Low Momentum Particle Correlations in Heavy-Ion Collisions

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Workshop on Particle Correlation and Femtoscopy

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Outline

- Two particle correlations (i.e. Deuteron-alpha correlations)
- Deblurring method for source function in two-particle correlations
- Summary

Particle Correlation Functions (CF)

Two particle CF

$$R(\boldsymbol{q}) + 1 = \frac{P_{12}(\boldsymbol{p_1}, \boldsymbol{p_2})}{P_1(\boldsymbol{p_1})P_2(\boldsymbol{p_2})}$$

Numerator- probability of coincident particle emission

Denomator- product of the probability of single emission

-Two-particle relative momentum ($\boldsymbol{q}$).

Identical particle:

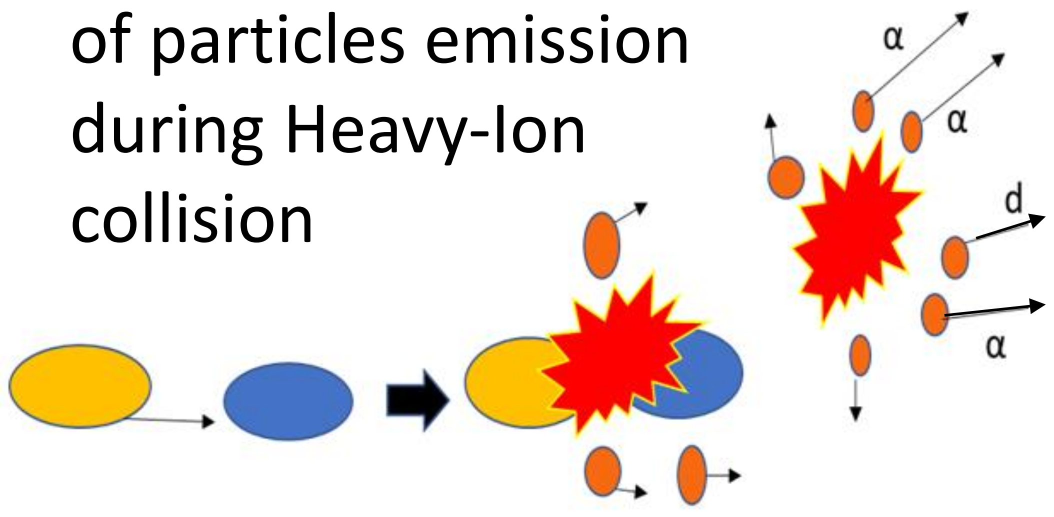
$\boldsymbol{q} = 0.5(\boldsymbol{p_1} - \boldsymbol{p_2})$

$\boldsymbol{p_{1,2}}$ momentum of 1, 2

Non-identical particle:

$\boldsymbol{q} = 0.5\,\mu\,(\boldsymbol{v_1} - \boldsymbol{v_2})$, μ is the reduced mass and velocity, $\boldsymbol{v_{1,2}}$

Pictorial representation of particles emission during Heavy-Ion collision



Pre-equilibrium fast emission

-Heavy particles may be emitted during collision time

-light particles decay from unstable intermediate nuclei.

- Charged particle are more likely to be measured.

CF Model

- Experimental definition
- $C_{Exp}(\mathrm{p}_1, \mathrm{p}_2) = R(\boldsymbol{q}) + 1 = \frac{P_{12}(\mathrm{p}_1, \mathrm{p}_2)}{P_1(\mathrm{p}_1)P_2(\mathrm{p}_2)}$

- Theoretical definition

$$C(\boldsymbol{q}) = \int d^3r \; \underbrace{S(\boldsymbol{r})} \; \underbrace{|\Psi(\boldsymbol{q}, \boldsymbol{r})|^2}$$

Source function and 2-particle wave function

Two-parameter Source Model:

The source model is given in terms of two parameters (λ, R_0)

$$\boldsymbol{S_{\lambda,R_0}(r) = \lambda S_{R_0}(r) + (1 - \lambda) S_{wide}(r)}$$

$\boldsymbol{S_{wide}(r)}$: particles emitted at wide relative separation

$$\boldsymbol{C_{TH}(q) = \lambda \int d^3r \, S_{R_0}(r)(|\Psi(q, r)|^2 - 1) + 1}$$

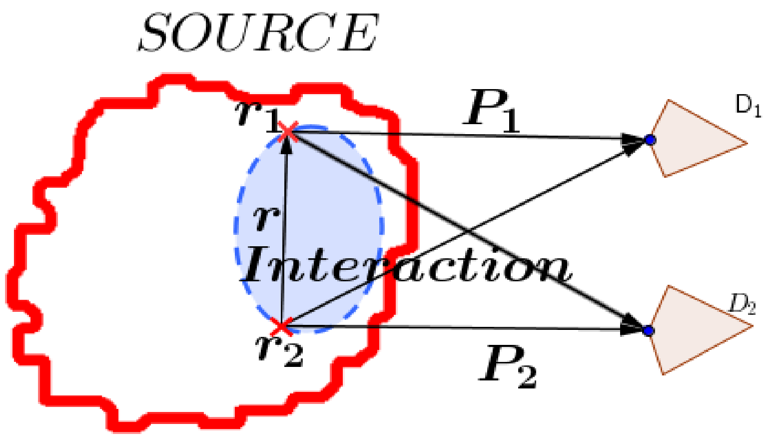
where $0 \leq \lambda \leq 1$: prob. of pair emission at short **r**

Our goals:

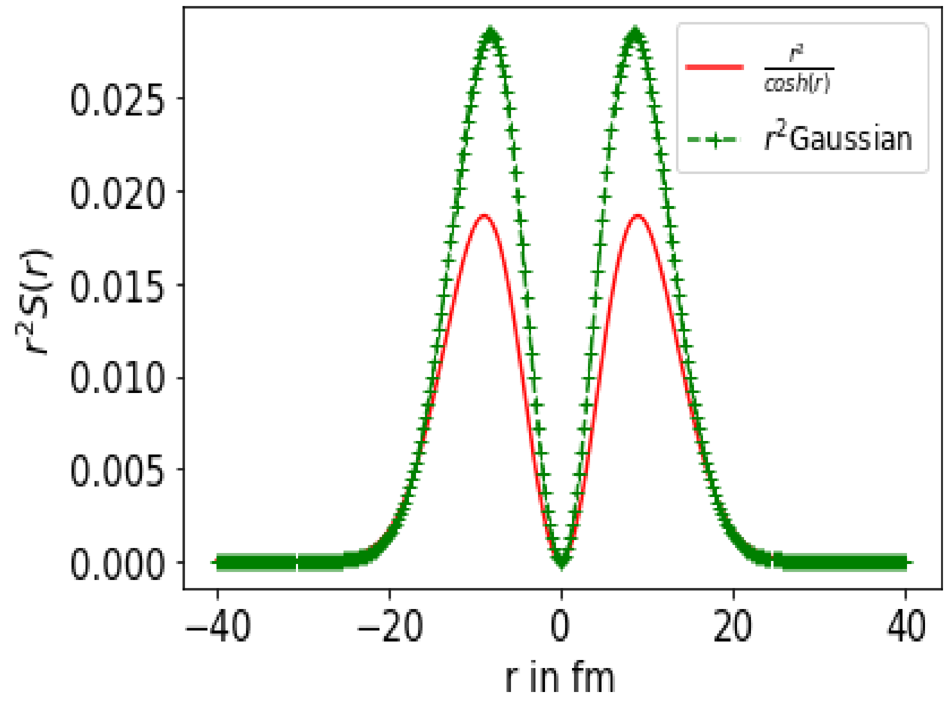
1. Study correlation between particles to learn about nuclear systems: one needs to understand interaction btn particles in the final state (i.e., one needs to determine $\Psi(\boldsymbol{q}, \boldsymbol{r})$)

2. To extract a reliable and realistic S(r) that characterizes nuclear reaction system.

Source Function(SF)



- Interactions and symmetrization, affecting the relative state of two particles on their way to the detectors, and encoded in the relative wave function, allow inferring characteristics of particle emission from correlation functions.
- Studying relative distribution of two-particle emission gives access to space-time characteristics of the emitting system.



Examples of SF: normalized Gaussian and $\propto \frac{1}{Cosh(r)}$

Method to estimate S(r):

-Comparing measured correlation C_{Exp} to C_{TH} one can extract $S(r)$

-Restoration of SF by Deblurring Method

- Or Imaging Restoration of SF (**D.A. Brown and P. Danielewicz, 1997**)

Wave function and Phase shift

Wave function ,

$$\Psi(\boldsymbol{q},\boldsymbol{r}) = \sum_l \frac{2}{\boldsymbol{qr}}(2l+1)i^l\, U_{l,\boldsymbol{q}}(\boldsymbol{r})P_l(\cos\theta)$$

Radial wave function, $U_{l,\boldsymbol{q}}(\boldsymbol{r})$ is a solution to the Radial Schrödinger Equation (SE):

$$\frac{d^2}{dr^2}U_{l,\boldsymbol{q}}(\boldsymbol{r}) = \left(\frac{2\mu}{\hbar^2}\left(V_f(r) + \frac{\hbar^2 l(l+1)}{2\mu r^2} - E\right)\right)U_{l,\boldsymbol{q}}(\boldsymbol{r})$$

$V_f(r)$: Coulomb potential + Nuclear interaction

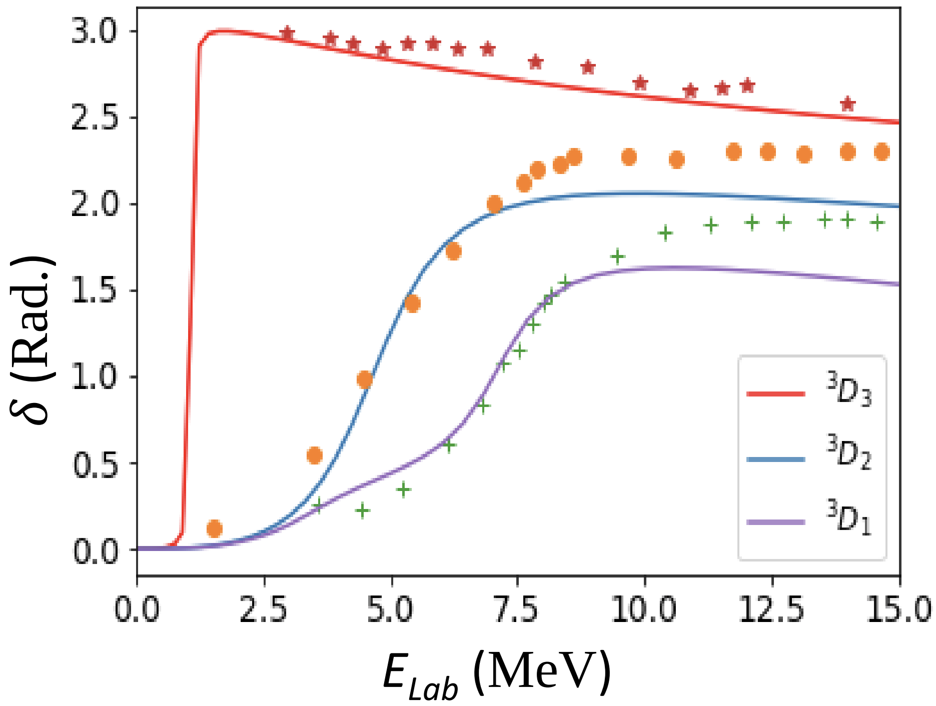
Then phase shifts are obtained by matching $U_{l,\boldsymbol{q}}(\boldsymbol{r})$ numerical and $U_{l,\boldsymbol{q}}(\boldsymbol{r})$ asymptotical solutions (i.e., logarithmic derivative)

We extract nuclear potential (Woods-Saxon form) by comparing theoretical phase shift to phase fit data,

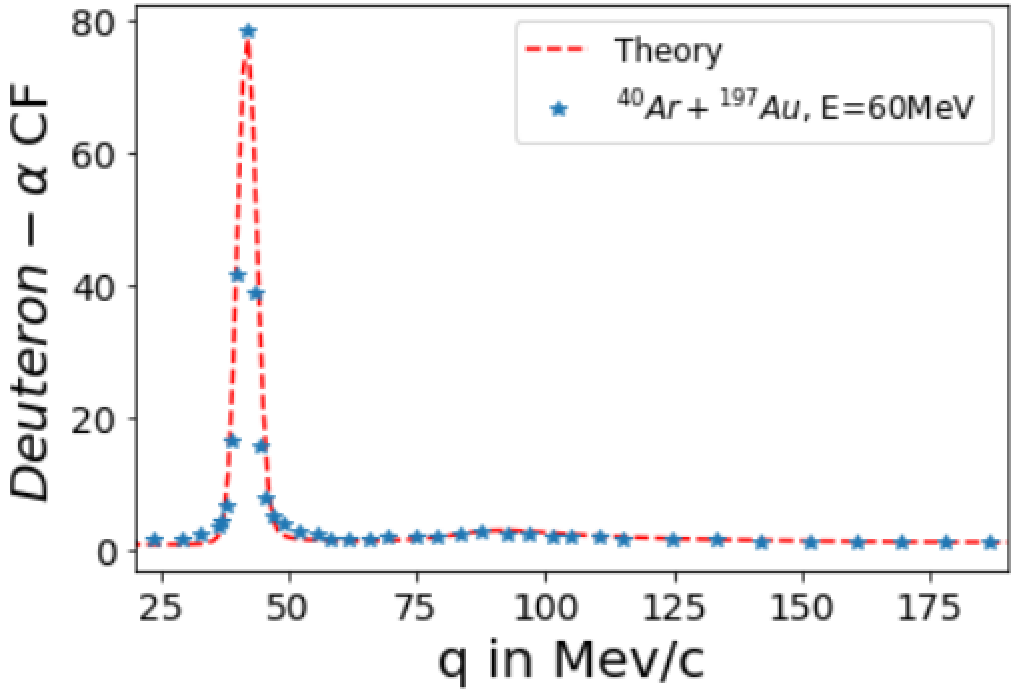
D-alpha phase shifts;

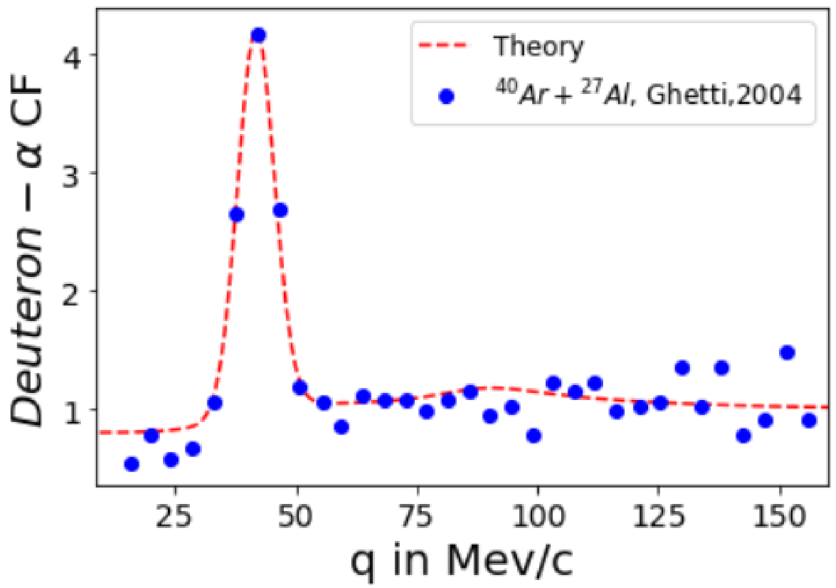
d-wave (**$^3D_3$,$^3D_2$,$^3D_1$ states are responsible of low energy region resonance in ^{6}Li).**

-Data from **(P. E. Shanley, 1969)**



Deuteron-Alpha CF





α-d CF with data(stars) from $^{40}Ar+^{197}Au$ reaction at E=60MeV/U (**Z. Fan Phd thesis, 1992).**

The estimated Gaussian source par: R=2.3 fm and λ=.32

α-d CF with data(stars) from $^{40}Ar+^{27}Al$ reaction at E=40MeV/U **(R. Ghetti, et al., 2004).**

The estimated Gaussian source par: R=6.3 fm and λ=.3

The peak at q=42 MeV/C corresponds to: $J=3^+$ state of ^{6}Li at E=2.186 decay.

The peak between 80-100 MeV/C is due to overlap of ^{6}Li state at E=4.31MeV ($^3d_1$) and E=5.6MeV ($^3d_2$)

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Deblurring

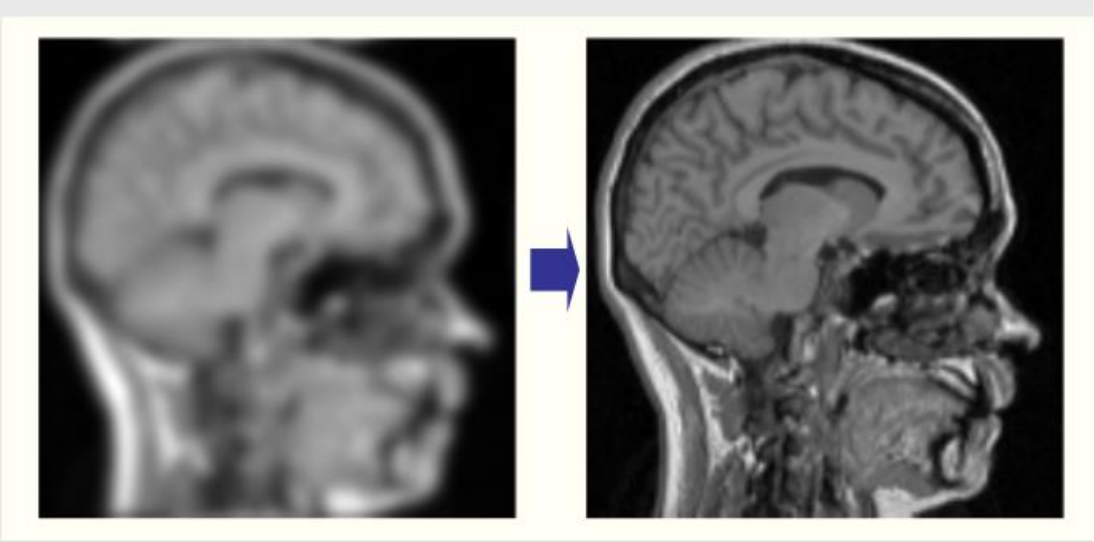
1. Deblurring in optical image processing

(a)Sharp (b) Blurred and noisy (c) Deblurred

Source: https://doi.org/10.1016/j.matpr.2020.11.076

2. Deblurring in Medicine

To locate a brain tumor.



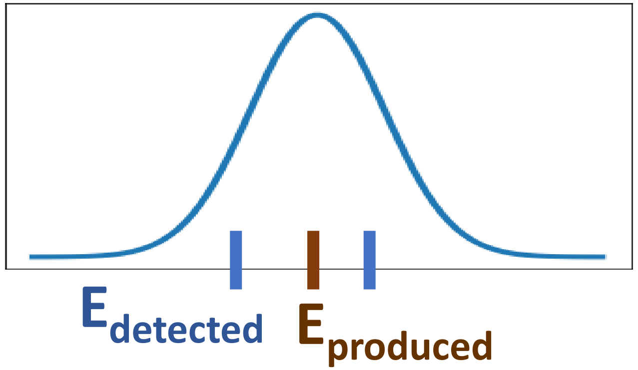
Source: https://slideplayer.com/slide/9530694/

Detector efficiency ε, n measured particle number, N actual number $n \simeq \varepsilon N$ probabilistically,

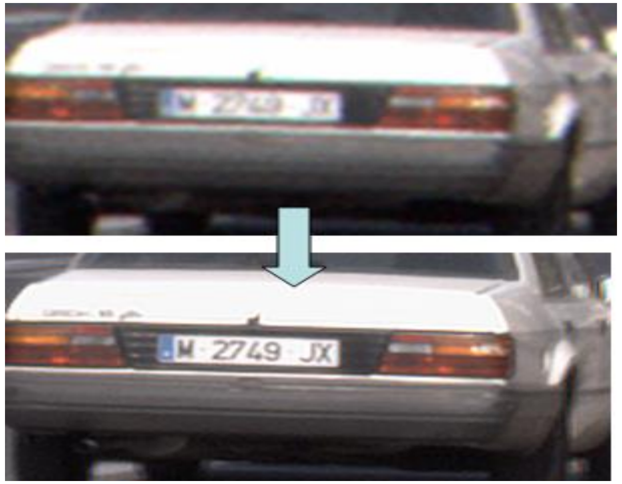
$$n(E_d) = \int dE'_d P(E_d|E'_d) N(E'_d).$$

with $P(E_d|E'_d)$ – probability to measure particle characteristic to be E_d when it is actually E'_d

Optical terminology: P - blurring or transfer function.



Schematic description of prob. density of energy measurements



Car plate identification

http://zoi.utia.cas.cz/restoration.html

Baye's theory for deblurring

Measured/blurred observable is defined as:

$$f(E_d) = \int H(E_d|E'_d)F(E'_d)dE'_d$$

In discretized form

$$f_i = \sum_j H_{ij}F_j$$

-We shall use discrete form in the derivation

-$\sum_j H_{ij} = 1$, implies $\sum_j f_j = \sum_j F_j = N$ all produced particle were detected

-f, H are know, except F

- Assume:

- $p(F_i) \sim \frac{F_i}{N}$, -probability that F_i occurs, and $p(f_i) \sim \frac{f_i}{N}$ -probability that F_i occurs .

- Then Baye's theory:
- $H_{ki} = p(F_k|f_i) = \frac{p(f_k|F_i)p(F_i)}{\sum_j p(f_k|F_j)\, p(F_j)}$,
- $p(F_k|f_i)$ - probability that F_k occurs given f_i and is a complement of $p(f_k|F_i)$

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Phys. Rev. C 105, 034608 (2022)

G. D'Agostini, Nucl. Instrum. Methods. Phys. Res. A 362, 487 (1995).

Richardson-Lucy (RL) Algorithm

From the previous page, we can show

$$F_i = F_i \sum_k \frac{H_{ki} f_k}{\sum_j H_{kj} f_j}$$

F: original signal (we want to restore)

RL-algorithm solves F_i iteratively;

$$F_i^{(n+1)} = \sum_j \frac{f_i}{f_i^n} T_{ji} F_i^n,$$

where

- $f^n(E) = \int H(E|E') F^n(E') dE'$,

- n = number of iterations and $F^0 > 0$, - we start with F^0=1
- $T_{ji} = \frac{W_j H_{ji}}{\sum_{j'} W_{j'} H_{j'i}}$ $\qquad \sum_j T_{ji} = 1$
- F^n, T_{ji}, f_i, and H_{ji} are kept positive

Add a noise term on the blurring model

$\hat{f}_i = \sum_j H_{ij} F_j + N_j \sim \mathcal{P}\left(\sum_j H_{ij} F_j\right)$, $\mathcal{P}$: statistical noise model

RL algorithm becomes:

$$F_i^{(n+1)} = \sum_j \frac{\hat{f}_j}{\hat{f}_j^n} T_{ji} F_i^n$$

Regularization to stabilize the solution:

$$I_i^n = \begin{cases} \frac{1}{1-\Lambda} & F_i^n < F_{\{i-1,i+1\}}^n \\ \frac{1}{1+\Lambda} & F_i^n > F_{\{i-1,i+1\}}^n \\ 1 & otherwise \end{cases}$$

$F_i^{(n+1)} = \sum_j \frac{\hat{f}_j}{\hat{f}_j^n} T_{ji} I_i^n F_i^n$, Λ is a constant

W. H. Richardson, JoSA 62, 55 (1972).

L. B. Lucy, the Astronomical journal 79, 745 (1974)

P. Danielewicz and M. Kurata-Nishimura, Physical Review C 105, 034608 (2022)

RL algorithm for source function

- Recall: $R(\boldsymbol{q}) = \int d^3r\, S(\boldsymbol{r})\, (|\Psi(\boldsymbol{q},\boldsymbol{r})|^2 - 1)$ and $C(\boldsymbol{q}) = \int d^3r S(\boldsymbol{r})|\Psi(\boldsymbol{q},\boldsymbol{r})|^2$.

- Write CF in discretized form: $R_i = \sum_{j=1}^{N} K_{ij} S_j$ where $\boldsymbol{K(r,q) = |\Psi(r,q)|^2 - 1}$ or $\boldsymbol{K(r,q) = |\Psi(r,q)|^2}$ for $R(\boldsymbol{q})$+1.
- Source function: $S \cong \sum_{j=1}^{N} S_j(\boldsymbol{r}) g_j(\boldsymbol{r})$, and $g_j(\mathrm{r}) = \begin{cases} 1 & r_{j-1} < r < r_j \\ 0 & otherwise \end{cases}$, [1,2]

- $K_{ij} = 4\pi \int_{r_{j-1}}^{r_j} dr\, r^2 K(q_i, r)$, Kernel Matrix (Transfer matrix in the deblurring terminology)
- RL algorithm one can calculate SF: R_i has some negative values but $C_i = R_i + 1$ is positive.

- $S_i^{(n+1)} = S_i^n \sum_j \frac{C_i}{C_i^n} \frac{|W_j K_{ji}|}{\sum_{j'} |W_{j'} K_{j'i}|}$, where $C_i^n = R_i^n + 1$, the absolute value prevent K to be negative.

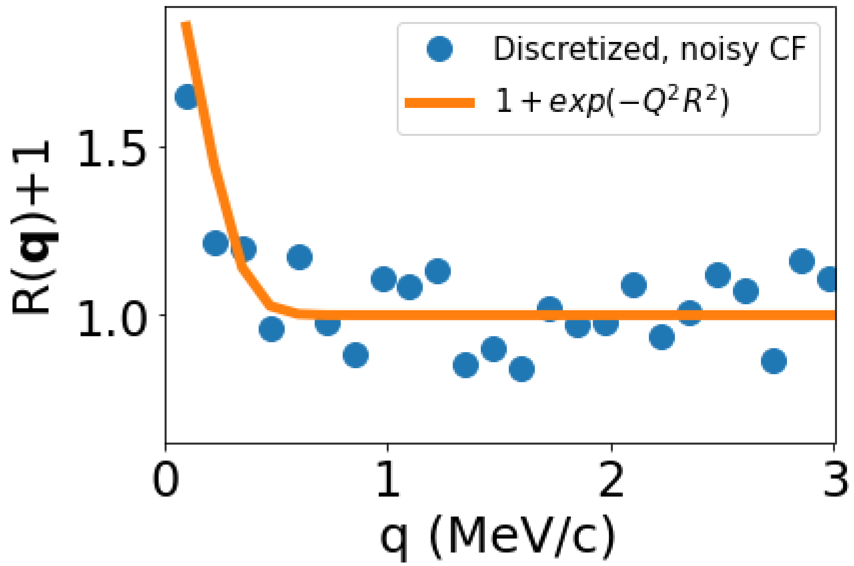
[1] *D.A. Brown and P. Danielewicz, Phys. Lett. B 398, 252 (1997)*

[2] *D.A. Brown and P. Danielewicz, Phys. Rev. C 64, 014902 (2001).*

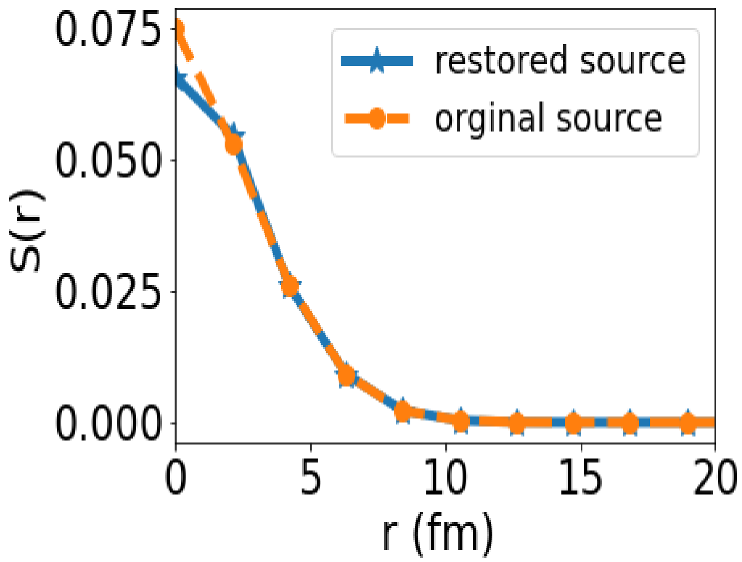
The ATLAS collaboration., Aad, G., Abajyan, T. *et al.* . *J. High Energ. Phys.* **2013,** 183 (2013)

Testing: Deblurring SF for $\pi^0 - \pi^0$ system

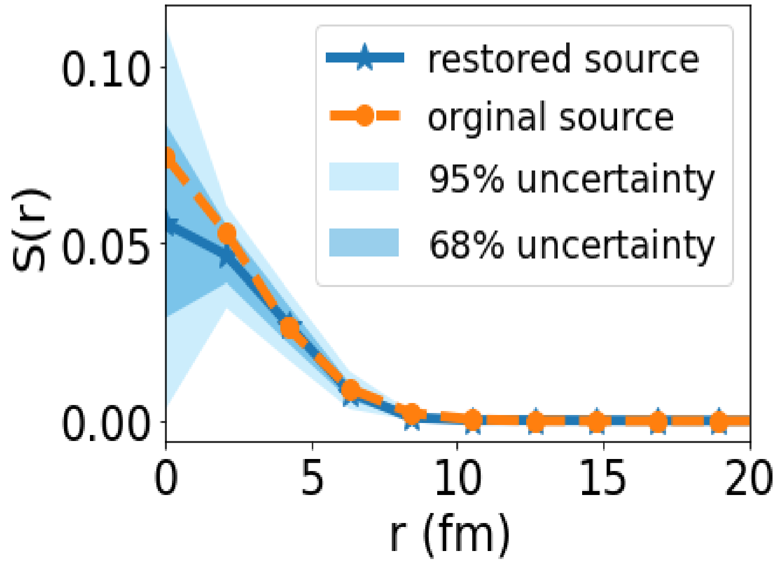
- $K(r,q) \approx \frac{\sin(rq)}{2rq}$ and $S(\boldsymbol{r}) \sim \exp\left(-\frac{\boldsymbol{r}^2}{2R_0^2}\right)$, Gaussian source function of radius R_0
- $\mathrm{R}(q) + 1 \approx 1 + \exp(-q^2 R_0^2)$ (***D.A. Brown and P. Danielewicz, 1997***)



Uncharged $\pi^0 - \pi^0$ Correlation function, $R_0 = 4$ fm



For smooth Correlation function

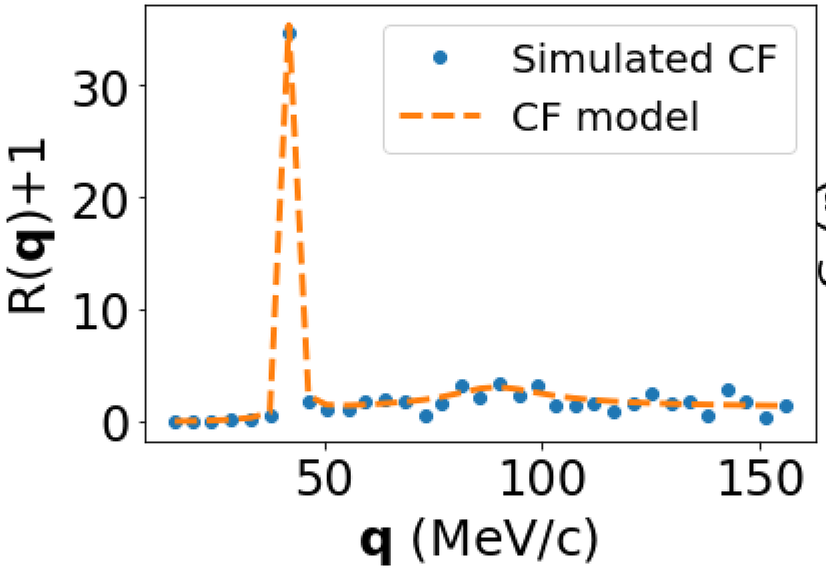


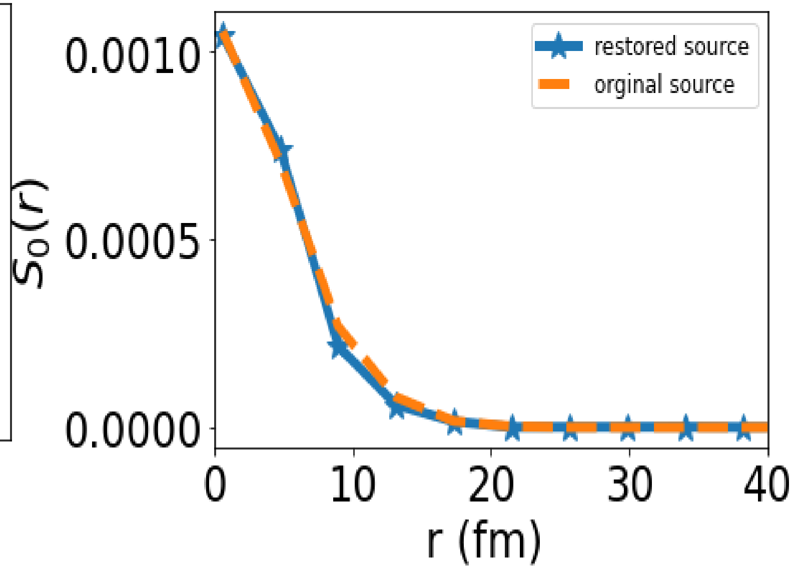
Deblurring S(r) for Noisy Correlation function

$d-\alpha$ source function recovering

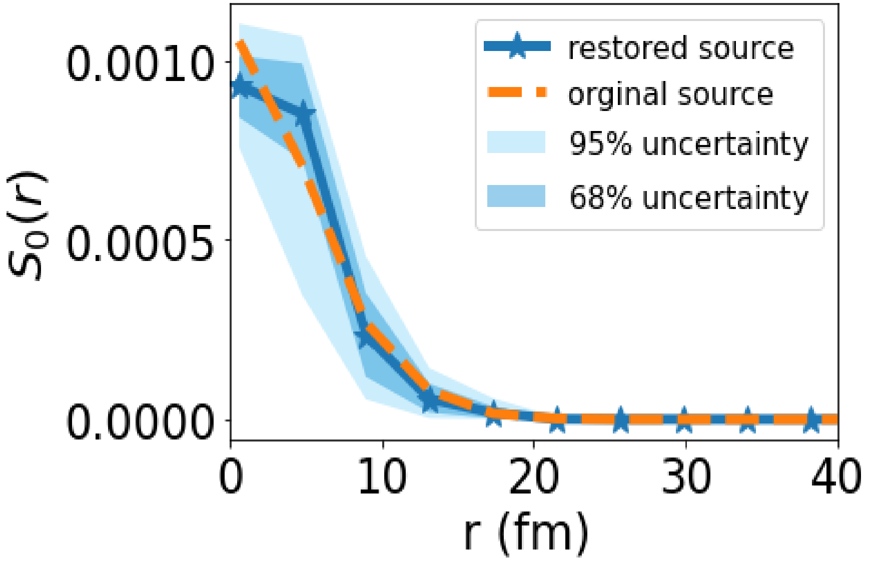
- This is the angle average SF
- Simulated $d-\alpha$ CF assuming Gaussian source function with radius, $R_0 = 6.5$ fm and $\lambda = 0.3$.

- Peaks corresponds to resonance of ^{6}Li - 2.26 MeV and 4.4 MeV.
- Restored (blue star line) and original (orange dashed) SF for $d-\alpha$ system.





For smooth Correlation function



Deblurring S(r) for Noisy Correlation function. With $\Lambda = 0.05$

The blue bands are uncertainties resulting from SF restoration for noisy CF.

Summary and outlook

- Deuteron-alpha correlation functions were studied, and we extracted the parameters of the source function.
- Discuss deblurring for Source function
- The TM is very different from the optics one:

    - For CF the TM is due to physics.

Outlook:

Deblurring d-alpha source function for experimental CF.

THANK YOU FOR YOUR ATTENTION!

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