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Regression analysis of experimental reaction cross-section data of ^{241}Am(n, 2n)^{240}Am

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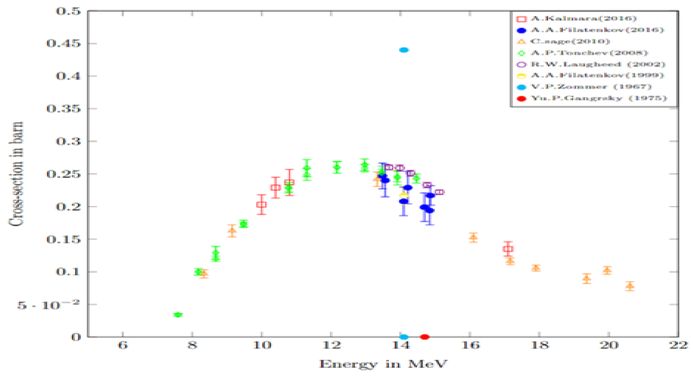
❑Why regression on experimental data ?

- In nuclear science, as more than one researcher performs the same experiment in different parts of different countries, they may get different cross section values for the same energy because each experimenter uses different methods or equipment, such situation may create ambiguity in the selection of data for future application.
- Also, experimental data usually do not cover the whole range of incident energy that are of interest.

❑Retrieval of Experimental data – EXFOR[1]



The scatter plot of experimental cross-section data for neutron reaction **^{241} Am (n, 2n) ^{240}Am**

❑Processing of Experimental data

I. Re-normalization of experimental data :

- **Re-normalization** process update the nuclear data used in the calculation with the current/present nuclear data available in various nuclear data libraries such as IRDFF-v1.05 and NuDat 3 database .

$$\sigma_{renorm} = \sigma_{old} \times \frac{A_{new}}{A_{old}}$$

- $Aold$ is the old values of attributes considered in the determination of cross section by the experimenters. $Anew$ is the recent values of attributes available in IRDFF-v1.05 and NuDat 3 database

II. Removal of an outliers from the experimental data

- Following methods are used:

✓ Studentized residual (r_i)

$$r_i = \frac{e_i}{\sqrt{MS_{res}(1-h_{ii})}}$$

✓Standardized residual (d_i)

$$d_i = \frac{e_i}{\sqrt{MS_{res}}}$$

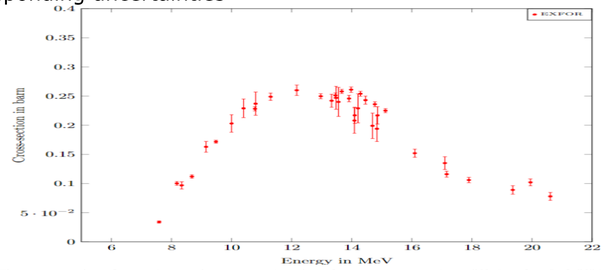
Where, h_{ii} is an element of hat matrix, $MS_{res}$is the mean value of the sum of the square of residual and e_i is the difference between observed value and predicated value.

- **II. Collapsing of Data point**

To replace multiple data points at the same neutron energy with a single value

$$X_{wav} = \frac{\sum_{i=1}^{N} w_i x_i}{\sum_{i=1}^{N} w_i} \qquad w_i = \frac{1}{\sigma_i^2} \qquad \sigma_{avg} = \frac{1}{\sqrt{\sum_{i=1}^{N} w_i}}$$

Where, $x_1, x_2, \ldots, x_N$ are measurements of a single quantity x and σ_i is corresponding uncertainties



The scatter plot of experimental cross-section data for neutron reaction **^{241} Am (n, 2n) ^{240}Am** after re-normalization, removal of outlie and collapsing of data points

❑Regression on processed experimental data

- **Polynomial Regression (PR)**

• fitted model equation with best degree (degree 5) ,

$$\hat{Y} = 57.69X^5 - 728.9743X^4 + 3671.405X^3 - 9219.378X^2 + 11550.5193X - 5781.27$$

- **Gaussian Process Regression (GPR)**

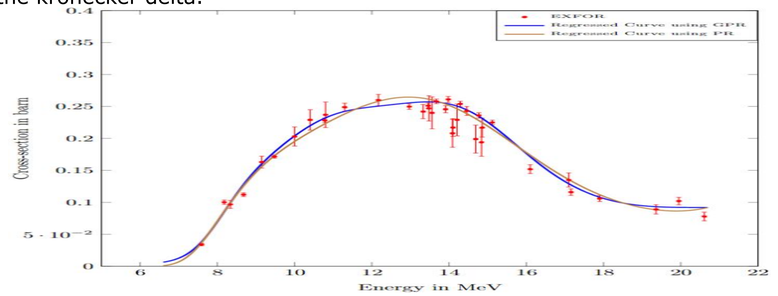
$$m_t(x) = K(X, X_t)\left[K(X_t, X_t) + \sigma_\epsilon^2 Corr\right]^{-1} y_t$$

$$K_t(x, x') = K(x, x')K(X_t, X_t)\left[K(X_t, X_t) + \sigma_\epsilon^2 Corr\right]^{-1}K(X_t, x')$$

Where, $m_t(x)$ and $K_t(x, x')$ are mean and covariance respectively, X_t is experimental data and x is the newly added point, σ_ϵ^2 is observation noise, y_t is observed value, $Corr$ is correlation matrix of the experimental data and $K()$ are kernels. following radial function is used for calculating kernel matrix

$$\mathbf{K}(\boldsymbol{x}, \boldsymbol{x}) = \boldsymbol{\sigma_f^2}\, \boldsymbol{exp}\left(-\frac{||x - x'||^2}{2\lambda^2}\right) + \boldsymbol{\sigma_n^2}\boldsymbol{\delta}$$

Where, σ_f^2 is variance, λ is length-scale , σ_n^2 is noise variance and δ is the kronecker delta.



The regressed curve using GPR and PR

❑Comparison between PR and GPR

	GPR	PR (Degree 5)
SS_{res}	0.003523	0.004322
R^2	0.9781769	0.973229

- SS_{res} is sum of square of the residual (SS_{res} should be minimum)
- R^2 is coefficient of determination (R^2 should be maximum.)

❑Conclusion

- Polynomial Regression and Gaussian Process Regression have been used For regression of the experimental cross-section data of reaction **^{241} Am (n, 2n) ^{240}Am.** Based on this statistical indices GPR is giving the best performance in comparison with the PR method.

❑Literature cited

1. IAEA-EXFOR Database available at http://www nds.iaea.org/ exfor.
2. E. Ostertagova, Modelling using polynomial regression, Procedia Engineering, Elsevier Ltd., no. 48, pp. 500506, 2012
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4. Hiroki Iwamoto (2020),"Generation of nuclear data using Gaussian process regression", Journal of Nuclear Science and Technology, 57:8, 932-938, DOI: 275 10.1080/00223131.2020.1736202.