Transfer Reactions and R Matrix

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Transfer Reactions and R Matrix

- What do we want to accomplish?
- Why are transfer reactions good for this?
- How do you do it?
- Details, computer codes, and so forth...
- Related topics: Trojan Horse Method, Surrogate Method.

What we would like to know:

- Recall our practical definition of a partial width:

$$\Gamma_c = 2N\gamma_c^2 P_c(a_c)$$

and Asymptotic Normalization Constant (ANC)

$$C_c = \frac{(2\mu_\alpha a_c)^{1/2}}{\hbar W_\ell(a_c)} N^{1/2}\gamma_c,$$

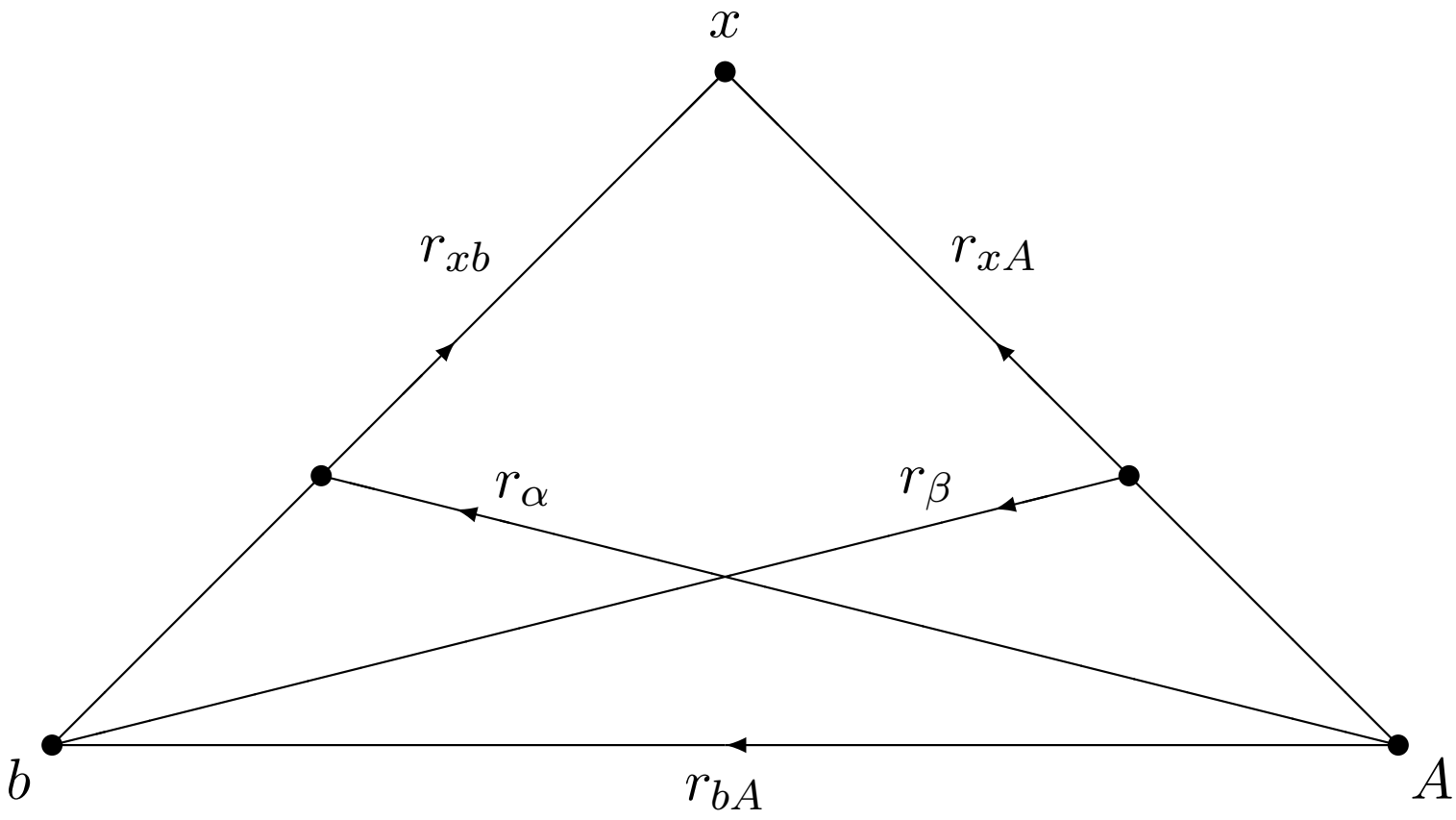
where $N^{-1} = 1 + \sum_c \gamma_c^2 \left.\frac{dS_c}{dE}\right|_{E_\lambda}$.

- E_x of levels.
- J^π of levels.
- Partial widths Γ_c or ANCs C_c of levels.
- Motivations could come from astrophysics, nuclear structure, applications, etc. . .

Transfer reactions can answer these questions

- An energy spectrum of the outgoing particles gives E_x with zero model dependence.
- The magnitude and shape of the differential cross section can supply information about the Γ_c and J^π, with the help of reaction theory.

Transfer Reaction Coordinates



For $a + A \to b + B$ and $a = b + x$ and $B = A + x$. We say "x is transferred from b to A to form B." Following G.R. Satchler, *Direct Nuclear Reactions*, (Oxford University Press, New York, 1983).

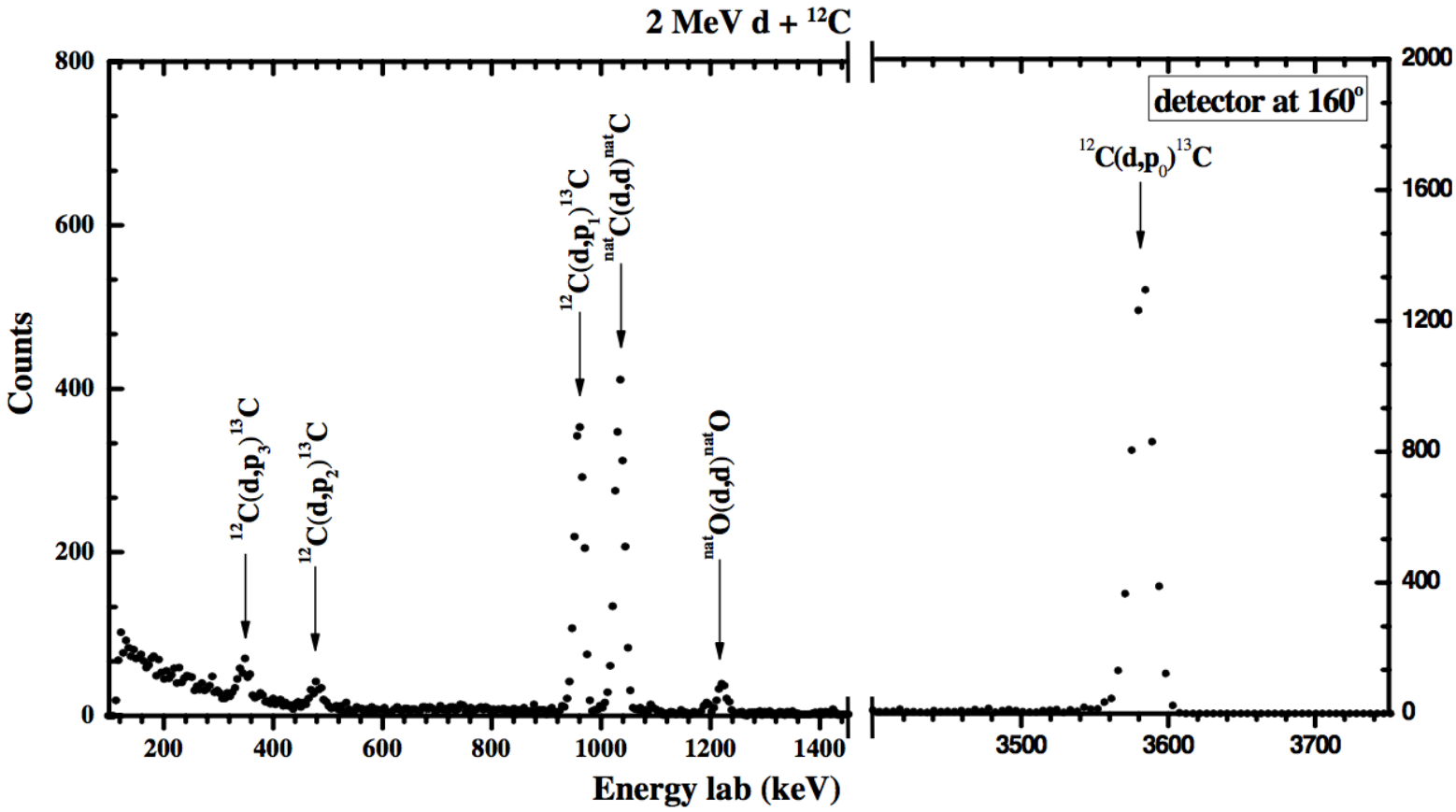
Example: $^{12}\mathrm{C}(d,p)^{13}\mathrm{C}$

- If we do the experiment by bombarding a $^{12}\mathrm{C}$ target with a deuteron beam and detecting the protons, the kinematics tells us what the residual excitation energy in $^{13}\mathrm{C}$ must have been.

- To connect with Satchler's notation:
  $a = d = n + p$ – beam
  $A = {}^{12}\mathrm{C}$ – target
  $x = n$ – transferred particle
  $b = p$ – detected particle
  $B = {}^{13}\mathrm{C} = {}^{12}\mathrm{C} + n$ - residual nucleus

- In the experiment, the residual nucleus $^{13}\mathrm{C}$ may be left in many different excited states, depending on how much energy is available.

Example: An actual $^{12}\mathrm{C}(d,p)^{13}\mathrm{C}$ Experiment



Particle energy spectrum for $E_d = 2.000$ MeV, and $\theta_{\mathrm{lab}} = 160°$, M.Kokkoris *et al.*, Nucl. Instrum. Mehtods B **254**, 10 (2007).

More about what you can learn

- Assume $a + A \rightarrow b + B$ and $a = b + x$ and $B = A + x$.
- The angular distribution may provide information about the J^π. This is very important, but I will not discuss it further.
- The reaction is mostly sensitive to the long-range cluster components of the projectile and final state at the nuclear surface and beyond, with little sensitivity to what goes on at short distances.
- This implies the cross section is sensitive to the reduced width γ_c for $\alpha = b + x$ in the target and $\alpha = A + x$ in the residual nucleus state.
- I will refer to b as the "projectile" and B as the "residual nuclear state" in what follows. Keep in mind that experiments can be done in many different ways.

Transfer Reaction Models

- Transfer reaction calculations assume simple two-cluster wavefunction components for the projectile and residual nuclear state. These are also known as single-particle wavefunctions, and are normalized to unity over all space (at least for bound states). More complicated structures can be implemented, but will not be considered here.
- In this picture, the transfer reactions is a three-body problem.
- The calculation can be done first-order perturbation theory, an approach known as the Distorted-Wave Born Approximation (DWBA).
- Optical potentials are required for $A + a$ and $B + b$.

Spectroscopic Amplitudes and Factors

- In general, a single-particle wavefunction needs to be renormalized by a *Spectroscopic Amplitude* $\mathcal{A}$ in order to reflect nuclear structure. One can also define the *Spectroscopic Factor* $S = |\mathcal{A}|^2$.
- It is common to have only a single component in the projectile and residual nuclear state. Then, in DWBA, the differential cross section is proportional $S_{bx}S_{Ax}$, where
  - S_{bx} is for configuration $b + x$ in the projectile a
  - S_{Ax} is for configuration $A + x$ in the residual B
- If the DWBA calculation is done with $\mathcal{A}_{bx} = \mathcal{A}_{Ax} = 1$, one has

$$\frac{d\sigma}{d\Omega} = S_{bx}S_{Ax}\left(\frac{d\sigma}{d\Omega}\right)_{\text{DWBA}}.$$

- $S_{bx}S_{Ax}$ may be determined by comparison to experiment.
- S_{bx} is likely to be reasonably well known in advance.

Multiple Components

- If there are multiple components in the projectile or residual nuclear state, that part of the wavefunction in the DWBA calculation should be constructed with spectroscopic amplitudes.
- The spectroscopic amplitudes can be defined to be real, but their relative signs can be important.
- For example, the deuteron in a (d, p) neutron transfer reaction could be modeled using both s- and d-wave components. This turns out to be negligible, unless one is measuring deuteron tensor analyzing power.

More About Spectroscopic Amplitudes and Factors

- Spectroscopic amplitudes and factors are *not* observable quantities.
- They depend on short-range physics, including the binding potentials used for $b + x$ and $A + x$ and the numbers of radial nodes assumed.
- They result naturally in Shell Model calculations.
- The spectroscopic amplitude can take into account isospin via

$$\mathcal{A}_{Ax} = (T_A m_A T_x m_x | T_B m_B) \mathcal{A}.$$

- You will often see the spectroscopic factor written as C^2S, where the square of the isospin Clebsch-Gordan coefficient C^2 has been factored out.

What Transfer Reactions do Measure

- To some approximation, the amplitude for $A + x$ at the nuclear surface is measured.
- This suggest that partial widths, ANCs, and R-matrix reduced widths are natural quantities to determine.
- Let's use the label c for the $A + x$ configuration, and assume its single-particle radial wavefunction is $v_c(r)$. We can then relate this to the R-matrix eigenfunction $u_c(r)$:

$$\mathcal{A}_c v_c(r) = N^{1/2} u_c(r),$$

  where N changes the normalization volume of the R-matrix wavefunction to all space.
- It is tempting to use this equation at $r = a_c$ to determine γ_c. **Resist.** For phenomenological R-matrix channel radii and the $A + x$ binding potential, it is unlikely that $v(r)$ has reached its asymptotic form at a_c.

Proceeding More Carefully

- Extend to larger radii (assuming the state is bound):

$$\mathcal{A}_c C_{\text{s.p.}} W_c(r) = N^{1/2} \left(\frac{2\mu a_c}{\hbar^2} \right)^{1/2} \gamma_c \frac{W_c(r)}{W_c(a_c)}.$$

- The quantity $C_{\text{s.p.}}$ is the single-particle ANC for w_c. It is provided by FRESCO.

- We thus have

$$C_c^2 = S_c C_{\text{s.p.}}^2 = \frac{2\mu a_c}{\hbar^2 W_c^2(a_c)} N \gamma_c^2.$$

- The annoying model dependence drops out of the product $S_c C_{\text{s.p.}}^2$. This can be confirmed with numerical experiments, e.g. with FRESCO.

Unbound States Present Several Complications

- The transfer calculation must be integrated over a range of energies (continuum bin states in FRESCO). Experimentalists will probably need an expert collaborator...

- If the state is not "narrow," one must consider background phase shifts and interference effects.

- For (d,p) or (d,n) reactions to narrow levels, the zero-range code DWUCK4 is a reasonable option. It calculates the cross section integrated over the extent of the resonance and also provides $\Gamma_{\text{s.p.}}$, such that

$$\Gamma_c = S_c \Gamma_{\text{s.p.}} = 2N\gamma_c^2 P_c(a_c).$$

- If the state is near the threshold, one can extract γ_c for a range of energies below the threshold and extrapolate to the physical energy.

Definition of Spectroscopic Factor

- There are two slightly different conventions for implementing the spectroscopic factor.
- One approach is the one used here, that assumes

$$\Gamma_c = S_c \Gamma_{\text{s.p.}} \quad \text{or} \quad C_c^2 = S_c C_{\text{s.p.}}^2.$$

- Another definition that is sometimes seen is

$$\gamma_c^2 = S_c \gamma_{\text{s.p.}}^2$$

  See M.H. McFarlane and J.B. French, Rev. Mod. Phys. **32**, 567 (1960) or F.C. Barker, Phys. Rev. C **56**, 3423 (1997).
- The two approaches are not equivalent due to the N factor.
- For connecting transfer reactions to R-matrix parameters, the first approach is definitely correct.
- There may be some contexts where the latter definition makes sense.

Common Transfer Reactions

- neutron transfer: (d, p)
- proton transfer: (d, n), $(^3\mathrm{He}, d)$
- α transfer: $(^6\mathrm{Li}, d)$, $(^7\mathrm{Li}, t)$
- For radioactive beam experiments, (d, p) and (d, n) have the largest cross sections and are preferred.
- Heavy ion transfers, such as $(^{13}\mathrm{C}, ^{12}\mathrm{C})$ and $(^{14}\mathrm{N}, ^{13}\mathrm{C})$, are possible.
- Transfer reactions at sub-Coulomb energies in the entrance and exit channels lead to minimized uncertainty from optical potentials.

Uncertainty Quantification

- Explore reasonable variations in the optical and binding potentials.
- Consider physics beyond DWBA: multi-step transfer, coupled channels, compound nuclear reactions,...
- Be careful when $S \ll 1$ or the DWBA cross section is small due to kinematic conditions.
- Consider performing a validation experiment for a state with known properties in nearby nucleus, using similar reaction kinematics.

A Modern Example

- ^{26}Al$(d,p)^{27}$Al, S.D. Pain *et al.*, Phys. Rev Lett. **114**, 212501 (2015).
- Goal: estimate Γ_p for states in ^{27}Si to constrain the ^{26}Al$(p,\gamma)^{27}$Si reaction rate stars.
- Approach:
  - determine S values for ^{27}Al states
  - convert to S values for mirror ^{27}Si states
  - convert to Γ_p
- This paper is a good example of what can be done.

Related Approaches

- Trojan Horse Method: A. Tumino *et al.*, Journal of Physics: Conference Series **665**, 012009 (2016).

  Idea: Measure $A + a \to C + d + b$ in "quasifree kinematics" where the nucleus b is spectator.

  It may then be possible to extract the cross section for $A + x \to C + d$ at low energies.

  Example: $^{16}\mathrm{O} + {}^{12}\mathrm{C} \to {}^{20}\mathrm{Ne} + \alpha + \alpha$ to study $^{12}\mathrm{C} + {}^{12}\mathrm{C} \to {}^{20}\mathrm{Ne} + \alpha$.

- Surrogate Reactions for (n, γ): J.E. Escher *et al.*, Phys. Rev. Lett. **121**, 052501 (2018).

  Idea: (n, γ) cross sections depend critically on the branching of compound-nuclear states to different channels.

  The surrogate approach uses a transfer reaction to populate the levels and allow the branching to be determined.

Thank you for your attention.