -19$ fm $\gg r_0 \approx 2.8$ fm
- In a wide range of energy is neutrons are fermions at unitarity

Nuclear reactions

- Many nuclear reactions with emissions of neutrons:
  - $^{3}H + {}^{3}H \rightarrow {}^{4}He + 2n$
  - $^{7}Li + {}^{7}Li \rightarrow {}^{11}C + 3n$
  - $^{4}He + {}^{8}He \rightarrow {}^{8}Be + 4n$
- Final-state neutrons can be considered as forming an "unnucleus" - a field in NRCFT
  - Regime of validity: kinetic energy of neutrons in their c.o.m. frame between $\hbar^2/ma^2 \sim 0.1$ MeV $\hbar^2/mr_0^2 \sim 5$ MeV

Few-neutron systems as unnuclei

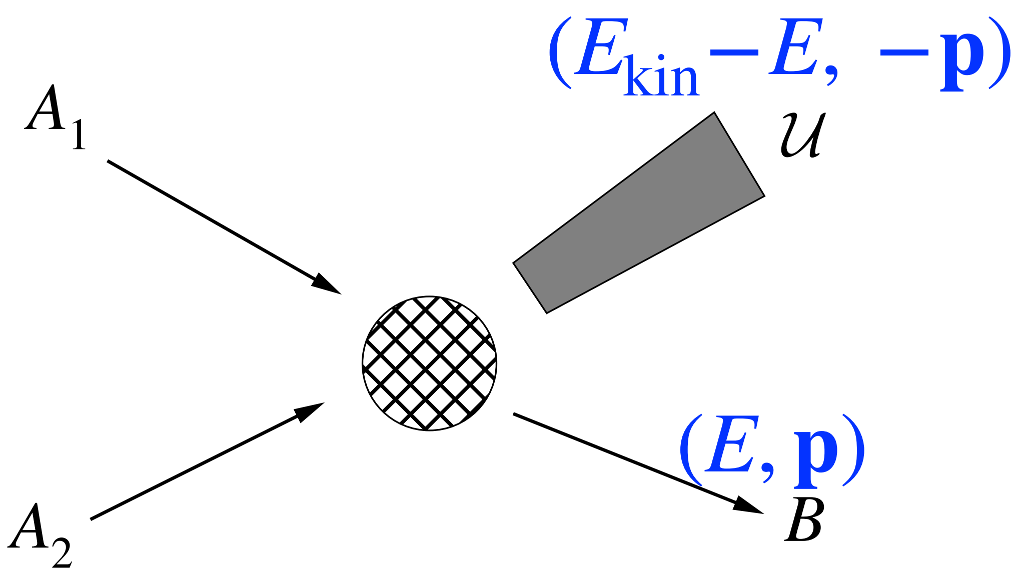


Factorization:

$$\frac{d\sigma}{dE} \sim |\mathscr{M}|^2 \sqrt{E_B} \times \mathrm{Im}\, G_{\mathscr{U}}(E_{\mathscr{U}}, \mathbf{p})$$

primary reaction has larger energy than final-state interaction

Rates of processes involving an unnucleus



$$E_{\text{kin}} = E + E_{\mathscr{U}}$$

- $$\frac{d\sigma}{dE} \sim |\mathscr{M}|^2 \sqrt{E} \times \underbrace{\mathrm{Im}\, G_{\mathscr{U}}(E_{\text{kin}} - E, \mathbf{p})}_{\displaystyle \left(E_{\text{kin}} - E - \frac{p^2}{2M_{\mathscr{U}}}\right)^{\Delta - \frac{5}{2}}}$$

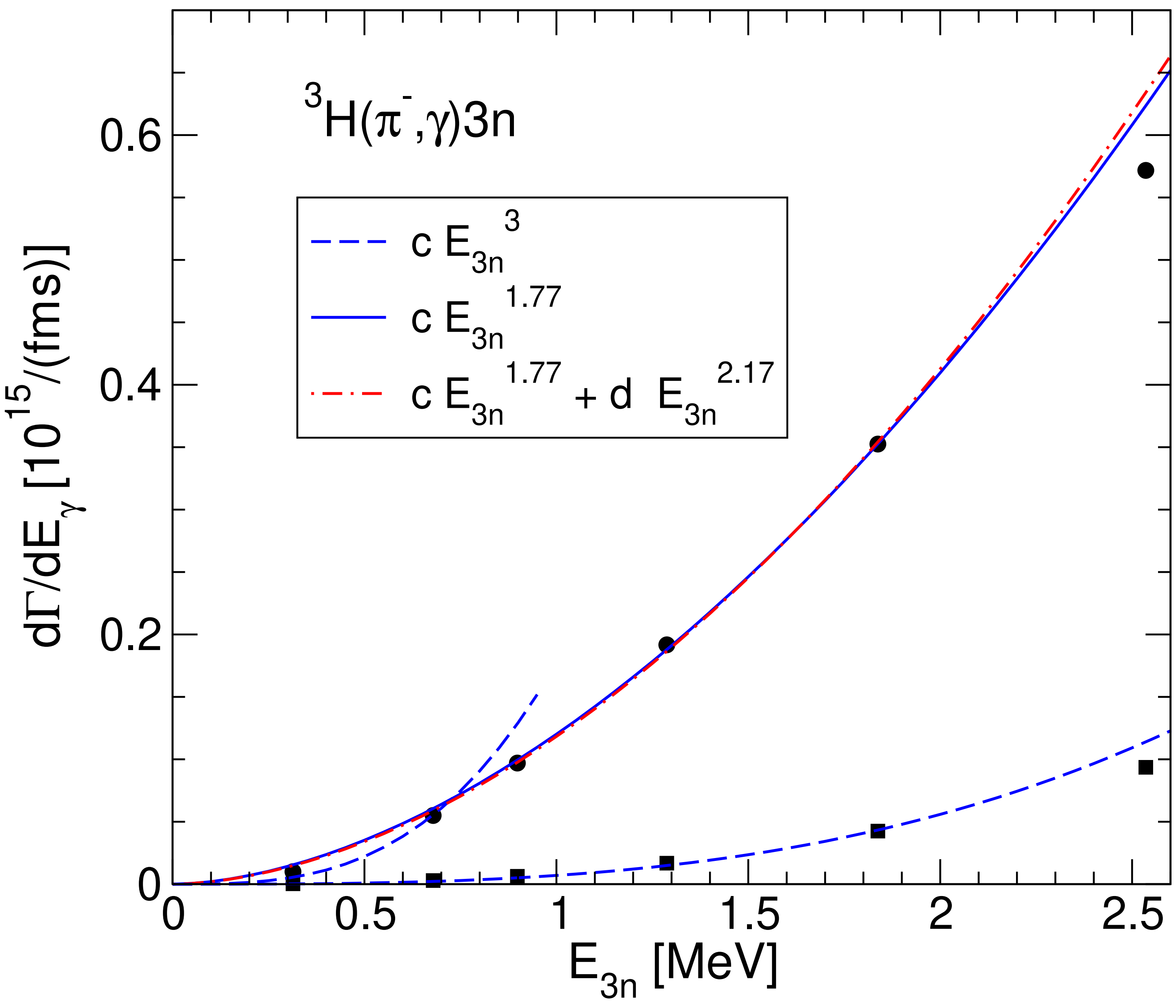
- Near end point: $\dfrac{d\sigma}{dE} \sim (E_0 - E)^{\Delta - \frac{5}{2}}$

Nuclear reactions

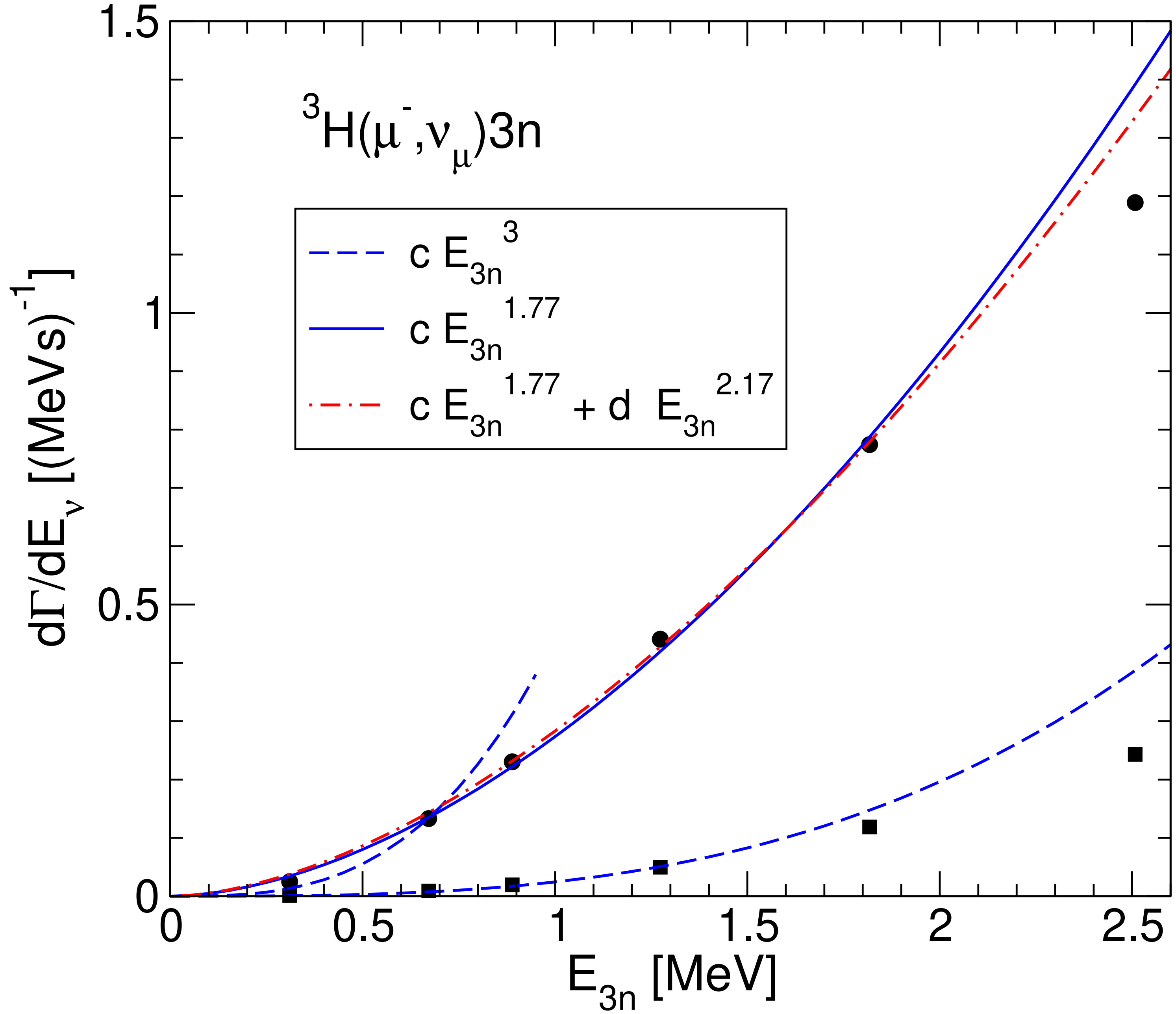
- $^3H + {}^3H \rightarrow {}^4He + 2n$ — $\alpha = -0.5$
- $^7Li + {}^7Li \rightarrow {}^{11}C + 3n$ — $\alpha = 1.77$
- $^4He + {}^8He \rightarrow {}^8Be + 4n$ — $\alpha = 2.5 - 2.6$

- Prediction:
  - $\frac{d\sigma}{dE} \sim (E_0 - E)^\alpha$
- Regime of validity: kinetic energy of neutrons in their c.o.m. frame between $\hbar^2/ma^2 \sim 0.1$ MeV $\hbar^2/mr_0^2 \sim 5$ MeV

Comparison with microscopic models



$$\pi^- + {}^3\mathrm{H} \rightarrow \gamma + 3n$$



$$\mu^- + {}^3\mathrm{H} \rightarrow \nu_\mu + 3n$$

Conclusion

- There is a nonrelativistic version of conformal field theory
- Example: fermions at unitarity
- Approximately realized by neutrons; leads to “unnuclear behavior” of differential cross sections near threshold
- (also in decay of multi-particle resonances Son, Stephanov, Yee 2112.03318)
- Almost conformal symmetry (logarithmic running coupling): weakly-bound two-neutron halo nuclei Hongo, DTS, 2201.09912

Thank you