LA-UR-22-27274

Properties of compound nucleus decay width predicted by Gaussian Orthogonal Ensemble nuclear reaction model

Y. Alhassid [1] G. Bertsch [2] T. Kawano [3] H. Sasaki [3]

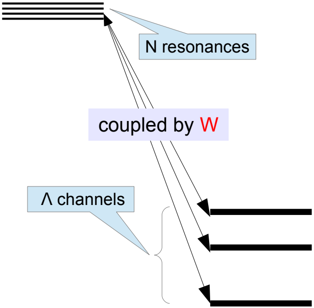
[1]Yale University

[2]University of Washington

[3]Los Alamos National Laboratory

Int. Conf. Nuclear Data for Science and Technology
LLNL (online event), 7/25 – 29, 2022

Compound Nucleus Cross Section Using GOE



$$S_{ab}^{(\mathrm{GOE})} = \delta_{ab} - 2i\pi \sum_{\mu\nu} W_{a\mu} \left(D^{-1}\right)_{\mu\nu} W_{\nu b} \quad (1)$$

$$D_{\mu\nu} = E\delta_{\mu\nu} - H_{\mu\nu}^{(\mathrm{GOE})} + i\pi \sum_{c} W_{\mu c} W_{c\nu} \quad (2)$$

$$\overline{H_{\mu\nu}^{(\mathrm{GOE})} H_{\rho\sigma}^{(\mathrm{GOE})}} = \frac{1}{N} (\delta_{\mu\rho}\delta_{\nu\sigma} + \delta_{\mu\sigma}\delta_{\nu\rho}) \quad (3)$$

- Perform ensemble average $\overline{|S_{aa}|^2}$ by realizations of $H^{(\mathrm{GOE})}$
- Transmission coefficients T_a are given by eigenvalues of WW^T
- Model parameters are T_a, N (number of resonance), and Λ (channel)

Decay Amplitude from GOE Hamiltonian

K-matrix Representation

$$K = \pi W^T \frac{1}{E - H^{\mathrm{GOE}}} W, \qquad K_{cc'} = \frac{1}{2} \sum_{\mu} \frac{\tilde{\gamma}_{\mu c} \tilde{\gamma}_{\mu c'}}{E - E_\mu} \tag{4}$$

S-matrix Pole Expansion

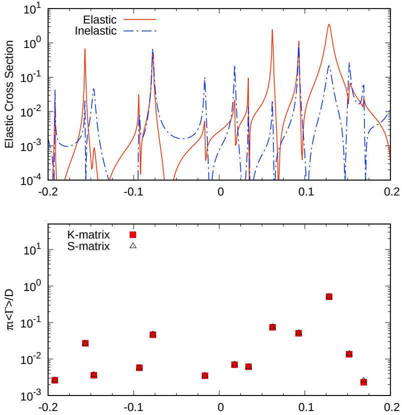
$$S = 1 - 2\pi i (CW)^T \frac{1}{E - C(H^{\mathrm{GOE}} - i\pi W W^T) C^T} CW, \qquad C^T C = 1 \tag{5}$$

$$S_{cc'} = \delta_{cc'} - i \sum_{\nu} \frac{\gamma_{\nu c} \gamma^*_{\nu c'}}{E - E_\nu + i\Gamma_\nu / 2} \tag{6}$$

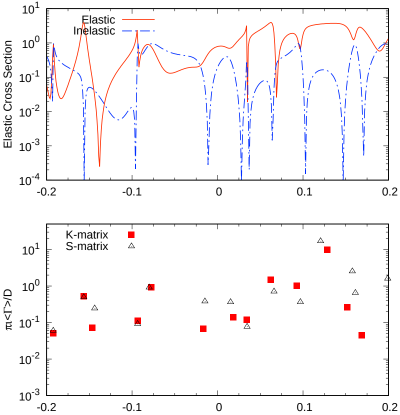
The decay width $\langle \Gamma_c \rangle$ to channel c is given by the ensemble average of $\tilde{\gamma}^2_{\mu c}$ or $\gamma^2_{\nu c}$

GOE Poles and Decay Widths, $\Lambda = 2$ and $N = 100$ Case

$T_a = T_b = 0.1$



$T_a = T_b = 0.9$



Single Channel Calculation (no inelastic scattering)

$$T_a \simeq 2\pi \frac{\langle \Gamma_a \rangle}{D} = 2x$$

where D is the average spacing

SPRT model by Noguere et al.

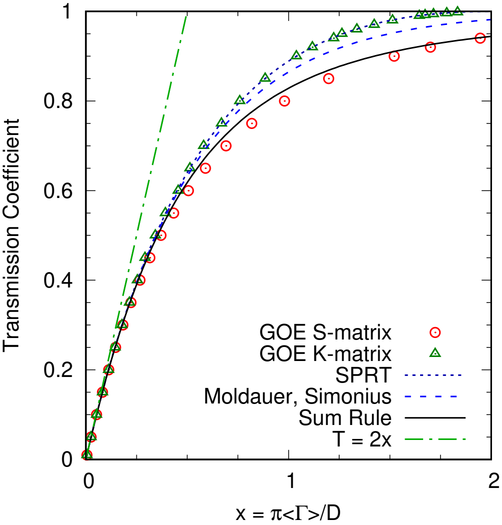
$$T^{\mathrm{SPRT}} = \frac{2x}{(1 + x/2)^2}$$

Moldauer and Simonius

$$T^{\mathrm{MS}} = 1 - e^{-2x}$$

Moldauer's sum rule

$$T^{\mathrm{SR}} = 2x\left(\sqrt{x^2+1} - x\right)$$



Relation between Width Γ and Cross Section σ

Cross Section Defined by S-matrix (Reference)

$$\sigma_0 = \langle|\delta_{ab} - S_{ab}|^2\rangle - \delta_{ab}\sigma^{\mathrm{dir}} \tag{7}$$

Cross Section Defined by Γ

$$\sigma_1 = \frac{2\pi}{D}\left\langle \frac{\Gamma_a\Gamma_b}{\sum_c \Gamma_c} \right\rangle \tag{8}$$

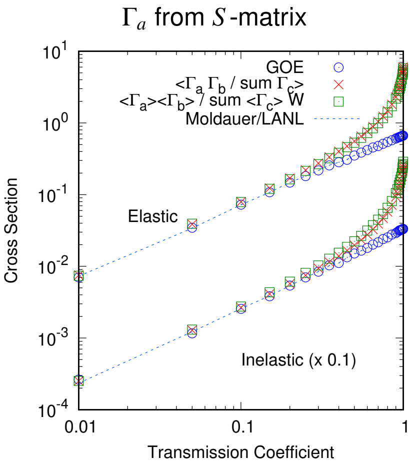
$$\sigma_2 = \frac{2\pi}{D}\frac{\langle\Gamma_a\rangle\langle\Gamma_b\rangle}{\sum_c\langle\Gamma_c\rangle}W_{ab} \tag{9}$$

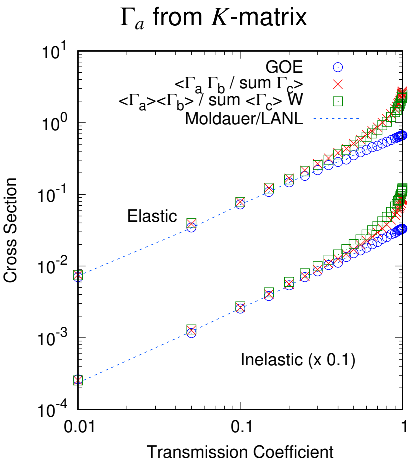
Cross Section Defined by Transmission Coefficient

$$\sigma_3 = \frac{T_aT_b}{\sum_c T_c}W_{ab} \tag{10}$$

The GOE statistical theory implies $\sigma_3 \simeq \sigma_0$

Results of $\Lambda = 2$ Calculation





× $= \sigma_1 = \langle \Gamma_a \Gamma_b / \sum \Gamma \rangle$ □ $= \sigma_2 = \langle \Gamma_a \rangle \langle \Gamma_b \rangle / \sum \langle \Gamma \rangle W_{ab}$

So, What's Happening?

Weak Coupling ($T_a \lesssim 0.3$)

$$\frac{2\pi}{D}\left\langle \frac{\Gamma_a \Gamma_b}{\sum_c \Gamma_c} \right\rangle = \frac{2\pi}{D} \frac{\langle \Gamma_a \rangle \langle \Gamma_b \rangle}{\sum_c \langle \Gamma_c \rangle} W_{ab} = \frac{T_a T_b}{\sum_c T_c} W_{ab} \tag{11}$$

Strong Coupling ($T_a \gtrsim 0.3$)

$$\frac{2\pi}{D}\left\langle \frac{\Gamma_a \Gamma_b}{\sum_c \Gamma_c} \right\rangle \simeq \frac{2\pi}{D} \frac{\langle \Gamma_a \rangle \langle \Gamma_b \rangle}{\sum_c \langle \Gamma_c \rangle} W_{ab} \neq \frac{T_a T_b}{\sum_c T_c} W_{ab} \tag{12}$$

Although many nuclear physics textbooks do not doubt this equality ...
The correct expression for the compound cross section is still

$$\sigma_{ab} = \frac{T_a T_b}{\sum_c T_c} W_{ab} \tag{13}$$

Concluding Remarks

- We investigated statistical properties of the scattering matrix associated with GOE Hamiltonians, using both K and S-matrix formalisms.
- We derived the compound nucleus cross sections in terms of the transmission coefficients into the channels or in terms of the corresponding decay widths.
- The compound cross section derived from the decay widths is ambiguous when the transmission coefficient is large (strong coupling).
- The formula expressed in terms of transmission coefficients valid for all coupling strengths and should be used to define the compound cross section.