Subject: Application-Dependent Multigroup Weighting applied to Critical Benchmarks

Background

Deterministic neutronics codes require multigroup (MG) nuclear data that are weighted averages of the continuous-energy cross sections. The proper weighting function is the energy-dependent angular flux, which is unavailable as it is a higher-fidelity solution to the full problem. Historically, LANL has used a problem-independent weighting function known as TD-4.

To attain application-appropriate cross sections, many reactor communities solve approximate problems for an energy-dependent scalar flux and use that for the weighting function. In this paper, we extend such a scheme to fast-spectrum critical assemblies, using infinite-medium problems to determine 618-group weighting functions for cross section collapse. We compare cross sections with various weightings for two criticality benchmark problems[1], PU-MET-FAST-001 (a bare plutonium sphere, aka Jezebel) and PU-MET-FAST-018 (a beryllium-reflected plutonium system, aka BeRP).

Methodology

We begin with cross sections defined on the LANL-618-group structure where TD-4 weighting is used within each fine group. We then test four weighting functions to produce cross sections on the LANL-30 group structure. Our weighting functions are compared in Fig. 1. For reaction x, the coarse-group (g) cross section is an average of the fine-group (f) cross sections with weighting function w_f:

$$\sigma_{x,g} = \frac{\sum\limits_{f\in g} \sigma_{x,f} w_f}{\sum\limits_{f\in g} w_f}. \tag{1}$$

We test the appropriateness of our weighting functions by comparing to a reference solution on four quantities of interest: 1) the criticality eigenvalue, $k_{\text{eff}} = F/(A+L)$; 2) the fission factor, $\eta = F/A$; 3) the nonleakage probability, $P_{NL} = A/(A+L)$; and 4) the neutron lifetime, $\tau = v^{-1}/(A+L)$, where $F = \overline{\nu\Sigma_f}$, $A = \overline{\Sigma_a}$, L is the leakage, and $v^{-1} = \overline{1/v}$, with v the neutron speed.

Results

We ran our four sets of 30-group cross sections on one-dimensional versions of the two benchmark problems. For reference solutions, we ran these problems with 618 groups. All results used transport-corrected P_3 scattering expansions. Space constraints preclude comparisons to MCNP with continuous-energy cross sections, though these were also done.

Table 1 shows errors in four quantities of interest for our four weighting functions. Bolded numbers indicate that the weighting spectrum was tailored to the problem. Errors in k_{eff} were below 270 pcm in all cases but neutron lifetime errors could exceed 25%. For the Jezebel problem, errors could be reduced below TD-4 errors by over 20x, except for k_{eff}, which benefited from error cancellation. For the BeRP problem, errors could be reduced by a factor of 2-4x compared to TD-4. TD-4 produced lower errors than flat weighting for criticality eigenvalue and fission factor but higher errors for non-leakage probability and neutron lifetime.

In this work, we have demonstrated the potential of application-dependent weighting functions, showing they reduce error compared to the TD-4 and flat weighting functions for fast criticality-benchmark problems. Interesting future research efforts include studying more complicated problems and tuning the zero-dimensional systems used to produce the weighting functions.

[1] NEA (2019), "ICSBEP Handbook 2019", *International Criticality Safety Benchmark Evaluation Project Handbook*, `https://doi.org/10.1787/e2703cd5-en`.

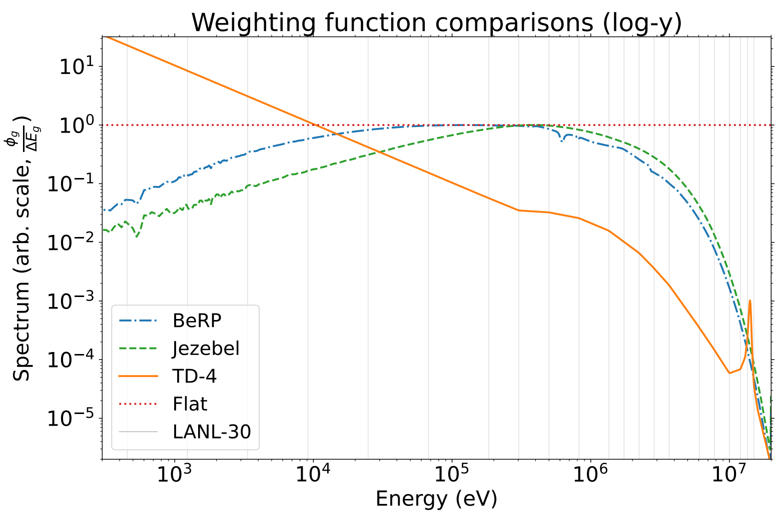

Figure 1: Weighting function comparisons plotted on a log-log scale. Absolute magnitude does not matter, only the shape / slope within a coarse group. Our coarse-group boundaries are given as grey vertical lines.

Table 1: Relative errors (pcm) in several quantities of interest for a 618-group reference compared to 30-group cross sections with four different weighting functions.

	k_{eff}	dk_{eff} (pcm) to 618g by spectrum			
Problem	**618g**	**BeRP**	**Jezebel**	**TD-4**	**Flat**
Jezebel	0.99880	8	**8**	19	234
BeRP	0.99595	**38**	123	228	268

	η	dη/η (pcm) to 618g by spectrum			
Problem	**618g**	**BeRP**	**Jezebel**	**TD-4**	**Flat**
Jezebel	3.04710	-40	**3**	-133	245
BeRP	3.16867	**282**	505	-681	1,004

	P_{NL}	dP_{NL}/P_{NL} (pcm) to 618g by spectrum			
Problem	**618g**	**BeRP**	**Jezebel**	**TD-4**	**Flat**
Jezebel	0.32779	48	**5**	152	-11
BeRP	0.31431	**-243**	-380	917	-728

	τ (sh)	dτ/τ (pcm) to 618g by spectrum			
Problem	**618g**	**BeRP**	**Jezebel**	**TD-4**	**Flat**
Jezebel	0.36657	527	**-14**	1,760	-34
BeRP	1.69070	**-14,600**	-18,400	27,700	-9,700