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Elastic $p+{}^{12}C$ Scattering Between 0.3 and 2 MeV*

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Absolute differential cross-sections of $p+{}^{12}C$ elastic scattering have been measured at $\theta_{cm}=89.1°$, 118.7°, 146.9° for bombarding energies between 0.3 and 2.0 MeV. Revised level parameters of the first three excited states in ${}^{13}N$ have been extracted with a R-matrix analysis. It is shown that the influence of the bound ground-state of ${}^{13}N$ has an appreciable effect on low-energy scattering. Recent predictions concerning Mott-Schwinger polarization are also discussed.

1. Introduction

Low energy elastic scattering of protons from ${}^{12}C$ below 2 MeV, i.e. in the region of the lowest three excited states in ${}^{13}N$ has been well investigated in the past. Several measurements of the differential cross-section [1–3] and of the proton polarization ([4] and refs. mentioned therein) have been reported. R-matrix parameters for the lowest three resonances in $p+{}^{12}C$ have been deduced [2, 3, 5]. These investigations aimed at obtaining the parameters of energy levels in the compound nucleus ${}^{13}N$.

Two recent developments have revived the interest in $p+{}^{12}C$ scattering at low energies. Firstly, a forward dispersion relation has been applied to $p+{}^{12}C$ scattering [6]. In the course of this work it was realized that previous investigations of the low-energy elastic scattering are not sufficient to determine the forward scattering amplitude below 2 MeV accurately enough.

Secondly, it has been proposed [4] to observe the Mott-Schwinger interaction in charged particle scattering as an interference effect near the lowest s-wave resonance in $p+{}^{12}C$. For such an investigation, the knowledge of the nuclear contribution to polarization is essential.

Since the published values for the level parameters of the three levels in question show discrepancies, it seemed advisable to reinvestigate low energy $p+{}^{12}C$ scattering. Advances in the techniques of measuring and interpreting experimental data allow for a substantial improvement of results obtained twenty years ago without the aid of solid state detectors and high-speed computers.

2. Experimental Procedure

We have measured absolute values of the $p+{}^{12}C$ differential scattering cross-section at three center-of-mass scattering angles ($\theta_{cm}=89.1°$, 118.7° and 146.9°) for bombarding energies between 0.3 MeV and 2.0 MeV. The experiment has been carried out at the Basel 3 MV Cockroft-Walton accelerator. The beam energy was calibrated by means of the ${}^{27}Al(p,\gamma){}^{28}Si$ resonance at $E_{p,\mathrm{Lab}}=504.88$ keV [7]. The uncertainty of the beam energy was 0.1 keV.

At the entrance of the scattering chamber, the beam was collimated by a 1×1 mm^2 slit. Self-supporting natural carbon foils of 5–10 μg/cm^2 thickness were used as targets. The target thickness was chosen such that the total energy loss in the target amounted to 1.3 to 1.7 keV for all the measurements. The scattered particles were detected by two surface-barrier detectors with entrance slits of 1.2×1.8 mm^2, located 84 mm from the scattering center. Thus the angular resolution was 1.5°. The detectors were cooled to −15 °C to improve their energy resolution. An overall resolution of 7.5 keV FWHM at 500 keV incident beam-energy for protons scattered in reflection to $\theta_{\mathrm{Lab}}=144°$ could be obtained. Thus, at back angles protons elastically scattered from ${}^{16}O$ and heavier contaminants in the target could be easily separated. At angles where the elastic peaks were not completely

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separated, the contribution of ^{16}O (typically 5 %) was subtracted on the basis of the back angle measurement of the same run. The beam current was adjusted such that the measured dead-time correction was always less than 3 %.

The ^{12}C target thickness was found to increase linearly with the accumulated amount of beam-current which passed through the target. This increase could be ascribed to the wrinkling of the target foil and to beam-induced deposition of carbon. By repeating otherwise identical measurements at different times this increase was measured and applied as a correction (in the order of 5–10 %). All runs were normalized to the total number of beam particles through the target, measured as integrated charge in a Faraday cup. In this manner, relative cross-sections were obtained.

Since the target thickness and the detector solid angles were not known accurately enough, we obtained an absolute calibration by comparing the measured $p+{}^{12}C$ yield to the yield for elastic scattering of α-particles from the same target. The latter process was shown to be Rutherford scattering to better than 99 % at the energies in question [8]. Therefore, the absolute cross-section for α-scattering is known with sufficient accuracy. This calibration was carried out at 1.0 and 2.0 MeV bombarding energy. The average charge state of the protons and the α-particles after passage through the target, needed for a normalization procedure which makes use of the integrated charge in the Faraday cup, was determined in a separate measurement.

An independent check on the absolute calibration was performed by considering that near 460 keV the nuclear part of the scattering is almost entirely determined by the s-wave resonance corresponding to the $J^\pi = 1/2^+$ first excited state of ^{13}N. In the absence of interfering background in any other partial wave it is easy to show that for any given scattering angle and energy the differential cross-section reaches a maximum for a certain s-wave phase-shift between 0 and π. Since at a resonance the phase-shift increases by π, the cross-section must reach this maximum value which in turn corresponds to a maximum in the excitation functions in Figure 1. Thus, from an approximate resonant s-wave phase-shift the absolute value of the differential cross-section at this maximum can be calculated. If the phase-shift is not known accurately enough, the procedure can be iterated. A comparison with the experimental maximum value was found to yield a normalization compatible with that obtained by α-scattering. It should be pointed out that this technique in general provides a useful absolute calibration of differential cross-section measurements with charged particles near threshold if scattering in all but one partial waves is negligible.

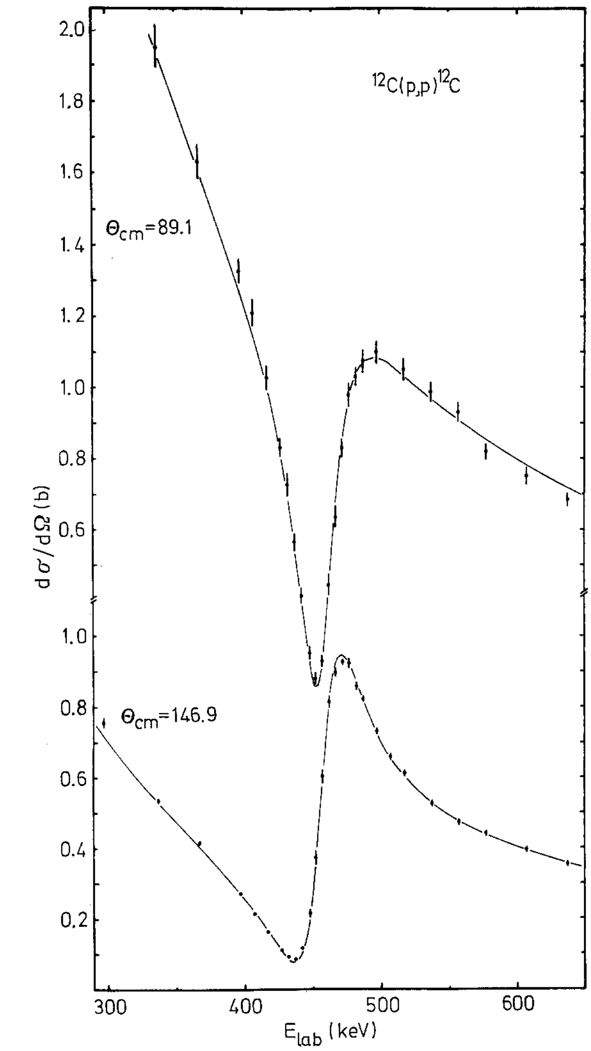

Fig. 1. Elastic scattering differential cross-section vs. the laboratory bombarding energy between 0.3 and 0.6 MeV. The solid line corresponds to the best fit parameters (Table 1) of an R-matrix analysis

Besides the statistical uncertainty, the errors of the differential cross-section measurements contain (i) an error in the subtraction of the ^{16}O contaminant contribution, if applicable, (ii) an error in the correction for changes in target thickness, (iii) an uncertainty of the scattering angle caused by geometrical uncertainties or a left/right-shift of the beam in the scattering chamber, (iv) an error of the absolute normalization and (v) an error due to a beam energy uncertainty of ± 0.5 keV, which is important when the cross-section varies rapidly with bombarding energy.

The contribution of proton elastic scattering from ^{13}C (1 % abundance in natural carbon) was of the order of or less than the error described above and was therefore neglected. The final absolute differential cross-sections are displayed in Figures 1 and 2. Nume-

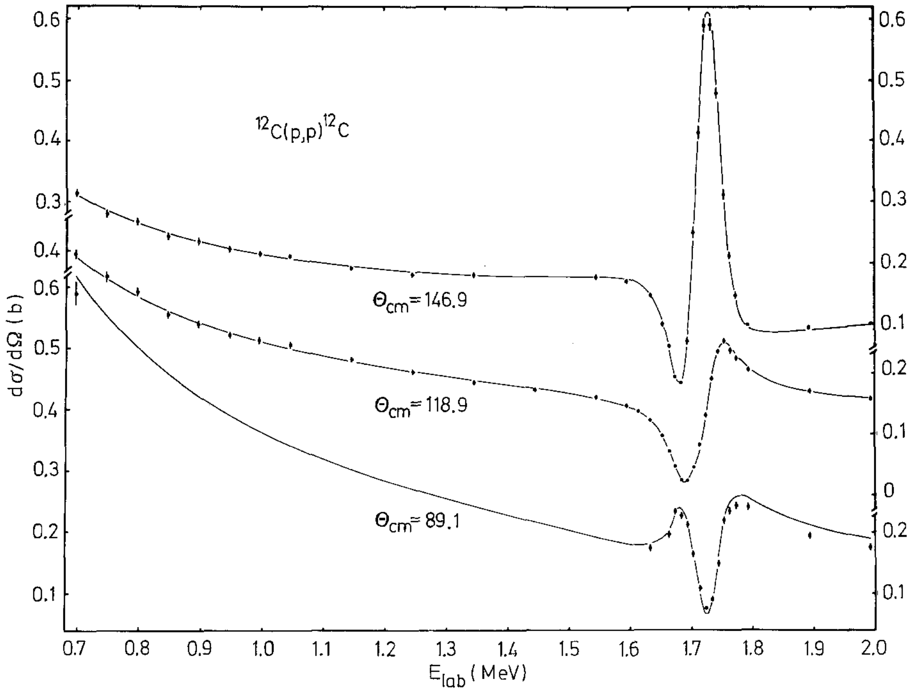

Fig. 2. Same as Figure 1 but between 0.6 and 2.0 MeV

rical values can be obtained from one of the authors upon request.

3. Analysis

The measured differential cross-sections were parametrized in terms of a number of isolated levels in the compound nucleus. The R-matrix formalism was used to determine level parameters from the observables. The nuclear phase-shift δ_l in a certain partial wave l is given [9] by

$$\delta_l = \arctan\left[\frac{\gamma_\lambda^2 P_l}{E_{\lambda l}+\Delta_{\lambda l}-E}\right] - \arctan\left[\frac{F_l}{G_l}\right]_{r=a}. \quad (1)$$

The evaluation of this expression (which is equivalent to [9], p. 273, Eq. (1.19)) depends on the interaction radius a and on the boundary condition B_l which are formal parameters of the R-matrix theory. The quantity B_l was chosen such that the level shift $\Delta_{\lambda l}$ ([9], p. 322, Eq. (1.11 b)) vanishes at resonance so that the characteristic energy $E_{\lambda l}$ becomes equal to the resonance energy $E_{R\lambda}$. The interaction radius a, the reduced width γ^2 and the resonance energy were treated as free parameters. Given the parameters of all the resonances which are to be included, δ_l can be calculated as a function of energy for arbitrary angular momentum l. The scattering amplitudes and, hence, the observables are then obtained (e.g. Ref. 10). A computer code has been written to simultaneously fit all our experimental differential cross-sections by varying the parameters γ_λ^2 and $E_{R\lambda}$ of the first three resonances in $p+{}^{12}C$ with $j^\pi = 1/2^+, 3/2^-$ and $5/2^+$ and the interaction radius a. Also included in the calculation, but not adjusted for best fit were the wide $3/2^+$ resonance near 6 MeV [11] and the $1/2^-$ resonance at -2.11 MeV. This latter resonance corresponds to the ^{13}N ground-state. Although the experimental widths of bound-states vanish because the penetrability is zero, the corresponding reduced widths γ^2 still are finite. Negative energy states therefore appear in an R-matrix expansion completely analogous to resonances at physical energies.

In this context arises the question what the reduced width of a negative energy resonance should be. In R-matrix theory the reduced width γ^2 is determined by the properties of the wave-function at the surface $r=a$. Assuming that at $r=a$ the bound-state wave function $\Psi(r)$ is given by its asymptotic form, we obtain

$$r\Psi(r) = N W_{-\eta,\, l+\frac{1}{2}}(2\kappa r) \quad (\text{for } r=a) \quad (2)$$

with $(\hbar\kappa)^2 = 2\mu E_B$

and $\eta = \mu \cdot \frac{Z_1 Z_2 e^2}{\hbar^2 \kappa}$.

Here, W denotes a Whittaker function, μ is the reduced mass and E_B the binding energy of the state. N is the asymptotic normalization of the single-particle wave-

Table 1. R-matrix parameters for ^{13}N levels near the $p+{}^{12}C$ threshold. All energies are quoted in the center-of-mass system

j^π	$E_{\lambda l}=E_{R\lambda}$ (MeV)	γ_λ^2 (MeV)	B_l	Γ_λ (keV)	Previous work	
					$E_{R\lambda}$ (MeV)	Γ_λ (keV)
$1/2^+$	0.424	3.237	−0.832	33	0.434[c], 0.426[b]	33[c], 32[b]
$3/2^-$	1.558	0.181	−1.005	55	1.566[c], 1.555[a]	51[c], 58[a]
$5/2^+$	1.604	1.899	−2.003	50	1.613[c], 1.600[a]	56[c], 68[a]
$3/2^+$	5.860[e]	1.4[e]	−1.0088	1,400	5.860[d]	1,400[d]
$1/2^-$	−2.11[e]	0.85[e]	−1.0027	—	—	—

[a] Ref. 2 [b] Ref. 3 [c] Ref. 5 [d] Ref. 11,
[e] These parameters have not been adjusted

function normalized such that

$$\int_0^\infty r^2\,\Psi^2(r)\,dr=1. \tag{3}$$

One then obtains for the reduced width of the bound-state

$$\gamma^2=\frac{\hbar^2}{2\mu a^2}N^2\mathscr{S}\frac{[W^2_{-\eta,l+\frac{1}{2}}(2\kappa r)]_{r=a}}{N^{-2}-\int_a^\infty W^2_{-\eta,l+\frac{1}{2}}(2\kappa r)\,dr}. \tag{4}$$

The spectroscopic factor $\mathscr{S}$ which measures the overlap of the $p+{}^{12}C$ (g.s.) with the ^{13}N (g.s.) wave-function appears as a factor in Equation (4) and has been set to $\mathscr{S}=0.53$, according to [12]. The normalization N can be taken from a Woods-Saxon potential wave-function calculated in a $p+{}^{12}C$ potential obtained from an analysis [13] of elastic electron scattering by ^{13}C. The potential depth was adjusted to reproduce the binding energy.

The results of the fitting procedure are revised level parameters for the first three excited states in ^{13}N. They are listed in Table 1, together with the level parameters which were not varied. The interaction radius a giving the best fit was found to be $a=4.0$ fm. This is in contrast to $a=4.77$ fm used as an unadjusted parameter in earlier work [2, 3, 5]. Since the definition of γ^2 depends on the radius a we have also calculated the experimental width Γ_λ, according to

$$\Gamma_\lambda=\frac{2\gamma_\lambda^2 P_l(E_{R\lambda})}{1+\gamma_\lambda^2\left.\dfrac{dS_l}{dE}\right|_{E=E_{R\lambda}}}, \tag{5}$$

which together with the resonance energy is compared to the results of previous work in Table 1.

The fit to the experimental data resulting from the parameters in Table 1 is shown as a line in Figures 1 and 2. The usual quantity χ^2 was used to measure the agreement between calculation and experiment. For the simultaneous fit to the 135 data points displayed in Figures 1 and 2 we achieved $\chi^2=198$. It is interesting to note that the $p+{}^{12}C$ polarization calculated from the parameters in Table 1 is in quantitative agreement with experimental analyzing powers [4] below 2 MeV, in spite of the fact that only cross-section measurements were considered in our analysis.

4. Discussion

As mentioned in the introduction, the R-matrix parametrization described in this paper was used in an analysis of $p+{}^{12}C$ scattering in the frame work of a Forward Dispersion Relation [6]. This type of analysis links the scattering amplitudes at all energies and is thus very sensitive to a consistent energy dependence of the amplitudes. This test clearly favours the present R-matrix parameters, probably because the influence of the ^{13}N ground-state was included in the analysis.

The second point of interest concerns the proposed [4] demonstration of Mott-Schwinger polarization as an interference effect in the neighbourhood of the $E_R=424$ keV $1/2^+$-resonance. With this purpose, angular distributions of the analyzing power in the energy range $0.37<E_p<0.6$ MeV have recently been measured [14]. The effect of the Mott-Schwinger polarization is expected to be observed as the difference between the experimental points and the purely nuclear analyzing power which can be calculated in the frame work of R-matrix theory.

It turns out to be essential to include in such a calculation the $1/2^-$-level at -2.11 MeV which corresponds to the ^{13}N ground-state. This is demonstrated in Figure 3. The displayed data points correspond to preliminary measurements [14]. The solid curve shows the analyzing power calculated from the R-matrix parameters in table 1 with the $1/2^-$ negative energy state included. If this $1/2^-$ bound-state is omitted in the analysis, but the remaining parameters again adjusted to the low energy cross-section-data, one obtains a different set of R-matrix parameters. With χ^2 increasing by 15 % this is still a reasonable fit to the low energy cross-section data, but the calculation of the analyzing power near 0.5 MeV (dotted line in Figure 3) shows that the prediction of this observable strongly depends on the influence of the ^{13}N ground-state.

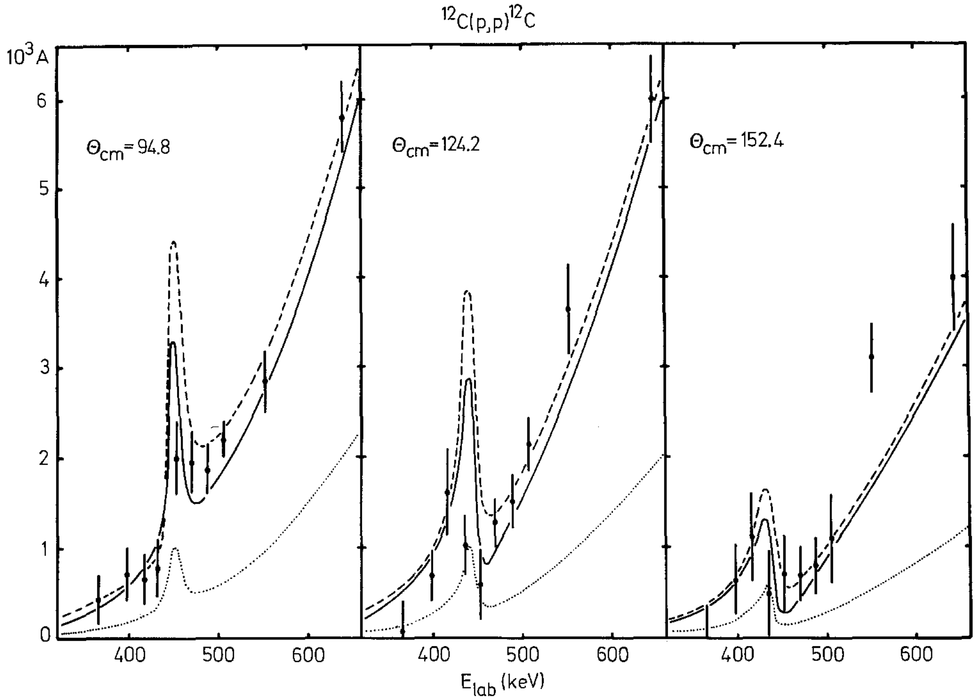

Fig. 3. Elastic scattering analyzing power between 0.3 and 0.6 MeV for three different scattering angles. The data points are taken from [14]. The dotted line corresponds to the best fit *R*-matrix parameters if the ${}^{13}N$ ground-state is neglected, the solid line is obtained with the *R*-matrix parameters of Table 1 and the dashed line results if in addition the Mott-Schwinger interaction is taken into account

The dashed curve in Figure 3 shows the prediction for the proton analyzing power when the Mott-Schwinger interaction is included. We used the best fit *R*-matrix parameters of Table 1 to calculate the nuclear scattering amplitudes, to which we coherently added the Mott-Schwinger amplitudes according to [15]. It is evident from the figure that the available data are not quite good enough to allow a distinction between the predictions with and without the Mott-Schwinger term. In addition, the reduced width (Eq. (4)) of the $1/2^-$ gound-state of ${}^{13}N$ is probably not known well enough to permit a definite conclusion even if considerably more precise data for the analyzing power $A(\theta)$ were available at any one energy. However, our calculations show that the relative slope A^{-1} $(dA(\theta_0)dE)$ of the analyzing power in certain energy regions is almost totally independent of the reduced width of the ${}^{13}N$ bound-state, while still depending quite sensitively on the Mott-Schwinger interaction. In particular, a precise measurement of the analyzing power ratio at an angle near 90° between 0.48 and 0.54 MeV might permit a conclusive experimental test of the presence of the Mott-Schwinger polarization.

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