Langevin approach to fission dynamics with Cassini shape parameterization

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Fission plays an important role in the study of nuclear data in the actinide region. The dynamical approach to fission using the multi-dimensional Langevin equation has been widely accepted as a practical method. In this approach, nuclear shapes are expressed with a few shape parameters and the fission process is described as their time evolution by the Langevin equation. The shape parameters determine the model space and the calculation results depend critically on them.

The two-center shell model (TSCM) is a widely used model, which has five deformation degrees of freedom: elongation, asymmetry of fragment mass, neck radius, and deformation of right and left fragments [1]. In many cases, the neck degree of freedom is not treated as a dynamical variable. TSCM has been successfully applied in the four-dimensional Langevin calculation and provides good agreement with the experimental data such as fragment mass and kinetic energy distributions [2]. One drawback of TSCM is that only quadrupole deformations of the fragments are taken into account. On the other hand, Cassini shape parameterization can describe the various shapes of deformed nuclei by modifying the original Cassini oval with the Legendre expansion [3]. The coefficients α_i of the Legendre polynomial are the shape parameters. It has been applied in the static approach using the microscopic-macroscopic method to calculate the potential energy of deformation and provided the fragment mass and kinetic energy distributions in the actinide and super-heavy nuclear regions [4, 5]. These studies show that we can select five parameters α, α_1, α_3, α_4, and α_6 among many Cassini parameters, corresponding to elongation, asymmetry of fragment mass, asymmetry of fragment shape, quadrupole deformation of fragments, and octupole deformation of fragments, respectively. However, its application to the dynamical treatment of fission was not realized.

In this study, we apply the Cassini shape parameterization to the dynamical calculation of fission using the multi-dimensional Langevin equation. We use the three- and the four-dimensional model space to calculate the fragment mass and kinetic energy distributions. As an example, we show the results for 14 MeV neutron-induced fission ^{235}U$(n,\, f)$ with three- and four-dimensional calculation. As it is well known, the fragment mass distribution is asymmetric with the mass number of the heavy fragment at the peak position being 136 [6]. It is considered to reflect the magic numbers $(Z,\, N) = (50,\, 82)$. Figure 1 shows the calculated fragment mass distributions for two cases: three-dimensional case with $\{\alpha,\, \alpha_1,\, \alpha_4\}$ and four-dimensional case with $\{\alpha,\, \alpha_1,\, \alpha_3,\, \alpha_4\}$. By including the shape asymmetric degree of freedom α_3, the peak position moves from 95+141 to 100+136, which gives a better reproduction to the experimental data. The corresponding shapes at scission are shown at the top of Fig. 1. In the three-dimensional calculation, the shape is given by $\{\alpha,\, \alpha_1,\, \alpha_4\} = \{1.0,\, 0.076,\, -0.036\}$, and in the four-dimensional calculation, it is given by $\{\alpha,\, \alpha_1,\, \alpha_3,\, \alpha_4\} = \{1.0,\, 0.015,\, -0.091,\, -0.013\}$. It should be noted that the mass asymmetry is determined by the combination of α_1 and α_3. In the former case, two fragments have the same deformation with a slightly prolate shape. In the latter case, the heavy fragment is almost a sphere and the light one is elongated. It is important to consider not only the mass asymmetry but also the shape asymmetry to study the asymmetric fission as ^{236}U. We examined the fission of Fm isotope as another example of actinide nuclei. For this element, it is known that the mass distribution in spontaneous fission changes drastically when we change the number of neutrons, i.e., it is asymmetric for $N = 156$ and is symmetric for $N = 158$ [7]. In the Langevin approach, we chose α_6 as the fourth degree of freedom, by which we can describe the super-long and the super-short configurations in the symmetric fission. We performed the L angevin calculation at very low excitation energy to simulate the spontaneous fission. It has been confirmed that the transition from the asymmetric to the symmetric fission occurs at the correct neutron number when we include α_6.

As a next step, we plan to extend the Langevin calculation to include five Cassini parameters $\{\alpha, \alpha_1, \alpha_3, \alpha_4, \alpha_6\}$. We will apply the five-dimensional calculation in the actinide region and compare the result with that of four-dimensional cases. Moreover, we will extend the system to super-heavy nuclei. It is suggested from the static approach that five Cassini parameters are necessary to describe the strongly deformed fragment that may appear in the fission of super-heavy nuclei [5]. It is important to confirm the mass and the shape of the fragment by the dynamical calculation using the Langevin equation with five Cassini parameters.

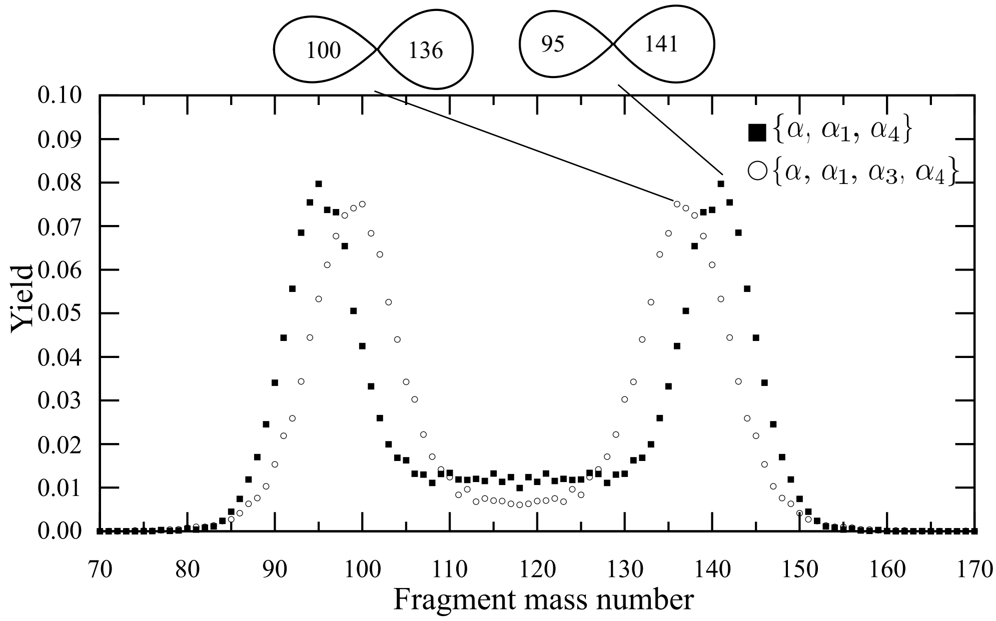

Figure 1. Calculated fragment mass distributions for fission of ^{236}U. Solid squares give the results using three parameters $\{\alpha, \alpha_1, \alpha_4\}$ and open circles give the results using four parameters $\{\alpha, \alpha_1, \alpha_3, \alpha_4\}$. At the top of the graph, we show the most probable shapes at scission for each cases.

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