Application-Dependent Multigroup Weighting applied to Critical Benchmarks

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July 2022

LA-UR-22-26638

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Outline

1. Introduction
   - Theory
   - Weights
   - Quantities of interest
2. Results
3. Conclusions

Using more appropriate weighting functions produce more accurate cross-section averages

Weighting functions

- Many transport methods require energy-averaged (MG) cross sections (XS)
- Appropriate XS need accurate weighting functions for XS averaging
- Exact XS are not possible in practice because exact weight is unknown, space/angle-dependent, and expensive
- Unlike reactors, critical assemblies often lack repeated geometries that make estimating these functions straightforward
- In this talk, we compare approximate, application-dependent weighting functions to LANL's application-independent TD-4 weighting function

XS averaging from fine-group (f) to coarse-group (g):

$$\sigma_g = \frac{\sum_{f \in g} \sigma_f w_f}{\sum_{f \in g} w_f}$$

We use an approximate, fast-running problem to determine application-dependent weight functions

To generate weights, we solve a 0D MG k-eigenvalue system with leakage approximated with an escape XS. We solve on the LANL-618 group structure (group index f):

$$\left(\Sigma_e + \Sigma_{t,f}\right)\phi_f = \sum_{f'} \Sigma_{s,f'\to f}\, \phi_{f'} + \frac{1}{k_{\text{eff}}} \sum_{f'} \chi_{f'\to f}\, \nu_{f'} \Sigma_{F,f'}\, \phi_{f'},$$

We run with an escape XS (Σ_e) that produces $k_{\text{eff}} = 1$.

We use the flux to weight from fine-group to coarse-group XS:

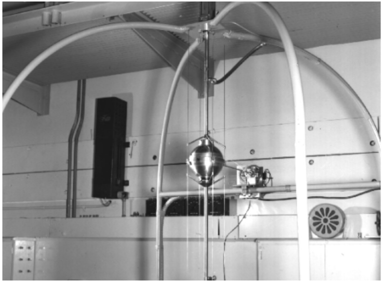
$$w_f = \phi_f,$$

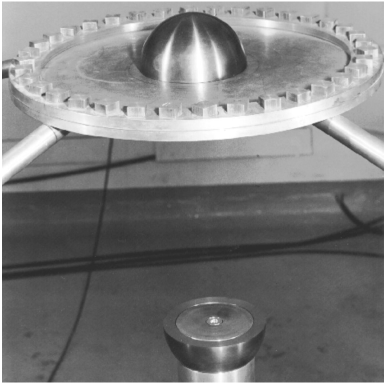
$$\sigma_g = \frac{\sum_{f\in g} \sigma_f\, w_f}{\sum_{f\in g} w_f}.$$

We study two fast, criticality-safety problems

Problem 1: Jezebel
ICSBEP: PU-MET-FAST-001
Description: Bare Pu sphere
Radius: 6.39157 cm

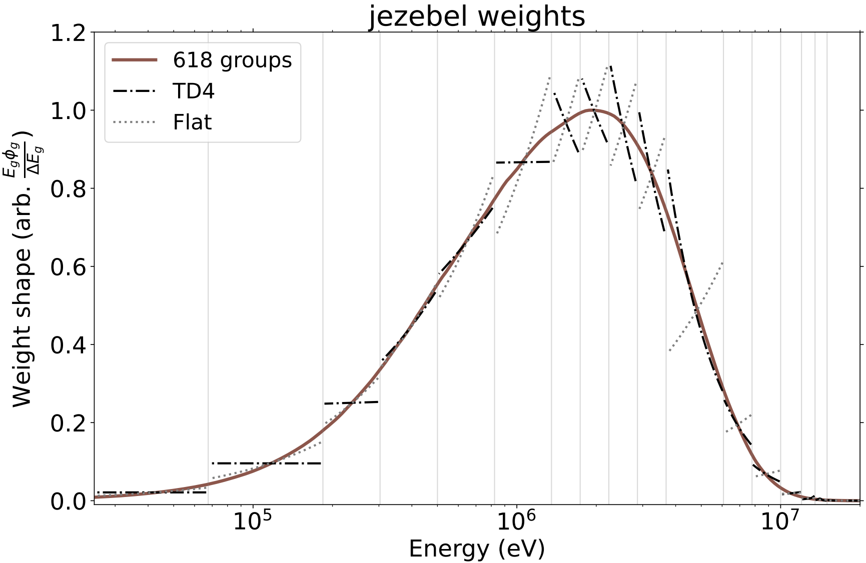
Problem 2: BeRP
ICSBEP: PU-MET-FAST-018
Description: Be-Reflected Pu sphere
Radii: Pu: 5.0419 cm, Be: 8.73 cm





Images from NEA/NSC/DOC(95)03/I

We compare different weighting functions (618-group)



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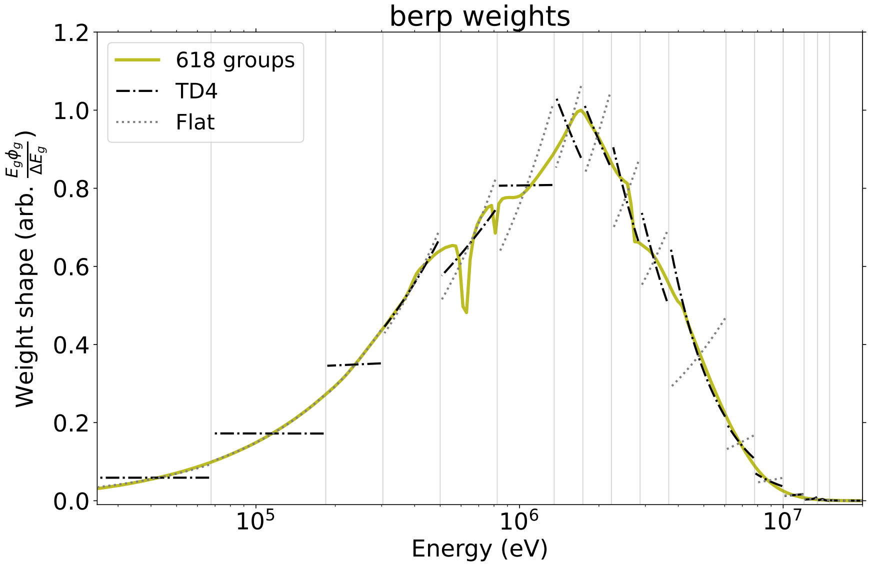

We compare several quantities of interest

All tests use the LANL-30 group structure

$$k_{\mathrm{eff}} = \frac{F}{A+L} = \eta\, P_{\mathsf{NL}} = \text{criticality eigenvalue (unitless)},$$

$$\eta = \frac{F}{A} = \text{fission factor (unitless)},$$

$$P_{\mathsf{NL}} = \frac{A}{A+L} = \text{nonleakage probability (unitless)},$$

$$\tau = \frac{v^{-1}}{A+L} = \text{neutron lifetime (time)},$$

with

$$A = \oint \mathrm{d}^3 r \sum_g \Sigma_{a,g}(\mathbf{r})\, \phi_g(\mathbf{r}) = \text{total absorption reaction rate},$$

$$F = \oint \mathrm{d}^3 r \sum_g \nu_g(\mathbf{r})\, \Sigma_{f,g}(\mathbf{r})\, \phi_g(\mathbf{r}) = \text{total fission-production reaction rate},$$

$$L = \int \mathrm{d}^2 r\, \hat{\mathbf{n}}(\mathbf{r}) \cdot \sum_g \mathbf{J}_g(\mathbf{r}) = \text{net out-leakage rate},$$

$$v^{-1} = \oint \mathrm{d}^3 r \sum_g \frac{\phi_g(\mathbf{r})}{v_g(\mathbf{r})} = \oint \mathrm{d}^3 r \sum_g N_g(\mathbf{r}) = \text{total number of neutrons},$$

We compare errors for different weighting functions

Using TD-4-weighted LANL-618 as the reference and comparing to LANL-30
All results are from 1D, spherical Partisn simulations (not infinite-medium)

	k_{eff}	dk_{eff} (pcm) to 618g by spectrum			
Problem	**618g**	**BeRP**	**Jezebel**	**TD-4**	**Flat**
Jezebel	0.99880	8	**8**	19	234
BeRP	0.99595	**38**	123	228	268

Quantity of interest: Criticality eigenvalue (using prompt $\bar{\nu}$)
Values: Errors in k_{eff} in pcm
↕ Rows: Problem
↔ Columns: Weighting spectrum
Bold values: Weighting spectrum **matches** problem

We compare QOI for various weighting functions

Using TD-4-weighted LANL-618 as the reference and comparing to LANL-30

All results are from 1D, spherical Partisn simulations (not infinite-medium)

	k_{eff}	dk_{eff} (pcm) to 618g by spectrum			
Problem	**618g**	**BeRP**	**Jezebel**	**TD-4**	**Flat**
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	η	dη/η (pcm) to 618g by spectrum			
Problem	**618g**	**BeRP**	**Jezebel**	**TD-4**	**Flat**
Jezebel	3.04710	-40	**3**	-133	245
BeRP	3.16867	**282**	505	-681	1,004

	P_{NL}	dP_{NL}/P_{NL} (pcm) to 618g by spectrum			
Problem	**618g**	**BeRP**	**Jezebel**	**TD-4**	**Flat**
Jezebel	0.32779	48	**5**	152	-11
BeRP	0.31431	**-243**	-380	917	-728

	τ (sh)	dτ/τ (pcm) to 618g by spectrum			
Problem	**618g**	**BeRP**	**Jezebel**	**TD-4**	**Flat**
Jezebel	0.36657	527	**-14**	1,760	-34
BeRP	1.69070	**-14,600**	-18,400	27,700	-9,700

Conclusions

Application-dependent weights enhance XS accuracy

- Multigroup (MG) cross sections (XS) are weighted averages
- LANL traditionally uses the TD-4 application-independent weighting function
- Many communities use application-dependent weighting functions
- Using an approximate application-dependent weighting function reduces errors in several relevant quantities of interest compared to a fine-group reference

Possible future directions

- Add more comparisons to continuous-energy Monte Carlo
- Try higher-fidelity approximate weighting functions
- Estimate the fine-group weighting function from a coarse-group calculation at nominal spatial resolution (not a 0D approximation)

Are there further questions from the audience?

Los Alamos Report LA-UR-22-26638. Funded by the Department of Energy at Los Alamos National Laboratory under contract 89233218NCA000001.

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We compare to continuous-energy Monte Carlo

Comparisons are between MCNP (w/ PT in URR) and two fine-group Partisn calculations

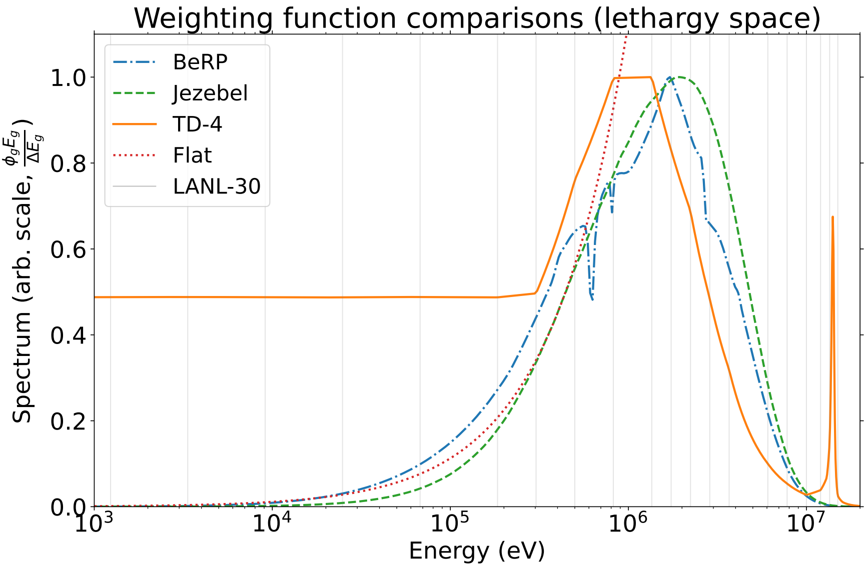
Comparisons to LANL-618 (no self-shielding)

	dk_{eff} (pcm)	$d\eta/\eta$ (pcm)	dP_{NL}/P_{NL} (pcm)
Jezebel	23	-21	44
BeRP	22	-152	173

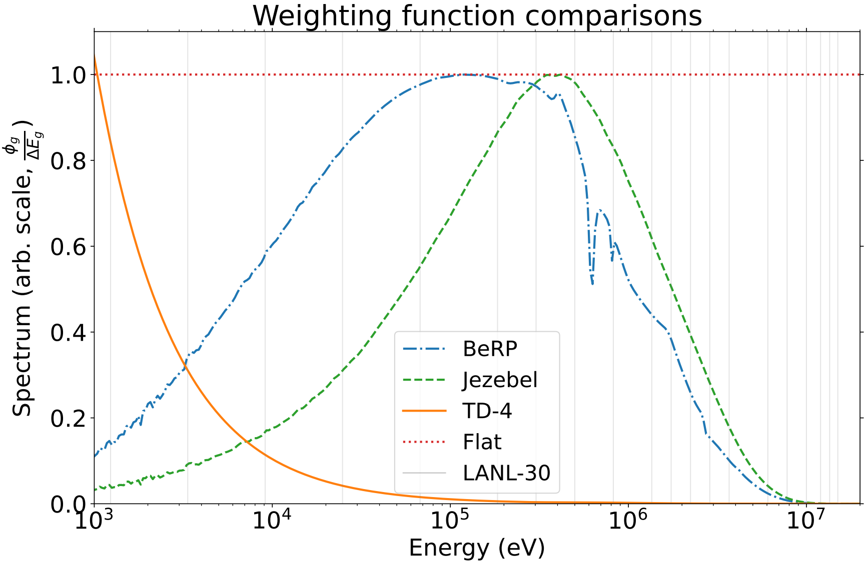
Comparisons to SALLER-261 (with self-shielding in RRR+URR)

	dk_{eff} (pcm)	$d\eta/\eta$ (pcm)	dP_{NL}/P_{NL} (pcm)
Jezebel	17	-9	26
BeRP	-4	-42	36

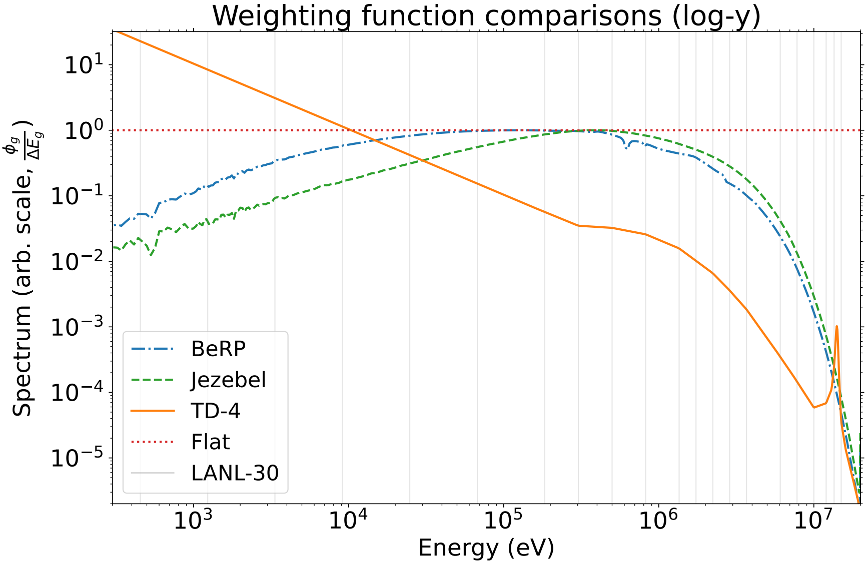
Plot of weights, linear scale, multiplied by E



Same weights, linear scale, without the factor of E



Same weights, log-scale, without the factor of E



Same weights, log scale, multiplied by E