Data Assimilation using Non-invasive Monte Carlo Sensitivity Analysis of Reactor Kinetics Parameters


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Introduction

- Reactor kinetics parameters are measured for integral experiments at the National Criticality Experiments Research Center (NCERC)
- These measured parameters can be used for criticality safety and validation of nuclear data
- Focus of this talk: (1) the prompt neutron decay constant ($\alpha$) and (2) the effective delayed neutron fraction ($\beta_{eff}$)
- Sensitivity/uncertainty analysis can be used to understand sources of uncertainty in reactor kinetics parameters [1]
- By understanding sources of uncertainty in nuclear data, targeted experiments can be performed to constrain nuclear data of interest
- EUCLID: Experiments Underpinned by Computational Learning for Improvements in nuclear Data
Theory

• Prompt Neutron Decay Constant:

$$\alpha = \frac{k_p - 1}{l}$$

• Effective Delayed Neutron Fraction:

$$\beta_{\text{eff}} = 1 - \frac{k_p}{k}$$

• Relative Sensitivity Coefficient:

$$S_{k,\sigma} = \frac{\sigma}{k} \frac{\partial k}{\partial \sigma}$$

• Relative Uncertainty:

$$\frac{\Delta k}{k} = \sqrt{S_{k,\sigma} C_{\sigma,\sigma} S_{k,\sigma}^T}$$

where $C_{\sigma,\sigma}$ is a covariance matrix of nuclear data $\sigma$ (e.g., $\sigma = ^{239}\text{Pu}(n,f)$)
Methods – Manual Perturbation

• A Compact ENDF (ACE) File Perturbation
• (+)/(-) Perturbations of ACE files will be used with Central Difference
• Sensitivity Coefficient calculation using central Difference Formula:

\[ S_{k,\sigma} = \frac{k_+ - k_-}{2k_0p} \]

(+) Perturbation \( k_+ \)  (-) Perturbation \( k_- \)  Unperturbed \( k_0 \)  Perturbation Size \( p \)

• Used KOPTS card of MCNP6.2 [3,4] to get reactor kinetics parameters
• Manually perturbed ACE Files using ACEtk
• ACEtk is a toolkit for reading and interacting with ACE nuclear data files
  https://github.com/njoy/ACEtk
Methods – Data Assimilation

• This work: target parameter = $k_{\text{eff}}$, experimental result = $\alpha$ or $\beta_{\text{eff}}$
• Nuclear data-induced uncertainty of neutron multiplication factor ($k_{\text{eff}}$):

$$\sqrt{S_{k,\sigma}C_{\sigma,\sigma}S_{k,\sigma}^T}$$

• After including experimental result, uncertainty an be reduced [5]:

$$\sqrt{S_{k,\sigma}C_{\sigma,\sigma}S_{k,\sigma}^T - R_{k_{\text{eff}}}(S_{\alpha,\sigma}C_{\sigma,\sigma}S_{k,\sigma}^T)}$$

where

$$R_{k_{\text{eff}}} = \frac{S_{k,\sigma}C_{\sigma,\sigma}S_{\alpha,\sigma}^T + X_{\text{mod},(k_{\text{eff}},\alpha)}}{S_{\alpha,\sigma}C_{\sigma,\sigma}S_{\alpha,\sigma}^T + X_{\text{exp},\alpha} + X_{\text{mod},\alpha}}$$

where $X_{\text{exp/mod}}$ is the covariance matrix of relative experimental/modeling errors with respect to parameter listed

• Results for both a priori and posterior uncertainties will be reported
Benchmarks & Simulation Inputs

PMF-1, Jezebel

PMF-8, Thor Core

PMF-6, Flattop, Pu Core

EUCLID, 8x1 Unit Model

EUCLID, 3x2 Unit Model
## Results – ICSBEP Benchmarks

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Nuclide</th>
<th>Reaction</th>
<th>Uncertainty (pcm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td>Exp. Meas.: None</td>
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<td>PMF-1, Jezebel</td>
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<td>(n,n)</td>
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<td>(n,f)</td>
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<td>PMF-8, Thor Core</td>
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<td>Total $\bar{\nu}$</td>
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### Results – Planned EUCLID Experiments

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<td>Exp. Meas.: None</td>
<td>Exp. Meas.: $\alpha$</td>
<td>Exp. Meas.: $\beta_{\text{eff}}$</td>
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<td>8x1 Unit Model</td>
<td>Pu-239</td>
<td>Total $\bar{\nu}$</td>
<td>310</td>
<td>295</td>
<td>244</td>
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</tbody>
</table>
Results – Planned EUCLID Experiments

239Pu(n,f)

Total Fission $\bar{\nu}$

Energy (MeV)

Sensitivity per Unit Lethargy

-3.0
-2.0
-1.0
0.0
0.1
0.2
0.3
0.4

-3.0
-2.0
-1.0
0.0
0.1
0.2
0.3
0.4

10^{-3} 10^{-2} 10^{-1} 10^{0} 10^{1}

3x2 $\alpha$
3x2 $\beta$
3x2 $k$
8x1 $\alpha$
8x1 $\beta$
8x1 $k$

Energy (MeV)
Conclusions

• Calculated sensitivity coefficients of prompt neutron decay constant ($\alpha$) and effective delayed neutron fraction ($\beta_{\text{eff}}$) to Pu-239 nuclear data using a newly available tool (ACEtk)

• Uncertainty calculations showed trends:
  (1) The prompt neutron decay constant can be used to reduce nuclear data-induced uncertainty in Pu-239(n,f)
  (2) The effective delayed neutron fraction can be used to effectively reduce nuclear data-induced uncertainty in Pu-239 total fission $\bar{\nu}$

• Contribution to EUCLID Mission: sensitivity/uncertainty analysis can be used to determine optimal responses to constrain targeted nuclear data

• Upcoming work includes investigating other nuclide-reaction pairs and response sensitivities, such as $\Delta p$, neutron leakage spectra, and reaction rate measurements to constrain nuclear data of interest
Acknowledgments

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References


Extra Slides – Manual Perturbation Comparison to Adjoint Weighting – PU-MET-FAST-001 (Jezebel)