EFFECTIVE FIELD THEORY ANALYSIS OF $^3\text{He} - \alpha$ SCATTERING DATA

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Plan:

- Motivation
- Build EFT for the scattering
- Discuss results, conclusions and future
MOTIVATION (I):

• Recent $^3$He $- \alpha$ scattering experiment using Scattering of Nuclei in Inverse Kinematics (SONIK) detector at TRIUMF facility, Canada
  • This experiment covers energy range 0.38 - 3.13 MeV in center of mass

• Important scales for EFT = $p_{typ}$ and $\Lambda$
  • $\alpha$-threshold for $^7$Be = 1.5874 MeV well separated from $S_{1n} = 10.677$ MeV and $S_{1p} = 5.6069$ MeV,
  • First excitation energy of the $\alpha$ particle $\sim$ 20 MeV
  • Corresponds to breakdown momentum of 150-200 MeV.
The uncertainties limit our ability to precisely test the solar models using neutrino data.
Motivation (II) contd:

To understand the capture reaction $^3$He($\alpha, \gamma$)$^7$Be phenomenologically

- 8 previous measurements but only few reliable low energy data available
  - Miller and Phillips (1958), Mohr et. al. (1993) - No error quantification
  - L. Chuang (1971) - 31 data points total at $E_{lab} = 2.98, 2.46, 1.72$ MeV
  - Roos et. al. (1980), Gorpinich et. al. (1992) - $E_{lab} > 10$ MeV
  - Spiger and Tombrello (1967) - $E_{lab} > 4.6$ MeV and errors not provided
  - Tombrello and Parker (1963) - Primary focus on level structure and resonances in $^7$Be
  - Barnard et. al. (1964) - $E_{lab} = 2.5 - 5.7$ MeV

Next: What we will do?
What we will do.

1. Construct t-matrix from Effective Field Theory

2. Borrow t-matrix from general theory of scattering

3. Match the powers of energy ($p^2$) between the t-matrices

4. Express the constants of EFT (LECs) with the effective range parameters of Effective Range expansion (ERPs)

5. Perform Bayesian MCMC sampling

6. Construct an Error model

7. Extract the ERPs as posterior pdf from Bayesian sampling
EFT of $^3\text{He} - \alpha$ system:

$$\mathcal{L} = \mathcal{L}_{\text{free}} + \mathcal{L}_{\text{Coulomb}} + \mathcal{L}_S + \mathcal{L}_{P_{1+}} + \mathcal{L}_{P_{1-}}$$

$^7\text{Be} = ^3\text{He} + \alpha$

$$\mathcal{O} \propto \sum_{\nu} c_{\nu} \left( \frac{p_{\text{typ}}}{\Lambda} \right)^{\nu}$$

$\alpha$-threshold for $^7\text{Be} = 1.5874$ MeV well separated from $S_{1n} = 10.677$ MeV and $S_{1p} = 5.6069$ MeV,

First excitation energy of the $\alpha$ particle $\sim 20$ MeV

Corresponds to breakdown momentum of 150-200 MeV.
Field picture of the $^3$He $- \alpha$ system (dimer formalism):

- Dashed $= \phi = \alpha$-field
- Solid $= \psi = ^3$He-field

Dyson Equation for the full dimer propagator

Full Coulomb propagator
EFT of $^3$He – α system (contd...):

\[
tagon{t}_L^\pm = (g_L^\pm)^2 \frac{\Gamma(2L+2)}{2^L \Gamma(L+1)} \left[ \frac{\Gamma(2L+2)}{2^L \Gamma(L+1)} \right]^2 \mathcal{C}_L^2(\eta)e^{2i\sigma_L p^{2L}} \]

\[
\Delta_L^\pm + \omega_L^\pm (E - \frac{\vec{P}^2}{2M}) + \Xi_L^\pm (E - \frac{\vec{P}^2}{2M})^2 + i\epsilon - \Sigma_L^\pm(E)
\]

EFT parameters (LECs) are:

• $g$ which is the vertex in the diagrams and the strength of interaction
• $\Delta$ is the bare dimer binding energy.
• $\omega$ is dimensionless constant while $\Xi$ has dimension of inverse energy.
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T-matrix from general theory of scattering/ effective range theory (ERT):

\[
T_L^\pm(E + i\epsilon) = -(2L + 1)\frac{2\pi}{\mu} \left[ \frac{\Gamma(2L + 2)}{2L\Gamma(L + 1)} \right]^2 \frac{C_L^2(\eta)e^{2i\sigma_Lp^{2L}}}{\left[ \frac{\Gamma(2L + 2)}{2L\Gamma(L + 1)} \right]^2 C_L^2(\eta)p^{2L+1}(\cot \delta_L^\pm - i)}
\]

ERT parameters (ERPs) are:

- Coefficients of \( p^2 \) apart from the numerical factors viz. \( a, r, P \) etc.
1. Construct t-matrix from Effective Field Theory

2. Borrow t-matrix from general theory of scattering

3. Match the powers of energy \( (p^2) \) between the t-matrices

4. Express the constants of EFT (LECs) with the parameters of Effective Range expansion (ERPs)

5. Perform Bayesian MCMC sampling

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7. Extract the ERPs as posterior pdf from Bayesian sampling
Match the t-matrices from EFT and ERT:

\[ t_0 = T_0 \]
\[ \frac{-1}{a_0} = -\frac{2\pi}{\mu(g_0)^2}(\Delta_0 - \Sigma_0 \text{div}) \]

\[ r_0 = -\frac{2\pi \omega_0}{\mu^2(g_0)^2} \]

\[ P_0 = -\frac{2\pi \Xi_0}{\mu^3(g_0)^2} \]

\[ t_1^\pm = T_1^\pm \]
\[ \frac{-1}{a_1^\pm} = -\frac{6\pi}{\mu(g_1^\pm)^2}(\Delta_1^\pm - \Sigma_0^0 \text{div}) \]

\[ r_1^\pm = -\frac{6\pi}{\mu(g_1^\pm)^2}(\omega_1^\pm - 2\mu \Sigma_1^1 \text{div}) \]

\[ P_1^\pm = -\frac{6\pi \Xi_1^\pm}{\mu^3(g_1^\pm)^2} \]

ERPs \equiv \text{ERPs (LECs)}

Next : What we have done so far?
1. Construct t-matrix from Effective Field Theory

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Next: p-wave bound states, ANC and WRC
Additional information from the bound state of $^7$Be:

- There are shallow p-wave bound states, respectively, the $\frac{3}{2}^-$ and $\frac{1}{2}^-$ channels i.e $T_1$ has poles at certain momentum.

\[
\left( -\frac{1}{a_1} + \frac{1}{2} r_1 (i\gamma_1)^2 + \frac{1}{4} P_L (i\gamma_1)^4 \right) - 2k_c (i\gamma_1)^2 (1 + (-i\eta_B)^2) H(-i\eta_B) = 0 \Rightarrow a_1 \equiv a_1(r_1, P_1)
\]

- From the relationship between ANCs and WRCs

\[ r_1 \equiv r_1(C_1, P_1) \]

- We now need to extract only following ERPs:

\[ a_0, r_0, C^{\pm}_1, P^{\pm}_1 \]
Details of Parameter space, Bayesian formulation and MCMC sampling

- The parameter space $\theta \in (a_0, r_0, P^+_1, P^-_1, C^+_1, C^-_1, \{f_\gamma\}, \Gamma_{\frac{7}{2}})$

- Bayes theorem: $p(\{\theta_i\} | D, I) = \frac{p(D | \{\theta_i\}, I) \ p(\{\theta_i\} | I)}{p(D | I)}$

- $p(D | \theta, I) \equiv \mathcal{L} = \frac{1}{\sqrt{(2\pi)^N\det(\Sigma^\text{expt} + \Sigma^\text{th})}} e^{-\chi^2/2}$

- $\chi^2 = \bar{r}^T(\Sigma^\text{expt} + \Sigma^\text{th})^{-1}\bar{r}$

- $r_\alpha = f_\gamma \ Y_\alpha - Y_\alpha$

- We used open source ‘emcee’ package which will perform MCMC sampling for us.
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The effect of tail of f-wave resonance:

- Validity region
  \[ p \in (60, 90) \text{ MeV} \equiv (2.0, 4.5) \text{ MeV} \]
  energy does avoid the \( \frac{7}{2}^- \) and \( \frac{5}{2}^- \) resonance level of \(^{7}\text{Be}\)
  \( \frac{7}{2}^- \) level peaks at 5.22 MeV, \( \frac{5}{2}^- \) level peaks at 9.02 MeV with width 1.9 MeV

- We will include the tail effects phenomenologically using the \( R \)-matrix formula
  \[
  \delta_L + \phi_L = \tan^{-1} \left[ -\frac{\Delta_L(E, \rho)}{S_L(E, \rho)} \frac{P_L(E, \rho)}{E_L + \Delta_L(E, \rho) - E} \right]
  \]

Next: power counting
Power counting hierarchy:

\[ y(p, \theta) = y_{\text{ref}}(p, \theta) \sum_{\nu} c_{\nu}(p, \theta) \left( \frac{p_{\text{typ}}}{\Lambda} \right)^{\nu} \]

- \( p_{\text{typ}} = \max\{q, p\} \); \( q \) is momentum transfer

Data: \( a_0 \) (fm), \( r_0 \) (fm), \( a_{1+} \) (fm\(^3\)), \( r_{1+} \) (fm\(^{-1}\)), \( P_{1+} \) (fm), \( a_{1-} \) (fm\(^3\)), \( r_{1-} \) (fm\(^{-1}\)), \( P_{1-} \) (fm), \( \Gamma_{\frac{7}{2}} \)

<table>
<thead>
<tr>
<th>( S+A_y )</th>
<th>( a_0 ) (fm)</th>
<th>( r_0 ) (fm)</th>
<th>( a_{1+} ) (fm(^3))</th>
<th>( r_{1+} ) (fm(^{-1}))</th>
<th>( P_{1+} ) (fm)</th>
<th>( a_{1-} ) (fm(^3))</th>
<th>( r_{1-} ) (fm(^{-1}))</th>
<th>( P_{1-} ) (fm)</th>
<th>( \Gamma_{\frac{7}{2}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>60(6)</td>
<td>0.78(2)</td>
<td>172(5)</td>
<td>-0.082(4)</td>
<td>1.50(4)</td>
<td>288(15)</td>
<td>-0.044(6)</td>
<td>1.65(6)</td>
<td>159(7)</td>
<td>407 16 0.86</td>
</tr>
</tbody>
</table>

\[ \frac{1}{a_0} / \zeta_0 \]
\[ \frac{1}{2} r_0 p^2 / \zeta_0 \]

\[ 1/a_{1+} / \zeta_1 \]
\[ \frac{1}{2} |r_{1+} p^2| / \zeta_1 \]
\[ \frac{1}{4} P_{1+} p^2 / \zeta_1 \]

\[ 1/a_{1-} / \zeta_1 \]
\[ \frac{1}{2} |r_{1-} p^2| / \zeta_1 \]
\[ \frac{1}{4} P_{1-} p^2 / \zeta_1 \]
Error Model and its validity:

- The first omitted term in the power counting series of \( y(p, \theta) \) will induce an error

\[
\Delta y(p, \theta) = y_{\text{ref}}(p, \theta)c_{\text{rms}}^{\nu_{\text{max}}+1}
\]

\[
c_{\nu}^{\text{rms}}(p_{\gamma}) = \sqrt{\frac{1}{N_\eta} \sum_\eta c_{\nu}^2(p_{\gamma}, \theta_\eta)},
\]

\( \Lambda = 200 \text{ MeV} \)

\( E_{\text{lab}} = 2.1 \text{ MeV} \)
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Findings and Results (I):

Bimodality of the truncated SONIK data:

Lower Region

Higher Region

Next : Add Ay
Remedy of the bimodality:

Spin-orbit splitting is not directly probed in the cross-section.

We add analysing power data by Boykin et. al. to the cross section data to resolve the sign of splitting.

\[
A_y(\theta) \equiv \frac{d\sigma}{d\Omega \uparrow} - \frac{d\sigma}{d\Omega \downarrow} - \frac{d\sigma}{d\Omega \uparrow} + \frac{d\sigma}{d\Omega \downarrow}
\]
All results henceforth include the effect of tail of f-wave resonance, theoretical error model and analysing power data
Results for the truncated SONIK data with $A_y$ included (I):

Cross sections at LO, NLO and NNLO:

<table>
<thead>
<tr>
<th>Energy (MeV)</th>
<th>LO</th>
<th>NLO</th>
<th>NNLO</th>
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<tbody>
<tr>
<td>0.7</td>
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<td>1.7</td>
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<td>3.6</td>
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<tr>
<td>4.3</td>
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$s$-wave $p$-wave $\nu$

<table>
<thead>
<tr>
<th>Energy (MeV)</th>
<th>$r_0$</th>
<th>$r_1^+$, $P_1^\pm$</th>
<th>$\frac{1}{a_0}$, $r_1^-$</th>
</tr>
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<tbody>
<tr>
<td>0.7</td>
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Results for the truncated SONIK data with $A_y$ included (II): $A_y$ and phase shifts:

- $\theta = 71.6^\circ$
- $\theta = 87.0^\circ$
- $\theta = 120.0^\circ$
Results from all the analysis we did and previous analyses:

- Higa et al. analyzed the data on radiative capture reaction $^3\text{He} + \alpha \rightarrow ^7\text{Be} + \gamma$ alongside Boykin phase shifts and extracted:
  
  \[
  a_0 = 21.6 \pm 3.4 \text{ fm} \\
  r_0 = 1.2 \pm 0.1 \text{ fm} \\
  P_1^+ = 1.59 \pm 0.03 \text{ fm} \\
  P_1^- = 1.74 \pm 0.05 \text{ fm}
  \]

- Premarathna and Rupak analysis of the capture reaction alongside phase shift gives:
  
  \[
  a_0 = 40^{+5}_{-6} \text{ fm} \\
  r_0 = 1.09^{+0.09}_{-0.1} \text{ fm}
  \]

- Zhang, Nollett and Phillips analysis of the capture reaction gives:
  
  \[
  a_0 = 60 \pm 6 \text{ fm} \\
  r_0 = 0.97 \pm 0.03 \text{ fm}
  \]
Our analysis vs previous analyses:

- Higa et. al. analyse the data on radiative capture reaction $^3\text{He} + \alpha \rightarrow ^7\text{Be} + \gamma$ alongside Boykin phase shifts.
- Premarathna and Rupak analyse the capture reaction alongside phase shift.
- Zhang, Nollett and Phillips analyse the capture reaction.
Summary, conclusion and future of $^3$He $- \alpha$ analysis:

- All the momentum-scale ERPs ($\frac{1}{a_0}, \frac{1}{a_1^\pm}, r_1^\pm$) are unnaturally small compared to what is expected on the basis of naive dimensional analysis with respect to $\Lambda$, while all the length-scale ERPs ($r_0, P_1^\pm$) tend to be natural when expressed in units of $1/\Lambda$.

- Analysing power data is required to resolve the sign of spin-orbit splitting at lower energies even though the contributions from higher partial waves get smaller. So, more analysing power data required.

- Analysis of both the scattering data together with the capture data seems imminent.

- We may include N3LO parameters in the analysis -parameters all of dimension fm$^3$ in the s-wave and p-wave channels.

- Include higher order electromagnetic effects to facilitate consistent extrapolation to lower energies.
Where to read the work?

- [https://arxiv.org/abs/2110.01451v1](https://arxiv.org/abs/2110.01451v1)

- Submitted to Journal of Physics G: Nuclear and Particle Physics
  🌟 Received referees’ comments
Thanks!
Comparison of central values (pictorial):