Pseudo-Wronskian

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Here is a quick note on matching the numerical solution calculated on a numerical grid to the asymptotic solution. Suppose we know the solution at $u(R)$ and $u(R+h)$ and that R is in the asymptotic region where

$$u(r) \propto kr \left[\cos\delta_\ell \, j_\ell(kr) - \sin\delta_\ell \, n_\ell(kr)\right]. \tag{1}$$

In the asymptotic region, one can easily shown that

$$\tan\delta_\ell = \frac{W(u,(kr)j_\ell)}{W(u,(kr)n_\ell)}, \tag{2}$$

where $W(v,w)$ is the Wronskian of the functions u and v:

$$W(v,w) = v\frac{dw}{dr} - \frac{dv}{dr}w. \tag{3}$$

One can also show

$$\tan\delta_\ell = \frac{u(R)\,k(R+h)\,j_\ell[k(R+h)] - u(R+h)\,(kR)\,j_\ell(kr)}{u(R)\,k(R+h)\,n_\ell[k(R+h)] - u(R+h)\,(kR)\,n_\ell(kr)}. \tag{4}$$

This latter approach avoids computing the numerical derivative of u and was communicated to me by Bob Wiringa when I was a graduate student, circa 1994. Note the Eq. (4) reduces to Eq. (2) in the limit that $h \to 0$. It is also described by Raynal [1], where he introduces the term *pseudo-Wronskian* for the Wronskian-like quantity calculated using a finite differences. The generalization to coupled channels is also provided.

[1] Jacques Raynal, *Notes on ECIS94*, Tech. Rep. CEA-N-2772 (CEA Saclay, 1994).