Exploring EDM enhancements in quadrupole-octupole interacting boson model

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Dipole-moment and violation of P and T-symmetries

\[ d = \frac{\langle d \cdot J \rangle}{J(J+1)} J \]

Observation of the dipole moment is an indication of parity and time-reversal violation.

**Limit on EDM in electron**
- Experiment: $10^{-27}$ e cm
- Standard model: $10^{-38}$ e cm
- Physics beyond SM: $10^{-28}$ e cm
Why is this interesting?

- Sensitive test of CP violation in the standard model
- Baryon asymmetry in the universe.
- Physics beyond standard model.

<table>
<thead>
<tr>
<th>system</th>
<th>EDM limit</th>
<th>SM</th>
</tr>
</thead>
<tbody>
<tr>
<td>e (electron)</td>
<td>$10^{-27}$</td>
<td>$10^{-40}$</td>
</tr>
<tr>
<td>n (neutron)</td>
<td>$3.0 \times 10^{-26}$</td>
<td>$10^{-32}$</td>
</tr>
<tr>
<td>$^{225}$Ra</td>
<td>$1.4 \times 10^{-23}$</td>
<td>$10^{-33}$</td>
</tr>
<tr>
<td>$^{199}$Hg</td>
<td>$7.4 \times 10^{-30}$</td>
<td>$10^{-33}$</td>
</tr>
</tbody>
</table>

Schiff Moment

- The nuclear dipole moment causes polarization of the electron cloud so that the total atomic dipole moment for point particles vanishes!
- For a finite size nucleus the screening is not complete. The net result is determined by the vector called the Schiff moment

\[ s = \frac{1}{10} \sum_a e_a r_a \left( r_a^2 - \frac{5}{3} \langle r_{ch}^2 \rangle \right) \]

\[ r_{ch}^2 = \frac{1}{eZ} \sum_a e_a r_a^2 \]

PT mixing NN-interaction

\[ W_{ab} = \frac{G}{\sqrt{2}} \frac{1}{2m} (\eta_{ab} \sigma_a - \eta_{ba} \sigma_b) \cdot \nabla_a \delta(r_a - r_b) \]

\[ W'_{ab} = \frac{G}{\sqrt{2}} \frac{\eta'_{ab}}{2m} (\sigma_a \times \sigma_b) \cdot [(p_a - p_b), \delta(r_a - r_b)]_+ \]

\[ W(r) = \frac{G}{\sqrt{2}} \frac{\eta}{2m} (\sigma \cdot \nabla) \rho(r) \]

PT mixing in a parity-doublet

\[ |\psi^-\rangle \]

Parity conservation

\[ \langle \psi^+ | \vec{S} | \psi^+ \rangle = 0 \]

\[ |\psi^+\rangle \]

Small parity violating interaction \( W \)

\[ |\psi\rangle = |\psi^+\rangle + \frac{\langle \psi^- | W | \psi^+ \rangle}{E_+ - E_-} |\psi^-\rangle \]

Perturbed ground state

\[ \langle \psi | \vec{S} | \psi \rangle = 2 \frac{\langle \psi^+ | \vec{S} | \psi^- \rangle \langle \psi^- | W | \psi^+ \rangle}{E_+ - E_-} \]

Non-zero Schiff moment
Single-particle parity-doublet

\[
H = \frac{p^2}{2m} + U
\]

\[
U(r) = U_0 \rho(r)
\]

\[
W(r) = \frac{G}{\sqrt{2}} \frac{\eta}{2m} (\sigma \cdot \nabla) \rho(r) = \frac{G}{\sqrt{2}} \frac{\eta}{2mU_0} i[\sigma \cdot p, H].
\]

\[
S = 2\langle j_+ | S_z | j_- \rangle \frac{\langle j_- | W | j_+ \rangle}{E_+ - E_-} = \frac{G}{\sqrt{2}} \frac{\eta}{mU_0} i\langle j_+ | S | j_- \rangle \langle j_- | \sigma \cdot p | j_+ \rangle
\]

No enhancement in close-lying states!

$^{199}\text{Hg}$ example

$\frac{1}{2}^+$

5.8 MeV

$\frac{1}{2}^-$

$\eta < 5 \times 10^{-4}$
Many-body enhancements

- Proximity of levels of opposite parity
- Many-body complexity
  - Chaotic dynamics
  - Collective dynamics (parity doublets)
- Solid state or molecular mechanisms
Parity-doublet octupole shape

Body-fixed frame:

\[ R(\theta) = R \left[ 1 + \sum \beta_i Y_{i0}(\theta) \right] \]

\[ S_{\text{in}} = \frac{9}{20\pi \sqrt{35}} eZ R^3 \beta_2 \beta_3 \]

Lab frame:

\[ S = 2\alpha S_{\text{in}} \frac{KM}{I(I+1)} \quad \alpha = 2 \frac{\langle I_- | W | I_+ \rangle}{E_+ - E_-} \]

\[ S \sim \frac{0.01I}{I+1} e\beta_2 \beta_3^2 Z A^{2/3} \frac{\eta G}{m r_0 (E_+ - E_-)} \]

Strongly enhanced Schiff moment

The path to a large Schiff moment

1. The quadrupole +octupole deformation leads to large intrinsic Schiff moments [1].
2. The parity doublets in the reflection asymmetric nuclei are very close in energy.
3. Parity and time reversal violating matrix element between the members of the doublet is relatively large.
4. Heavy nuclei - large Z.

Candidates $^{223}\text{Ra}, ^{225}\text{Ra}, ^{223}\text{Rn}, ^{221}\text{Fr}, ^{223}\text{Fr}$

Questions:

- Is enhancement possible in dynamical regions of soft shape?

Soft octupole mode in a deformed nucleus

\[ S \sim \frac{0.01 I}{I + 1} e \beta_2 \beta_3^2 Z A^{2/3} \frac{\eta G}{m r_0 (E_+ - E_-)} \]

Soft octupole \[ \beta_3^2 \rightarrow \langle \beta_3^2 \rangle \]

Soft dipole + octupole?

Soft dipole + soft octupole?

Vibration-rotation shape transition?

Coupling between phonons?

Interacting Boson Model (IBM)

\[
(b_{\ell_1}^\dagger \tilde{b}_{\ell_2})_{LM} = \sum_{m_1m_2} (-)^{\ell_2-m_2} C_{\ell_1m_1 \ell_2-m_2}^{LM} b_{\ell_1m_1}^\dagger b_{\ell_2m_2}
\]

\[
J_\mu = \sum_\ell \sqrt{\frac{\ell(\ell+1)(2\ell+1)}{3}} (b_\ell^\dagger \tilde{b}_\ell)_{1\mu}
\]

\[b_{0,0} \equiv s \quad b_{2,m} \equiv d_m\]

Consistent Q model

\[
H^{(d)} = \omega_2 \left( (1-\eta) \sum_m d_m^\dagger d_m - \frac{\eta}{4N} \sum_M Q_M^\dagger (\chi) Q_M (\chi) \right)
\]

\[
Q_M (\xi) = \left[ (s^\dagger \tilde{d})_{2M} + (d^\dagger \tilde{s})_{2M} - \chi \frac{\sqrt{7}}{2} (d^\dagger \tilde{d})_{2M} \right]
\]


IBM shape triangle

\( \eta \)

\( U(5) \) vibrational

\( E_4 / E_2 \)

\( E_4 \)

4^+

\( E_2 \)

2^+

0^+

SU(3)

rotational

oblate

\( \gamma - \text{unstable} \)

\( \chi \)

rotational

prolate
IBM: Quadrupole moment of the $2^+$

$E_4$ $4^+$

$E_2$ $2^+$

$0^+$
f-boson \quad b_{3,m} = f_m

\[
H = H^{(d)} + H^{(f)} + H^{(\alpha)}
\]

\[
H^{(f)} = \omega_3 \sum_m f_m^\dagger f_m
\]

\[
H^{(\alpha)} = \frac{\alpha}{\sqrt{N}} \sum_M \left[ (f^\dagger \tilde{f})_{2M}^\dagger Q_M(\chi) \right]
\]

Angular momentum

\[
J_M = \sqrt{10}(d^\dagger \tilde{d})_{1M} + \sqrt{28}(f^\dagger \tilde{f})_{1M}
\]

Schiff operator

\[
S_M = (f^\dagger \tilde{d})_{1M} - (d^\dagger \tilde{f})_{1M}
\]
Quadrupole-octupole coupling

\[ H = \omega_2 \sum_m d_m^+ d_m^+ + \omega_3 \sum_m f_m^+ f_m + \alpha \sum_m \left[ (f_2^+ f_3^+)^{2m} d_m + (f_2^+ f_3^+)^{2m} d_m^+ \right] \]

\[ [d_m, H] = \omega_2 \sum_m d_m + \alpha \sum_m (f_2^+ f_3^+)^{2m} \]

\[ \langle d_\mu \rangle = -\frac{\alpha}{\omega_2} \langle (f_2^+ f_3^+)^{2\mu} \rangle \]

\[ E_{3^-} = \omega_3 - \frac{5 \alpha^2}{7 \omega_2} \]

Mueller, et.al. PRC 73, (2006) 014316
V. Zelevinsky Phys. At. Nucl. 72, 1107 (2009)

Coupling between modes, three-body force?
Quadrupole-octupole collectivity

Coupling between modes, three-body force?

Mueller, et.al. PRC 73, (2006) 014316
V. Zelevinsky Phys. At. Nucl. 72, 1107 (2009)
$B=0.7 \text{ MeV}$

Cocks, et.al. PRL 78, 2920
Quadrupole-octupole coupling

\[ \chi = \eta \]

\[ \alpha = 0 \quad \alpha = 1 \]
Quadrupole-octupole coupling (revisited)

\[ E_{3^-} = \omega_3 - \frac{5 \alpha^2}{7 \omega_2} \]

From Xe \( \alpha = 0.8 \text{ MeV} \)
Schiff moment \( \langle 2^+ || S || 3^- \rangle \)
Schiff moment \[ \langle 2^+ || S || 3^- \rangle \]
Prolate to oblate transition

\[ \eta \}

\[ \chi \]

\[ \epsilon \]

\[ \frac{1}{\beta^2} \]
Particle-core coupling

\begin{align*}
6^+ & \quad \text{d}^{3/2} & \quad 3/2^+ \\
3^- & \quad \text{d}^{5/2} & \quad 1/2^+ \\
4^+ & \quad & \quad 7/2^- \\
2^+ & \quad & \quad 433 \text{ keV} \\
0^+ & \quad \text{g}^{9/2} & \quad 9/2^+ \\
& & \quad 7/2^+ \\
\text{IBM} & & \text{s.p.}
\end{align*}
Particle-core coupling

\[ L + j = I \quad S_{\text{tot}} = S + s \]

\[
\langle (L'j)I||S||(Lj)I \rangle = (2I + 1)(-1)^{L'+j+I+1} \left\{ \begin{array}{ccc} L & j & I \\ I & 1 & L' \end{array} \right\} \langle L'||S||L \rangle
\]

\[
\langle (L'j')I'||S_{\text{tot}}||(Lj)I \rangle = g \langle 3^-||S||2^+ \rangle
\]

| A  | \( I \) | \( g_{9/2} \otimes 2^+ \) | \( g_{9/2} \otimes 3^- \) | vc       | \(|g|\)  |
|-----|--------|----------------------------|--------------------------|----------|--------|
| \(^{217}\text{Ra}\) | 9/2    | 0.14                       | 0.999                    | -0.888   | 0.124  |
| \(^{219}\text{Ra}\) | 7/2    | 0.08                       | 0.98                     | -0.835   | 0.065  |
| \(^{221}\text{Ra}\) | 5/2    | 0.25                       | 0.86                     | -0.670   | 0.144  |
| \(^{217}\text{Rn}\) | 9/2    | 0.11                       | 0.999                    | -0.888   | 0.098  |
| \(^{219}\text{Rn}\) | 5/2    | 0.27                       | 0.98                     | -0.670   | 0.177  |
| \(^{221}\text{Rn}\) | 7/2    | 0.04                       | 0.98                     | -0.835   | 0.033  |
Explicit PT-violation

\[ H^{(\beta)} = \beta \sum_M (d^\dagger \tilde{d})^\dagger \left[ (f^\dagger \tilde{s})_{3M} - (s^\dagger \tilde{f})_{3M} \right] \]

\[ X_\alpha = \lim_{\beta \to 0} \frac{1}{\beta} \langle a \parallel S \parallel a \rangle \]
Dipole moment of excited states

\[ X_\alpha = \lim_{\beta \to 0} \frac{1}{\beta} \langle \alpha || S || \alpha \rangle \]
The “minimal” Shell Model Hamiltonian

Two-level system with single-particle states of opposite parity \( j_+ \) and \( j_- \).

Parity Mixing operator

\[
W = \sum_m a_{m+}^\dagger a_{m-} + \sum_m a_{m-}^\dagger a_{m+}
\]

Schiff vector

\[
S_\mu = \sum_{m m'} \langle 1 \mu, j m' | j m \rangle (a_{m+}^\dagger a_{m'}^- + a_{m-}^\dagger a_{m'}^+ )
\]

\( j = 17/2 \)

\( \varepsilon \)

\( j = 17/2^+ \)
Dynamics

• Single-particle gap:
• Pairing: P-pair operator
• Quadrupole interaction: $Q_2$ -phonon
• Octupole interaction: $Q_3$ –phonon

$$H = \epsilon_1 N_1 + \epsilon_2 N_2 - G P^\dagger P - \kappa_2 Q_2^\dagger Q_2 - \kappa_3 Q_3^\dagger Q_3$$

\[ H = \epsilon_1 N_1 + \epsilon_2 N_2 - G P^\dagger P - \kappa_2 Q_2^\dagger Q_2 - \kappa_3 Q_3^\dagger Q_3 \]

\[ -\frac{5\kappa_2}{2} \sum_{\mu} \left[ Q_{2\mu}^\dagger (jj) Q_{2\mu} (jj) + Q_{2\mu}^\dagger (j' j') Q_{2\mu} (j' j') \right] \]

\[ Q_{\lambda\mu}^\dagger (j_1 j_2) = \sum_{m_1 m_2} (-)^{j_1-m_1} \begin{pmatrix} j_1 & \lambda & j_2 \\ -m_1 & \mu & m_2 \end{pmatrix} a_{j_1 m_1}^\dagger a_{j_2 m_2} \]

\[ \epsilon = G = 0.2 \]

\[ j = j' = 17/2 \]

Spectra of the two-level model
Spectra of the two-level model
Spectra of the two-level model
Spectra of the two-level model

\[ \chi_3 = 1 \]

\[ \chi_2 \]

<table>
<thead>
<tr>
<th>0.0</th>
<th>0.5</th>
<th>1.0</th>
<th>1.5</th>
<th>2.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.5+</td>
<td>8.5+</td>
<td>8.5+</td>
<td>8.5-</td>
<td>8.5+</td>
</tr>
<tr>
<td>7.5-</td>
<td>8.5-</td>
<td>8.5+</td>
<td>8.5-</td>
<td>8.5+</td>
</tr>
<tr>
<td>5.5-</td>
<td>7.5+</td>
<td>8.5+</td>
<td>8.5-</td>
<td>8.5+</td>
</tr>
<tr>
<td>9.5+</td>
<td>6.5-</td>
<td>10.5-</td>
<td>10.5-</td>
<td>8.5-</td>
</tr>
<tr>
<td>6.5-</td>
<td>6.5+</td>
<td>6.5-</td>
<td>6.5+</td>
<td>6.5-</td>
</tr>
</tbody>
</table>

\[ \chi_3 = 1 \]

(0, 1)
Degenerate levels
level interchange symmetry

• Even particle system can be classified with the additional symmetry
• Odd particle system: all states are degenerate parity doublets.

\[ I : (j \leftrightarrow j') \]
\[ \mathcal{P}I = (-)^N I \mathcal{P} \]
Degenerate levels

- Even particle system can be classified with the additional symmetry.
- Odd particle system: all states are degenerate parity doublets.
Two-level $j=17/2$ model
Schiff and Weak Operators
$G=0.2 \ \kappa_3=1$

Left: $N=6$ system

• Schiff operator $<2|S|3>$
• Excitation energy of $2^+$ and $3^-$ states

Right: $N=7$ system

• Energy of the g.s. doublet $17/2^+$ and $17/2^-$
• Schiff operator $<17/2^+|S|17/2^->$
• Weak Matrix element $<17/2^+|W|17/2^->$
• Enhancement product
Two-level j=17/2 model
Schiff and Weak Operators
\( G=0.2 \) \( \kappa_3=1 \)

Left: \( N=6 \) system
- Schiff operator \( <2|S|3> \)
- Excitation energy of \( 2^+ \) and \( 3^- \) states

Right: \( N=7 \) system
- Energy of the g.s. doublet \( 17/2^+ \) and \( 17/2^- \)
- Schiff operator \( <17/2^+|S|17/2^-> \)
- Weak Matrix element \( <17/2^+|W|17/2^-> \)
- Enhancement product
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