Widths and Isospin

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Widths, Isospin, and Mirror Symmetry

- What determines if a partial width is large or small?
- Isospin and Mirror Symmetry.
- How to use this in a practical way.
- Thomas-Ehrman Effect.

What Determines Widths?

- Recall our practical definition of a partial width:

$$\Gamma_c = \frac{2\gamma_c^2 P_c}{1 + \sum_{c'} \gamma_{c'}^2 \left. \frac{dS_{c'}}{dE} \right|_{E_R}}.$$

- The key pieces are P_c and γ_c.
- The penetration factor P_c is strongly impact by the angular momentum and Coulomb barriers.
- The choice of channel radius also makes a difference, with larger $a \rightarrow$ larger P_c.
- The value of γ_c also matters. This comes from nuclear structure.

How larger can γ_c be?

- One defines the Wigner Limit:

$$\gamma_W^2 = \frac{3\hbar^2}{2\mu a^2} \quad \text{or} \quad \frac{\hbar^2}{\mu a^2}.$$

- Both definitions appear in the literature.

- Then one requires $\gamma^2 \leq \gamma_w^2$, which is observed to be true in practice.

- Where does it come from?
  It is based on $\gamma_c^2 = \frac{\hbar^2}{2\mu a} u_c^2(a_c)$ and reasonable assumptions about $u_c(r)$.

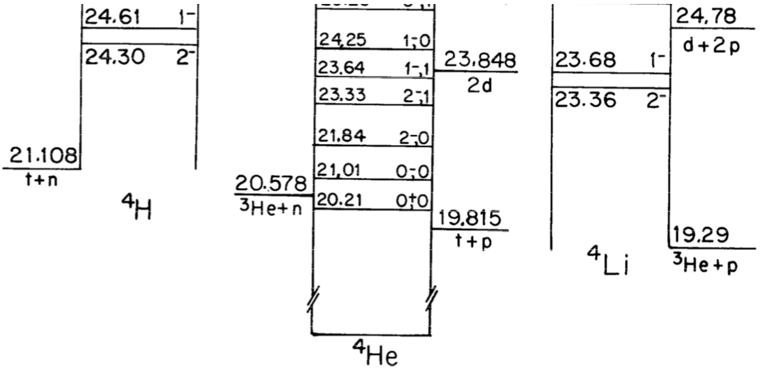
- It becomes useless of you let a be "too large."

- For example, the possible $^7\mathrm{Be} + d$ resonance of Cybert and Pospelov, Int. J. Modern Phys. E, **21**, 1250004 (2012), which required $a \approx 10$ fm.

Isospin in Nuclear Physics

- In nuclear physics, one can think about isospin as the nuclear for being about the same for neutrons and protons.
- Isospin is approximately conserved in nuclear physics.
- Quantum numbers: T and $T_3 = (N - Z)/2$.
- Isospin symmetry is broken by the Coulomb interaction.

$A = 4$ Isobar Diagram



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What can we do with this in R-matrix

- States which are members of the same isospin multiplet are, in *isospin space*, the same state.
- In general, a particle pair α will consist of a pair of nuclei a and b, with (T_a, m_a) and (T_b, m_b).
- The reduced width varies across the multiplet according to a Clebsch-Gordan coefficient:

  $$\gamma_{ab} = (T_a m_a T_b m_b | T m)\gamma,$$

  where T is the isospin of the multiplet and $m = m_a + m_b$.
- Can be used to relate partial widths amongst the multiplet.

Width Scaling

- If we label two members of a multiplet by 1 and 2, we have

$$\frac{\Gamma_1}{(T_{a1}m_{a1}T_{b1}m_{b1}|Tm_1)^2 P_\ell(1)} = \frac{\Gamma_2}{(T_{a1}m_{a2}T_{b1}m_{b2}|Tm_2)^2 P_\ell(2)},$$

  where the correction in the denominator of $1 + \Sigma_c \ldots$ has been dropped.

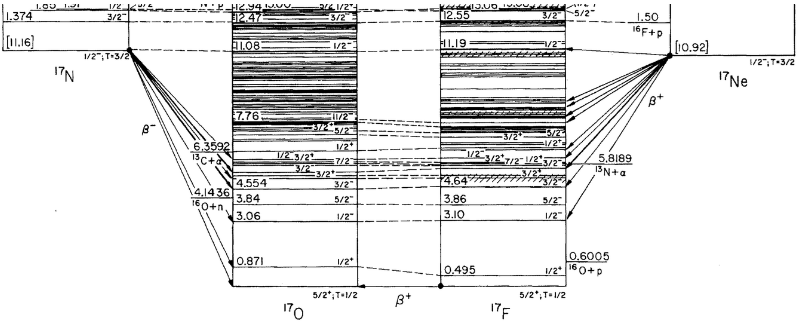
- The same channel radius *must* be used on both sides.

- The experimental energies should be used for calculating the penetration factors whenever possible.

- For mirror nuclei, we have $N \leftrightarrow Z$ and the Clebsch-Gordan factors drop out.

- Sometimes, a level will be bound in one nucleus, but unbound in another. What happens then?

Another Example: $A = 17$



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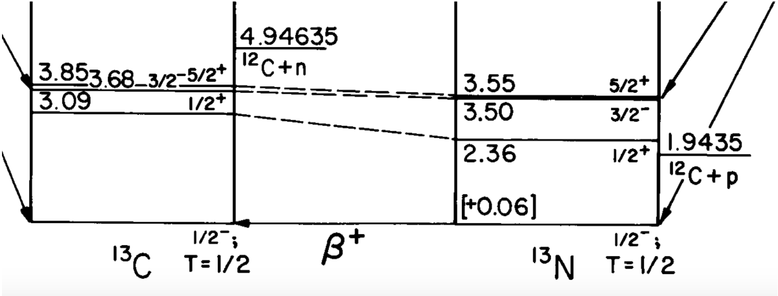
$A = 13$ Isobar Diagram



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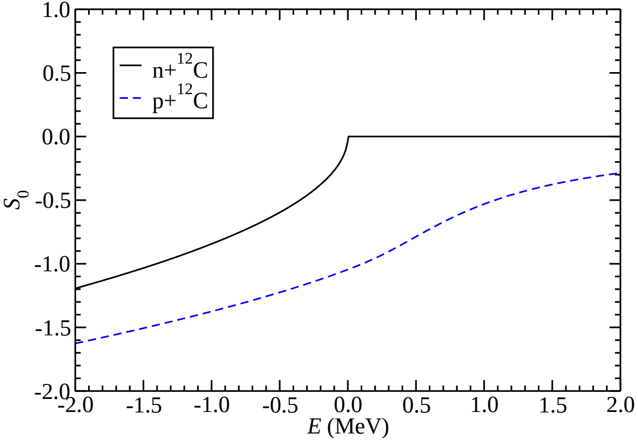
Can we understand why levels are bound in ^{13}C but unbound in ^{13}N?

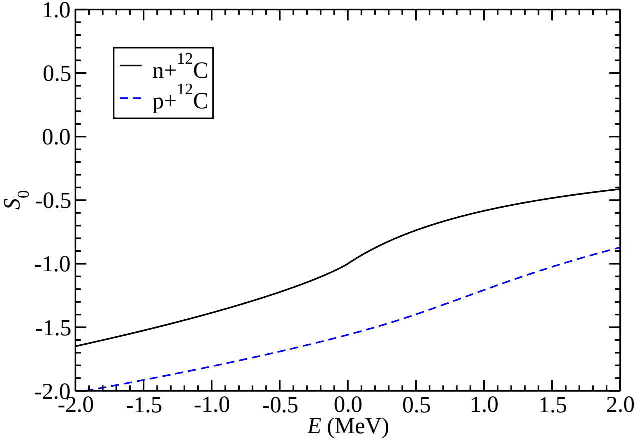
Thomas-Ehrman Shift

- Use the R-matrix idea: the wavefunctions are about the same inside the channel radius.
- This suggest the logarithmic derivatives should be the same.
- What does this say about the separation energy?
- The separation energies (energies relative to threshold) must be different!
- See J.B. Ehrman, Phys. Rev **81**, 412 (1951) and R.G. Thomas, Phys. Rev **88**, 1109 (1952).
- A full understanding of such energy differences is quite complicated, and still an interesting research topic.

s-wave Shift Function at $a = 4$ fm



p-wave Shift Function at $a = 4$ fm



What about Photon Widths?

- The same ideas apply.
- For the penetration factor, we assume $P_c \propto E_\gamma^{2L+1}$.
- This ammounts to assuming the the reduced transiton probability can be the same.

Common Applications

- The use of information from mirror nuclei is very common.
- Example: $^{17}\mathrm{F}(p, \gamma)^{18}\mathrm{Ne}$.
- When I was a graduate student the nucleus $^{18}\mathrm{Ne}$ was poorly known, because it 2 steps off the stability line.
- But the mirror nucleius $^{18}\mathrm{O}$ is stable, and it's states can be easily studied using $^{17}\mathrm{O}(d, p)$.
- Now, width radioactive beams, it can be studied directly.
- The translation of partial widths is straightforward.
- The translation of energies is more challenging.
- We also need to remember that Mirror and Isospin symetry are approximate.

Thank you for your attention.